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Theoretical Approach to Treatment of Non-Newtonian Forces*)

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We consider the set of gravitational and unified field theories that predict the deviation on Newton's law. The main attention is paid to mechanisms of new forces origin and particles which can be regarded as the mediators of new type of interaction. The all known to us theories are divided into eight classes and relation and discrepancy between them are discussed.

Contents

§ 1. Introduction

Gravitation was the first to be studied but up till now it seems to be less understood. Moreover, the constant of gravitational coupling is known with less accuracy (10^{-4}) than other fundamental constants (many atomic constants are known up to 10^{-6} $\sim 10^{-8}$).

Einstein's theory of gravitation based on general covariance and equivalence principles[1),2)] stayed aside from the main road of particle and field theory. The

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classical examples of testing the principle of universal free fall (equivalence principle) are given in Refs. 3)~5). Red shifts of photons measured by Pound and Rebka[6] became another confirmation of this principle in Einstein theory.

Though some attempts were made in the formulation of gravitation as a gauge theory of external symmetries it was not possible to reproduce exactly the Einstein theory in this scheme.

In the middle of the 70's such a representation of basic interactions was formed.

Table I.

INTERACTION	RADIUS cm	CHARACTERISTIC PARTICLES	PARTICLES-MEDIATORS
Strong	10^{-3}	Quarks	Gluons
Electromagnetic	∞	All charged	Photons
Weak	10^{-16}	Leptons	Vector bosons
Gravitational	∞	All particles	Gravitons

At the same time it became evident that further development of physics demands not the further separation of interactions on types, symmetries and ranges but a search for general principles and construction of unified quantum theory of fundamental interactions.

The era of Grand Unification, Supersymmetries, Strings and Multidimensional theories came.

With the extension of symmetries new notions and conceptions appeared, new particles were introduced, in particular superpartners of many existing particles. The construction of gauge approach to gravitational interactions led to alternative geometrized theories of gravitation and supergravitation. Such predictions as the proton decay in GUT models and renormalization of supergravity drew attention of physicists for a long period.

At the background of big successes in these fields and in the hopes of appearance of the "theory of everything" some predictions related to macrophysics such as a new short-range interaction due to vector boson exchange from extended supergravity and the new spin-torsion short range interaction remained nearly unnoticed.

And only continuous results of geophysicists obtaining different values of the gravitational constant in mines from those got in a laboratory and some other experimental and theoretical considerations (see Refs. 7) and 8)) draw at last the attention of theoreticians and more than 50 groups of experimental people to a thorough investigation of phenomena called now the fifth force problem or a possible violation of Newton's law or a new interaction problem.

Experiments for a search of possible deviations from Newton's law are planned and realized at different ranges: of the order of millimeters and centimeters, from hundreds meters to several kilometers, and from tens to hundreds kiloparsec. Probably mechanisms of generating of intermediate forces are also different in these ranges.

One may find now a large variety of different theoretical schemes and a lot of contradicting experimental data in this field. It means only that we are lacking of good reliable theoretical scheme in frames of which it may be possible to explain the existing data or may be some of the results are wrong and need to be clarified.

It is our aim here to systematize the existing theoretical approaches.

We analyze different theories leading to new possible additional interactions or a violation of Newton's law. Here we present seven big groups of theories. Partially this classification is based on the fact that massless gravitons are mediators of gravitational interaction, that they obey second order equations and interact with matter with a constant strength, defined by the value of the Newtonian gravitational constant G and the speed of light c. Evidently, a violation of any of these conditions may lead to deviations from Newton's law. It may appear in additional terms reflecting a different nature of gravitational field source without a changing in nature of mediator and preserving the long-range interaction or it will lead to the short-range forces. Everything will depend on the model. In each example we shall try to describe mechanisms of a non-Newtonian interaction both on classical and quantum levels.

So, we dwell upon such theories:

1. Theories with massive gravitons interacting with matter directly. Among such theories are theories with cosmological constant and bimetric theories in which one of "metrics" corresponds to the tensor massive field $\varphi_{\alpha\beta}$ (see, for example Ref. 9)).

2. Theories with an effective gravitational constant, where $G \to G_0 \cdot f(\boldsymbol{x}, t)$. Among them are different scalar-tensor theories. Additional non-gravitational fields (scalar, spinor, vector) do not interact with matter directly but interact with gravitons and influence the metric.

3. Theories with torsion, where the only modification is the introduction of an asymmetric affine connection: The antisymmetric part of the connection is a tensor called the Cartan torsion and is related to the density of intrinsic angular momentum.

4. Theories with higher derivatives, where a gravitational field satisfies fourth and higher order field equations. Let us illustrate such theories by the following example. Let the massless scalar field obey such an equation:

$$(a_1 \Box^2 + a_2 \Box)\varphi = 0\,, \tag{1·1}$$

then the propagator may be written as

$$\boldsymbol{D}(p^4) = \frac{1}{a_1 p^4 + a_2 p^2} = \frac{1}{p^2 + A_1} + \frac{1}{A_2 p^2}\,. \tag{1·2}$$

One of the terms describes a massless mode and another massive one. So in such a model both short-range and long-range forces are present.

5. Theories in which besides gravitons there are other mediators of the gravitational interaction. Among them are supergravity, gauge theory of gravitation (gravitational theory with torsion), nonmetric theories of gravitation (Weyl's theory with $\nabla_\alpha g_{\mu\nu} \neq 0$). In these theories new dynamical fields interact directly with space-time characteristics of matter (energy-momentum tensor, hypermoment, spin and dilaton currents).

6. Theories with long-range forces of Van-der-Waals type. The interaction potential is formed due to a joint action of all four types of interactions.

7. Theories with a nonlinearity induced by one of the four types of fundamental interactions. Here we have the possibility to interpret the nonlinearity as an appearance of a mediator mass.

8. Empirical theories and models where the mechanism of deviations from Newton's law is unknown.

Following the above-mentioned classification we shall try to review known results limiting ourselves only to static potentials. Of course, study of dynamical sources of non-Newtonian interactions and corresponding forces may bring some new results.

Even if new interactions with intermediate ranges are not discovered we hope that their search will influence a further development of fundamental interactions theory and also contribute to our knowledge of the gravitational field of the Earth. We believe also that the list of references will help readers to extend our way of presentation of the subject.

§ 2. Theories with massive gravitons

First we start from the simplest possibility of an introduction of a graviton mass —existence of a cosmological constant. Its value is still arbitrary.

The Lagrangian of such a theory is

$$L=R-2\Lambda+L_{\text{mat}}(\varphi, \partial\varphi, g)\,. \tag{2·1}$$

Corresponding Einstein equations are

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}=\frac{8\pi G}{c^4}T_{\mu\nu}\,. \tag{2·2}$$

Solution of (2·2) in the static spherically symmetric case outside a source leads to

$$g_{00}=1-\frac{2GM}{c^2 r}-\frac{\Lambda r^2}{3}\,. \tag{2·3}$$

In a weak field approximation the gravitational potential is

$$V(r)=-\frac{GM}{r}-\frac{\Lambda c^2 r^2}{6} \tag{2·4}$$

and the gravitational force is

$$F=-\frac{GMm}{r^2}+\frac{c^2\Lambda m}{3}r\,. \tag{2·5}$$

This simple analysis does not give the correct answer whether these forces are short-range or not. In order to get answer let us treat (2·2) in a weak field approximation using the Lorentz gauge. Then

$$\frac{1}{2}\,\Box h_{\mu\nu}+\Lambda h_{\mu\nu}=\frac{8\pi G}{c^4}\left(T_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}T\right)+\Lambda\eta_{\mu\nu}\,. \tag{2·6}$$

For an isolated source and in a spherically symmetric static case we have

$$h_{00}=-\frac{2GM}{c^2r}\{1-\alpha\exp(-r/\lambda)\}\,, \tag{2·7}$$

where $\lambda^{-1}=\sqrt{2\Lambda}$, $\alpha=\int d^3x\Lambda$. If one decomposes the corresponding force

$$F=-\frac{GMm}{r}\left\{\frac{1}{r}+\frac{\alpha}{\lambda}\exp(-r/\lambda)\right\} \tag{2·8}$$

into series one obtains

$$F=-\frac{GMm}{r^2}(1+\alpha)+\frac{GMm\alpha}{r\lambda}-\frac{GMm\alpha}{\lambda^2}+\cdots. \tag{2·9}$$

So, it is seen that there exists a certain constant force which acts on all bodies and does not depend on ranges, and other contributions are essential at different distances. For example, the exponential term is not important at distances $r\gg\Lambda$ but at ranges of the order of Λ we cannot neglect it.

After such a formal introduction of a graviton mass we must remember that it is usually supposed that $|\Lambda|=|\Lambda_0|\cong10^{-52}\mathrm{m}^{-2}$ according to dynamics of astronomical sources. Then $r_0=\lambda\cong10^{26}$m or is close to the radius of the visible Universe. This, without doubt, far exceeds any of existing suggestions about the possible range of the Yukawa gravitational potential.

There are two ways to make this mechanism viable. First, let us consider a global scalar field (cosmological) with many "minima", whose potential energy may be represented by Fig. 1.

Due to spontaneous symmetry breaking $U(\varphi)$ may take a set of values $\{U(\varphi_n{}^{\mathrm{ext}})\}$ which induces corresponding Λ_n.

Similar hypothesis was discussed in Ref. 10), but a mechanism of forming a discreet set of Λ_n was not pointed out. It may be obtained in the frames of quantum cosmology.[11)]

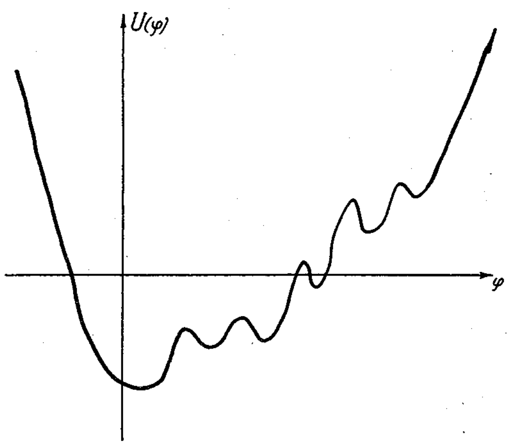

Fig. 1.

Let us consider $V(r)$ as a classical limit of a quantum process of a massive graviton exchange. Decomposing the action functional into series over small deviations $h_{\mu\nu}=g_{\mu\nu}-\eta_{\mu\nu}$ in the Lorentz gauge, we get

$$S=\int d^4xh^{\mu\nu}P_{\mu\nu\alpha\beta}(\Box+\lambda)h^{\alpha\beta}\,, \tag{2·10}$$

where λ is a graviton de Broglie length. The graviton propagator will be

$$\boldsymbol{D}_1(x-y)=\frac{1}{(2\pi)^4}\int d^4pe^{ip(x-y)}$$

$$\times\frac{1}{p^2+\lambda+i\varepsilon}. \tag{2·11}$$

After performing calculations using Gupta's method[12),13)] one obtains a gravitational potential with the Yukawa additional term.

Another realistic mechanism of a gravitational mass generation is a dimensional reduction. We mean here not only a transition from a $(4+N)$-dimensional representation of interaction fields to a usual 4-dimensional space-time but also a transition to 3-dimensional gravity and account of ST-noninvariant contributions induced by Chern-Simons terms in 4-dimensional theories.

The main idea of a dimensional reduction in Kaluza-Klein theories is the exclusion of a dependence on internal space coordinates. Practically all these schemes lead to an appearance of a scalar field and a cosmological constant,[14)-18)] whose value is different from Λ_0 as demanded by astrophysical limits. Possibly, experimental data on hypothetical gravitational interactions will turn out as new tests of multidimensional theories.

Graviton mass generation arises also when topological Chern-Simons terms are included in the Lagrangian[19),20)]

$$S_{\rm cs}=-\frac{1}{4k^2m_0}\int d^3x\varepsilon^{abc}\left[\omega^{ij}{}_aR_{ijbc}+\frac{2}{3}\omega^{ij}{}_a\omega_j{}^k{}_b\omega_{kic}\right]. \tag{2·12}$$

Under the action of a 3-dimensional rotation group the Lorentz connections are transformed according to

$$'\omega^i{}_{ja}(x')=(L^i{}_n\omega^n{}_{kb}(x)L^{-1k}{}_j+L^i{}_n\partial_bL^k{}_j)\cdot\frac{\partial x'^b}{\partial x^a}. \tag{2·13}$$

When finite transformations $L^i{}_j$ are done the action changes over

$$\Delta S=\int d^3x\left(-\frac{2\pi^2}{k^2m_0}\right)\cdot W(L), \tag{2·14}$$

where $W(L)$ is a winding number which is equal to zero only for homotopically trivial transformations continuously deformed to unity. For homotopically nontrivial transformations $W(L)\in\pi_3(SO(3,1))\equiv Z$, i.e., the manifold of integer numbers. Under these transformations the phase of a gravitational field generating functional does not change only if $m_0=\pi/k^2n$ and $n\in Z$. In all other cases the phase changes and a gravitational mass appears. So, it is another-topological-mechanism of a gravitational field mass generation.

Another possibility to introduce massive graviton comes from the consideration of strong gravity. In this case[21),22)] the mass term in the Lagrangian density becomes:[22)]

$$L=-\sqrt{g}\frac{M^2}{4k_1{}^2}(f^{\mu\nu}-g^{\mu\nu})(f^{\alpha\beta}-g^{\alpha\beta})(g_{\mu\alpha}g_{\nu\beta}-g_{\mu\nu}g_{\alpha\beta}), \tag{2·15}$$

where, in the hypothesis of tensor-meson dominance, $f^{\mu\nu}$ is normalized so that $f^{\mu\nu}=\eta^{\mu\nu}+k_1F^{\mu\nu}$ (here k_1 is a coupling constant with dimension $[k_1]=[\text{mass}]^{-1}$ and $F^{\mu\nu}$ is a pure spin-2 field).

The mass term is not diagonal with respect to the fields $F^{\mu\nu}$ and $g^{\mu\nu}$. The mixing term provides a direct coupling between the massive and massless spin-2 fields. The non-diagonal term of the mass term implies that the interaction eigenstates corresponding to $F^{\mu\nu}$ and $g^{\mu\nu}$ are not states of definite mass: So every spin-2 particle produced in the interaction is to be regarded as a combination of massive and massless states, corresponding to the eigenvalues of the mass matrix.

This effect has two interesting consequences: The first is that any hadronic reaction in which spin-2 eigenstates of strong interactions are produced can be regarded also as an effective source of gravitational radiation[22] and the second consequence is that the emission of gravitational waves with sufficiently high energy should be accompanied by the production of a flux of neutrinos and electromagnetic radiation.[22]

§ 3. Theories of gravitation with effective gravitation constant

Among theories of gravitation in which the gravitational "constant" depends on coordinates the scalar-tensor theory occupies a peculiar place because, from one side, it unites the theory of gravitation with the so-called Mach's principle[23)~26)] and from other side, extends a symmetry to the group of conformal transformations.

Let us consider the appearance of non-Newtonian terms in such a theory. We start from the Lagrangian in the form[27]

$$L=\sqrt{-g}\{h(\varphi)R+l(\varphi)g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi+2\lambda(\varphi)\}+L_{\mathrm{mat}}\,, \tag{3·1}$$

where

$$L_{\mathrm{mat}}=L[\psi^2(\varphi)g,\,\chi(x),\,\partial_\mu\chi(x)\cdots]\,. \tag{3·2}$$

As it was shown in Ref. 27) this rather general Lagrangian through the redefining of an interval may be transformed to

$$L_0=\sqrt{-g}\{R(g)-ng^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi+2\lambda\}+L_{\mathrm{mat}}(\bar{\psi}^2,\,\cdots)\,, \tag{3·3}$$

where for the Jordan-Brans-Dicke theory [24),25)]

$$\bar{\psi}^2\propto\exp\left\{-\varphi\left(\omega+\frac{3}{2}\right)^{-1/2}\right\},\quad \lambda=0\,,\quad n=1\,, \tag{3·4}$$

and for the Einstein theory $\varphi=$const and for the Hanlon theory[28]

$$h(\varphi)=\varphi\,,\quad l(\varphi)=1\,,\quad 2\lambda(\varphi)=-m^2 3(2\varphi_0)^{-1}(\varphi-\varphi_0)\,, \tag{3·5}$$

φ_0 is a certain constant. For variant with conformal scalar field see Ref. 18).

Equations of motions are obtained via variational procedure and take the form

$$R^\mu{}_\nu-\frac{1}{2}\delta^\mu{}_\nu R-n\left(\delta^\alpha{}_\nu g^{\mu\beta}-\frac{1}{2}\delta^\mu{}_\nu g^{\alpha\beta}\right)\partial_\alpha\varphi\partial_\beta\varphi-\delta^\mu{}_\nu\lambda=\frac{8\pi G}{c^4}T^\mu{}_\nu\,, \tag{3·6a}$$

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\alpha}\left(\sqrt{-g}\,g^{\alpha\beta}\frac{\partial\varphi}{\partial x^\beta}\right)+n\frac{d\lambda}{d\varphi}=-n\frac{8\pi G}{c^4}\frac{1}{\psi}\frac{d\psi}{d\varphi}\,T\,, \tag{3·6b}$$

$$T^{\mu\nu}\sqrt{-g}=\frac{\delta L_{\rm mat}}{\delta g^{\mu\nu}}\,. \tag{3·7}$$

For a perfect fluid

$$T^{\mu}{}_{\nu}=\psi^4\Big\{\Big(\rho+\frac{p}{c^2}\Big)\psi^2 u^{\mu}u_{\nu}-p\delta^{\mu}{}_{\nu}\Big\}\,, \tag{3·8}$$

where ρ is a density, p is a pressure and u^{μ} is a 4-velocity.

Let us study a limit of a weak field, then

$$\varphi=\overset{(0)}{\varphi}+\overset{(1)}{\varphi}+\overset{(2)}{\varphi}+\cdots\,, \tag{3·9a}$$

$$g_{\mu\nu}=\eta_{\mu\nu}+\overset{(1)}{g}_{\mu\nu}+\overset{(2)}{g}_{\mu\nu}+\cdots\,, \tag{3·9b}$$

$$\lambda(\varphi)=\lambda_0+\lambda_0'\overset{(1)}{\varphi}+\cdots\,, \tag{3·9c}$$

$$\psi(\varphi)=1+\psi_0'\overset{(1)}{\varphi}+\cdots\,, \tag{3·9d}$$

$$T^{\mu}{}_{\nu}=\overset{(1)}{T}{}^{\mu}{}_{\nu}+\cdots\,, \tag{3·9e}$$

where $\overset{(0)}{\varphi}=$const, $\lambda_0'=(d\lambda/d\varphi)|_{\varphi=\overset{(0)}{\varphi}}$ and so on.

The choice $\lambda_0=0$[28] corresponds to zero cosmological constant or a graviton mass. After taking into account the gauge condition $\partial_{\mu}(\overset{(1)}{g}{}^{\mu\nu}-(1/2)\eta^{\mu\nu}\overset{(1)}{g})=0$, which defines the harmonic system of coordinates in the first approximation, Eqs. (3·6a, b) in the same approximation take the form

$$\Box\overset{(1)}{g}_{\mu\nu}=-\frac{16\pi G}{c^4}\Big(\overset{(1)}{T}_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}\overset{(1)}{T}\Big)\,, \tag{3·10a}$$

$$\Box\overset{(1)}{\varphi}+n\lambda''\overset{(1)}{\varphi}=2\pi n\frac{G}{c^4}\psi_0' T\,. \tag{3·10b}$$

Solutions of these equations are

$$\overset{(1)}{g}_{\mu\nu}=-\frac{2\pi G}{c^4}\int d^3x'dt'd\omega\frac{\overset{(1)}{T}_{\mu\nu}(x')-\frac{1}{2}\eta_{\mu\nu}\overset{(1)}{T}(x')}{|\boldsymbol{x}-\boldsymbol{x}'|}e^{(\omega(t-t')+(\omega/c)|\boldsymbol{x}-\boldsymbol{x}'|)}\,, \tag{3·11}$$

$$\overset{(1)}{\varphi}=-n\frac{G}{\pi c^4}\psi_0'\int d^3x'dt'd\omega\frac{\overset{(1)}{T}(x')}{|\boldsymbol{x}-\boldsymbol{x}'|}\exp\Big[\Big(\frac{\omega^2}{c^2}-n\lambda_0''\Big)\Big]|\boldsymbol{x}'-\boldsymbol{x}|\,. \tag{3·12}$$

In order to obtain a gravitational potential we consider the interaction of a particle with a gravitational field. As usual, the particle Lagrangian is

$$L_{\rm part}=\frac{16\pi G}{c^2}m\int\psi(\varphi)\Big(g_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds}\Big)^{1/2}\delta^4(x-x(s))ds\,, \tag{3·13}$$

where $x^k(s)$ is a world line coordinate of a particle. An equation of motion is derived by variation of an action and has a form of geodesic in a space-time metric $\psi^2 g_{\mu\nu}$ with a mass depending on coordinates

$$\frac{d}{ds}\Big(\psi^2 g_{\mu\nu}\frac{dx^{\nu}}{ds}\Big)=\frac{1}{2}\psi^2\frac{dx^{\alpha}}{ds}\frac{dx^{\beta}}{ds}g_{\alpha\beta,\mu}+\frac{c^2}{\psi}\frac{d\psi}{d\varphi}\partial_{\mu}\varphi\,. \tag{3·14}$$

Using $c^2=\psi^2 u_\mu u^\mu$ one gets

$$V(r)=c^2\left(\frac{1}{2}\overset{(1)}{g}_{00}+\psi_0'\overset{(1)}{\varphi}\right). \tag{3·15}$$

Comparing with the equation of motion in the Newtonian case, we get the potential energy

$$V(x)=-G\int d^3x\rho_0(x')\frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|}\{1+2n(\psi_0')^2 e^{-\sqrt{n\lambda_0''}|x-x'|}\}. \tag{3·16}$$

This result was obtained in Ref. 28), but due to a special choice of functions $h(\varphi)$, $l(\varphi)$ and $\lambda(\varphi)$ the explicit form of $g_{\mu\nu}$ is

$$\overset{(1)}{g}(x)=\frac{4G}{c^4}\int d^3x'\frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|}\left[T_{\mu\nu}(x')-\frac{1}{2}\eta_{\mu\nu}T(x')\left(1-\frac{1}{3}e^{-m|x-x'|}\right)\right]. \tag{3·17}$$

In a static limit both results are equivalent and lead to the potential

$$V(r)=-\frac{GM}{r}(1+\alpha e^{-mr}). \tag{3·18}$$

Such potential was used in Refs. 29) and 30) for a study of physical properties of the theory.

Models with conformal and several scalar fields are useful modifications of this scheme.[24)]

Here we must state that methods of treating a non-Newtonian potential in scalar-tensor theories are to a certain extent limited as one cannot consider $V(r)$ as a quantum limit of one-graviton and one-scalaron exchanges. There are no technical difficulties in calculating $S_2=T(:L_{\rm int}(g,\varphi)::L_{\rm int}(g,\varphi):)$ but in its nature $\varphi(x)$ is a global classical field describing a change in intensity of gravitational interaction in different regions of space-time and is not interacting with matter in some variants of the theory. So, in such an approach the scalar field plays the role of a certain "source" of a graviton mass.

Another type of a theory is a vector-tensor theory with the Lagrangian

$$L_1=\sqrt{-g}\{A_\alpha A^\alpha R(g)+L(A,\partial A)+L_{\rm mat}(\psi,\partial\psi,\cdots)\}, \tag{3·19}$$

where a scalar field is replaced by a vector one. One of negative consequences of this model is a space-time anisotropy.

Tensor-tensor theory is also of this class representing a certain variant of bimetric theories, whose Lagrangian may be taken as

$$L_2=\sqrt{-g}\left\{R(g)-\frac{1}{4}(\nabla_\alpha\varphi_{\mu\nu})^2+\frac{1}{2}\nabla_\alpha\varphi_{\mu\nu}\nabla^\nu\varphi^{\mu\alpha}-\frac{1}{2}\nabla_\alpha\varphi\nabla^\nu\varphi_\nu{}^\alpha+\frac{1}{4}(\nabla_\alpha\varphi)^2\right.$$
$$\left.-\frac{1}{4}m^2\varphi_{\mu\nu}\varphi^{\mu\nu}+\frac{1}{4}m^2\varphi^2\right\}+L_{\rm int}(g,\varphi,\psi), \tag{3·20}$$

where $\varphi_{\mu\nu}$ is a massive tensor meson and ψ are material fields. We want to stress that such a theory does have sense as mesons exist in nature. They are f-mesons with spin-parity $J^P=2^+$, mass $m_f=1270$ MeV and life time $\Delta t\leq 10^{-20}$ s. Despite the fact that these particles are resonances, in principle, there may be mechanisms where they serve as mediators of gravitational interaction. This hypothesis has been discussed for a certain period in connection with the problem of the strong gravity[33),34)]

Here we dwell upon the mechanism of a gravitational short-range interaction appearance suggested by Fujii.[35)] Ideas lying in the base of his approach are related to the development of a current algebra and include the treatment of an energy-momentum tensor as a current in hadron physics. Scale symmetry is an important one in the high energy physics. Scale transformations have the meaning of a length unit change. In a general case the unit of length may change from point to point arbitrarily. Naturally, the scale invariance demand modifies physical laws. For example, for the dilaton current conservation, which is caused by Noether's theorem, due to transformations $x_\mu \to x'_\mu \cong (1-\varepsilon)x_\mu$, it is necessary to consider the so-called "improved energy-momentum tensor", namely $\Theta_{\mu\nu}=T_{\mu\nu}+A\partial_\mu\partial_\nu+Bg_{\mu\nu}\Box$. Here $T_{\mu\nu}$ is the canonical energy-momentum tensor. As the energy-momentum tensor is a source of a gravitational field a substitution of $\Theta_{\mu\nu}$ instead of $T_{\mu\nu}$ into Einstein equations for obtaining their solutions or using for calculation of two particle interaction potential via one-graviton exchange does not lead to substantial changes in a static potential. In the dynamical case an exchange of virtual gravitons gives finite corrections but does not change the functional form of the potential dependence with range.

Essential changes will be due to a scale invariance connected with the introduction of a new dilaton field. One way of a realization of scale invariance is a construction of the theory with zero masses and dimensionless constants. Another way valid for a theory with massive particles is an introduction of a new elementary particle-dilaton which is deeply connected with gravitation.

As an illustration let us consider a realization of scale invariance in a matrix element of an energy-momentum tensor for two particles states. The trace of this element will be[36)]

$$\langle p_2|\Theta^\mu{}_\mu|p_1\rangle=2m^2F(q^2)+q^2\left(3G(q)-\frac{1}{2}F(q^2)\right). \qquad (3\cdot21)$$

From scale invariance when $q=p_1-p_2=0$ and $F(0)=1$ one obtains

$$m^2=\lim_{q^2\to 0}\frac{3}{2}q^2G(q^2). \qquad (3\cdot22)$$

So, we have two alternatives

a) $m^2=0$ and $G(0)$ is finite,

b) $m^2\neq 0$ and $G(q^2)$ has a pole at $q=0$ because of an existence of the massless particle-0^+ dilaton.

Let us single out the state 0^+ which is supposed to be unique, and denote it by σ, then

$$\langle 0|\Theta^\mu{}_\mu|\sigma\rangle=\mu_\sigma{}^2f_\sigma, \quad \langle p'|p\sigma\rangle=mg, \qquad (3\cdot23)$$

where g is dimensionless and μ is a scale of scaling breakdown in the exact limit of a scale symmetry. Then

$$2m = f_\sigma g + \mathcal{D}(\mu_\sigma^2) \tag{3·24}$$

and for baryons

$$m_B = f_\sigma g_{\sigma B} + \mathcal{D}(\mu_{\sigma B}^2)\,. \tag{3·25}$$

So, the existence of dilatons leads to a certain relation between masses of interacting particles and constants, i.e., relation of Goldberger-Treiman type.

Breaking of scale invariance and consequent appearance of a dilaton mass lead to intermediate short-range forces. Let us consider a one-graviton exchange in the presence of a massive dilaton whose mass appeared as a result of a chiral symmetry breaking.[36] The interaction potential is calculated similarly to Gupta's method[12),13)] and is an average over states, i.e. of nucleons. Then, as it is shown in Ref. 35), diagrams of such an exchange are sums of three diagrams,

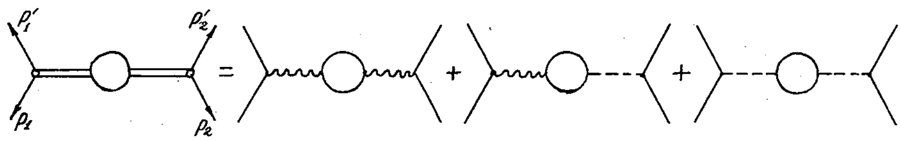

Fig. 2.

whose wave line is a graviton, dashed line is a dilaton and continuous line is a nucleon. The potential energy will be the sum of three terms corresponding to three graphs

$$V = V_1 + V_2 + V_3\,, \tag{3·26}$$

where

$V_1 = \hbar^2 \langle T_{\mu\nu} \boldsymbol{D}^{\mu\nu\alpha\beta} T_{\alpha\beta} \rangle$ is the one graviton exchange,

$V_2 = f_{\bar{N}N\sigma} \langle T_{\mu\nu} \boldsymbol{D}^{\mu\nu}{}_{(\sigma)} \rangle$ is the exchange of a graviton with creation of a dilaton,

$V_3 = f_{\bar{N}N\sigma} \boldsymbol{D}_{(\sigma\sigma)} f_{\bar{N}N\sigma}$ is the one dilaton exchange.

Here $f_{\bar{N}N\sigma}$ is an interaction constant of nucleons with dilatons.

Fujii supposed[35)] that the dilaton mass is of the order of $[G_\infty(\alpha')^{-2}]^{1/2}$, where α' is a universal slope of a Regge trajectory. This mass is a combination of gravitational and strong interaction constants and is a manifestation of deep relations between them which may be seen as a deviation from Newton's law at ranges ≈ 30 m[35)] due to Yukawa type forces.

The above-mentioned theory one may consider as a theory with additional mediators of a gravitational interaction. But as a scale symmetry is a part of more general conformal symmetry we treat it as a modification of scalar-tensor theories.

§ 4. Theories with torsion

We would like to speak of torsion because we think that it is a necessary generalization of the Einstein theory of relativity that is a generalization of the Riemannian geometry of space-time. This generalization goes back to a slight modification of the Einstein theory proposed in 1922~23 by Cartan.[37)~40)]

According to Trautman "the Einstein-Cartan theory is the simplest and the most natural modification of the original Einstein theory of gravitation".[41)] But why one tries to propose alternative or more general gravitation theories when, as we know, the general theory of relativity is the simplest gravitation theory which is in agreement with all experimental facts in the domain of macrophysics including the more recent experiments on time-delay, with radar experiments on Mercury and Venus and other sophisticated experiments in the Solar system?

The motivations are mainly of theoretical character: In fact we are, at first sight, in two very different words: On one hand we have at a microscopical level the strong and weak interactions while the gravitational interaction is the weakest and seems not to play any role; on the other hand all known interactions, but not gravitation, that is strong, weak and electromagnetic, are well described within the fromework of relativistic quantum field theory in the flat Minkowski space-time. So, it seems that gravitation has no effects when we are concerned with elementary particle physics. But today we know that this is not true: In fact if we consider the quantum theory in a curved space-time instead that in the flat Minkowski space-time, we have some very important new effects (fifth force [42)], neutron interferometry,[43)] etc.), and, moreover, when we go at a microphysical level, that is when we are concerned with elementary particle physics, we realize that the role of gravitation becomes very important and necessary for instance when we have to do with the early universe. In the very early universe the cosmological problem is strictly connected with elementary particle physics. But then we must pay attention to such a question: When we consider together general relativity and elementary particle physics, the latter described by the quantum field theory, we are obliged to take into account not only the mass of elementary particles, but also the spin. In fact the elementary particles are characterized not only by mass but also by spin which occurs in units of $\hbar/2$. Mass and spin are two elementary and independent original concepts: As a mass distribution in a space-time is described by the energy-momentum tensor, so a spin distribution is described in a field theory by the spin density tensor. As the mass is connected with the curvature of space-time, the spin will be connected with another geometrical property of the space-time so that we must modify consequently the general theory of relativity in order to connect this new geometrical property with the spin density tensor. In this way we are led to the notion of torsion. In fact all elementary particles can be classified by means of the irreducible unitary representation of the Poincaré group and can be labelled with translation part of the Poincaré group, while spin is connected with the rotational part. In a classical field theory, mass corresponds to a canonical stress-energy-momentum tensor, and spin to a canonical spin tensor. The dynamical relation between the stress-energy-momentum tensor and curvature is expressed in general relativity by Einstein equations; one feels here a need for an analogous dynamical relation including spin density tensor. So, as this is impossible in the framework of the general relativity we are forced to introduce this new geometrical property that is called torsion. We can say that as the mass is responsible for curvature, spin is responsible for torsion. We now will see from a formal point of view in which way we must modify the general relativity theory and how it represents a slight modification of this theory: In fact the main point is to assume an affine connection asymmetric instead of the symmetric connection that we have in the Einstein theory (the Christoffel symbols). Torsion is in fact connected with antisymmetric part of the affine connection as we shall see.

To sum up: When we are dealing with a microphysical realm we find that, besides the mass, the spin comes into play and then has to be considered as the source of a gravitational field (as the mass): Remember that when we speak of a gravitational field, we mean a field inseparably coupled to the geometry, that is, we speak of the structure of the space-time. In analogy to the mass where the energy-momentum tensor is coupled to the metric, we will expect that the spin-density tensor is coupled to some geometrical quantity of the space-time (a quantity that should relate to the rotational degrees of freedom in a space-time).

In this way we are led to a generalization of a Riemannian space-time. We shall see very briefly which are the new geometrical properties of the torsion as all the development will follow very near the structure of general relativity. We may also notice that Cartan proposed to relate the torsion tensor to the density of intrinsic angular momentum in 1922,[37)~40)] well before the introduction of the modern concept of spin by Uhlenbeck and Goudsmit in 1925.[44)] This fact may have been the reason for which Cartan hypothesis has been ignored for so long time, and when it has been reconsidered, it was connected not with the spin but with electromagnetic field in the attempt to unify the gravitational and electromagnetic interaction. At last it should be noted that with the introduction of torsion, with this small departure from Einstein's general relativity, the field equations in empty space are the same so that the majority of its experimental verifiable consequences for the Solar system cannot be distinguished from the predictions of general relativity. On the other hand we will see that we have many important consequences: For instance when we apply this Einstein-Cartan theory in the ambit of the cosmology, especially in the early universe; or when considering superdense objects as neutron stars or black holes where we have strong internal magnetic fields which may give rise to alignment of spins or vice versa we will see that the torsion interaction may align spins giving rise to magnetic field. So, we will consider the problem of magnetic field in the universe, the neutrino magnetic momentum, the connection of torsion with quantum effects, the problem of cosmological constant, to quote only some of the problems where torsion appears to be important. But let us, first of all, introduce torsion, that introduces spin in general relativity as a dynamical quantity.

4A. *Riemann-Cartan geometry*

Torsion as the antisymmetric part of an asymmetric affine connection was introduced by Cartan in 1922~25.[37)~40)] The definition of Cartan torsion is then the antisymmetric part of the affine connection $\Gamma^{\alpha}{}_{\mu\nu}$ that is

$$Q^{\alpha}{}_{\mu\nu}=\Gamma^{\alpha}{}_{[\mu\nu]}=\frac{1}{2}(\Gamma^{\alpha}{}_{\mu\nu}-\Gamma^{\alpha}{}_{\nu\mu}) \tag{4·1}$$

and has a tensor character: Cartan had some idea that it might be connected with the intrinsic angular momentum of matter but only later it became clear that the energy-momentum tensor of massive spinning fields, as for instance Dirac field, must be asymmetric.

Now we know that the Einstein field equations show how the stress energy-momentum tensor of matter generates a curvature of space-time. But is only this tensor the source of geometry? Or whether some other dynamical quantities which influence the geometry of the space-time exist?

We know that a tetrad field is determined in each point of the space-time by a metric tensor up to the six-parameter group of rotations and it is possible to treat gravitation as a compensating field of the group of coordinate transformations and tetrad rotations (in a similar way to the treatment of the electromagnetic field as a compensating field of the group of gauge transformations of charged fields).

In this way one can introduce the spin as a dynamical quantity in the theory of gravitation and one is led to an extension of the geometric structure of the space-time. Cartan's idea about a relation between torsion and spin may be supported considering that in special relativity theory we have to do with the Poincaré group.[41] The Poincaré group is a semi-direct product of the Lorentz group and the group of translations. Now the mass is connected with the translation part of the Poincaré group while the spin is connected with the rotational part. In classical field theory mass corresponds to a canonical stress-energy-momentum tensor and spin to a canonical spin tensor. In general relativity the dynamical relation between the stress-energy-momentum tensor and curvature is expressed by the Einstein equations so one may think that it must be an analogous dynamical relation including spin tensor, a relation that would connect the spin tensor with another geometric property of the space-time. This obviously is not possible in the ambit of general relativity so that one is necessitated to modify the theory in order to introduce torsion and relate it to the spin tensor.

When this is done one arrives at an interesting link between the theory of gravitation and the special relativity: In fact one can show that torsion is related to translations in the tangent space of the manifold in a similar fashion as the curvature is related to Lorentz transformations. If L_{mat} denotes the material Lagrangian density in general relativity we have for the dynamical stress-energy-momentum tensor

$$T^{\mu\nu}=\frac{2}{\sqrt{-g}}\frac{\delta L_{\mathrm{mat}}}{\delta g_{\mu\nu}}. \qquad (4\cdot2)$$

Now if L depends not only on the tensor $g_{\mu\nu}$ and its derivatives but also on a new geometrical quantity $K^{\alpha}{}_{\mu\nu}$, we have the following dynamical definition of spin

$$S^{\alpha}{}_{\mu\nu}=\frac{1}{\sqrt{-g}}\frac{\delta L_{\mathrm{mat}}}{\delta K_{\mu\nu}{}^{\alpha}}. \qquad (4\cdot3)$$

$K^{\alpha}{}_{\mu\nu}$ is called the contorsion tensor and, as we shall see later on, it is connected with the torsion tensor defined in (4·1)

$$K^{\alpha}{}_{\mu\nu}=Q^{\alpha}{}_{\mu\nu}+Q_{\mu\nu}{}^{\alpha}+Q_{\nu\mu}{}^{\alpha}\,, \qquad (4\cdot4)$$

so that we can write for the affine connection

$$\Gamma^{\alpha}{}_{\mu\nu}=\{^{\alpha}{}_{\mu\nu}\}+K^{\alpha}{}_{\mu\nu}\,. \tag{4·5}$$

It is usual to denote the Riemann-Cartan space-time of the Einstein-Cartan theory (when an affine asymmetric connection is introduced) as U_4 to distinguish it from the Riemann space-time (with a symmetric affine connection) which is denoted by V_4. It is then possible to go from V_4 to U_4 simply substituting everywhere the asymmetric affine connection in the place of Christoffel symbols. When $K^{\alpha}{}_{\mu\nu}=0$ we are back in the Riemannian space V_4.

Consider now in U_4 a differentiable manifold in 4 dimensions in which a metric $g_{\mu\nu}(x)$ is defined which provides the distance ds between two nearby points x^{μ} and $x^{\mu}+dx^{\mu}$

$$ds^2=g_{\mu\nu}(x)dx^{\mu}dx^{\nu} \tag{4·6}$$

and in which also an affine connection $\Gamma^{\alpha}{}_{\mu\nu}$ is defined which provides the infinitesimal variation of a vector B^{μ} during the parallel transport from x^{μ} to $x^{\mu}+dx^{\mu}$

$$\delta B^{\mu}=-\Gamma^{\mu}{}_{\alpha\beta}(x)B^{\beta}dx^{\alpha}\,. \tag{4·7}$$

Now if B^{μ} are the components of the vector in x^{μ} and $B^{\mu}+dB^{\mu}$ the components in $x^{\mu}+dx^{\mu}$, we have

$$dB^{\mu}-\delta B^{\mu}=\frac{\partial B^{\mu}}{\partial x^{\alpha}}dx^{\alpha}+\Gamma^{\mu}{}_{\alpha\beta}B^{\beta}dx^{\alpha} \tag{4·8}$$

and is easy to see that the antisymmetric part of the affine connection, that is $\Gamma^{\mu}{}_{[\alpha\beta]}$, is a tensor.

4B. *Some physical consequences*

First of all we can note an analogy between torsion and magnetic field. In fact the spin-torsion interaction Lagrangian in a Riemann-Cartan space is[45]

$$L_{\rm int}=-\frac{1}{2}S^{\alpha\mu\nu}K_{\nu\mu\alpha}\,. \tag{4·9}$$

In the case of a Dirac particle we have

$$S^{\mu\alpha\beta}=\varepsilon^{\mu\alpha\beta\nu}S_{\nu} \tag{4·10}$$

(where $\varepsilon^{\mu\alpha\beta\nu}$ is the completely antisymmetric symbol) and the interaction Lagrangian contains the axial-vector part of the torsion tensor

$$L=-S_{\nu}\check{Q}^{\nu} \tag{4·11}$$

(where $\check{Q}_{\nu}=(1/3!)\varepsilon^{\nu\alpha\beta\mu}Q_{\alpha\beta\mu}$); going to the particle rest system ($S^{\alpha}\equiv(0,\boldsymbol{S})$) the interaction energy between spin $\boldsymbol{S}$ and torsion is

$$E=-\boldsymbol{SQ} \tag{4·12}$$

and one can see the formal analogy with the interaction energy of a magnetic moment

$\boldsymbol{\mu}$ in a constant magnetic field $\boldsymbol{H}$, namely,

$$E = -\boldsymbol{\mu}\boldsymbol{H}\,. \tag{4·13}$$

We can now understand that torsion induces spin alignment as well as a magnetic field and if each spin is associated with a magnetic moment, torsion can also produce a magnetic field inside a matter.

Applying these considerations to the early universe, we can give a theoretical basis [see Ref. 46)] to the Blackett law[47)]

$$S = qU\,, \tag{4·14}$$

(where S is an intrinsic angular momentum, U is a magnetic moment of stars and q is a universal constant) finding for q the value

$$q \sim \frac{c}{(\alpha G)^{1/2}}\,, \tag{4·15}$$

(where c is the light velocity, α is the fine-structure constant and G is the gravitational constant) which is in remarkable agreement with astrophysical data ($q = 1.3\cdot 10^{15}\mathrm{g}^{1/2}\,\mathrm{cm}^{-1/2}$).[48)] Moreover torsion by including alignment of spins (and then of the intrinsic magnetic moments) of particles in the primordial matter, can produce a magnetic field[49)]

$$H = \frac{8\pi}{3c}\sqrt{2\alpha G}\,\sigma \tag{4·16}$$

(where σ is the spin density). By hypothesis of tensor-meson dominance, hot hadronic matter undergoes strong gravitational interactions[50)] with a strong coupling constant $G_f = \hbar/mc^2 \cong 10^{38}G$ so that G must be replaced by G_f in Eq. (4·16).

Thus, for a neutron star[51)] with all nucleon spins aligned by the strong gravity spin-torsion interaction, Eq. (4·16) would give a magnetic field $H = 10^{15}$ G.

Another interesting consequence that also shows the link between torsion and magnetism is the possibility to have a magnetic moment for neutral particles. In particular one can show that a neutrino magnetic moment can be present also for zero mass neutrinos[52)] and this can resolve the solar-neutrino puzzle[53)] and one can some predictions as regards experiments with ^{37}Cl and ^{71}Ga.[53)]

Also the possible presence of a fifth force can find an explanation through torsion.[42)]

The presence of torsion can be directly tested with a proposed experiment which uses an interferometric apparatus with polarized neutrinos.[43)]

At last we would like to mention that through torsion we can have a way to go towards quantization of length and time: In fact if we consider a small closed circuit and write

$$l^{\alpha} = \oint Q^{\alpha}{}_{\mu\nu}\,dA^{\mu\nu} \tag{4·17}$$

(where $dA^{\mu\nu} = dx^{\mu} \wedge dx^{\nu}$ is the area element enclosed by the loop), then l^{α}, having a dimension of length, represents the closure failure, i.e., torsion has an intrinsic geometric meaning: It represents the failure of the loop to close analogous to the crystal case; in other words in the geometrical description of crystal dislocations or defects, it is known that torsion plays the role of defect density in the limit of dislocations having a continuous distribution.[54),55)] As torsion is related to the intrinsic spin, if we connect torsion to the fundamental unit of intrinsic spin $\hbar$, we find that the defect in space-time topology should occur in multiples of the Planck length,[56)] so that we can write

$$\oint \mathbf{Q} dA = n(\hbar G/c^3)^{1/2} \tag{4·18}$$

and for the same reason, considering the fourth component of (4·17), we have also an analogous situation as regards time, namely,

$$t = (1/c)\oint \mathbf{Q} dA = n(\hbar G/c^5)^{1/2} \tag{4·19}$$

which gives a minimum unit of time $\neq 0$ (for $n=1$).

This quantum of time or minimal unit of time implies a limiting frequency of $f_{\max} = (\hbar G/c^5)^{1/2}$ giving the convergence of Feynman integral on QED.

We mentioned here the Einstein-Cartan theory as an example of the extended gravitational theories, that was the first theory in which the deviations from Newton's law were presented. This theory is the particular variant of the Poincaré gauge theory and the last will be discussed in § 6 in detail.

§ 5. Theories with higher derivatives and quantum gravitational corrections to Newton's law

Quantum nature of matter fields gives rise to two important effects—vacuum polarization and particle creation by external gravitational fields. The first may be taken into account by introduction of quadratic in curvature terms into the Lagrangian. The study of quantum gravitational effects was started in Refs. 58)~60) and was developed in Ref. 61).

The problem of quantum corrections first arose in relation to the singularity problem as near Planck scales quantum fluctuations of matter fields may be dominant. Self-consistent treatment of these corrections in cosmology and also the account of spontaneous breaking of symmetry effect in a gravitational field led to a number of important results, such as inflational models of the Universe[62)~64)] and possibility of a bounce, i.e., change of contraction to expansion at non-zero value of a scale factor.[65)]

Quantum fields of a different tensor dimension lead to effective Lagrangian with additional terms of the type R^2 and $R_{\mu\nu}R^{\mu\nu}$ (in other variant to R^2 and $C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}$, where $C_{\mu\nu\alpha\beta}$ is the Weyl tensor). Corresponding equations of the gravitational field will be of the fourth order. This may lead, from one hand, to better convergence of Green's function[66)] and renormalizability of the theory[67),68)] and from another to appearance of massive modes in the spectra and as a consequence to short-range forces. As an example of explicit separation and obtaining of irreducible graviton-field components, let us study a case similar to Refs. 69) and 70) where the Lagrangian was suggested

$$L = \sqrt{-g}\{\gamma k^{-2}R + \alpha R_{\mu\nu}R^{\mu\nu} - \beta R^2\}\,. \tag{5·1}$$

Let the metric be static and spherically symmetric

$$ds^2=B(r)dt^2-A(r)dr^2-r^2(d\theta^2+\sin^2\theta d\varphi^2)\,. \qquad (5\cdot2)$$

Then, for a point particle described by energy-momentum tensor $T_{\mu\nu}=\delta^0{}_\mu\delta^0{}_\nu M\delta^3(0)$ the gravitational potential is[69]

$$V(r)=-\frac{k^2M}{8\pi\gamma r}+\frac{k^2M}{6\pi\gamma}\frac{e^{-m_2r}}{r}-\frac{k^2M}{24\pi\gamma}\cdot\frac{e^{-m_0r}}{r}\,, \qquad (5\cdot3)$$

where $m_2=\gamma^{1/2}(\alpha k^2)^{-1/2}$ and $m_0=\gamma^{1/2}[2(3\beta-\alpha)k^2]^{-1/2}$. But the static potential obtained as a solution of the corresponding equations does not tell us which components of a metric field are responsible for short-range forces. In order to find it out one must decompose a graviton propagator into states using spin projectors.[71)~73)] The construction of spin projectors and extracting of irreducible components is based on a construction of irreducible representations of the Lorentz group. Here we shall write only the explicit form of a graviton propagator for the theory with Lagrangian $(4\cdot1)$[67)]

$$\boldsymbol{D}_{\mu\nu\alpha\beta}(p)=-\frac{i}{(2\pi)^4}\left\{\frac{2P^{(2)}_{\mu\nu\alpha\beta}(p)}{p^2(\alpha k^2p^2+\gamma)}-\frac{2P^{(os)}_{\mu\nu\alpha\beta}(p)}{p^2\left[(3\beta-\alpha)k^2p^2+\frac{1}{2}\gamma\right]}+\frac{2\varDelta P^{(1)}_{\mu\nu\alpha\beta}(p)}{p^4k^2}\right.$$

$$\left.-\frac{\varDelta(3P^{(os)}_{\mu\nu\alpha\beta}(p)-\sqrt{3}[P^{(osw)}_{\mu\nu\alpha\beta}(p)+P^{(ows)}_{\mu\nu\alpha\beta}(p)]+P^{(ow)}_{\mu\nu\alpha\beta}(p))}{p^4k^2}\right\}\,, \qquad (5\cdot4)$$

where $P^{(\cdots)}_{\mu\nu\alpha\beta}$ are projectors which single out corresponding irreducible subspaces.

In a Lorentz gauge all terms proportional to $\varDelta$ disappear and in the action of interacting matter and gravitational fields only projectors with states 2^+ and 0^+ will remain. This linearized action will take the form

$$S=\int d^4x\left\{-\frac{1}{4}h^{\mu\nu}(\alpha k^2\Box-\gamma)\Box P^{(2)}_{\mu\nu\alpha\beta}h^{\alpha\beta}\right.$$

$$\left.+\frac{1}{4}h^{\mu\nu}(2(3\beta-\alpha)k^2\Box-\gamma)\Box P^{(os)}_{\mu\nu\alpha\beta}h^{\alpha\beta}+\frac{1}{2}h^{\mu\nu}T_{\mu\nu}\right\}\,, \qquad (5\cdot5)$$

where $T_{\mu\nu}$ is an energy-momentum tensor of matter fields.

Let us introduce

$$h_{\mu\nu}=\varphi_{\mu\nu}+\psi_{\mu\nu}+\eta_{\mu\nu}\chi+2m_2{}^{-2}\partial_\mu\partial_\nu\chi \qquad (5\cdot6)$$

and put it into $(5\cdot5)$. After simple calculations taking into account the conservation law we obtain

$$S=\int d^4x\left\{\gamma L_E(\varphi)-\gamma L_E(\psi)+\frac{\gamma m_2{}^2}{4}(\psi_{\mu\nu}\psi^{\mu\nu}+\psi^\alpha{}_\alpha\psi^\lambda{}_\lambda)\right.$$

$$\left.-\frac{3\gamma}{2}(\partial_\mu\chi\partial^\mu\chi+m_0{}^2\chi^2)+\frac{k}{2}(\varphi_{\mu\nu}+\psi_{\mu\nu}+\eta_{\mu\nu}\chi)T^{\mu\nu}\right\}\,, \qquad (5\cdot7)$$

where

$$L_E(h)=-\frac{1}{2}h^{\mu\nu}(\eta_{\mu\alpha}\eta_{\nu\beta}+\eta_{\mu\beta}\eta_{\nu\alpha}+\eta_{\mu\nu}\eta_{\alpha\beta})\Box h^{\alpha\beta}\,. \tag{5·8}$$

So, it is evident that $h_{\mu\nu}$ contains states 2^+ and 0^+. One of the modes, i.e., 2^+ is massless, another ($\psi_{\mu\nu}$) is massive but with a negative energy. The last one is a massive scalaron with a positive energy. Without doubt one should not treat these states as separate particles as they are components of a graviton field interacting with matter. The form (5·7) is convenient for Gupta type calculations[12),13)] as in this case

$$\boldsymbol{D}_{\mu\nu\alpha\beta}=\boldsymbol{D}^{(\varphi)}_{\mu\nu\alpha\beta}+\boldsymbol{D}^{(\psi)}_{\mu\nu\alpha\beta}+\boldsymbol{D}^{(\chi)}_{\mu\nu\alpha\beta}\,, \tag{5·9}$$

where $\boldsymbol{D}^{(\varphi)}_{\mu\nu\alpha\beta}$ is the same as in the Einstein theory and

$$\boldsymbol{D}^{(\psi)}_{\mu\nu\alpha\beta}=\frac{1}{4}(\eta_{\alpha\mu}\eta_{\nu\beta}+\eta_{\mu\beta}\eta_{\nu\alpha}+\eta_{\mu\nu}\eta_{\beta\alpha})\frac{1}{(2\pi)^4}\int d^4p\frac{e^{-ipx}}{p^2-m_2{}^2} \tag{5·10}$$

and

$$\boldsymbol{D}^{(\chi)}_{\mu\nu\alpha\beta}=\eta_{\mu\nu}\eta_{\alpha\beta}\frac{1}{(2\pi)^4}\int d^4p\frac{e^{-ipx}}{p^2+m_0{}^2}\,. \tag{5·11}$$

It is easy to show that the interaction potential of two particles in a static case will be exactly (5·3). But now it is clear that the first term is caused by a one-graviton exchange, the second one is an "exchange" of ghost states and the third is an exchange of massive particles. We point out that massive ghosts lead to a repulsion potential.

An introduction of quadratic terms into Lagrangian is caused by quantum corrections. We reproduce here exact calculations of corrections to Newton's law in a background Schwarzschild solution.[74),75)] When $(2GM/r)\ll 1$ one may write

$$ds^2=\left(1-\frac{2GM}{r}\right)dt^2-\left(1+\frac{2GM}{r}\right)dr^2-r^2d\Omega^2\,. \tag{5·12}$$

Gravitational field may be decomposed into the sum of classical and quantum parts as

$$g^{\mu\nu}=\eta^{\mu\nu}+k(h^{\mu\nu}_{(\mathrm{cl})}+h^{\mu\nu}_{(\mathrm{q})})\,, \tag{5·13}$$

where $h^{\mu\nu}_{(\mathrm{cl})}$ obeys linearized Einstein equations whose solutions are

$$h^{00}_{(\mathrm{cl})}=\frac{4GM}{r}\,,\quad kh^{ij}_{(\mathrm{cl})}=0\quad \text{for}\quad i\neq j\,. \tag{5·14}$$

Quantum part in momentum representation will be

$$h^{\mu\nu}_{(\mathrm{q})}=\boldsymbol{D}^{\mu\nu\alpha\beta}\mathcal{P}_{\alpha\beta\gamma\delta}h^{\gamma\delta}_{(\mathrm{cl})}\,, \tag{5·15}$$

where $\boldsymbol{D}^{\mu\nu\alpha\beta}$ is a graviton field propagator and $\mathcal{P}_{\alpha\beta\gamma\delta}$ is a graviton field proper energy propagator. Using Ward-Slavnov-Takahashi identity one may single out the finite part. Then its real part will take the form[76),77)]

$$\begin{aligned}\mathcal{P}_{\alpha\beta\gamma\delta}(p^2)=&\mathcal{P}_1(p^2)p^4\eta_{\alpha\beta}\eta_{\gamma\delta}+\mathcal{P}_2(p^2)p^4(\eta_{\alpha\gamma}\eta_{\beta\delta}+\eta_{\alpha\delta}\eta_{\beta\gamma})\\&+\mathcal{P}_3(p^2)p^2(\eta_{\alpha\beta}p_\gamma p_\delta+\eta_{\gamma\delta}p_\alpha p_\beta)+\mathcal{P}_4(p^2)p^2(\eta_{\alpha\gamma}p_\beta p_\delta+\eta_{\alpha\delta}p_\beta p_\gamma+\eta_{\beta\delta}p_\alpha p_\gamma)\\&+\mathcal{P}_5(p^2)p_\alpha p_\beta p_\gamma p_\delta\,.\end{aligned} \tag{5·16}$$

From the W-S-T identity the conditions on $\mathcal{P}_i(\cdots)$ follow

$$\mathcal{P}_2+\mathcal{P}_4=0\,, \quad 4(\mathcal{P}_1+\mathcal{P}_2-\mathcal{P}_3)+\mathcal{P}_5=0\,. \tag{5·17}$$

Using dimensional technique to single out finite terms, one obtains

$$\mathcal{P}_i=k^2\left(a_i\log\frac{p^2}{\mu_i}+b_i\right), \quad i=1,\cdots,5\,, \tag{5·18}$$

where μ is a dimensional parameter and a and b are numbers.

Substituting (5·15)~(5·18) into (5·4) and taking into account $h^{00}_{(\mathrm{cl})}(p)=(\pi/2M)\delta(P_0)(1/|\boldsymbol{p}|^2)$, we get

$$h^{\mu\nu}_{(\mathrm{q})}(p)=h^{\alpha\beta}_{(\mathrm{cl})}\Big[2\mathcal{P}_2(p^2)p^2\delta^{\mu}{}_{\alpha}\delta^{\nu}{}_{\beta}-\Big(\mathcal{P}_1(p^2)+\mathcal{P}_2(p^2)+\frac{1}{2}\mathcal{P}_3(p^2)\Big)p^2\eta^{\mu\nu}\eta_{\alpha\beta}$$
$$+\mathcal{P}_3(p^2)p^{\mu}p^{\nu}\eta_{\alpha\beta}\Big]\,. \tag{5·19}$$

It is evident now that there are three types of contributions to $h^{\mu\nu}_{(\mathrm{q})}$, namely, $\delta^3(\boldsymbol{r})$, $r^{-3}\log(\mu r)$, r^{-3}. The first is a consequence of a contact interaction conservation: One may expect Lamb type effects in an atom here. The second may be eliminated using suitable gauge. Finally, the third is very essential and leads to such changes in a metric:

$$ds^2_{\mathrm{ren}}=\left(1-\frac{r_0}{r}-\alpha\frac{r_0L_P{}^2}{r^3}\right)c^2dt^2-\left(1+\frac{r_0}{r}+\beta\frac{r_0L_P{}^2}{r^3}\right)dr^2$$
$$-r^2\left(1+\beta\frac{r_0L_P{}^2}{r^2}\right)d\Omega^2\,, \tag{5·20}$$

where $r_0=(2GM/c^2)$, and $L_P=(G\hbar/c^3)^{1/2}\cong10^{-33}$ cm is a Planck length, $\alpha>0$, $\beta>0$.

Many-loop contributions will be of the order higher than (L_P/r). Possible sum of this infinite set was analyzed in Ref. 75).

So, it is evident that quantum nature of the gravitational interaction and of matter fields as well leads to a change in Newton's law. But due to smallness of quantum effects one may talk here only about ranges that are less than in scalar-tensor theories.

§ 6. Extended theories of gravitation

In connection with attempts of including gravitation into the unified scheme of all fundamental physical interactions among viable theories from the point of view of their correspondence to basic quantum postulates, i.e., local gauge invariance and renormalizability, supergravity and gauge theory of gravitation are more often used. In both these models not only short range potentials but also repulsion and antigravity are predicted and obtained. Let us discuss both theories in more detail.

6.A. *Supergravity and extended supergravity*

Supergravity was first suggested in Refs. 78) and 79) as a natural development of a global extension of Poincaré group by spinor generators in Refs. 80) and 81). One may find a lot about these models in Refs. 82)~85). Here we pay attention only to one fact necessary for our aims which, maybe, is one of the most important predictions of supergravity.

Supersymmetrization of the gravitational interaction led to prediction of a possible existence of a graviton superpartner—massless particle of spin 3/2—gravitino. Attempts to unify all four types of fundamental interactions in a frame of a supersymmetry created the so-called extended supergravity schemes with different internal symmetry groups ($SU(N)$, $SO(N)$, $\cdots$, $N=1, 2, \cdots, 8$). As a separate field of study we point out also the interaction of supersymmetric theories of internal symmetries with a gravitational field. These investigations led in particular to the existence of deviations from Newton's law.

Appearance of the "vector"gravitation was pointed out in Refs. 86), and 87) where $N=2$ supergravity is combined with global supertransformations. Rather complicated Lagrangian[86] contains a number of blocks: $N=2$ supergravity Lagrangian, a massless sector of globally supersymmetric matter, Noether supercurrents, massive terms and selfinteracting fields. A set of vector fields $A_\mu{}^a (a=1, 2)$ is introduced in the global supersymmetry sector, which interact with fundamental ones according to $D_\mu \psi_j = (\delta_{ij}\partial_\mu + km\varepsilon^i{}_{aj}A_\mu{}^a)\psi_i$, where k is the gravitational constant and m is a mass of a boson or fermion field. $A_\mu{}^a$ are fields with strengths $F^a_{\mu\nu} = \partial_\mu A_\nu{}^a + \partial_\nu A_\mu{}^a$, and the coupling $g=km$ is caused by the supersymmetry properties of the Lagrangian as a whole.[87]

Using simpler model we calculate here the gravitational potential in such theories. Let the Lagrangian of supermultiplet interacting with a gravitational field be

$$L = -\det|h_\mu{}^a|\left\{-\frac{1}{4k^2}h^{a\mu}h^{b\nu}R_{ab\mu\nu} - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + g^{\mu\nu}(D_\mu\varphi^i)^*(D_\nu\varphi^i) - m_i{}^2\varphi_i^*\varphi_i + \sum\psi^k(i\gamma^\mu \overleftrightarrow{D}_\mu - m_k)\psi^k\right\}, \tag{6·1}$$

where

$$D_\mu\varphi^i = \partial_\mu\varphi^i + ig\varepsilon^i{}_{aj}A_\mu{}^a\varphi^j\,, \quad D_\mu\psi^i = \nabla_\mu\psi^i - ig\varepsilon^i{}_{aj}A_\mu{}^a\psi^j\,.$$

The interaction between matter fields (boson plus fermion) is described by

$$L_{\text{int}} = g^{\mu\nu}T_{\mu\nu} + A_\mu{}^a J_\mu{}^a\,, \tag{6·2}$$

where $T_{\mu\nu} = (\delta L(\varphi, \psi, \cdots)/\delta g^{\mu\nu})$ and $J_\mu{}^a = (\delta L(\varphi, \psi, \cdots)/\delta A_\mu{}^a)$ is a vector current.

Gravitational field propagator in a Lorentz gauge takes the usual form and the vector graviton propagator in the gauge $\partial_\mu A^{a\mu}=0$ is $\boldsymbol{D}^{ab}_{\mu\nu} = \delta^{ab}(\eta^{\mu\nu}/p^2)$. Then in a tree approximation the second order matrix element is

$$S^{(2)} = \frac{16\pi G}{p^2}\left\{2\left(\overset{(1)}{T}_{\mu\nu}\overset{(2)}{T}_{\mu\nu} - \frac{1}{2}\overset{(1)}{T}_\alpha{}^\alpha\overset{(2)}{T}_\beta{}^\beta\right) - \frac{g^2}{4\pi G}\overset{(1)}{J}_\mu{}^a\overset{(2)}{J}_\mu{}^b\right\}. \tag{6·3}$$

Due to Refs. 86) and 87) in order to make the vector interaction short-ranged it is necessary to break symmetry spontaneously demanding $\langle\varphi\rangle \cong 1$ GeV. Then the interaction potential of two particle multiplets must contain the Yukawa term. To a certain extent it was an inevitable step as under the condition $g=km$ the massless vector field led to the large value of the violation of the equivalence principle $\delta|\boldsymbol{w}|/|\boldsymbol{w}| \cong 1.5\ 10^{-5}((Z_1/A_1)-(Z_2/A_2))$ at any range ($|\boldsymbol{w}|$ is a free fall acceleration, Z is a number of protons, A is the atomic number). From experiments of EPF type it follows that $\delta w/w \leq 10^{-11}$.[9)]

Symmetry breaking ensures the appearance of a vector boson mediators mass $m_A = km_\varphi\langle\varphi\rangle$. Then taking $m_\varphi \cong 1\,\mathrm{GeV}$ we get $m_A \cong 10^{-19}\,\mathrm{GeV}$. The gravitational potential near the Earth will be

$$V(r) = -\frac{G}{r}\left\{MM' - M_0'M_0'\exp\left(-\frac{r}{R}\right)f\left(\frac{R}{R_0}\right)\right\} \tag{6·4}$$

with $R_0 \cong (1/m_A)$ and $f(x)=(3/x^3)(xchx - shx)$. The meaning of this form factor we discuss later. $M=Z(M_P+m_e)+(A-Z)M_N$, $M_0=Z(2m_u+m_d+m_e)+(A-Z)\cdot(m_u+2m_d)$ are atoms masses depending on the composition.

Here we want to stress two important moments: 1) dependence of the repulsion potential on a matter structure; 2) relation between the coupling constant of matter with antigravity and the mass of interacting particles. In this model $g_i = km_i$ relation was used, in $N=8$ supergravity $g=\pm km$.[88)~90)] It is evident that this relation is not universal[90)] and may take different forms in different models. From one side it is a new possibility for experimental studies to test unified models predictions, from another side it is a hint to a possible composite nature of new forces.

6.B. *Gauge theory of gravity*

The principle of local gauge invariance is now the leading principle of fundamental interactions physics. It introduces universally fields—mediators of interactions—which are known as gauge or compensating fields. From geometrical point of view they are connections in the principal bundle. Due to this duality theories of strong, weak and electromagnetic interactions may be formulated as a geometrical terms.[91)] But the Einstein theory of gravitation in which a metric is a dynamical variable is not compatible with such a scheme. Attempts to reformulate it as a Yang-Mills type theory[92)~97)] led to basic changes in gravitational field description preserving the correspondence with main results of GR.

Gauge theory of gravitation happened to be richer in its content than the gauge theory of internal symmetries and GR. Different variants of the gauge approach, choice of a symmetry group are still being discussed.[95)~97)] As the Poincaré group $P_{10} = T(4)\subset SO(3,1)$ plays a fundamental role in the elementary particle physics we choose here this variant. We point out also that supergravity is a gauge theory of the super-Poincaré group.

The kinematics of gauge P_{10}-gravity may be found in Refs. 95)~98). Here we remind only that independent dynamical variables of a gravitational field in this theory are local Lorentz connection $\omega^a{}_{b\mu}(x)$ and tetrad coefficients $h^a{}_\mu(x)$. Field strengths of these two gauge fields are the curvature tensor

$$R^a{}_{b\mu\nu} = \partial_\mu\omega^a{}_{b\nu} - \partial_\nu\omega^a{}_{b\mu} + \omega^a{}_{c\mu}\omega^c{}_{b\nu} - \omega^a{}_{c\nu}\omega^c{}_{b\mu}\,, \tag{6·5}$$

and the torsion tensor components

$$Q^a{}_{\mu\nu}=\partial_\mu h^a{}_\nu-\partial_\nu h^a{}_\mu+\omega^a{}_{c\mu}h^c{}_\nu-\omega^a{}_{c\nu}h^c{}_\mu\,. \tag{6·6}$$

So, the background manifold for the gauge P_{10}-gravitation will be the Riemann-Cartan space-time U_4 which is characterized by two independent objects: By metric $g_{\mu\nu}(x)$ related to tetrad coefficients via usual formula

$$g_{\mu\nu}=\eta_{ab}h^a{}_\mu h^b{}_\nu\,,\quad h^a{}_\mu h_b{}^\mu=\delta^a{}_b\,, \tag{6·7}$$

and a torsion tensor which is the antisymmetric part of connection

$$Q^\lambda{}_{\mu\nu}=h_a{}^\lambda Q^a{}_{\mu\nu}=\frac{1}{2}(\Gamma^\lambda{}_{\mu\nu}-\Gamma^\lambda{}_{\nu\mu})\,, \tag{6·8}$$

where Γ is the connection in U_4 compatible with the metric in a sense that $\nabla_\alpha g_{\mu\nu}=0$ and represented via a set of variables (h, ω) universally

$$\Gamma^\lambda{}_{\mu\nu}=h_{a\mu}h^b{}_\nu\omega_{ab}{}^\lambda+h^a{}_\mu\partial_\nu h_a{}^\lambda\,. \tag{6·9}$$

Due to metricity condition

$$\Gamma^\lambda{}_{\mu\nu}=\{^\lambda{}_{\mu\nu}\}+Q^\lambda{}_{\mu\nu}+Q_{\mu\nu}{}^\lambda+Q_{\nu\mu}{}^\lambda\,, \tag{6·10}$$

where $\{^\lambda{}_{\mu\nu}\}$ are Cristoffel coefficients.

Despite the fact that kinematic quantities (h, ω), (g, Γ) and (g, Q) are connected by (6·6～6·10) dynamics of theories based on these different choices of vaiables may happen to be different. Analysis of a spin states spectra leads to such a conclusion. In order to find out a spin content of a theory it is necessary to decompose small perturbations of a flat background $(\lambda^a{}_\mu=h^a{}_\mu-\delta^a{}_\mu, \omega^a{}_{b\mu})$, $(h_{\mu\nu}=g_{\mu\nu}-\eta_{\mu\nu}, \Gamma^\lambda{}_{\mu\nu})$, $(h_{\mu\nu}, Q^\lambda{}_{\mu\nu})$ onto subspaces of Lorentz group irreducible representations characterized by a definite spin-parity. As an illustration here we show spin states obtained in Refs. 72), 73), 99), where J^P is a spin parity

$$\lambda^a{}_\mu\rightarrow(1^-0^+)\otimes(1^-0^+)=(2^+)\oplus 2(1^-)\oplus 2(0^+)\oplus(0^-)\,,$$

$$h_{\mu\nu}\rightarrow(1^-0^+)\otimes(1^-0^+)=(2^+)\oplus 2(1^-)\oplus 2(0^+)\oplus(1^+)\,,$$

$$\omega_{ab\mu}\rightarrow(1^-0^+)\otimes(1^-0^+)\otimes(1^-0^+)=(2^+)\oplus 2(1^+)\oplus(0^+)\oplus(2^-)\oplus(1^-)\oplus(0^-)\,,$$

$$\Gamma^\lambda{}_{\mu\nu}\rightarrow(3^-)\oplus(2^+)\oplus(2^-)\oplus(1^+)\oplus(1^-)\oplus(0^+)\oplus(0^-)\,.$$

$Q^\lambda{}_{\mu\nu}$ spectrum coincides with the spectrum of a local Lorentz connection. Here the symmetry with respect to interchange of tetrads indices was not supposed.

Before analyzing particle interactions we note that one may expect (in analogy with electodynamics) that repulsion effects and short range forces may appear. They will be defined not only by spectra of fields of mediators but also by sources and the choice of the gravitational Lagrangian.

If we want to obtain equations of not more than second order, it is necessary to choose as a Lagrangian of the gauge theory of gravitation the sum of geometrical invariants formed from traces of curvature and torsion of not more than the second power.

The model Lagrangian consists of 11 terms

$$L_{P_{10}}=\Lambda+\frac{1}{16\pi G}\{R+Q_{\lambda\mu\nu}(a_1Q^{\lambda\mu\nu}+a_2Q^{\nu\mu\lambda}+a_3g^{\lambda\nu}Q^{\alpha\mu}{}_{\alpha})$$
$$+R^{\alpha\beta\mu\nu}(b_1R_{\alpha\beta\mu\nu}+b_2R_{\mu\nu\alpha\beta}+b_3R_{\alpha\mu\beta\nu}+b_4R_{\alpha\mu}g_{\beta\nu}+b_5R_{\mu\alpha}g_{\nu\beta}+b_6g_{\mu\alpha}g_{\nu\beta}R)\}\,. \qquad (6\cdot 11)$$

For such a theory there exist a principal possibility of the Lagrangian choice with smaller number of parameters free from ghosts and tachyons. This is not a subject of our analysis.

At first in Ref. 100) the possibility of repulsive short-range forces appearance in a gauge theory of gravitation was indicated. Besides intensity and range of these new forces mainly depended on the source structure. Investigation of the particular case of type (6·11) Lagrangian in the frames of PPN-formalism has brought the author to the conclusion that parameter γ characterizing a space curvature caused by the presence of an isolated mass, depends in such a theory only on the source structure and it can be both more or less than 0. We remind that in the Einstein theory it is strictly $\gamma=1$. By this it was supposed that both attraction and repulsion are possible depending on the sign of γ. As a consequence of such a phenomenon a hypothesis about a backward shift of planets perihelious moving around "anti-stars" (stars consisting mainly of antimatter) was stated.

In the frames of P_{10}-theory a potential of gravitational interaction that is $(g_{00}-1)$-component was considered in Refs. 101)~103). Here we shall discuss based on these works the appearance of the short-range forces starting from the general Lagrangian (6·11) and a point source of a gravitational field. Leaving in the weak field limit only small perturbations in equation of a free gravitational field (see Refs. 101)~103)), we get the following general solution for a central symmetry case[103)]

$$V(r)=-\frac{GM}{r}\left\{1+\alpha_1\exp\left(-\frac{r}{\lambda_1}\right)+\alpha_2\exp\left(-\frac{r}{\lambda_2}\right)\right\}, \qquad (6\cdot 12)$$

where

$$\alpha_1=-\frac{4(4+2a_1+a_2)}{3(2a_1+a_2)}, \quad \lambda_1=-\frac{4[4(b_1+b_2)+2b_3+b_4+b_5]}{3\alpha_1} \qquad (6\cdot 13\text{a})$$

and

$$\alpha_2=\frac{2a_1+a_2+3a_3-2}{3(2a_1+a_2+3a_3)}, \quad \lambda_2=\frac{4(b_1+b_2+b_4+b_5)+2b_3+12b_6}{2\alpha_2}. \qquad (6\cdot 13\text{b})$$

From this it becomes clear that depending on signs of α_i, λ_i both attraction and repulsion are possible in the theory. With such a great number of arbitrary parameters only the most general conclusions about the interaction could be made. The first thing that should be mentioned is that for any gravitational field source and among them for a spinless one, the potential (6·12) takes place. In assuming that $b_i(i=1\cdots 5)$ are more than zero demand $\alpha_1>0$ is becoming obvious and as a consequence either $a_1>0$ and $a_2>0$ or $(2a_1+a_2)>0$. On the other hand, from the demand $b_i>0$ $(i=1,\cdots,6)$ either $(2a_1+a_2+3a_3)>8$ (or <8) immediately follows. Depending on the sign ($\gtrless$) we shall have an effect of either Yukawa repulsion or only attraction.

In the solutions obtained there is no obvious dependence of intensity and range of Yukawa forces on a structure of matter source. However, if we suppose that a Lagrangian is induced by vacuum effects of interacting particle multiplets as shown in Refs. 104) and 105), constants a_i, b_i will depend on a multiplets structure. This is not a fact of dependence on baryon and lepton numbers but this fact should be accepted as an indication to a possible structure of a new interaction charge.

At the same time the question of which components of a field spectrum are specifically responsible for an appearance of the Yukawa component has remained unclear. This question is not an idle one because according to a more or less adopted opinion a new type of a short-range interaction is caused by an exchange of either scalar or vector bosons (graviscalar or graviphoton). Besides, we have analyzed only results in the system (h, ω) of dynamic variables. As it is seen from the above-mentioned spectrum for variables (g, Γ) modes of spin-parity 3^- and 2^- are allowed. Perhaps these "new mediators" of the gravitational interaction will be a prominent contribution to a violation of Newton's law.

In our opinion at the given stage the most reasonable is to pay attention to a spin origin of Yukawa contributions that is to consider the behaviour of a half-integer spin particles interacting minimally with a gravitational field and also backward spin influence on a gravitational field. In order to underline the role of a spin-torsion interaction we restrict ourselves by the case $g_{\mu\nu}=\eta_{\mu\nu}$ and consider the interaction of only Dirac particles for which

$$L_D=\bar{\psi}\gamma^\mu(\partial_\mu+\gamma_5\check{Q}_\mu)\psi\,, \tag{6·14}$$

where

$$\check{Q}_\mu=3!\varepsilon_{\mu\nu\alpha\beta}Q^{\nu\alpha\beta}\,, \tag{6·15}$$

is the pseudo-trace of a torsion tensor.

For simplicity we shall restrict the considered gravitational Lagrangian and shall proceed from

$$L_0=\sqrt{-g}\left\{\Lambda+\frac{1}{k^2}R+\frac{\lambda_1}{4}R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}-\lambda_2R_{\mu\nu}R^{\mu\nu}+\frac{\lambda_3}{4}R^2\right\}. \tag{6·16}$$

The Lagrangian corresponds to some special choice of a_i and b_i. Further we shall follow the ideology of Refs. 106) and 107). Because of the fact that spin interacts only with a pseudo-trace it will be enough to keep in (6·16) only pseudo-vector components of torsion, the Lagrangian for that will be

$$L_0=-\frac{1}{24k^2}\check{Q}_\alpha\check{Q}^\alpha-\frac{\lambda_2}{9}(\partial_\nu\check{Q}_\mu-\partial_\mu\check{Q}_\nu)(\partial^\nu\check{Q}^\mu-\partial^\mu\check{Q}^\nu)$$
$$+\frac{\lambda_1}{18}\left(\partial_\alpha\check{Q}_\beta\partial^\alpha\check{Q}^\beta+\frac{1}{2}(\partial_\alpha\check{Q}^\alpha)^2\right). \tag{6·17}$$

Adding to the Lagrangian (6·17) divergent term $(\lambda_1/18)\partial_\sigma(\partial_\alpha\check{Q}^\alpha\check{Q}^\sigma-\partial_\alpha\check{Q}^\sigma\check{Q}^\alpha)$ we shall reduce it to

$$L=-\frac{1}{24k^2}\check{Q}^\alpha\check{Q}_\alpha-\frac{1}{9}(\lambda_2-\lambda_1)(\partial_\mu\check{Q}_\nu-\partial_\nu\check{Q}_\mu)(\partial^\mu\check{Q}^\nu-\partial^\nu\check{Q}^\mu)+\frac{\lambda_1}{12}(\partial_\sigma\check{Q}^\sigma)^2\,. \quad (6\cdot18)$$

Pseudo-vector $\check{Q}^\alpha$ can be represented as a sum $\check{Q}_\alpha=q_\alpha+\partial_\alpha\chi$, where $\partial_\alpha q^\alpha=0$ and $\Box\chi=\partial_\alpha\check{Q}^\alpha$. Then, putting these expressions into (6·18), we shall have

$$L=-\frac{1}{24k^2}\check{q}_\alpha\check{q}^\alpha-\frac{1}{9}(\lambda_2-\lambda_1)(\partial_\mu\check{q}_\nu-\partial_\nu\check{q}_\mu)(\partial^\mu\check{q}^\nu-\partial^\nu\check{q}^\mu)$$
$$+\frac{\lambda_1}{12}(\Box\chi)^2-\frac{1}{24k^2}\partial_\alpha\chi\partial^\alpha\chi\,. \quad (6\cdot19)$$

Denoting $\Box\chi+(1/k^2\lambda_1)\chi=\varphi$ we shall write $\chi=\psi+k^2\lambda_1\varphi$ and

$$L=-\frac{1}{24k^2}\check{q}_\alpha\check{q}^\alpha-\frac{1}{9}(\lambda_2-\lambda_1)(\partial_\mu\check{q}_\nu-\partial_\nu\check{q}_\mu)(\partial^\mu\check{q}^\nu-\partial^\nu\check{q}^\mu)$$
$$-\frac{1}{24k^2}\partial^\alpha\psi\partial_\alpha\psi+\frac{1}{24k^2\lambda_1}\psi^2-\frac{1}{12}\lambda_1^2k^2\partial^\alpha\varphi\partial_\alpha\varphi\,. \quad (6\cdot20)$$

We note that we could choose the gauge for a torsion vector $\partial_\alpha\check{Q}^\alpha=0$ at once, excluding by that scalar modes. But it should not be done because as is shown in Ref. 99) pseudo-vector part of a torsion field comprises 1^+ and 0^- spin particles, that is confirmed by a corresponding projectors choice. The suggested decomposition underlines the fact of these states division into independent fields. Unlike Refs. 106) and 107) we shall show that it is necessary to choose either $\psi=0$, or suppose φ=const, as the original field of the torsion pseudo-vector comprises four degrees of freedom, but by condition $\partial_\alpha\check{q}^\alpha=0$ one of the degrees of freedom is eliminated and only three degrees of freedom are left. It is obvious that a simultaneous consideration of all three fields ($\check{q}^\alpha$, ψ, φ) violates the condition of the total number degrees of freedom conservation. So, in reality we have two possible choices for a torsion field Lagrangian

$$L_1=-\frac{1}{24k^2}\check{q}_\alpha\check{q}^\alpha-\frac{1}{9}(\lambda_2-\lambda_1)(\partial_\mu\check{q}_\nu-\partial_\nu\check{q}_\mu)(\partial^\mu\check{q}^\nu-\partial^\nu\check{q}^\mu)$$
$$-\frac{1}{24k^2}\partial^\alpha\psi\partial_\alpha\psi+\frac{1}{24k^2\lambda_1}\psi^2 \quad (6\cdot21)$$

or

$$L_2=-\frac{1}{24k^2}\check{q}_\alpha\check{q}^\alpha-\frac{1}{9}(\lambda_2-\lambda_1)(\partial_\mu\check{q}_\nu-\partial_\nu\check{q}_\mu)(\partial^\mu\check{q}^\nu-\partial^\nu\check{q}^\mu)$$
$$-\frac{1}{12}\lambda_1^2k^2\partial^\alpha\varphi\partial_\alpha\varphi\,. \quad (6\cdot22)$$

The first Lagrangian will correspond to two massive torsion modes of unit and zero spins, obeying equations

$$\Box\check{q}^\alpha+\left\{\frac{3}{k^2}(\lambda_1-\lambda_2)\right\}\check{q}^\alpha=0\,, \quad (6\cdot23\text{a})$$

$$\Box\phi+\frac{1}{k^2\lambda_1}\phi=0\,. \tag{6·23b}$$

The second Lagrangian describes a massless particle of zero spin obeying the equation

$$\Box\varphi=0\,, \tag{6·23c}$$

and a massive vector mode obeying (6·23a).

As a clear proof and for the sake of convenience new fields will be introduced by changing norms

$$\varphi\to\frac{1}{\lambda_1}\sqrt{\frac{6}{k^2}}\varphi\,,\quad \phi\to\sqrt{6k^2}\phi\,,\quad \check{q}^{\alpha}=\frac{3}{\sqrt{\lambda_2-\lambda_1}}\check{q}^{\alpha}\,. \tag{6·24}$$

Interaction Lagrangian of torsion field with Dirac particles spin $\check{S}^{\mu}=\bar{\psi}\gamma^{\mu}\gamma^5\psi$, that can be represented as $\check{S}^{\mu}=\check{s}^{\mu}+\partial^{\mu}\phi$, where $\partial_{\mu}\check{s}^{\mu}=0$ and $\phi=\bar{\psi}\gamma^5\psi$ and $\check{s}^{\mu}=\psi\bar{\gamma}^{\mu}\gamma^5\psi$ will get the following expression, corresponding to the considered earlier decomposition of the torsion field into transversal and longitudinal parts

$$L^{(1)}_{\rm int}=L_{\check{q}\leftrightarrow\check{s}}+L_{\phi\leftrightarrow\phi}\,, \tag{6·25a}$$

$$L^{(2)}_{\rm int}=L_{\check{q}\leftrightarrow\check{s}}+L_{\varphi\leftrightarrow\phi}\,. \tag{6·25b}$$

Using the Gupta method[12),13)] for calculation of the interaction potential of two spins at the account of a torsion exchange we shall have[107)]

$$V^{(\varphi)}(\boldsymbol{p})=-\frac{3}{8}\cdot\frac{k^2m^2}{E^2}(\boldsymbol{\sigma}_1\boldsymbol{p})(\boldsymbol{\sigma}_2\boldsymbol{p})\frac{1}{\boldsymbol{p}^2}\,, \tag{6·26a}$$

$$V^{(\phi)}(\boldsymbol{p})=-\frac{3}{8}\cdot\frac{k^2m^2}{E^2}(\boldsymbol{\sigma}_1\boldsymbol{p})(\boldsymbol{\sigma}_2\boldsymbol{p})\frac{1}{\boldsymbol{p}^2+m_{\varphi}{}^2}\,, \tag{6·26b}$$

$$V^{(q)}(\boldsymbol{p})=-\frac{1}{16(\lambda_1-\lambda_2)E^2}\cdot\sum_{i=1}^{4}R_i(\boldsymbol{p})\frac{1}{\boldsymbol{p}^2+m_q{}^2}\,, \tag{6·26c}$$

where m is a mass of spinor particles, E is an energy, $\boldsymbol{p}=\boldsymbol{p}_1'-\boldsymbol{p}_1=\boldsymbol{p}_2'-\boldsymbol{p}_2$ is momenta difference before scattering and after it in the center of mass system, $m_{\varphi}{}^2=(1/k^2\lambda_1)$ is the square of scalar torsion mass, $m_q{}^2=1/(2k^2(\lambda_2-\lambda_1))$ is the square of vector torsion mass.

After Fourier transformation and determination of the explicit dependence of interaction on coordinates one can make a conclusion that just massive torsion modes are responsible for the appearance of non-Newtonian forces in the gravity law. This theoretical analysis gives the spin-gravitational nature of the new type interaction. Finishing this section we should note that a spin-torsion interaction constant differs from the Newtonian gravitational constant. Moreover, it depends in a great deal on a structure of matter and as for fermions the existence of Regge trajectories connecting a particle spin with its mass is very essential. The constant of spin-gravitational interaction for a given multiplet is universally connected with a Regge trajectory slope.[43)] This qualitative analysis proposes an answer to the question: "Is it possible in the frames of a geometrized theory of gravitation to calculate a theoretical dependence of the new charge on initial parameters, proceeding from the Regge theory?". Most likely, yes, but to this we shall devote a special work.

§ 7. Van-der-Waals forces between hadrons and quarks

Nature of interaction forces between quarks and hadrons has been studied for a long time. For the present a generally accepted scheme is the quantum chromodynamics in which gluon fields multiplets (Yang-Mills vector fields, transforming by adjoint representation of a coloured symmetry group) serve as strong interaction mediators. At the same time hadrons being neutral consist of charged particles between which attractive and repulsive electromagnetic forces act. In some models of strong interaction a field of gluons spread is limited by hadron dimensions (bag model) and, naturally, no suppositions about interactions at distances larger than 10^{-13} cm are acceptable. On the other side if we consider chromodynamics as a field theory, in which forces range is not limited initially, but should be found in a calculations process, the hypothesis of possibility of hadrons interaction at large distances would not seem unacceptable. On the contrary the appearance of long-range potentials during a Feynman amplitude evaluation, intensity of which however is very quickly decreases with a distance, is not forbidden and can be explained in the frames of Van-der-Waals forces scheme, in analogy with molecular physics. Indeed, the Coulomb potential is a result of one-photon exchange between charged point particles. If particles (atoms) are electrically neutral and spinless, then short-range forces appear between them. The last does not forbid also the existence of long range of the type $V_{12}(r)\propto -C_{12}r^{-6}$ that is the result of multiphoton exchange with accuracy of the order e^4 [108] in the tree approximation. What is the reason of a possible short-range force between hadrons?

Hadron being a bound state of quarks turns out to be a coloured singlet, so if a one-gluon exchange could be possible, this charge would not provide quarks confinement. Naturally, a matrix element of one-gluon exchange between two hadrons (see Fig. 3) (as well as baryons or mesons) is $\langle HH'|L_{\rm int}|HH'\rangle=0$ because this element is proportional to $({\rm tr}\lambda_a)^2$ and $\lambda_a\in SU(3)$ are traceless matrices.[109]

Two meson interaction energy was considered in Refs. 110) and 111) and for two baryons the analogous investigation was performed in Ref. 110). It is necessary to mention that there is no answer to the question about potential energy of hadron interactions. A detailed analysis of this situation is given in Refs. 112)~114). Interaction potential appears due to the multi-gluon exchange. Following Ref. 113) we shall demonstrate a scheme of an interaction potential calculation. Energy of interaction can be written in the form

$$E_{\rm int}=\frac{1}{2}\sum_{p}\boldsymbol{j}_{\mu p}(M_p{}^{\mu\nu})^{-1}\boldsymbol{j}_{\nu p}\,, \qquad (7\cdot1)$$

where $M_p{}^{00}=\boldsymbol{p}^2$ and (for proton)

$$j_\mu=q\delta_{\mu0}[\delta^3(\boldsymbol{x}-\boldsymbol{x}_1)-\delta^3(\boldsymbol{x}-\boldsymbol{x}_2)]+q\delta_{\mu0}[\delta^3(\boldsymbol{x}-\boldsymbol{x}_3)-\delta^3(\boldsymbol{x}-\boldsymbol{x}_4)] \qquad (7\cdot2)$$

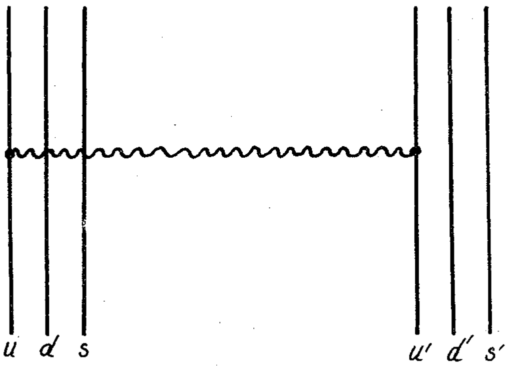

Fig. 3.

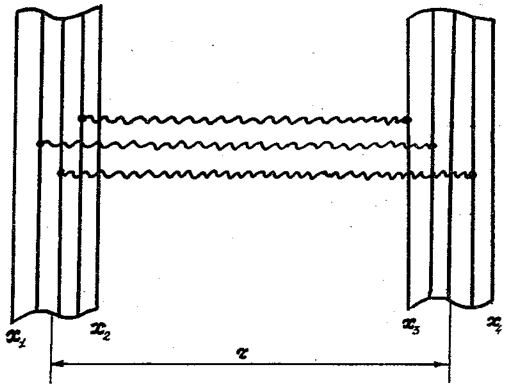

Fig. 4.

is a fundamental particle current (q is some charge) in the region $[x_1, x_2]$ and $[x_3, x_4]$ (see Fig. 4).

Interaction energy consists of Coulomb potential and QCD-potential that is

$$V(r) \cong -\alpha\left(\frac{C_1}{C_A}\right)\frac{r_0{}^5}{r^6}, \tag{7·3}$$

where $\alpha = (g^2/4\pi)$, g is the QCD interaction constant, $C_1 = 1(T_a T_a)$, $C_A \delta_{ab} = f_{acd} f_b{}^{cd}$ and r_0 is some dimensional quantity constructed from fundamental constants. The given formulae is a consequence of two gluon exchange for $L_{\text{int}} = gA_\mu{}^a \bar{\psi}\gamma^\mu T_a \psi$. This is not a unique possible form of interaction. In Ref. 112) the interaction of kind $L_{\text{int}} = hM^{-3}\bar{\psi}\psi F^a_{\mu\nu}F^{a\mu\nu}$ was considered, where h is a nuclear interaction constant and M is hadron mass. It was mentioned that a functional form of $V(r)$ depends on a number of gluons participating in the exchange. We only note that for two-gluon exchange $V_2(r) \propto r^{-7}$, for three-gluon exchange $V_3(r) \propto r^{-1}\sum_{l=0}^{2} a_l(\log r)^l$ and for four-gluon exchange $V_4(r) \propto r\sum_{l=0}^{3} b_l(\log r)^l$. It is rather difficult to sum all these contributions. And it is not our aim. One point is obvious that QCD-long-range potential in a general case can be represented as

$$V(r) \cong \lambda_n\left(\frac{r_0}{r}\right)^n \hbar c(N_1 N_2), \tag{7·4}$$

where N_i is a number of protons in a chosen sample, n is some number and λ_n is a constant characterising interaction intensity. Because it is not possible to get the exact value for n coming from a field treatment of the strong interaction theory different authors study cases for n from $1+\delta$ up to $n=7$ [109)~114)] (here δ is not always an integer). This type of a potential can be used as a good phenomenological approximation to a study of interaction between quarks, baryons and mesons.

Due to the fact that r_0 and λ_n depend from one side on a number of gluons, participating in the exchange and from the other side they are functions of spin properties, this process interaction intensity is evaluated as comparable with single-gluon exchange at distances of the order 10^{-8} cm and it is extremely small in comparison with electromagnetic interaction on molecular scales; it is prevailing inside elementary particles and it may be comparable with gravitational one at distances of the order 1000÷10000 meters.

We think that gluon exchange may be one of mechanisms of new intermediate-range forces. Unfortunately, similar analysis for multigraviton exchange is not done. If one takes seriously new long-range forces as Van-der-Waals ones then it is necessary to take into account contributions from all fields—photon, graviton and gluon type.

§ 8. Non-linear vector field

The main source of deviation from Newton's law is the presence in mediators spectrum of massive graviphotons or graviscalars leading to a potential of the form $V=V_0+ar^{-1}\exp(-r/d)$, where $d=\hbar/mc$ and m is a graviphoton or graviscalar mass. We shall discuss here the study of a potential of the form $V=\varepsilon r^{-(1+\sigma)}$, that is caused by exchange of non-linear vector particles. By the form of the potential we shall reconstruct an equation for vector particles.

Assume that $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ and $A_0=V(r)$ then in a static case $\boldsymbol{E}=\boldsymbol{\nabla} V$ and by direct calculation it is easy to find that $\boldsymbol{E}$ satisfies the equation

$$V\cdot(\boldsymbol{\nabla}\boldsymbol{E})=\left(-\frac{\sigma}{1+\sigma}\right)\boldsymbol{E}^2 . \tag{8·1}$$

Now we can restore a covariant form of non-linear field equations. Remembering that $\boldsymbol{E}=(F_{01}, F_{02}, F_{03}\}$ we find that the equations which $F_{\mu\nu}$ satisfies will be

$$\partial_\nu F^{\mu\nu}=-\beta\frac{A^\mu(F_{\alpha\beta}F^{\alpha\beta})}{A_\sigma A^\sigma}, \tag{8·2}$$

where $\beta=\sigma/2(1+\sigma)$.

If $\sigma\ll 1$ the inverse square law is fulfilled quite well, because $\beta\ll 1$ and the non-linear additional terms are small. We assume that $(F_{\alpha\beta}F^{\alpha\beta}/A_\nu A^\nu)\cong l^2$ at large distances, here l is a constant value, that corresponds to soliton-like solutions of (8·2). It is evident that in such an approximation Eq. (8·2) becomes the Proca equation for massive vector particles with mass $m\cong\beta l^2$.[115]

This purely qualitative analysis cannot serve a reasonable ground for considering a phenomenology of non-linear vector fields that is why we regard here theoretical field models in which similar non-linear combinations appear with necessity. We shall mention two cases, namely, low-energy limit in a field theory of strings[116] and as well a self-consistent gravitational theory of Einstein-Cartan with electromagnetic field as a gravitational source.[117] Let us consider the last model in more detail.

The action functional describing an interaction of affine-metrical gravitational field with an electromagnetic one and spinless matter is

$$S=\int d^4x\sqrt{-g}\left\{\frac{1}{16\pi G}R(g)+\frac{1}{\gamma}Q_{\alpha\mu\nu}(Q^{\alpha\mu\nu}+Q^{\mu\nu\alpha}+Q^{\nu\mu\alpha})\right.$$
$$\left.-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+L_{\text{mat}}(\varphi, g, A, \cdots)\right\}, \tag{8·3}$$

where

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+2Q^\lambda{}_{\mu\nu}A_\lambda$$

and $Q^\lambda{}_{\mu\nu}$ is a torsion tensor already mentioned in §4.B.

We shall not consider here a full system of field equations, but pay attention to the spin-torsion relation. Variation of (8·3) with respect to torsion tensor leads to the equation

$$Q^\lambda{}_{\mu\nu}+\delta^\lambda{}_\mu Q_\nu-\delta^\lambda{}_\nu Q_\mu=\gamma(A_\mu F_\nu{}^\lambda-A_\nu F_\mu{}^\lambda)\,. \tag{8·4}$$

Solving this equation with respect to a torsion field we shall get

$$Q^\lambda{}_{\mu\nu}=2\gamma\delta^\lambda{}_{[\mu}F_{\nu]}{}^\alpha\frac{A_\alpha}{2(1-\gamma^2A^2)}+2\gamma A_{[\mu}F_{\nu]}{}^\lambda+4\gamma^2A^\lambda A_{[\mu}F_{\nu]}{}^\sigma\frac{A_\sigma}{2(1-\gamma^2A^2)}\,, \tag{8·5}$$

and $A^2=A_\mu A^\mu$, $Q_\mu A^\mu=0$.

Substituting the expression for torsion tensor in terms of the electromagnetic strength we shall get an interaction of matter field with non-linear vector field being described by the Lagrangian

$$L_{\rm nl}=-\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}+\tilde{F}_{\sigma\beta}\tilde{F}^{\sigma\alpha}A_\alpha A^\beta\,, \tag{8·6}$$

where

$$\tilde{F}_{\mu\nu}=F_{\mu\nu}+\frac{2\gamma A_{[\mu}F_{\nu]}{}^\alpha A_\alpha}{1+\gamma^2A^2}\,, \tag{8·7}$$

and also with a gravitational field in the Einstein theory. The total Lagrangian will be now

$$L_{\rm tot}=\frac{1}{16\pi G}R(g)+L_{\rm nl}+L_{\rm mat}(\varphi, g, A, \cdots)\,. \tag{8·8}$$

In such a theory a potential interaction proportional to r^{-1} is due to graviton exchange and additional terms $r^{-(1+\sigma)}$ are due to non-linear vector boson exchange. The field equations are not exact copy of (8·2) but the first term on the right-hand side will be surely of this type.

§ 9. Multidimensional models

The existing increased interest in multidimensional gravity is largely stimulated by studies in superstring theories.[118] One of the potential windows to the multidimensional world is possible variation of the gravitational constant G (see, e.g., Refs. 119) ~122)). Constraints on G variation[123)~124)] imply constraints on the cosmological parameters.

We consider the Einstein equations

$$R_{MN}=0 \tag{9·1}$$

in the D-dimensional manifold

$$M=M^{(4)}\otimes M_1\otimes\cdots\otimes M_n\,;\quad \dim M_i=N_i\,;\quad D=4+\sum_{i=1}^{n}N_i\,, \tag{9·2}$$

where $M^{(4)}$ is the ordinary space-time and M_i are Ricci-flat manifolds with the intervals $ds^2_{(i)}$; $i=1,\cdots,n$. We seek solutions of (9·1) such that $M^{(4)}$ is static, spherically symmetric, while all the scale factors $\exp(\beta_i)$ of the internal spaces M_i depend on the radial coordinate u, i.e., the D-metric is

$$ds^2_{(D)}=\exp[2\gamma(u)]dt^2-\exp[2\alpha(u)]du^2-\exp[2\beta(u)]d\Omega^2$$
$$-\sum_{i=1}^{n}\exp[2\beta_i(u)]ds^2_{(i)}\,, \tag{9·3}$$

where $d\Omega^2=d\theta^2+\sin^2\theta d\varphi^2$ is the standard S^2 metric.

If we denote $\gamma=\beta_{-1}$, $N_{-1}=1$ and $\beta=\beta_0$, $N_0=2$, and choose the harmonic radial coordinate u such that $\alpha=\sum_{\nu=-1}^{n}\beta_\nu N_\nu$, the Einstein equations (9·1) can be written in the form

$$\sum_{\nu=-1}^{n}N_\nu(-\beta''_\nu+\alpha'\beta'_\nu-(\beta'_\nu)^2)=0\,, \tag{9·4a}$$

$$\beta''_i=0,\ i=-1,1,\cdots,n\,, \tag{9·4b}$$

$$\beta''_0=\exp[2\alpha-2\beta_0]\,. \tag{9·4c}$$

This set of equations is easily solved, so that

$$\exp[\alpha-\beta_0]=\frac{L}{\sinh(ku)}\,,\quad \beta_i=A_iu\quad (i=-1,1,\cdots,n)\,,$$

$$k,A_i=\text{const}\,,\quad 2k^2=(A_{-1}+\sum_{i=1}^{n}N_iA_i)^2+A_{-1}^2+\sum_{i=1}^{n}N_iA_i^2\,, \tag{9·5}$$

where nonessential integration constants (leading to arbitrary coefficients by dt^2 and $ds^2_{(2)}$) are omitted. After the substitution

$$u=-\frac{1}{2k}\log\left(1-\frac{2k}{R}\right),\quad A_{-1}=-ka\,,\quad A_i=ka_i\,,\quad (i=1,\cdots,n) \tag{9·6}$$

the metric is brought to its final form

$$ds^2_{(D)}=\left(1-\frac{L}{R}\right)^a dt^2-\left(1-\frac{L}{R}\right)^{-a-b}dR^2-\left(1-\frac{L}{R}\right)^{1-a-b}R^2d\Omega^2$$
$$-\sum_{i=1}^{n}\left(1-\frac{L}{R}\right)^{a_i}ds^2_{(i)}\,, \tag{9·7}$$

where $L=2k$ and

$$b=\sum_{i=1}^{n}N_ia_i\,. \tag{9·8}$$

The constants $a, a_1, \cdots, a_n$ satisfy the relation

$$(a+b)^2+a^2+\sum_{i=1}^{n}N_i a_i^2=2. \tag{9·9}$$

In the special case $n=1$ this solution was considered earlier in Refs. 125) and 126).

The mass of the system is found from the asymptotic of g_{00}, namely $m=(aL/2G)$.

At large distances from a massive body then four-dimensional post-Newtonian metric in a vast class of gravitational theories can be written in the form

$$g_{00}=1-2U+2\beta U^2+O(U^3), \tag{9·10a}$$

$$g_{0j}=0, \tag{9·10b}$$

$$g_{ij}=-\delta_{ij}(1+2\gamma U+O(U^2)),\ i,j=1,2,3. \tag{9·10c}$$

The constants β and γ varying from theory to theory can be determined via the classical relativistic effects, namely, light deflection and secular perihelion shift.

To compare metric (9·7) (the first line) with (9·10) let us transform it to the isotropic radial coordinate, so that its spatial part becomes conformally flat:

$$ds_{(4)}^2=A(r)dt^2-B(r)(dr^2+r^2d\Omega^2), \tag{9·11a}$$

$$R=r\cdot\left(1+\frac{L}{4r}\right)^2, \tag{9·11b}$$

$$A(r)=\left(\frac{1-\dfrac{L}{4r}}{1+\dfrac{L}{4r}}\right)^{2a},\quad B(r)=\left(1+\frac{L}{4r}\right)^4\cdot\left(\frac{1-\dfrac{L}{4r}}{1+\dfrac{L}{4r}}\right)^{2-2a-2b}. \tag{9·11c}$$

Expanding A and B in powers of L/r at large r and comparing (9·11) with (9·10), we get

$$\beta=1,\quad \gamma=1+\frac{a}{b},\quad 2Gm=aL. \tag{9·12}$$

From the observational constraint on γ [124] one easily obtains the corresponding constraint on the solution parameters

$$\frac{b}{a}=\gamma-1=(-0.7\pm1.7)\cdot10^{-3} \tag{9·13}$$

which can be satisfied, e.g., for sufficient small $a_1, \cdots, a_n$. This constrains a certain segment from the parameter ellipsoid (9·9). For $n=1$, we have

$$aN=(-0.7\pm1.7)\cdot10^{-3},\quad a\approx1-\frac{1}{2}a_1N_1. \tag{9·14}$$

For $n>1$, a_i are not inevitably small.

Evidently the constraint corresponds only to the field of the Sun whose gravitational field was tested. For massive bodies the integration constants can be arbitrary.

The influence of high dimensions on the test particle motion can be demonstrated through the reduction of Kaluza-Klein geodesic equation[127)]

$$q^A \nabla_A q^B = 0 \tag{9·15}$$

to the following 4-dimensional equation of motion

$$\frac{dp^\mu}{ds} + \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta = \sqrt{16\pi G}\frac{\sqrt{\varphi}}{\chi} g^{\mu\alpha} q_a F^a_{\alpha\beta} p^\beta + \frac{1}{2}(\varepsilon_4 g^{\mu\alpha} + p^\mu p^\alpha)\left(\frac{\partial_\alpha \varphi}{\varphi} - 2\frac{\partial_\alpha \chi}{\chi}\right), \tag{9·16}$$

where $\Gamma^\mu_{\alpha\beta}$ are Christoffel symbols, $F^a_{\alpha\beta}$ is the gauge field strength and $\varphi = |\det \varphi_{ab}|$ is the Kaluza-Klein dilaton field.

The general solution of 7-dimensional Kaluza-Klein theory equations for the interval (9·11) was obtained in Ref. 127). We do not consider here the details of this very interesting work. But we discuss only the form of gravitational potential. The surface analysis of the motion equations tells us that in the presence of new forces caused by Kaluza-Klein dilaton field the motion of a neutral test particle is no longer described by a geodesic. For this case the equation of motion is

$$\frac{dp^\mu}{ds} + \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta = \frac{1}{2}(\varepsilon_4 g^{\mu\alpha} + p^\mu p^\alpha)\frac{\partial_\alpha \varphi}{\varphi}. \tag{9·17}$$

So that the Newtonian potential $V(r)$ will be[127)]

$$V(r) = -\frac{1}{2}\varphi^{-1/2}(g_{00} - 1). \tag{9·18}$$

It is evident that the additional force is caused by Kaluza-Klein dilaton field.

§ 10. Non-Newtonian forces and dark matter problem

Suppositions that Newtonian gravitation does not quite adequately describe a situation in astrophysics appeared rather long ago.[128)] The basis for such assuming, that is suppositions on the violation of the inverse square law for scales larger than that of the Solar system, was difference in evaluation of galaxies mass densities, necessary for preventing of their run way and estimation that was based on the "mass-luminosity" observable ratio. The luminosity of a star and galaxy can be measured and we can get a relation of "luminosity unit" and "mass unit". For example, this ratio is assumed equal to 1 for the Sun. The masses of nearest stars can be determined through the measuring of the rotation velocity, since their dynamics are well known. The visible regions of our galaxy consist of the stars with the "mass-luminosity" ratio of the order of 10. We can assume that for the elliptic galaxies this ratio also takes place, at any rate for their central parts.

Only a few years ago the rotation velocities for a most part of galaxies were measured through the investigation of neutral hydrogen radiation in radio spectra. This rotation has been found both for galaxy nuclei and for external regions of the galaxy disks. It was a very surprising result, because the rotation velocity was a constant for any regions. Due to previous galaxy models it was believed that boundary part of galaxies should have less density. Then the rotation velocity should be less. But this statement contradicts the observable data, because the

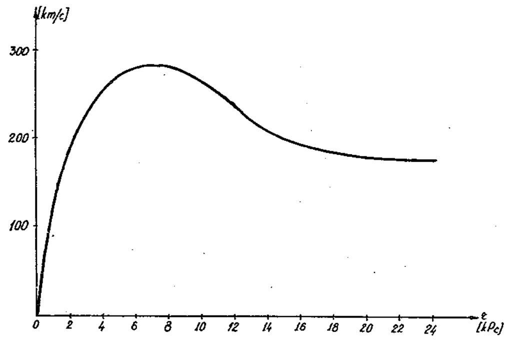

Fig. 5.

rotation velocity is changed only slightly and this change is pictured in Fig. 5.

The majority of astronomers suppose that a consequence of this fact is a much larger value of the "mass-luminosity" relation. Consequently galaxies should have a larger mass than it was considered earlier and true "mass-luminosity" ratio should be of order of 30 or even more. It was the reason of the dark matter hypothesis (invisible matter that does not radiate electromagnetic waves), however possessing a mass different from zero. At the present time there are no convincing examples of this matter structure.

In connection with difficulties of the dark matter hypothesis realization in nature some authors have considered a modification of the gravitational law at distances from 10 kPc up to 100 kPc.[129)~131)]

Modified attraction potential

$$V(r) = -\frac{m_1 m_2}{r} G_\infty \left(1 + \alpha \log \frac{r}{\lambda}\right) \qquad (10\cdot1)$$

was suggested without any fundamental theoretical basis in Refs. 129) and 131). This proposition appeared not to be a unique one and alongside with it there have been studied a repulsion potential of the Yukawa type.

In the recent work[131)] physical consequences of $1/r$ component in the gravitational force were analyzed. In particular it was shown that in this case "mass-luminosity" ratio will be in the limits of 3 to 7 for various galaxies.

Unfortunately the above-mentioned modification of the gravitation law satisfies not all experimental requirements. So it does not explain the relation $L \propto v^4$ connecting luminosity and galaxies matter velocities. Theoretical evaluations give $L \propto v^2$ that is true only for an inconsiderable number of spiral-type galaxies. In Ref. 131) the potential was constructed that satisfies the law $L \propto v^4$, however, a number of difficulties with an explanation of matter density distribution in galaxy was left.

In spite of successes achieved by theoretician-astrophysicists we should state that this field of hypothesis on intermediate range forces existence has not acquired yet a status of a theory, but it is still on a stage of phenomenological search. Such a poor examination of this important hypothesis is probably connected with our inadequate knowledge of the structure of stars and their evolution.

§ 11. Concluding remarks

A number of generalized gravitation theory models predict with necessity the Yukawa type of additional forces. To point out uniquely the nature and mediators of these forces is not possible. More preferable to our opinion is that the source of Newton's law violation is a new macroscopic interaction whose mediators are massive bosons of spin 0^+ and unity. This source seems to be "complex", i.e., a combination of fundamental charges (baryon, lepton, isospin etc.). Maybe among them are such characteristics as color charge, strangeness and other quantum numbers, characterizing elementary particles. Partially the answer to this question was done in Ref. 132) in the frames of $SU(3)\times SU(2)\times U(1)$ unified theory.

Numerous experiments up till now did not give the answer to the question whether we have a new type of interaction, these forces are new or they are combinations of already known ones as maybe all these experiments are erroneous.

In some papers new laboratory experiments using the Casimir effect[133] are suggested for testing possible interaction dependence of two-electroneutral bodies on their form and composition.

We believe that studies of new possible interactions may be an important part of theoretical and experimental investigations of unified scheme viability. In many cases obtained up till now limits on the Yukawa type contributions are upper limits. In many theories considered α (and also λ) are functions of primary and sometimes unknown constants (as in the gauge theory of gravitation). So limits on α and λ may lead to limits on known and unknown charges. Direct measurements of unified theories parameters, i.e., search for superpartners of many particles and clarifying of QCD parameters are rather problematic and in any case too expensive as one should project and build accelerators of more than 1000 GeV. We are sure that expriments searching "the fifth or other forces" with increased accuracy may serve as good tests for verifying the predictions of Grand Unified Theories.

References

1) A. Einstein, Ann. der Phys. **49** (1916), 760.
2) D. Hilbert, Nachrichten K. Geselschaft Wiss., Göttingen, Math. -Phys. Klasse **3** (1915), 395.
3) R. V. Eötvös, D. Pekar and E. Fekete, Ann. der Phys. **68** (1922), 11.
4) P. G. Roll, R. Krotkov and R. H. Dicke, Ann. of Phys. **26** (1964), 236.
5) V. B. Braginsky and V. I. Panov, ZhETP **61** (1971), 873 (in Russian).
6) R. V. Pound and G. A. Rebka, Phys. Rev. Lett. **4** (1960), 337.
7) F. D. Stacey et al., Rev. Mod. Phys. **59** (1987), 157.
8) E. Fischbach et al., Ann of Phys. **182** (1988), 1.
9) N. Rosen, Phys. Rev. **57** (1940), 147.
H. -J. Treder, *Gravitationstheorie und Equivalenzprinzip* (Akademie-Verlag-Berlin, 1971).
10) E. Recami and V. Tonin-Zanchin, Nuovo Cim. **B105** (1989), 701.
11) V. N. Melnikov, G. D. Pertsov and K. P. Stanukovich, in *Problems of Gravitation and Elementary Particle Theory* (Moscow, Energoatomizdat, 1985), p. 41.
12) S. N. Gupta, *Quantum Electrodynamins*, ed. Gordon and Breach Sc. Pub. (NY-Paris-London, 1977), p. 198.
13) B. M. Barker, S. N. Gupta and R. D. Haraez, Phys. Rev. **149** (1966), 1027.
14) J. Scherk and J. H. Schwarz, Nucl. Phys. **B153** (1987), 61.
15) I. Ya. Aref'eva and I. F. Volovich, Uspekhi Fiz. Nauk **46** (1985), 655 (in Russian).
16) M. W. Kalinowski, Acta Phys. Austr. **27** (1986), 45.
17) V. N. Melnikov, K. A. Bronnikov and V. D. Ivaschuk, Nuovo Cim **102** (1988), 209.
18) K. P. Stanukovich and V. N. Melnikov, *Hydrodynamics, Fields and Constants in the Theory of Gravitation* (Moscow Energoatomizdat, 1983) (in Russian).
19) S. Deser, R. Jackiw and S. Templeton, Ann. of Phys. **140** (1982), 379.
20) J. J. Van der Bij, R. D. Pisarski and S. Rao, Phys. Lett. **B179** (1986), 87.
21) C. Isham, A. Salam and J. Strathdee, Phys. Rev. **D3** (1971), 867.

22) V. De Sabbata and M. Gasperini, Gen Rel. Grav. **18** (1986), 669.
23) P. Jordan, Z. Phys. **157** (1959), 112.
24) C. Brans and R. H. Dicke, Phys. Rev. **124** (1961), 925.
25) S. Deser, Ann. of Phys. **59** (1970), 248.
26) Y. L. Anderson, Phys. Rev. **D3** (1971), 1689.
27) R. V. Wagoner, Phys. Rev. **D1** (1970), 3209.
28) Y. O'Hanlon, Phys. Rev. Lett. **29** (1972), 137.
29) A. Zee, Phys. Rev. Lett. **42** (1979), 417.
30) Y. Fujii, Nature (Phys. Sci.), **234** (1971), 5.
31) V. N. Melnikov and A. G. Radynov, Probl. Theor. Grav. Elem. Part., **14** (1984), 73; Energoatomizdat (in Russian).
32) J. Scherk and J. H. Schwarz, Phys. Lett. **B82** (1979), 60.
33) C. J. Isham, Phys. Rev. **D3** (1971), 1805.
34) C. Sivaram and C. Sinha, Lett. Nouvo Cim. 8 (1973), 324.
35) Y. Fujii, Phys. Rev. **D9** (1974), 874.
36) V. De Alfaro, S. Fubini, G. Furlan and C. Rosetti, *Currents in Hadron Physics* (North-Holland Pub. Comp., Amsterdam-London, 1973).
37) E. Cartan, Comp. Rend. 174 (1922), 437, 593.
38) E. Cartan, Ann. Ec. Norm. **40** (1923), 325.
39) E. Cartan, Ann Ec. Norm. **41** (1924), 1.
40) E. Cartan "Sur les Variétés á connexion affine et la theorie de la relativité generalisée" (Gauthier-Villars, Paris, 1955).
41) A. Trautman, Nature **242** (1973), 7.
42) V. De Sabbata and C. Sivaram, Int. J. Theor. Phys. **29** (1990), 1.
43) V. De Sabbata, P. I. Pronin and C. Sivaram, Int. J. Theor. Phys. **30** (1991), 1671.
44) G. E. Uhlenbeck and S. A. Goudsmit, Physica **5** (1925), 266.
45) V. De Sabbata and M. Gasperini, Lett. Nouvo Cim. **27** (1980), 133.
46) V. De Sabbata, *Gravitational Measurements, Fundamental Meteorology and Constants* ed. V. De Sabbata and V. N. Melnikov (Klumer Academic Publishers, Dordrecht), vol. 230 (1988), p. 115.
47) P. M. S. Blackett, Nature **159** (1947), 658.
48) S. P. Sirag, Nature, **278** (1979), 358.
49) V. De Sabbata and C. Sivaram, Nuovo Cim. **102B** (1988), 107.
50) V. De Sabbata and P. Razzaki, Lett. Nuovo Cim. **20** (1977), 525.
51) V. De Sabbata and M. Gasperini, Lett. Nuovo Cim. **27** (1980), 289.
52) V. De Sabbata and C. Sivaram, Nuovo Cim. **105B** (1990), 603.
53) V. De Sabbata and C. Sivaram, Nuovo Cim. **106B** (1991), 873.
54) D. K. Ross, Int. J. Theor. Phys. **28** (1989), 1333.
55) B. A. Bilby, R. B. Bullough and E. Smith, Proc. Roy. Soc. London **A231** (1955), 263.
56) V. De Sabbata and C. Sivaram, *Modern Problems of Theoretical Physics* (Festschriftt, for Professor D. Ivanenko) ed. P. I. Pronin and Yu. N. Obukhov (World Sci. Singapore 1991), p. 143.
57) V. De Sabbata and D. V. Gal'tsov, (1992) to be published.
58) L. Parker, Phys. Rev. Lett. **21** (1968), 562; Phys. Rev. **183** (1969), 1057.
59) A. A. Grib and S. G. Mamaev, Yadern. Phys. **10** (1969), 1276 (in Russian).
60) Ya. B. Zeldovich and A. A. Starobinsky, ZhETP **61** (1971), 2161 (in Russian).
61) A. A. Grib, S. N. Mamaev and V. M. Mostepanenko, *Quantum Effects in Strong External Fields* (Atomizdat, Moscow, 1980) (in Russian).
62) A. A. Starobinsky, Pisma v ZhETF **30** (1979), 719 (in Russian).
63) A. H. Guth, Phys. Rev. **D23** (1981), 347.
64) A. D. Linde, Phys. Lett. **B99** (1981), 391.
65) V. N. Melnikov and S. V. Orlov, Phys. Lett. **A70** (1970), 263.
66) W. Thirring, Phys. Rev. **79** (1950), 703.
67) K. S. Stelle, Phys. Rev. **D16** (1977), 953.
68) J. Julve and P. Tonin, Nuovo Cim. **B46** (1978), 137.
69) K. S. Stelle, Gen. Rel. Grav. 9 (1978), 353.
70) P. L. Poznanin, Izvestiya VUZov **6** (1975), 15 (in Russian).
71) R. J. Rivers, Nuovo Cim. **34** (1964), 386.
72) P. van Nieuvenhuizen, Nucl. Phys. **B60** (1973), 478.
73) E. Sezgin and P. van Nieuvenhuizen, Phys. Rev. **D21** (1980), 326.
74) M. Duff, Phys. Rev. **D9** (1974), 1837.
75) G. A. Vilkovisky and V. P. Frolov, *Proceedings of the Second Seminar "Quantum Gravity", Moscow 13-15 October 1981,* p. 155 (in Russian).
76) D. M. Capper, G. Leibrand and M. Ramon Medrano, Phys. Rev. **D8** (1973), 4320.

77) D. M. Capper and M. Ramon Medrano, Phys. Rev. **D9** (1974), 1641.
78) D. Z. Freedan, P. van Nieuvenhuizen and S. Ferrara, Phys. Rev. **D13** (1976), 3214.
79) S. Deser and B. Zumino, Phys. Lett. **B62** (1976), 335.
80) Yu. A. Golfand and E. P. Likhtman, Pisma v ZhETF **13** (1971), 459 (in Russian).
81) D. V. Volkov and V. P. Akulov, Pisma v ZhETF **13** (1971), 459 (in Russian).
82) P. van Nieuvenhuizen, Phys. Rep. **68** (1981), 189.
83) E. Cremmer and B. Julia, **B159** (1979), 121.
84) *"Supergravity* 81", ed. S. Ferrara and J. G. Taylor (Cambridge Univ. Press, 1982, Cambridge-London-New York).
85) J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton Univ. Press, Priceston, 1983).
86) C. K. Zachos, Phys. Lett. **B76** (1978), 320.
87) J. Scherk, Phys. Lett. **B88** (1979), 265.
88) J. Scherk and J. H. Schawarz, Phys. Lett. **B82** (1979), 60.
89) E. Cremmer, J. H. Schwarz and J. Scherk, Phys. Lett. **B84** (1979), 83.
90) J. Scherk, *Unification of Fundamental Particle Interaction,* ed. S. Ferrara (Paris-New York, 1981), p. 352.
91) M. Daniel and C. M. Viallet, Rev. Mod. Phys. **52** (1980), 175.
H. J. Bernstein and A. V. Phillips, Scientific American **245** (1981), 94.
D. D. Ivanenko, P. I. Pronin and G. A. Sardanashvily, *Group, Geometrical and Topological Methods in Field Theory* (Moscow Univ. Publ. Hours., Moscow, 1983) (in Russian).
92) R. Utiyama, Phys. Rev. **101** (1956), 1597.
93) T. Kibble, J. Math. Phys. **2** (1959), 212.
94) S. Brodsky, D. Ivanenko and G. Sokolik, ZhETF, **41** (1961), 1307 (in Russian).
95) D. D. Ivanenko, P. I. Pronin and G. A. Sardanashvily, *Gauge Theory of Gravity* (Moscow Univ. Publ. Hous., Moscow) (in Russian).
96) V. N. Ponomariev, A. O. Barvinsky and Yu. N. Obukhov, *Geometrodynamical Methods and Gauge Treatment of Gravitational Interaction* (Energoatomizdat, Moscow, 1985) (in Russian).
97) D. Ivanenko and G. Sardanashvily, Phys. Rep. **94** (1983), 1.
98) E. W. Mielke, *Geometrodynamics of Gauge Fields* (Akademie-Verlag, Berlin, 1987).
99) P. A. Baikov, N. F. Nelipa and S. S. Ostapchenko, *Ghost- and Tachyons-Free Gauge-Invariant Affine and Projective Lagrangians*, Preprint (1985) NIIYaF MGU.
100) M. Camenzind, Phys. Rev. Lett. **35** (1975), 1188.
101) J. K. Kim and Y. Yoon, Phys. Lett. **A121** (1987), 213.
102) M. Gasperini, Phys. Lett. **B205** (1988), 517.
103) V. N. Ponomariev, Yu. N. Obukhov and V. Zytnikov, Gen. Rel. Grav. **19** (1990), 731.
104) I. L. Buchbinber and I. L. Shapiro, Phys. Lett. **B151** (1985), 203.
105) P. I. Pronin, in *Quantum Mechanics in Curved Space-time,* ed J. Audretsch and V. De Sabbata (Plenum Publishing Corporation New York-London-Washington-Boston, 1990), p. 517.
106) P. L. Poznanin, Izvestiya VUZov **1** (1975), 116 (in Russian).
107) P. L. Poznanin, Izvestiya VUZov **4** (1975), 134 (in Russian).
108) F. London, Z. Phys. **63** (1930), 245.
109) R. S. Willey, Phys. Rev. **D18** (1978), 270.
110) P. M. Fishbane and M. T. Grisaru, Phys. Lett. **B74** (1978), 93.
111) T. Appelquist and W. Fischler, Phys. Lett. **B77** (1978), 405.
112) Y. Fujii and K. Mima, Phys. Lett. **B79** (1978), 138.
113) S. Matsuyama and H. Miyazawa, Prog. Theor. Phys. **61** (1979), 942.
114) G. Feinberg and J. Sucker, Phys. Rev. **D20** (1979), 1717.
115) G. L. Murphy, Int. J. Theor. Phys. **28** (1989), 923.
116) A. A. Tseytlin and E. S. Fradkin, Phys. Lett. **B207** (1986), 321.
117) V. N. Ponomariev and E. V. Smetanin, Vestnik Mosk. Gos. Univ. ser. Fiz. Astr. 6 (1979), 15.
118) M. Green, J. Schwarz and E. Witten, *Superstring Theory* (Cambridge Univ. Press. 1986).
119) Y. -S. Wu and Z. Wang, Phys. Rev. Lett. **57** (1986), 1978.
120) K. -I. Maeda, Mod. Phys. Lett. **3** (1988), 243.
121) V. D. Ivashchuk and V. N. Melnikov, Nuovo Cim. **B102** (1988), 131.
122) K. A. Bronnikov, V. D. Ivashchuk and V.N. Melnikov, Nuovo Cim. **B102** (1988), 209.
123) R. W. Hellings et al., Phys. Rev. Lett. **102** (1990), 209.
124) R. W. Hellings, Peport GR-10(1983), Padua, Italy.
125) M. Yoshimura, Phys. Rev. **D34** (1986), 1021.
126) R. C. Mayers, Phys. Rev. **D35** (1987), 455.
127) Y. M. Cho and D. H. Park, Nuovo Cim. **B105** (1990), 817.
128) A. Finzi, Mon. Not. R. Astron. Soc. **27** (1961), 21.
129) M. Milgrom, Astrophys. J. **270** (1983), 284, 365, 371.

130) P. H. Sanders, Astron. Appl. **136** (1966), 121; **154** (1966), 135.
131) J. R. Kuhn and L. Kruglyak, Astrophys. J. **313** (1987), 1.
132) P. Fayet, Phys. Lett. **B277** (1989), 127.
133) V. M. Mostepanenko and I. Yu. Sokolov, Phys. Lett. **A132** (1988), 313.