Theoretical calculation of energy received by semi-spherical solar collector

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Abstract. Solar energy is widely used as an environmentally friendly energy source in many countries, Latvia among them. However, geographical and climatic conditions in Latvia, low maximal height of the sun and maximal irradiance, as well as comparatively great cloudiness, present particular challenges. Long days and the long path of the sun in summertime means traditional constructions of solar collectors are not efficient enough for use in Latvia.

A new principle in the construction of the solar collector has been developed: a solar collector with the absorber in the shape of a semi-sphere. It has advantages in comparison with the traditional fixed flat-plate model: it effectively receives both direct and diffuse solar radiation all day. In addition, it is durable against the destructive impact of the wind.

This article reports theoretical calculations of received energy which have been carried out, based on continual visible position of the sun in the sky. Results of these calculations have shown that on a clear sunny day, a semi-spherical collector with the base of 1 m^2 receives 1.3 times more energy than a flat-plate collector positioned at optimal (approx. 40^o) inclination, and 1.6 times more than the flat horizontal one. In cloudy conditions, the difference is greater because of greater diffused radiation.

Such a semi-spherical solar collector has been constructed. Experimental investigation of the received heat energy from the semi-spherical solar collector will be carried out in field conditions this summer.

Key words: Solar collector, radiation, effective area, semi-spherical absorber

INTRODUCTION

Solar water heating and electricity production systems are used in many countries, mostly in southern regions, but the use of solar energy is possible also in Latvia (Ziemelis et al. 2004, Kancevica et al. 2006). However, in Latvia, because of its geographical and climatic conditions, different conditions for the use of solar energy exist:

1) Long summer days with a long path of the sun, making it important to receive the solar energy in wide aperture;
2) Small maximal height of the sun and maximal irradiance;
3) Frequent great cloudiness, which decreases the direct radiation but increases the diffused radiation.

Therefore solar energy receivers currently in use are insufficient in Latvia, and newly designed constructions are required for more efficient use of the diffused radiation.

Additionally, for better elaboration and evaluation of new constructions, more precise, complete and convenient methods for calculation and forecasting of the received energy are required.

MATERIALS AND METHODS

Solar radiation consists of direct and diffused energy. At the first approximation, the diffused radiation can be considered as constant and independent of orientation of the receiving surface. Direct radiation can be calculated depending on elevation and azimuth of the sun as it strikes a surface.

Solar energy at the upper border of the atmosphere is considered constant and equal to the solar constant S_0 = 1367 W m^{-2}. At the ground level the direct radiation *I*, received by the perpendicular to the sun rays surface, depends on the lucidity of the atmosphere *P* and on the air mass *m*, which depends only on the elevation of the sun, from which the well known coherence can be calculated

$$I = S_0 \cdot P^m \tag{1}$$

then the direct radiation received by the flat surface, oriented at any angle to the sun, can be calculated

$$I' = I \cdot \cos\alpha , \tag{2}$$

where α - angle at which solar rays fall to the surface.

For the calculation of the received energy by any (curved) surface, a method for an effective area is offered in this article. The effective area *L* is the visible area of the receiving surface from the direction of the sun. Then the energy received by this surface is

$$E = I \cdot L , \tag{3}$$

where *I* – direct radiation from formula (1) can be calculated, and does not depend on the receiving surface.

For evaluation of the lucidity of the atmosphere and the diffused radiation, both can be considered as constant at the first approximation. Calculated hemi-spherical energy received by the flat surface has been compared with the one measured.

Measurements of received energy in Riga from 1 March - 31 October 2006 were carried out, measuring energy received by two surfaces, one fixed so that it would be perpendicular to solar rays at noon, and one tracking the sun throughout the day (always perpendicular).

It has been found that in Latvian conditions the lucidity of the atmosphere is 0.78 and the diffused radiation is 150W m^{-2}. Solar coordinates are calculated using the method given in our publication (Pelece et al. 2007); the air mass is figured using the method given by Kasten and Young (Kasten & Young 2000).

With the mentioned values of the lucidity of the atmosphere and the diffused radiation, a good coincidence between calculated and measured energy is obtained (Fig. 1).

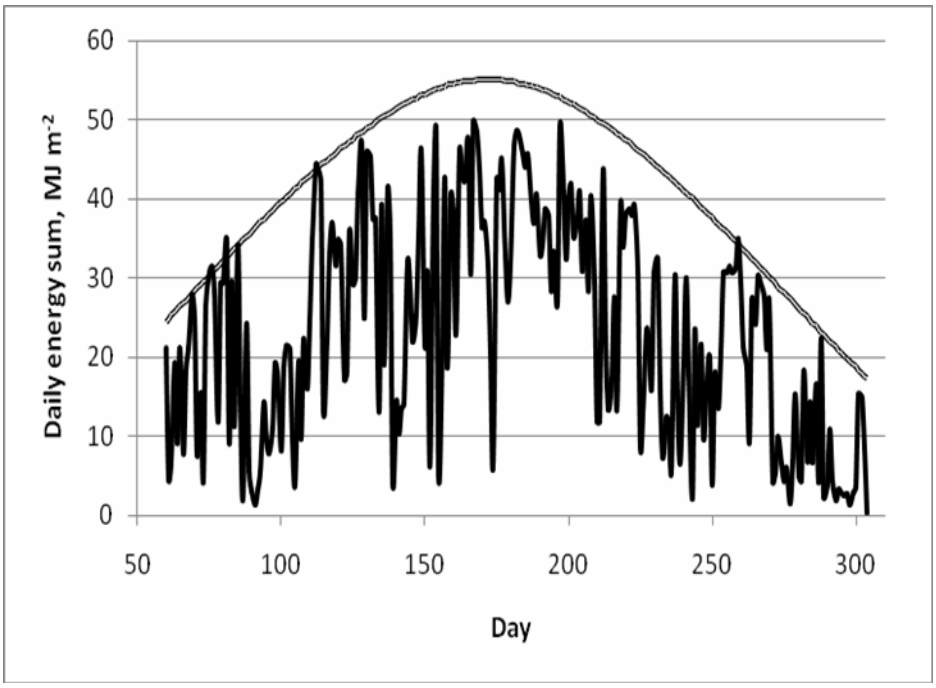

Fig. 1. Measured (———) and calculated (═══) daily sums of received energy by tracking the sun receiver in 2006.

It must be taken into account here that the calculated energy is at clear-sky conditions, while the measured one is in actual cloudy conditions, therefore the measured energy can be smaller than the calculated amount, but not greater.

RESULTS AND DISCUSSION

Value of the energy received by particular surface (formula 3) can be characterized by an effective area. For flat surface with area *S* the effective area is

$$L = S \cdot \cos\alpha \,. \tag{4}$$

where α - angle, at which solar rays fall to the surface.

For Latvian conditions, a better option than the traditional flat plate solar collector would be the collector with a half-sphere as an absorbing surface (Iljins et al., 2007) (Fig. 2).

Fig. 2. Semi-spherical solar collector.

The effective area of a spherical surface can be calculated as follows: a normal vector $\vec{n}$ of the surface at any one point (Fig.3.) can be represented in polar coordinates

$$\vec{n} = \sin\theta \cdot \cos\varphi \cdot \vec{i} + \sin\theta \cdot \sin\varphi \cdot \vec{j} + \cos\theta \cdot \vec{k} \tag{5}$$

where $\vec{i}; \vec{j}; \vec{k}$ are unity orts in Dekart coordinate system.

Direction of solar rays can be represented by the vector $\vec{l}$. Because the x axis can be chosen in the vertical plane of solar rays, it can be expressed as

$$\vec{l} = \cos\delta \cdot \vec{i} + \sin\delta \cdot \vec{j} \tag{6}$$

where δ - elevation of the sun.

Then cosines of the angle between normal vector and direction of the sun rays are expressed by scalar product of vectors $\vec{n}$ and $\vec{l}$

$$\cos\beta = \vec{n} \cdot \vec{l} = \sin\theta \cdot \cos(\delta - \varphi) \tag{7}$$

Energy received by the surface area element is

$$dE = Ir^2 \cos^2\theta \cos\beta , \tag{8}$$

where r is radius of the sphere and $r^2 \sin\theta$ is Jacobian.

$$dE = Ir^2 \sin^2\theta \cdot \cos(\delta - \varphi) d\theta d\varphi . \tag{9}$$

For calculation of all the energy received by the surface this expression must be integrated over the entire irradiated surface, *i.e.,* by coordinate θ from 0 to π and by φ from 0 to $\pi/2-\delta$ (Fig.3).

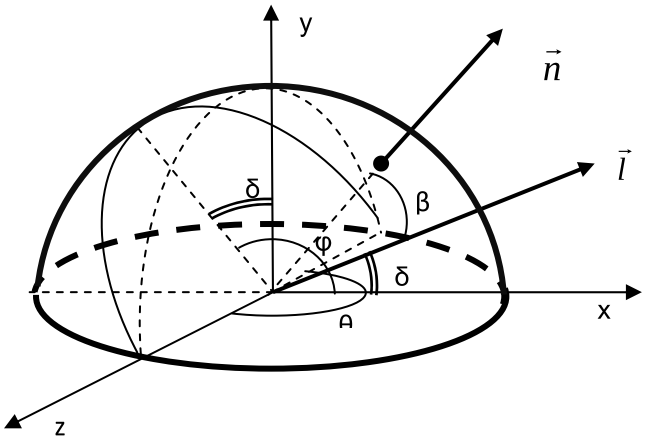

Fig. 3. Entities for calculation of energy received by spherical surface.

$$E = I \cdot r^2 \cdot \int_0^{\pi} d\theta \int_0^{\pi/2+\delta} \sin^2\theta \cos(\delta-\varphi) d\varphi \qquad (10)$$

After integration we obtain

$$E = I \cdot r^2 \cdot \frac{\pi}{2}(1+\sin\delta), \qquad (11)$$

Effective area of the spherical surface

$$L = \frac{\pi}{2} r^2 (1+\sin\delta). \qquad (12)$$

Comparison of diurnal changes of the effective area of a flat 1 m^2 large surfacepositioned at optimal 40^o inclination (Pelece et al. 2007) with that of the semi-spherical surface, having the base area 1 m^2 (Fig. 4) shows that only precisely at noon is the effective area of the flat surface larger. During the rest of the day the effective area of the spherical surface is considerably larger than that of the flat surface.

Calculations of the energy received from 1 March-31 October (Fig. 5) show that at completely clear sky conditions the obtained energy by the semi-spherical surface with the base area 1 m^2 will be 9.9 GJ, which is 1.3 times more than that of the flat 1 m^2 large surface positioned at optimal 40^o angle (7.4 GJ), and 1.6 times bigger than that of the flat horizontal 1 m^2 large surface (6.2 GJ).

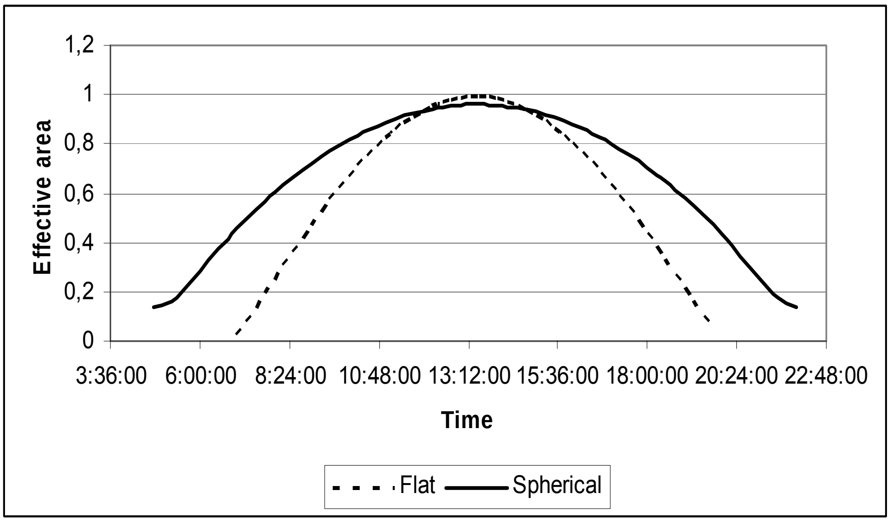

Fig. 4. Calculated effective areas of flat 40^o declined surface (- - - -) and semi spherical surface (———) on 22 June.

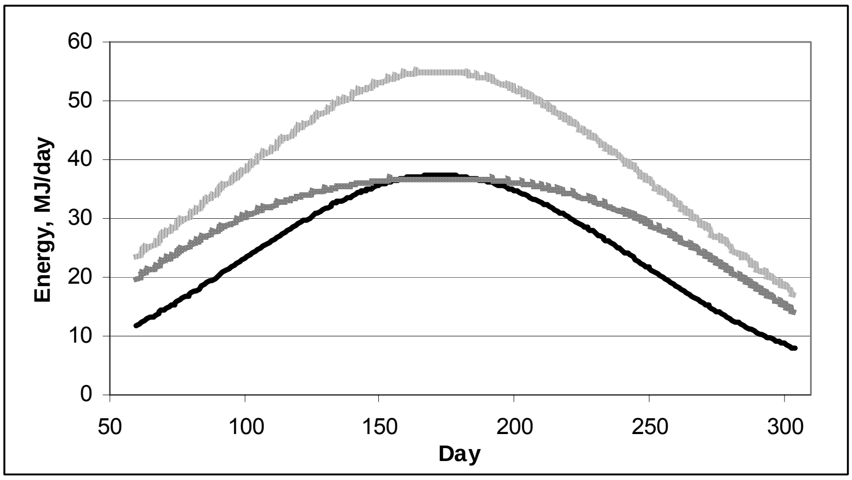

Fig. 5. Calculated daily energy sums of flat 1 m^2 large horizontal (▬), flat 1 m^2 large 40^o declined ( ) and spherical with base area 1 m^2 ( ) surfaces at clear- sky conditions.

CONCLUSIONS

The theoretically calculated energy sum received by the semi-spherical collector with the base area of 1 m^2 is 1.3 times greater than that of the optimally declined 1 m^2

large flat surface, and 1.6 times greater than that of the flat plane 1 m^2 large horizontal collector.

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