Theoretical Investigation and Computation of Time Dependent CW NMR Blood-flow Signal for the Estimation of Blood Flow Parameters

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A new approach is described here for Continuous Wave Nuclear Magnetic Resonance (CW NMR) application to quantification of blood flow in human blood vessels. To study the time dependence of the flow signal generated by the flowing blood spins, a simple model of CW NMR excitation scheme is used for an accurate estimation of blood steady velocity, V_0, the peak pusatile velocity, V_{Pulse}^{0} and the blood vessel cross-section. Starting with the time dependent Bloch equations, we have generated an equation that presents a CW NMR blood signal, whose profile corresponds to the human cardiac cycle. Using a spin-spin relaxation time of 0.1 s, blood steady velocity of 20 cm/s (typical of a healthy human) and a peak pulse velocity of 30 cm/s, our model provides results very close to those obtained by conventional NMR pulse machines and other modality like the Ultrasound Doppler (US) technique. The application of our CW NMR model technique for the determination of blood flow parameters of importance covers a wide range of variations seen in human patients.

1. Introduction

Although pulsed methods using conventional imagers provides useful theoretical and experimental information of blood flow rates, such instrumentation are expensive when only knowledge of blood flow rate [1,2,3,4,5] and other relevant parameters are desired.

MRI modalities essentially rely on the strong influence of spins on the amplitude and phase of MR imaging signal. Unfortunately, the results obtained from many MRI blood flow velocity techniques are difficult to interpret since the factors that affect the MRI signal dependence flow pattern may not be completely identified. Subsequently, the complex flow patterns measured by emerging methods of MRI can possibly be confused with ever-present measurement of artifacts [6]. The accuracy of the flow measurement depends strongly on the artifact suppression and the phase correction procedure applied to the data after its acquisition.

It is desirable to find a suitable, simple and easily affordable NMR technique so that the measurement of NMR signal strength can yield knowledge of the blood flow rates in human patients despite the finite size of blood vessels, small magnetic field inhomogeneity ($\approx$ 1 mG), static tissue signal from tissue surrounding the blood vessel, variation of effective T_2 relaxation time from patient to patient, and such factors.

Using such techniques, one should be able to obtain a reliable estimate of blood flow rates and other medically relevant parameters, such as blood vessel cross-sections and T_2 relaxation times.

It has been stated before that in pulse flow method, it is difficult to correlate the detected signal strength with the flow rates because of the presence of magnetic field inhomogeneities. CW NMR has an advantage over the pulse flow NMR in that the field inhomogeneity (FI) of the order of 1 mG for $B_0 \approx 0.1 - 1$ T is not critical to the accuracy of the method [7,8,9,10,11]. For accurate estimation of blood flow rates from pulsed NMR signal, the homogeneity requirement is 1 part in 10^8. The cost implication of such a highly homogenous magnet is too high.

However, in a CW NMR system that uses a single detector coil overlapping the excitor coil (or the single excitor-detector coil), the estimation of steady blood flow is impeded by the overwhelming influence of static tissue signal on the time-independent steady flow signal. Such systems cannot separate the contribution of the steady flow signal due to flowing blood spins from the signal that itself is due to static tissue surrounding the blood vessels. The continuous wave signal also depends largely on the physical parameter of the CW system. One major problem that has not allowed for an appreciable research in the CW NMR system is the mathematical complexities involved, which of course is the development of exact algorithm that will provide useful solution to

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Bloch equations. Tackling these problems (static tissue, system parameters) no doubt will make CW NMR very useful for accurate measurement of blood velocity as an indicator of heart and vessel disease [11].

2. CW NMR signal dependence on flow rates

Our goal is to work out a technique that will facilitate the quantification of blood flow rates using a CW NMR system. To do this, we shall consider a model system of excitor and detector, which has a simple geometrical arrangement as in Fig. 1.

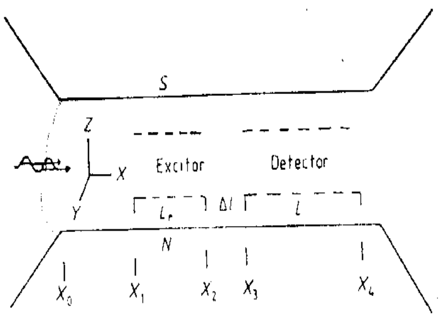

Fig. 1: Diagram of the model CW NMR Excitation Scheme with Separate Movable Detection System for accurate estimation of V_0, V^0_{pulse} and also the total cross-section of the blood vessel. L_e is the length of the excitor coil and Δl is the separation of the excitor and the detector coil whose length is L.

The CW NMR excitation is carried out over the excitor coil of length Le = X_2-X_1. A fluid (blood) flowing in from the left is assumed to be magnetized to an equilibrium value M_0 before entering the excitor coil. Static tissue in the excitor coil region is also subjected to CW excitation. The B_0 field inhomogeneity should be less than the magnitude of the radio frequency (rf) B_1 field of the rf excitation for our scheme to be successful. The requirement is far less stringent than that required for an ideal pulse NMR spectrometer [11]. The detector coil is separated from the excitor coil by distance Δl in our model.

The theoretical analysis of the dependence of NMR signal on flow rates begins with a model system in which B_1 is assumed to be: (i) zero or negligible inside the detector coil; and (ii) finite inside the excitor coil. However, a highly reduced value of B_1 in the detector coil does not alter the conclusion reached with the model system. Beyond the excitation coil region, the detector coil receives the signal from the blood excited in the excitor coil region. The detection coil can be positioned at different distances ΔL from the excitor coil. For case (i), we consider the signal as a result of precessing transverse magnetization M_y of the flowing spins and is dependent on both the flow velocity and on the T_2 relaxation time.

Earlier, De [11], under some assumptions, carried out theoretical analysis of the CW NMR signal dependence on blood flow parameters such as steady blood velocity, pulsatile peak velocity and T_2 relaxation time. In this article, we have carried out *ab initio* analysis of clinical importance from a more fundamental concept.

The x,y,z components of the magnetization are given by the Bloch equations [12,11], which may be written as

$$\frac{dM_x}{dt} = -\frac{M_x}{T_2} \tag{1}$$

$$\frac{dM_y}{dt} = \gamma[M_z B_1(x)] - \frac{M_y}{T_2} \tag{2}$$

$$\frac{dM_z}{dt} = -\gamma M_y B_1 + \frac{M_0 - M_z}{T_1} \tag{3}$$

Where, all x, y, z are of the rotating frame of reference. The z-axes is along the laboratory *Z* axis, whereas the *x* and *y* axes form an angle ωt with the laboratory *X* and *Y* axes, respectively. All other symbols have their earlier definitions.

In the CW NMR under investigation, we assume that the strength of $B_1(x)$ is such that M_z does not differ appreciably from M_0. Then, Eqn. 2 describing the rate of change in magnetization along the y – axis is given by

$$\frac{\delta M_y}{\delta t} = -\frac{M_y}{T_2} - V(t).\,grad\, M_y + \gamma M_0 B_1 S \tag{4}$$

Where, $V(t)$ is along the laboratory X axis and S is called the instrument factor. S = 1 for $X_1 < X < X_2$ and S = 0 for $X > X_2$. In this work, using a time retarded concept, a form of solution of Eqn. 4 that gives the flow signal I_{FS}, is given as

$$I_{FS}(t) = \omega\beta \int_{t-\tau}^{t} V(t') M_y(t - t' - T_0') \exp\left(-(t - t' + T_0')/T_2\right) dt' \tag{5}$$

Here, I_{FS} is the CW NMR signal resulting from spins flowing with time dependent velocity and T_0' and τ are given by the following:

$$\int_{t-\tau}^{t} V(t')dt' = L \tag{6}$$

$$\int_{t-t'-T'}^{t-\tau} V(t')dt' = \Delta L \tag{7}$$

$$M_y(t-t'-T_0') = \gamma M_0 B_{10} T_2 \left(1-\exp\left(-\frac{L_e}{V(t-t'-T_0')T_2}\right)\right) \tag{8}$$

Where, ω is the angular frequency generated by the rF signal and β is the blood vessel cross-section. The time $t = 0$ is counted from the moment when a fresh pulse enters the coil.

In equation (8), V is a function of $(t-t'-T_0')$. Note that $V(t-t'-T_0')$ is the velocity of blood bolus at time, $(t-t'-T_0')$. Eqns. 5 - 8 can be handled only through numerical analysis in order to understand the dependence of CW NMR signal on steady and pulsatile flow rates, T_2 relaxation time, vessel cross-section, and so on.

Since blood is made up of both steady and pulsatile flow, *V(t′)*, the blood velocity is given by

$$V(t') = V_0 + V_p(t') \tag{9}$$

In this study, in order to arrive at analytical expression for the CW NMR signal as a function of these parameters, we proceed as follows:

$$V_p(t') = \left(V^0_{pulse}/0.365\right)\exp(-4t')\sin(-1.5\pi t') \tag{10}$$

Eqns. 9 and 10 have been clearly explained by De [11].

It would be interesting to derive the explicit expression for I_{FS} (Eqn. 5) with flow velocity given by equations (9) and (10). Thus substituting the expressions for $V(t')$ and $M_y(t-t'-T_0')$ in Eqn. 5, yields

$$I_{FS} = \omega\beta C\int_{t-\tau}^{t}\left[\left(V_0+\left(\frac{V^0_{Pulse}}{0.365}\right)\exp(-4t')\sin(-1.5\pi t')\right)\gamma M_0 B_{10}T_2\left(1-\exp\left(-\frac{L_e}{V(t-t'-T_0')T_2}\right)\right)\exp\left(-\frac{(t-t'+T_0')}{T_2}\right)\right]dt' \tag{11}$$

$$I_{FS}(t) = \omega\beta C\left\{\int_{t-\tau}^{t}\gamma M_0 B_{10}T_2 V_0 dt' + \int_{t-\tau}^{t}\left[\left(\frac{V^0_{pulse}}{0.365}\right)\exp(-4t')\sin(-1.5\pi t')\gamma M_0 B_{10}T_2\left(1-\exp\left(-\frac{L_e}{\mathrm{V(t-t'-T_0')}T_2}\right)\right)\exp\left(-\frac{(t-t'+T_0')}{T_2}\right)\right]dt'\right\} \tag{12}$$

$$I_{FS}(t) = \omega\beta C\gamma M_0 B_{10}T_2\left(V_0 t'\right)\Big|_{t-\tau}^{t} + \omega\beta C\gamma M_0 B_{10}T_2\frac{V^0_{pulse}}{0.365}\int_{t-\tau}^{t}\left[\exp(-4t')\sin(-1.5\pi t')\left(1-\exp\left(-\frac{L_e}{\mathrm{V(t-t'-T_0')}T_2}\right)\right)\exp\left(-\frac{(t-t'+T_0')}{T_2}\right)\right]dt' \tag{13}$$

$$I_{FS} = \omega\beta C\gamma M_0 B_{10}T_2 V_0\tau + K\int_{t-\tau}^{t}\left[\exp(-4t')\sin(-1.5\pi t')\left(1-\exp\left(-\frac{L_e}{\mathrm{V(t-t'-T_0')}T_2}\right)\right)\exp\left(-\frac{(t-t'+T_0')}{T_2}\right)\right]dt' \tag{14}$$

Where, $K = \omega\beta C\gamma M_0 B_{10}T_2\left(\frac{V^0_{pulse}}{0.365}\right)$ and *C* is a Constant equals to unity that has been introduced to ascribe a unit of E.M.F to the flow signal. Now,

$$\exp(-4t')\sin(-1.5\pi t') = \left(\frac{\exp(-i1.5\pi t') - \exp(i1.5\pi t')}{2i}\right)\exp(-4t') = \frac{\exp(-i1.5\pi t' - 4t') - \exp(i1.5\pi t' - 4t')}{2i} \quad (15)$$

Also,

$$\left[1 - \exp\left(-\frac{L_e}{V(t-t'-T_0^{'})T_2}\right)\right]\exp\left[-\frac{(t-t'+T_0^{'})}{T_2}\right] = \exp\left[-\frac{(t-t'+T_0^{'})}{T_2}\right] - \exp\left[-\frac{(L_e + (t-t'+T_0^{'})V)}{VT_2}\right] \quad (16)$$

For convenience, $V = V(t - t' - T_0^{'})$.

From Eqn. 14, we obtained the integral:

$$\int_{t-\tau}^{t}\left[\exp(-4t')\sin(-1.5\pi t')\left(1 - \exp\left(-\frac{L_e}{V_{(t-t'-T_0^{'})}T_2}\right)\right)\exp\left(-\frac{(t-t'\_T_0^{'})}{T_2}\right)\right]dt' =$$

$$\int_{t-\tau}^{t}\left[\frac{\exp(-i1.5\pi t' - 4t') - \exp(i1.5\pi t' - 4t')}{2i}\right]\left[\exp\left(-\frac{(t-t'+T_0^{'})}{T_2}\right) - \exp\left(-\frac{(L_e + (t-t'+T_0^{'})V)}{VT_2}\right)\right]dt' \quad (17)$$

Simplifying Eqn. 17 gives:

$$\frac{1}{2i}\left[\begin{array}{l}\int_{t-\tau}^{t}\exp\left(-\frac{(t+T_0^{'})}{T_2}\right)\exp\left(\frac{(-i1.5\pi T_2 - 4T_2 + 1)t'}{T_2}\right)dt' - \int_{t-\tau}^{t}\exp\left(-\frac{(t+T_0^{'})}{T_2}\right)\exp\left(\frac{(i1.5\pi T_2 - 4T_2 + 1)t'}{T_2}\right)dt' \\ -\int_{t-\tau}^{t}\exp\left(-\frac{(L_e + (t+T_0^{'})V)}{VT_2}\right)\exp\left(\frac{(-i1.5\pi VT_2 - 4VT_2 + 1)t'}{VT_2}\right)dt' + \int_{t-\tau}^{t}\exp\left(-\frac{(L_e + (t+T_0^{'})V)}{VT_2}\right)\exp\left(\frac{(i1.5\pi VT_2 - 4VT_2 + 1)t'}{VT_2}\right)dt'\end{array}\right] \quad (18)$$

Since $V = V(t - t' - T_0^{'})$, the integration of Expression (18) is a difficult task in the numerator.

The average value, $\langle V\rangle$, within the excitor, is considered such that

$$\langle V\rangle = \frac{1}{\tau}\int_{t-\tau}^{t} V(t - t' - T_0^{'})dt' \quad (19)$$

and

$$V(t - t' - T_0^{'}) = V_0 + \left(V_{Pulse}^{0}/0.365\right)\exp\left(-(t - t' - T_0^{'})\right)\sin 1.5\pi(t - t' - T_0^{'}) \quad (20)$$

However, V in subsequent expressions denote $\langle V\rangle$ of Eqn. 19. Note that $\langle V\rangle$ is a function of t, τ and $T_0^{'}$ but not of t'.

The integral in (18) can be expressed as

$$I = \frac{1}{2i}\left[\begin{array}{l}C_1\int_{t-\tau}^{t}\exp\left(-\frac{(i1.5\pi T_2 + 4T_2 - 1)t'}{T_2}\right)dt' - C_1\int_{t-\tau}^{t}\exp\left(\frac{i1.5\pi T_2 - 4T_2 + 1)t'}{T_2}\right)dt' \\ -C_2\int_{t-\tau}^{t}\exp\left(-\frac{i1.5\pi VT_2 + 4VT_2 + -V)t'}{VT_2}\right)dt' + C_2\int_{t-\tau}^{t}\exp\left(\frac{(i1.5\pi VT_2 - 4VT_2 + V)t'}{VT_2}\right)dt'\end{array}\right] \quad (21)$$

Where,

$$C_1 = \exp\left(-\frac{(t + T_0'}{T_2} \right) \tag{22}$$

and

$$C_2 = \exp\left(-\frac{(L_e + (t + T_0')}{VT_2} \right) \tag{23}$$

are treated as constant coefficients independent of t'. Performing the integration of Eqn. 21 after simplifying the last two terms gives

$$I = \frac{1}{2i}\left[\begin{array}{l} -\dfrac{C_1T_2}{(i1.5\pi T_2 + 4T_2 - 1)}\exp\left(-\dfrac{(i1.5\pi T_2 + 4T_2 - 1)t'}{T_2} \right)\Bigg|_{t-\tau}^{t} - \dfrac{C_1T_2}{(i1.5\pi T_2 - 4T_2 + 1)}\exp\left(\dfrac{(i1.5\pi T_2 - 4T_2 + 1)t'}{T_2} \right)\Bigg|_{t-\tau}^{t} \\ +\dfrac{C_2T_2}{(i1.5\pi T_2 + 4T_2 - 1)}\exp\left(-\dfrac{(i1.5\pi T_2 + 4T_2 - 1)t'}{T_2} \right)\Bigg|_{t-\tau}^{t} + \dfrac{C_2T_2}{(i1.5\pi T_2 - 4T_2 + 1)}\exp\left(\dfrac{(i1.5\pi T_2 - 4T_2 + 1)t'}{T_2} \right)\Bigg|_{t-\tau}^{t} \end{array} \right] \tag{24}$$

Now, we insert the limits in Eqn. 24 so that we obtain

$$I = \frac{1}{2i}\left\{ \begin{array}{l} -\dfrac{C_1T_2}{(i1.5\pi T_2 + 4T_2 - 1)}\left[\exp\left(-\dfrac{(i1.5\pi T + 4T_2 - 1)t}{T_2} \right) - \exp\left(-\dfrac{(i1.5\pi T_2 + 4T_2 - 1)(t-\tau)}{T_2} \right) \right] \\ -\dfrac{C_1T_2}{(i1.5\pi T_2 - 4T_2 + 1)}\left[\exp\left(\dfrac{(i1.5\pi T - 4T_2 + 1)t}{T_2} \right) - \exp\left(\dfrac{(i1.5\pi T_2 - 4T_2 + 1)(t-\tau)}{T_2} \right) \right] \\ +\dfrac{C_2T_2}{(i1.5\pi T_2 + 4T_2 - 1)}\left[\exp\left(-\dfrac{(i1.5\pi T + 4T_2 - 1)t}{T_2} \right) - \exp\left(-\dfrac{(i1.5\pi T_2 + 4T_2 - 1)(t-\tau)}{T_2} \right) \right] \\ +\dfrac{C_2T_2}{(i1.5\pi T_2 - 4T_2 + 1)}\left[\exp\left(\dfrac{(i1.5\pi T - 4T_2 + 1)t}{T_2} \right) - \exp\left(\dfrac{(i1.5\pi T_2 - 4T_2 + 1)(t-\tau)}{T_2} \right) \right] \end{array} \right\} \tag{25}$$

Eqn. 25 can further be expanded to give

$$I = \frac{1}{2i}\left\{ \begin{array}{l} -\dfrac{C_1T_2}{(i1.5\pi T_2 + 4T_2 - 1)}\left[\exp\left(-\dfrac{(i1.5\pi T + 4T_2 - 1)t}{T_2} \right) - \exp\left(-\dfrac{(i1.5\pi T_2 + 4T_2 - 1)t}{T_2} \right)\exp\left(\dfrac{(i1.5\pi T + 4T_2 - 1)\tau}{T_2} \right) \right] \\ -\dfrac{C_1T_2}{(i1.5\pi T_2 - 4T_2 + 1)}\left[\exp\left(\dfrac{(i1.5\pi T - 4T_2 + 1)t}{T_2} \right) - \exp\left(\dfrac{(i1.5\pi T_2 - 4T_2 + 1)t}{T_2} \right)\exp\left(-\dfrac{(i1.5\pi T_2 - 4T_2 + 1)\tau}{T_2} \right) \right] \\ +\dfrac{C_2T_2}{(i1.5\pi T_2 + 4T_2 - 1)}\left[\exp\left(-\dfrac{(i1.5\pi T + 4T_2 - 1)t}{T_2} \right) - \exp\left(-\dfrac{(i1.5\pi T_2 + 4T_2 - 1)t}{T_2} \right)\exp\left(\dfrac{(i1.5\pi T_2 + 4T_2 - 1)\tau}{T_2} \right) \right] \\ +\dfrac{C_2T_2}{(i1.5\pi T_2 - 4T_2 + 1)}\left[\exp\left(\dfrac{(i1.5\pi T - 4T_2 + 1)t}{T_2} \right) - \exp\left(\dfrac{(i1.5\pi T_2 - 4T_2 + 1)t}{T_2} \right)\exp\left(-\dfrac{(i1.5\pi T_2 - 4T_2 + 1)\tau}{T_2} \right) \right] \end{array} \right\} \tag{26}$$

Because of the occurrence of common exponentials in the terms, Eqn. 26 can be written with some factors, as

$$I = \frac{1}{2i}\left\{ \begin{array}{l} -\dfrac{C_1T_2}{(i1.5\pi T_2 + 4T_2 - 1)}\left(1 - \exp\left(\dfrac{(i1.5\pi T + 4T_2 - 1)\tau}{T_2} \right) \right)\exp\left(-\dfrac{(i1.5\pi T_2 + 4T_2 - 1)t}{T_2} \right) \\ -\dfrac{C_1T_2}{(i1.5\pi T_2 - 4T_2 + 1)}\left(1 - \exp\left(-\dfrac{(i1.5\pi T_2 - 4T_2 + 1)\tau}{T_2} \right) \right)\exp\left(\dfrac{(i1.5\pi T - 4T_2 + 1)t}{T_2} \right) \\ +\dfrac{C_2T_2}{(i1.5\pi T_2 + 4T_2 - 1)}\left(1 - \exp\left(\dfrac{(i1.5\pi T_2 + 4T_2 - 1)\tau}{T_2} \right) \right)\exp\left(-\dfrac{(i1.5\pi T + 4T_2 - 1)t}{T_2} \right) \\ +\dfrac{C_2T_2}{(i1.5\pi T_2 - 4T_2 + 1)}\left(1 - \exp\left(-\dfrac{(i1.5\pi T_2 - 4T_2 + 1)\tau}{T_2} \right) \right)\exp\left(\dfrac{(i1.5\pi T - 4T_2 + 1)t}{T_2} \right) \end{array} \right\} \tag{27}$$

Rationalizing the denominators and simplifying Eqn. 27, gives:

$$
\begin{aligned}
I = &-\frac{C_1T_2\left[1.5\pi T_2 + i(4T_2-1)\right]}{4.5\pi^2 T_2{}^2 + 2(4T_2-1)^2}\left(1-\exp\left(\frac{(i1.5\pi T_2 + 4T_2 - 1)\tau}{T_2}\right)\right)\exp\left(-\frac{(i1.5\pi T_2 + 4T_2-1)t}{T_2}\right) \\
&-\frac{C_1T_2\left[1.5\pi T_2 - i(4T_2-1)\right]}{4.5\pi^2 T_2{}^2 + 2(4T_2-1)^2}\left(1-\exp\left(-\frac{(i1.5\pi T_2 - 4T_2 + 1)\tau}{T_2}\right)\right)\exp\left(\frac{(i1.5\pi T_2 - 4T_2+1)t}{T_2}\right) \\
&+\frac{C_2T_2\left[1.5\pi T_2 + i(4T_2-1)\right]}{4.5\pi^2 T_2{}^2 + 2(4T_2-1)^2}\left(1-\exp\left(\frac{(i1.5\pi T_2 + 4T_2 - 1)\tau}{T_2}\right)\right)\exp\left(-\frac{(i1.5\pi T_2 + 4T_2-1)t}{T_2}\right) \\
&+\frac{C_2T_2\left[1.5\pi T_2 - i(4T_2-1)\right]}{4.5\pi^2 T_2{}^2 + 2(4T_2-1)^2}\left(1-\exp\left(-\frac{(i1.5\pi T_2 - 4T_2 + 1)\tau}{T_2}\right)\right)\exp\left(\frac{(i1.5\pi T_2 - 4T_2+1)t}{T_2}\right)
\end{aligned}
\tag{28}
$$

The expression, given by Eqn. 28, can be written in a compact form as:

$$
I = \frac{S_1(C_2-C_1)T_2\left[1.5\pi T_2 + i(4T_2-1)\right]}{4.5\pi^2 T_2{}^2 + 2(4T_2-1)^2} + \frac{S_2(C_2-C_1)T_2\left[1.5\pi T_2 - i(4T_2-1)\right]}{4.5\pi^2 T_2{}^2 + 2(4T_2-1)^2}
\tag{29}
$$

Where,

$$
S_1 = \left(1-\exp\left(\frac{(i1.5\pi T_2 + 4T_2 - 1)\tau}{T_2}\right)\right)\exp\left(-\frac{(i1.5\pi T_2 + 4T_2 - 1)t}{T_2}\right)
\tag{30}
$$

$$
S_2 = \left(1-\exp\left(-\frac{(i1.5\pi T_2 - 4T_2 + 1)\tau}{T_2}\right)\right)\exp\left(\frac{(i1.5\pi T_2 - 4T_2 + 1)t}{T_2}\right)
\tag{31}
$$

The expression in Eqn. 28 contains both the imaginary and real components. Since it is only the real part that makes contribution to the NMR signal of interest, it is important to extract it from the equation. The following procedure could be adopted.

Let $D = 4.5\pi^2 T_2{}^2 + 2(4T_2-1)^2$, $\alpha_1 = C_1T_2 1.5\pi T_2$ and $\alpha_2 = C_1T_2(4T_2-1)$.

Also, let $\theta_1 = \dfrac{1.5\pi T_2\tau}{T_2}$ and $\beta_1 = \dfrac{(4T_2-1)\tau}{T_2}$.

If we let

$A_1 = \left(\dfrac{\alpha_1}{D} + \dfrac{i\alpha_2}{D}\right) = (a+ib)$, the first part of Eqn. 28 can be written as

$$
A_1\left(1-e^{\beta_1}\times e^{i\theta_1}\right)e^{-\left(\frac{i\theta_1 t}{\tau}+\frac{\beta_1 t}{\tau}\right)} =
$$

$$
A_1\left(1-B_1e^{i\theta_1}\right)e^{-\left(\frac{i\theta_1 t}{\tau}+\frac{\beta_1 t}{\tau}\right)}
\tag{32}
$$

Where, $B_1 = e^{\beta_1}$ The exponential 'e' has been adopted for convenience. Using the Euler's relation,

$$
\begin{aligned}
&A_1\left(1-B_1e^{i\theta_1}\right)e^{-\left(\frac{i\theta_1 t}{\tau}+\frac{\beta_1 t}{\tau}\right)} = \\
&A_1\left(1-B_1\cos\theta_1 - iB_1\sin\theta_1\right)e^{-\frac{\beta_1 t}{\tau}}\times e^{-\frac{i\theta_1 t}{\tau}} \\
&= A_1\left(1-B_1\cos\theta_1 - i\sin\theta_1\right)e^{-\frac{\beta_1 t}{\tau}}\times \\
&\left(\cos\frac{\theta_1 t}{\tau} - i\sin\frac{\theta_1 t}{\tau}\right);
\end{aligned}
$$

which gives, on evaluation, the equation

$$A_1 e^{-\frac{\beta_1 t}{\tau}}\left[(1-B_1\cos\theta_1)\cos\frac{\theta_1 t}{\tau} - iB_1\sin\theta_1\cos\frac{\theta_1 t}{\tau} - i(1-B_1\cos\theta_1)\sin\frac{\theta_1 t}{\tau} - B_1\sin\theta_1\sin\frac{\theta_1 t}{\tau}\right]$$

$$= A_1 e^{-\frac{\beta_1 t}{\tau}}\left(\cos\frac{\theta_1 t}{\tau} - B_1\cos\theta_1\cos\frac{\theta_1 t}{\tau} - iB_1\sin\theta_1\cos\frac{\theta_1 t}{\tau} - i\sin\frac{\theta_1 t}{\tau} + iB_1\cos\theta_1\sin\frac{\theta_1 t}{\tau} - B_1\sin\theta_1\sin\frac{\theta_1 t}{\tau}\right) \quad (33)$$

Substituting the expression $A_1 = (a+ib)$ in Eqn. 33 and expanding the resulting expression would give the real part of the first term of Eqn. 28 as

$$I_{R1} = ae^{-\frac{\beta_1 t}{\tau}}\left(\cos\frac{\theta_1 t}{\tau} - B_1\cos\theta_1\cos\frac{\theta_1 t}{\tau} - B_1\sin\theta_1\sin\frac{\theta_1 t}{\tau}\right) + be^{-\frac{\beta_1 t}{\tau}}\left(B_1\sin\theta_1\cos\frac{\theta_1 t}{\tau} + \sin\frac{\theta_1 t}{\tau} - B_1\cos\theta_1\sin\frac{\theta_1 t}{\tau}\right)$$

and could further be written as

$$I_{R1} = ae^{-\frac{\beta_1 t}{\tau}}\left[\cos\frac{\theta_1 t}{\tau}(1-B_1\cos\theta_1) - B_1\sin\theta_1\sin\frac{\theta_1 t}{\tau}\right] + be^{-\frac{\beta_1 t}{\tau}}\left[\sin\frac{\theta_1 t}{\tau}(1-B_1\cos\theta_1) + B_1\sin\theta_1\cos\frac{\theta_1 t}{\tau}\right]. \quad (34)$$

The same procedure can be adopted to get the real part of the second term of Eqn. 28. But now we set $A_2 = \left(\frac{\alpha_1}{D} - \frac{i\alpha_2}{D}\right) = (a-ib)$. Working the same way as with the first term, the real part can be separated as

$$I_{R2} = ae^{-\frac{\beta_1 t}{\tau}}\left(\cos\frac{\theta_1 t}{\tau} - B_1\cos\theta_1\cos\frac{\theta_1 t}{\tau} - B_1\sin\theta_1\sin\frac{\theta_1 t}{\tau}\right) - be^{-\frac{\beta_1 t}{\tau}}\left(-B_1\sin\theta_1\cos\frac{\theta_1 t}{\tau} - \sin\frac{\theta_1 t}{\tau} + B_1\cos\theta_1\sin\frac{\theta_1 t}{\tau}\right)$$

Which, also can be written

$$I_{R2} = ae^{-\frac{\beta_1 t}{\tau}}\left[\cos\frac{\theta_1 t}{\tau}(1-B_1\cos\theta_1) - B_1\sin\theta_1\sin\frac{\theta_1 t}{\tau}\right] + be^{-\frac{\beta_1 t}{\tau}}\left[\sin\frac{\theta_1 t}{\tau}(1-B_1\cos\theta_1) + B_1\sin\theta_1\cos\frac{\theta_1 t}{\tau}\right] \quad (35)$$

Eqn. 35 is, of course, the same as Eqn. 34. The contribution of the real parts by the first two terms of Eqn. 28, considering their signs, can therefore be obtained by adding Eqns. 34 and 35 as

$$I_{1,2} = -2e^{-\frac{\beta_1 t}{\tau}}\left\{a\left[\cos\frac{\theta_1 t}{\tau}(1-B_1\cos\theta_1) - B_1\sin\theta_1\sin\frac{\theta_1 t}{\tau}\right] + b\left[\sin\frac{\theta_1 t}{\tau}(1-B_1\cos\theta_1) + B_1\sin\theta_1\cos\frac{\theta_1 t}{\tau}\right]\right\} \quad (36)$$

We now evaluate the real part of the last two terms of Eqn. 28. To do this, we let

$D = 4.5\pi^2 T_2{}^2 + 2(4T_2 - 1)^2$, $\alpha_3 = C_2 T_2 1.5\pi T_2$ and $\alpha_4 = C_2 T_2(4T_2 - 1)$.

Also, let $\theta_1 = \theta_2 = \frac{1.5\pi T_2 \tau}{T_2}$ and $\beta_1 = \beta_2 = \frac{(4T_2 - 1)\tau}{T_2}$.

Let $A_3 = \left(\frac{\alpha_3}{D} + \frac{i\alpha_4}{D}\right) = u + iv$. The third term of Eqn. 28 can be written as

$$\left(\frac{\alpha_3}{D} + \frac{i\alpha_4}{D}\right)\left(1 - e^{i\theta_2 + \beta_2}\right)e^{-\left(\frac{i\theta_2 t}{\tau} + \frac{\beta_2 t}{\tau}\right)} = A_3\left(1 - B_2 e^{i\theta_2}\right)e^{-\left(\frac{i\theta_2 t}{\tau} + \frac{\beta_2 t}{\tau}\right)} \quad (37)$$

Where, $B_2 = e^{\beta_2}$. Following the previous steps, the real part is given as

$$I_{R3} = ue^{-\frac{\beta_2 t}{\tau}}\left[\cos\frac{\theta_2 t}{\tau}\left(1 - B_2\cos\theta_2\right) - B_2\sin\theta_2\sin\frac{\theta_2 t}{\tau}\right] + be^{-\frac{\beta_2 t}{\tau}}\left[\sin\frac{\theta_2 t}{\tau}\left(1 - B_2\cos\theta_2\right) + B_2\sin\theta\ \cos\frac{\theta_2 t}{\tau}\right]. \quad (38)$$

The same procedure can be adopted to get the real part of the fourth term of Eqn. 28, but now we set

$$A_4 = \left(\frac{\alpha_3}{D} - \frac{i\alpha_4}{D}\right) = \left(u - iv\right),$$

so that the term can be written as

$$A_4\left(1 - e^{\beta_2} \times e^{-i\theta_2}\right)e^{\left(\frac{i\theta_2 t}{\tau} - \frac{\beta_2 t}{\tau}\right)} = A_4\left(1 - B_2 e^{-i\theta_2}\right)e^{\left(\frac{i\theta_2 t}{\tau} - \frac{\beta_2 t}{\tau}\right)}.$$

The real part can be separated as before which can similarly be written as

$$I_{R4} = ue^{-\frac{\beta_2 t}{\tau}}\left[\cos\frac{\theta_2 t}{\tau}\left(1 - B_2\cos\theta_2\right) - B_2\sin\theta_2\sin\frac{\theta_2 t}{\tau}\right] + ve^{-\frac{\beta_2 t}{\tau}}\left[\sin\frac{\theta_2 t}{\tau}\left(1 - B_2\cos\theta_2\right) + B_2\sin\theta_2\cos\frac{\theta_2 t}{\tau}\right] \quad (39)$$

The combined real contribution of the third and fourth terms, like in the first two terms and considering their signs, give

$$I_{3,4} = 2e^{-\frac{\beta_2 t}{\tau}}\left\{u\left[\cos\frac{\theta_2 t}{\tau}\left(1 - B_2\cos\theta_2\right) - B_2\sin\theta_2\sin\frac{\theta_2 t}{\tau}\right] + v\left[\sin\frac{\theta_2 t}{\tau}\left(1 - B_2\cos\theta_2\right) + B_2\sin\theta_2\cos\frac{\theta_2 t}{\tau}\right]\right\} \quad (40)$$

The real part of Eqn. 28 can now be written as $I = I_{1,2} + I_{3,4}$, so that

$$\begin{aligned} I = &-2e^{-\frac{\beta_1 t}{\tau}}\left\{a\left[\cos\frac{\theta_1 t}{\tau}\left(1 - B_1\cos\theta_1\right) - B_1\sin\theta_1\sin\frac{\theta_1 t}{\tau}\right] + b\left[\sin\frac{\theta_1 t}{\tau}\left(1 - B_1\cos\theta_1\right) + B_1\sin\theta_1\cos\frac{\theta_1 t}{\tau}\right]\right\} \\ &+ 2e^{-\frac{\beta_2 t}{\tau}}\left\{u\left[\cos\frac{\theta_2 t}{\tau}\left(1 - B_2\cos\theta_2\right) - B_2\sin\theta_2\sin\frac{\theta_2 t}{\tau}\right] + v\left[\sin\frac{\theta_2 t}{\tau}\left(1 - B_2\cos\theta_2\right) + B_2\sin\theta_2\cos\frac{\theta_2 t}{\tau}\right]\right\} \end{aligned} \quad (41)$$

Substituting the values of $\beta_1, \beta_2, a, b, u, v, \theta_1, \theta_2, B_1$ and B_2 in Eqn. 41 above, the expression I will be given as

$$\begin{aligned} I = &-2\exp\left(-\frac{(4T_2-1)t}{T_2}\right)\left\{\begin{aligned}&\frac{1.5\pi C_1 T_2^2}{4.5\pi^2 T_2^2 + 2(4T_2^2-1)^2}\left[\cos 1.5\pi t\left(1 - \exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\cos 1.5\pi\tau\right) - \exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\sin 1.5\pi\tau\sin 1.5\pi t\right] \\ &+ \frac{C_1 T_2^2(4T_2-1)}{4.5\pi^2 T_2^2 + 2(4T_2^2-1)^2}\left[\sin 1.5\pi t\left(1 - \exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\cos 1.5\pi\tau\right) + \exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\sin 1.5\pi\tau\cos 1.5\pi t\right]\end{aligned}\right\} \\ &+ 2\exp\left(-\frac{(4T_2-1)t}{T_2}\right)\left\{\begin{aligned}&\frac{1.5\pi C_2 T_2^2}{4.5\pi^2 T_2^2 + 2(4T_2^2-1)^2}\left[\cos 1.5\pi t\left(1 - \exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\cos 1.5\pi\tau\right) - \exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\sin 1.5\pi\tau\sin 1.5\pi t\right] \\ &+ \frac{C_2 T_2^2(4T_2-1)}{4.5\pi^2 T_2^2 + 2(4T_2^2-1)^2}\left[\sin 1.5\pi t\left(1 - \exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\cos 1.5\pi\tau\right) + \exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\sin 1.5\pi\tau\cos 1.5\pi t\right]\end{aligned}\right\} \end{aligned} \quad (42)$$

We now substitute for I, i.e., Eqn. 42 in Eqn. 14 to arrive at the final CW NMR signal due to blood flow. For the purpose of simplification and computation of CW NMR signal, we assumed a particular case where $\Delta l = 0$. In this case, T_0' from Eqn. 7 turns out to be zero. Noting the expressions for C_1 and C_2, respectively, in Eqns. 22 and 23 and $K = \omega\beta C\gamma M_0 B_{10} T_2 \left(\frac{V_{pulse}^0}{0.365}\right)$, the final result is given as:

$$
\begin{aligned}
I_{FS}(t) = {} & \omega\beta C\gamma M_0 B_{10} T_2 V_0 \tau + \left[\frac{2\omega\beta C\gamma M_0 B_{10} T_2}{4.5\pi^2 T_2^2 + 2(4T_2^2 - 1)^2}\left(\frac{V_{Pulse}^0}{0.365}\right)\exp\left(-\frac{(4T_2-1)}{T_2}\right)\right]\left\{-1.5\pi T_2^2 \exp\left(-\frac{(t+T_0')}{T_2}\right).\right. \\
& \left[\cos 1.5\pi t\left(1-\exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\cos 1.5\pi\tau\right) - \exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\sin 1.5\pi\tau \sin 1.5\pi t\right] - T_2(4T_2-1)\exp\left(-\frac{(t+T_0')}{T_2}\right). \\
& \left[\sin 1.5\pi t\left(1-\exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\cos 1.5\pi\tau\right) + \exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\sin 1.5\pi\tau \cos 1.5\pi t\right] + 1.5\pi T_2^2 \exp\left(-\frac{L_e + (t+T_0')V}{VT_2}\right). \\
& \left[\cos 1.5\pi t\left(1-\exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\cos 1.5\pi\tau\right) - \exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\sin 1.5\pi\tau \sin 1.5\pi t\right] + T_2(4T_2-1)\exp\left(-\frac{L_e + (t+T_0')V}{VT_2}\right). \\
& \left.\left[\sin 1.5\pi t\left(1-\exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\cos 1.5\pi\tau\right) + \exp\left(\frac{(4T_2-1)\tau}{T_2}\right)\sin 1.5\pi\tau \cos 1.5\pi t\right]\right\}
\end{aligned} \tag{43}
$$

3. Calculation of I_{FS}

To calculate the actual value of the NMR signal flow given by Eqn. 43 above, the T_2 for blood, the time τ which the signal has remained in the detector, the time T_0' which it traverses the separation between the excitor and the detector, the time dependent blood flow velocity and of course V_{pulde}^0, the peak of the pulse velocity should be known. The T_2 relaxation time for blood ranges from 0.1s to 0.15s, while V_{pulse}^0 changes have been given by [10] and [11]. To know the values of τ when the detector receives signal in the present model, Eqn. 6 should be evaluated and known values of L and duration in which the blood bolus spends in the system are used for the computation. This proceeds as follows.

From the Eqn. 6, we have

$$\int_{t-\tau}^{t} V(t')dt' = L$$

Substituting the expression for $V(t')$ in Eqns. 9 and 10 and performing the integration taking the limits of integration into consideration, we have

$$
L = V_0\tau + \frac{V_{pulse}^0}{0.365}\left[\begin{array}{l}\left\{\dfrac{e^{-4t}}{2.25\pi^2+16}\right\}\{(-1.5\pi)\cos(-1.5\pi t) + 4\sin(-1.5\pi t)\} \\ -\left\{\dfrac{e^{-4(t-\tau)}}{2.25\pi^2+16}\right\}\{(-1.5\pi)\cos(-1.5\pi(t-\tau)) + 4\sin(-1.5\pi(t-\tau))\}\end{array}\right] \tag{44}
$$

Using Eqn. 44, with values of L between 10 cm and 20 cm, while t was taken from 0.0s to 1.6s, different sets of value of τ were obtained. The NMR blood flow signal I_{FS} in Eqn. 43 can then be evaluated under different conditions of the CW NMR detection system.

4. Discussion of result

The result obtained at a T_2 = 0.1 s, steady blood velocity of 20 cm/s and V_{ppulse}^0 = 30 cm/s is given in Fig. 2. It shows a close similarity with the results of earlier experimental work [13, 14, 2] and [15].

Figs. 5 and 6 show some of their results using two-dimensional pulsed NMR scanner. Significantly, all the results (previous and our present model) picture the flow pattern of the cardiac cycle. The first part indicating a rise in signal defines the diastole, while the second is the systolic decay. Our result also clearly agrees with the known fact that the cycle decays to zero in about 1 s. Another striking result of our present CW NMR model is the peak signal, a little above 0.2 s, which is consistent with other methods mentioned above. A little deviation however is that CW-NMR signal shows a faster decay in the systolic region. This has to do with the values of τ , the time which the signal spends in the detector coil.

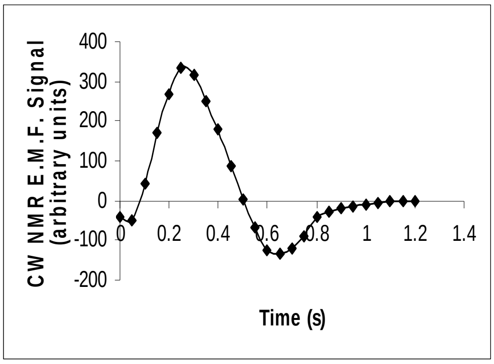

Fig. 2: Blood flow curve obtained from the new CW-NMR detecting system obtained with a T_2 = 0.1 s, blood steady velocity of 20 cm/s and V^0_{pulse} = 30 cm/s.

The variation of the CW NMR signals with different steady flow velocity values from 5 cm/s to 45 cm/s for a given peak pulse velocity $V^0_{Pulse} = 30$ cm/s is given in Fig. 3. These plots show that though the signal amplitude reduces at low velocities the signal itself decays less sharply than at higher velocities. This also shows that the time dependent CW NMR peak-to-peak signal depends not only on V^0_{Pulse}, a fact that has already been established [11], but also on blood steady velocity V_0 . This is a significant result though overlooked by earlier workers. Fig. 4 shows the changes in the signal amplitude as the V^0_{pulse} changes from 15 cm/s to 45 cm/s. The signal strength decreases sharply with decreasing pulse peak velocity, V^0_{pulse}, a result, which as stated above, is in agreement with earlier work.

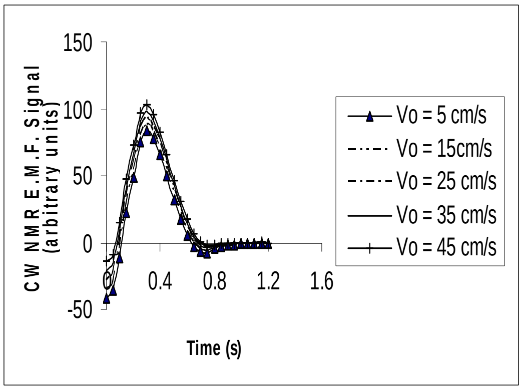

Fig. 3: The variation CW NMR time dependent signal with time as the steady velocity changes from 5 cm/s to 45 cm/s. The curve with the lowest peak represents the least velocity, while that with the highest peak represents the highest velocity in the group; V^0_{pulse} = 30 cm/s, T_2 = 0.15s.

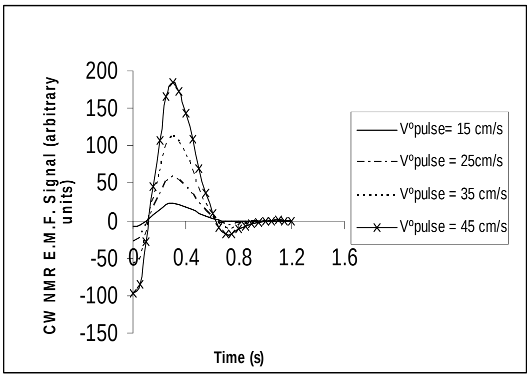

Fig. 4: The Variation of CW NMR signal with time at pulse peak velocity of V^0_{pulse} 15 cm/s, 25 cm/s, 35 cm/s and 45 cm/s. The signal increases with increasing V^0_{pulse} values. The steady blood velocity = 5 cm/s, T_2 = 0.15 s.

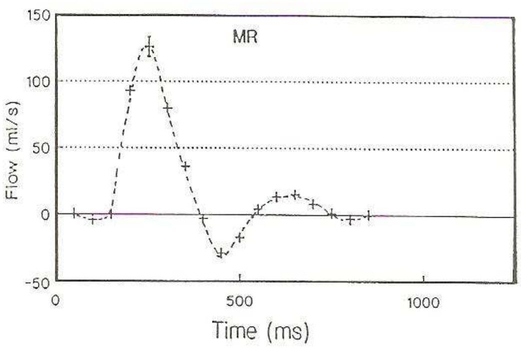

Fig. 5: Time behavior of the aortic blood rate measured with MR of a volunteer (measured by [13]).

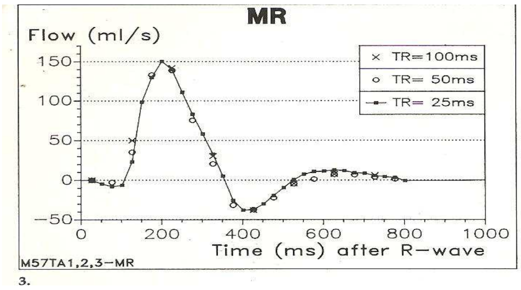

Fig. 6 Temporary MR imaging flow curve of blood flow in Abdominal aorta of healthy volunteer measured with TRs within 1 hour shows the physiologic reproducibility and the importance of a high sampling rate to detect fast changes of the flow [2].

5. Conclusion

We have shown in this research work that CW NMR can be a very useful technique that can be used for quantification of blood flow rate. The success proves that nuclear magnetic resonance being a very significant tool in medical diagnosis can be made an affordable machine for developing countries if the CW NMR method of blood flow estimation is fully exploited a situation which has hitherto not been given much attention because of the mathematical complexities involved. The theoretically simulated time dependent CW NMR signal has agreed with the experimentally measured blood flow signal by other workers. This shows that it is possible, based on our approach, to construct CW NMR blood flow meter that can yield results in agreement with those measured by sophisticated methods employing MRI technology.

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Received: 23 June, 2008
Revised: 29 September, 2009
Accepted: 6 October, 2009