

Theory Overview of the Pentaquark Baryons

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YITP workshop on Multi-quark Hadrons; Four, Five and More?

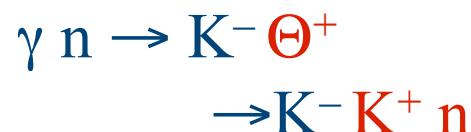
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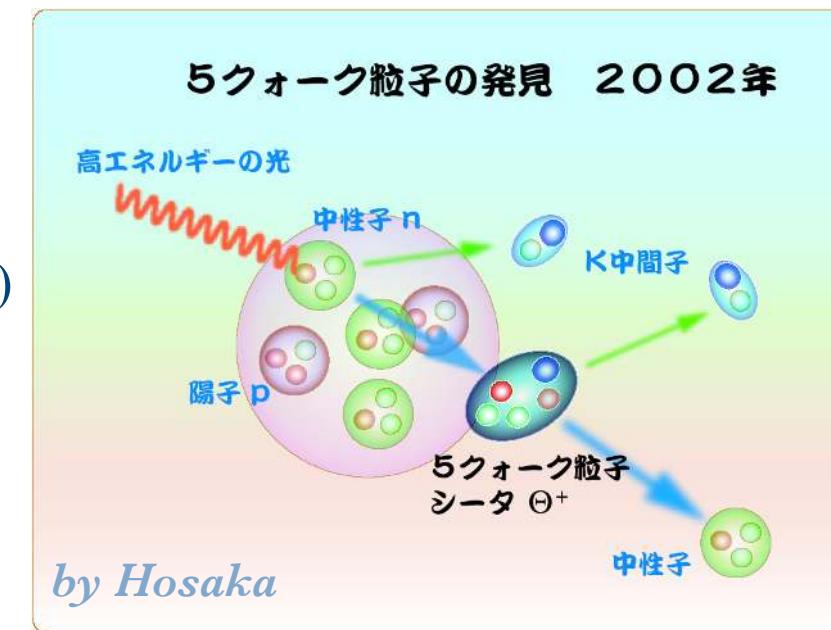
Discovery of Θ^+ at SPring-8

T. Nakano et. al. (LEPS collaboration), Phys. Rev. Lett. 91, 012002 (2003)



SPring-8 tagged γ ($E_\gamma < 2.4$ GeV)

Penta-quark $u^2 d^2 \bar{s}$
 $M = 1540 \pm 10$ MeV
 $\Gamma < 25$ MeV (4.6σ)



Confirmed by ITEP (DIANA), JLAB (CLAS), ELSA (SAPHIR),





2/18/04

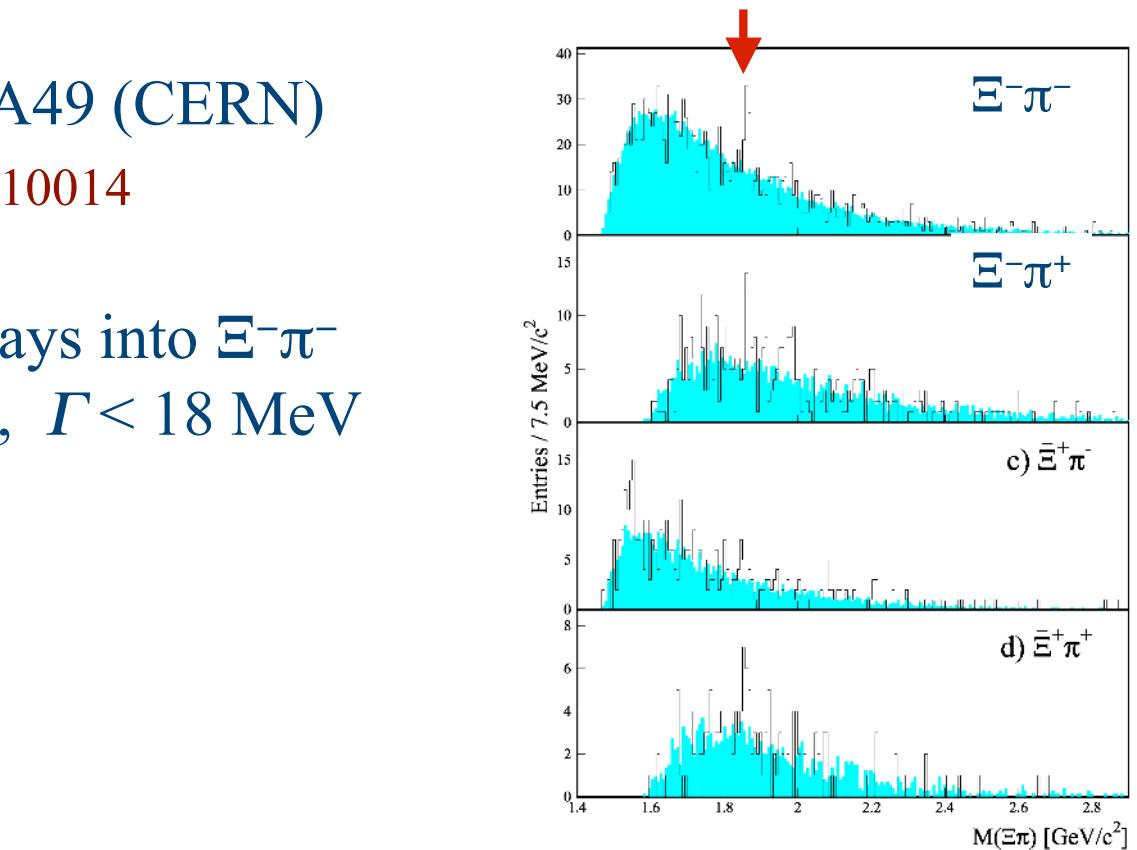
Ξ^- (I=3/2)

Ξ^{--} found by NA49 (CERN)

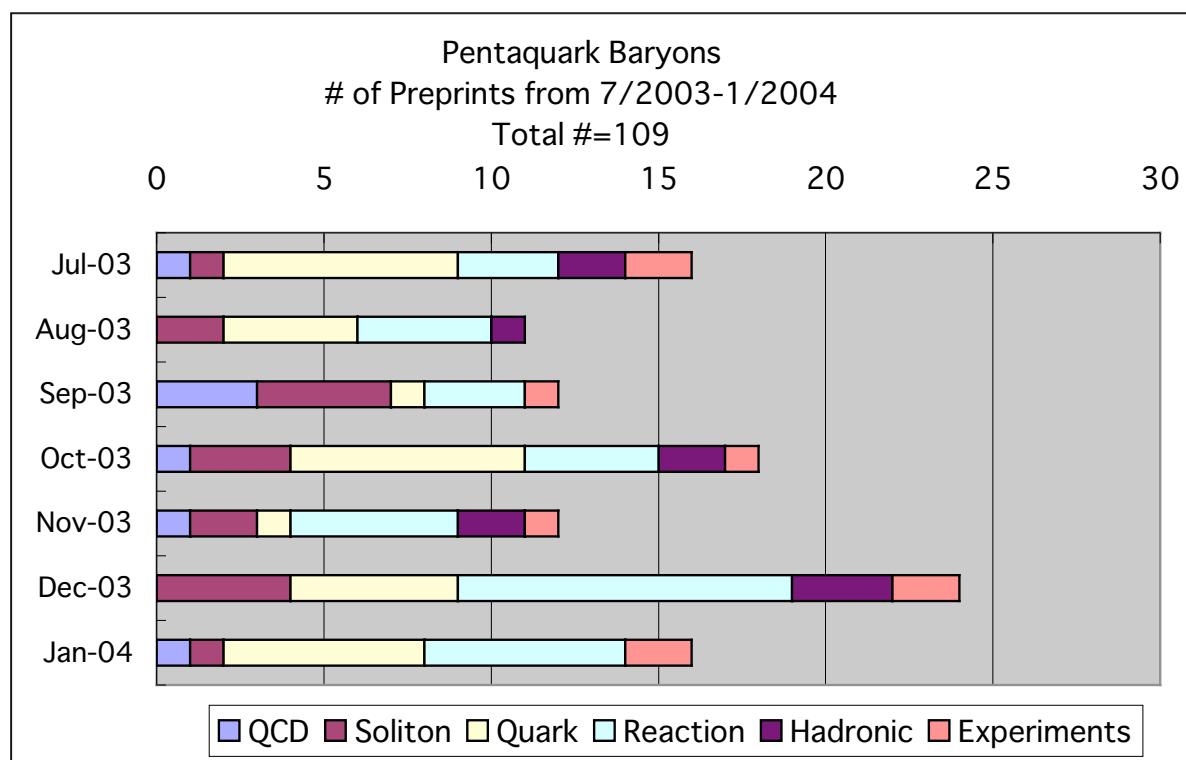
hep-ex/0310014

Ξ^{--} (ssddu) decays into $\Xi^-\pi^-$
 $M = 1.862 \text{ GeV}, \Gamma < 18 \text{ MeV}$

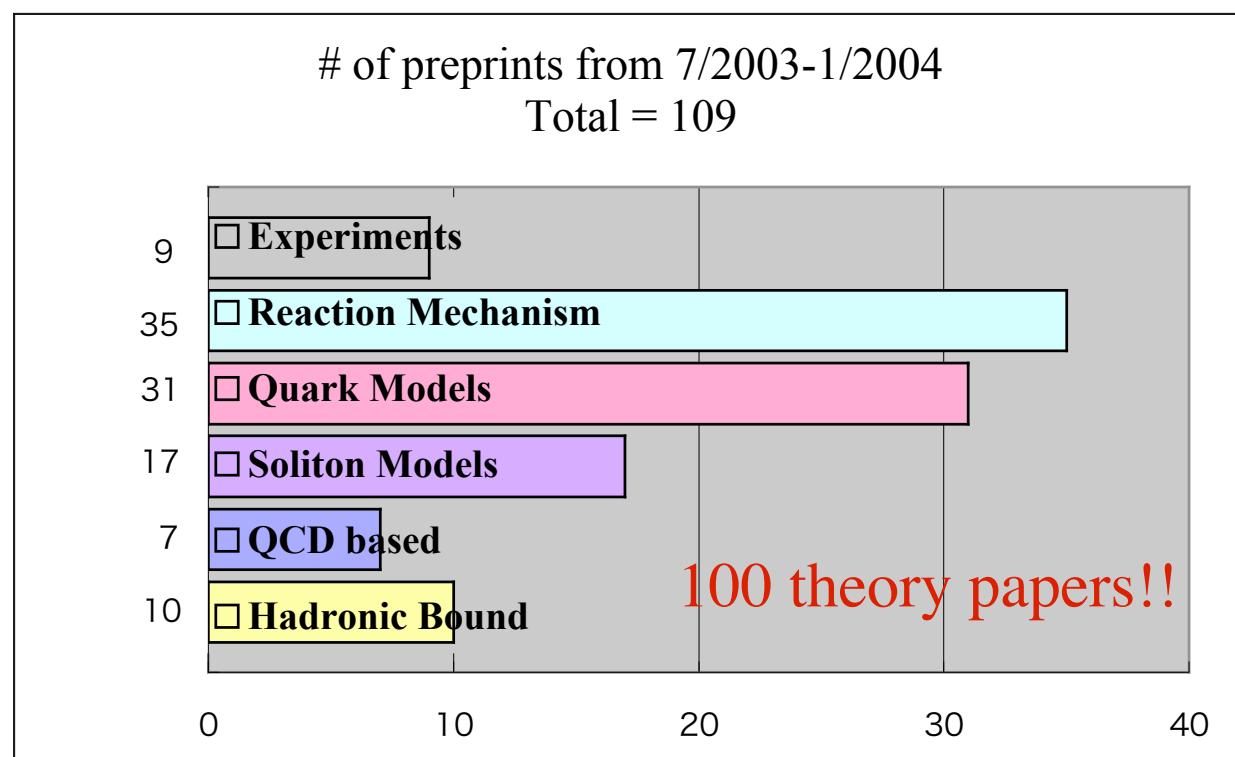
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Papers



Papers



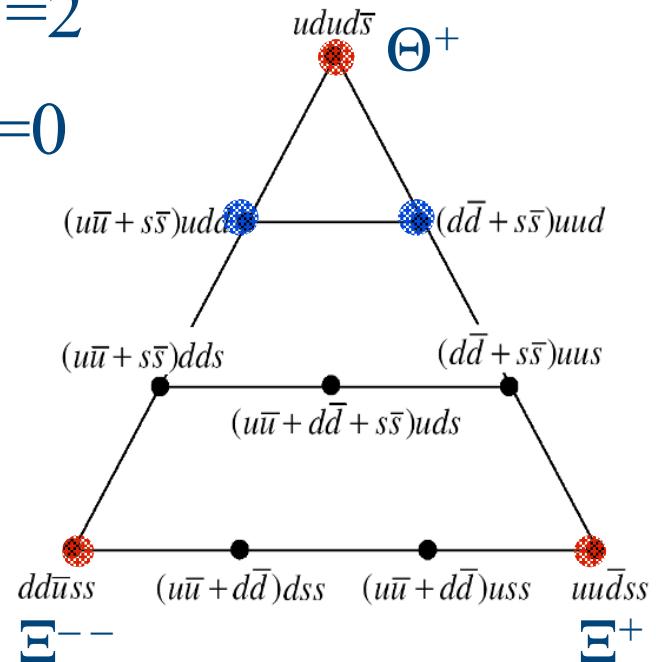
2/18/04

YITP workshop on Multi-quark Hadrons; Four, Five and More?

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Facts of Θ^+

- $M \approx 1540 \text{ MeV}, \Gamma < 9 \text{ MeV}$
- Strangeness = +1, $B=1 \rightarrow Y=2$
- No pK⁺ ($I_3=+1$) state $\rightarrow I=0$
- No lower Str = +1 states
This is the ground state!
- Simplest SU(3) irrep. $\rightarrow 10^*$
unique for ≤ 5 quarks



Problems

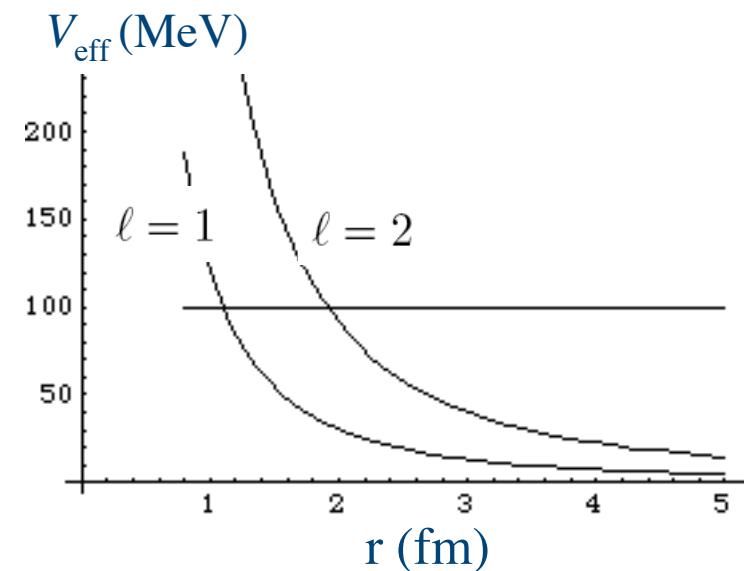
- ◆ Mass too low
 - Most *conventional* calculations of $u^2d^2\bar{s}$ give higher masses
- ◆ Width (to KN) too narrow

100MeV above the threshold
centrifugal barrier

$$V_{\text{eff}}^{KN} \sim \frac{\ell(\ell+1)}{2\mu r^2} \simeq \frac{k^2}{2\mu}$$

$$\mu(KN) \sim 324\text{MeV} \quad k \sim 1.3\text{fm}^{-1}$$

$$kr = 1 \text{ at } r \sim 0.8 \text{ fm}$$



Theories for Θ^+

- ◆ Soliton models (*aka* Skyrmion) *17 papers*
 - Excited states in SU(3) Skyrmion: 10^* irrep.
Diakonov, Petrov, Polyakov, Zeit. Phys. A359 (1997) 305
positive parity: $1/2^+$
 $\Theta^+ = 1530 \text{ MeV}, \Gamma < 15 \text{ MeV} \rightarrow 30 \text{ MeV}$ (Jaffe)
 - K^+ Skyrmion BS approach : compared to rigid rotor approach
Itzhaki, Klebanov, Ouyang, Rastelli, hep-ph/0309305
WZW term is repulsive (attractive) for K^+ (K^-) Callan, Klebanov (1985)
 $1/2^+$ most likely, but not enough to make a bound state
 - In general, many (sharp) resonances predicted
 - Significant $1/N_c$ corrections expected
esp. for excited states as they are subleading in $1/N_c$



Theories for Θ^+

Chiral soliton model

Diakonov, Petrov, Polyakov, Zeit. Phys. A359 (1997) 305

$N^*(1710)$ is assumed to be in the $J^\pi=1/2^+$: 10^*

$$M = (1890 - 180 Y) \text{ MeV} \rightarrow \Theta^+ = 1530 \text{ MeV}$$

$$\Gamma < \cancel{15 \text{ MeV}} \rightarrow 30 \text{ MeV (Jaffe)}$$

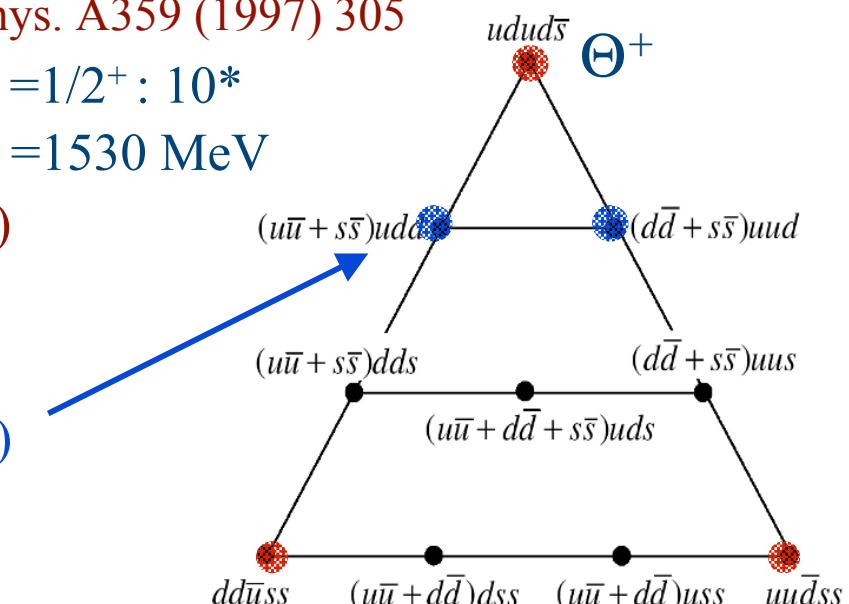
SU(3) symmetry requires

a penta-quark N^* ($I=1/2$, $Y=1$, 10^*)
 $(2/3) \bar{s}s + (1/3) (\bar{u}u + \bar{d}d)$

Other candidates

$(1/2)^- : 1535/1650$ $(1/2)^+ : 1440/1710$

ideal mixing with an octet (Jaffe, Wilczek, PRL 91,55)



Theories for Θ^+

- ◆ Quark Models *31 papers*
 - pentaquark spectroscopy
 - conventional or exotic (ex. diquarks)
- ◆ QCD *7 papers*
 - Lattice QCD
 - QCD sum rule
 - Others (ex. χ PT, $1/N_c$ expansion)
- ◆ Production Mechanisms *35 papers*
 - determining the quantum numbers, ex spin, parity
 - cross sections, polarization
 - new reactions to produce pentaquarks



Theories for Θ^+

- ◆ Hadronic bound/resonances *10 papers*

$\text{KN} (\text{K}^+\text{n}, \text{K}^0\text{p}) I=0, L=0$ interaction is weak and repulsive

$$\text{K}^+\text{n weak repulsive} \quad a_0^{K+n} = -\frac{M_N m_K}{8\pi f_\pi^2(M_N + m_K)} + O(m_K^2)$$

$$\text{K-p strong attractive} \quad a_0^{K-p} = \frac{M_N m_K}{4\pi f_\pi^2(M_N + m_K)} + O(m_K^2)$$

Other possibilities

$\text{NK}\pi$ bound state $E_{\text{th}} = 1.57 \text{ GeV}$

Width must be strongly suppressed for $L=0$ bound states.

Bicudo, Marques, hep-ph/0308073 did not find a bound state.





Quark Models

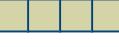
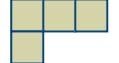
- ◆ Symmetry
- ◆ Dynamics
 - Conventional (*traditional, uncorrelated*)
 - Exotic ideas (*correlated, chiral*)

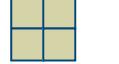


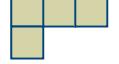
Θ^+ Symmetry

♦ quatre-quark $(u^2d^2)_{I=0, C=3}$

$$I=0 \quad \begin{array}{|c|c|}\hline \textcolor{brown}{\square} & \textcolor{brown}{\square} \\ \hline \end{array} \times C=3 \quad \begin{array}{|c|c|c|}\hline \textcolor{brown}{\square} & \textcolor{brown}{\square} & \textcolor{brown}{\square} \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline \textcolor{brown}{\square} & \textcolor{brown}{\square} & \textcolor{brown}{\square} \\ \hline \end{array} + \begin{array}{|c|c|c|}\hline \textcolor{brown}{\square} & \textcolor{brown}{\square} & \textcolor{brown}{\square} \\ \hline \end{array} \quad (\text{no } \begin{array}{|c|c|}\hline \textcolor{brown}{\square} & \textcolor{brown}{\square} \\ \hline \textcolor{brown}{\square} & \textcolor{blue}{\square} \\ \hline \end{array})$$

if orbital =  $L=0 \leftrightarrow$ spin  $S=1$ only

if orbital =  $L=1 \leftrightarrow$ spin  $S=0$

 $S=1$



Θ^+ Symmetry

- ◆ possible low energy $(u^2d^2)_{I=0, C=3}$ states

		<i>spin-color</i>	<i>spin-flavor</i>
$L=0, S=1, J=1$	1^+		$+ \dots$ 
$L=1, S=0, J=1$	1^-		$+ \dots$ 

- ◆ possible quantum numbers for Θ^+

$$u^2d^2(1^+) + \bar{s}(1/2^-) = 1/2^-, 3/2^- \leftrightarrow \text{KN } S\text{-wave}$$

$$u^2d^2(1^-) + \bar{s}(1/2^-) = 1/2^+, 3/2^+ \leftrightarrow \text{KN } P\text{-wave}$$



Quark Models

- ◆ Orbital excitation energy
How large is $\Delta E(\Delta L=1) = E(L=1) - E(L=0)$?

- ◆ Spin-dependent interaction(s)
hyperfine (HF) interaction
How large is $\Delta E(\Delta S=1) = E(S=1) - E(S=0)$?



Constituent Quark

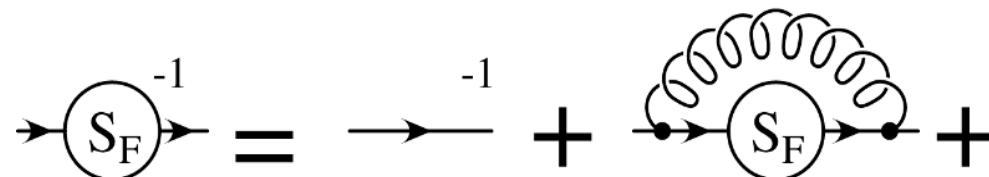
- ◆ Dyson-Schwinger equation

$$S_F^{-1}(p) = S_{F0}^{-1}(p) - \Sigma(p)$$

$$S_{F0}^{-1} = \not{p} - m$$

$S_F^{-1} = A(p)\not{p} - B(p)$ → effective mass generated
dynamical chiral symmetry breaking

dressed quark propagator



gluon



Constituent Quark

- ◆ Conserved currents are not renormalized.
 I, Y, C charges do not change.
- ◆ Constituent quark mass (\sim single particle energy in the bag)
 $m_{u, d} \approx 360 \text{ MeV}$
 $m_s \approx 540 \text{ MeV}$
$$\left. \begin{array}{l} \Sigma m_q = 1080 \text{ MeV for } N/\Delta \\ = 1260 \text{ MeV for } \Lambda/\Sigma/\Sigma^* \\ = 1980 \text{ MeV for } u^2d^2s \end{array} \right\} + \text{Hyperfine interaction (spin dependent)}$$
- ◆ Assume that the residual interactions are weak.

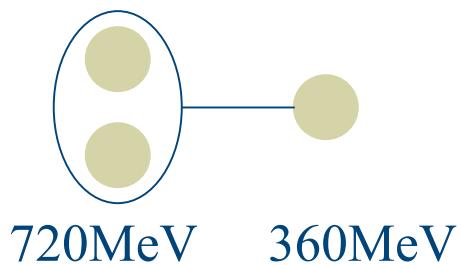


Quark Models

- ◆ Estimate of orbital excitation energy

$N^*(1535) - N(940)$: $\Delta E \sim 600$ MeV (incl. difference in HF int.)

Kinetic energy contribution for $\Delta L=1$



$$\mu = 240 \text{ MeV}$$

$$\left\langle \frac{\ell(\ell+1)}{2\mu r^2} \right\rangle \simeq 450 \text{ MeV} \quad \text{for } \langle r^2 \rangle^{1/2} \sim 0.6 \text{ fm}$$

ex. di-quark models



HF interaction in Baryon

Single particle motion

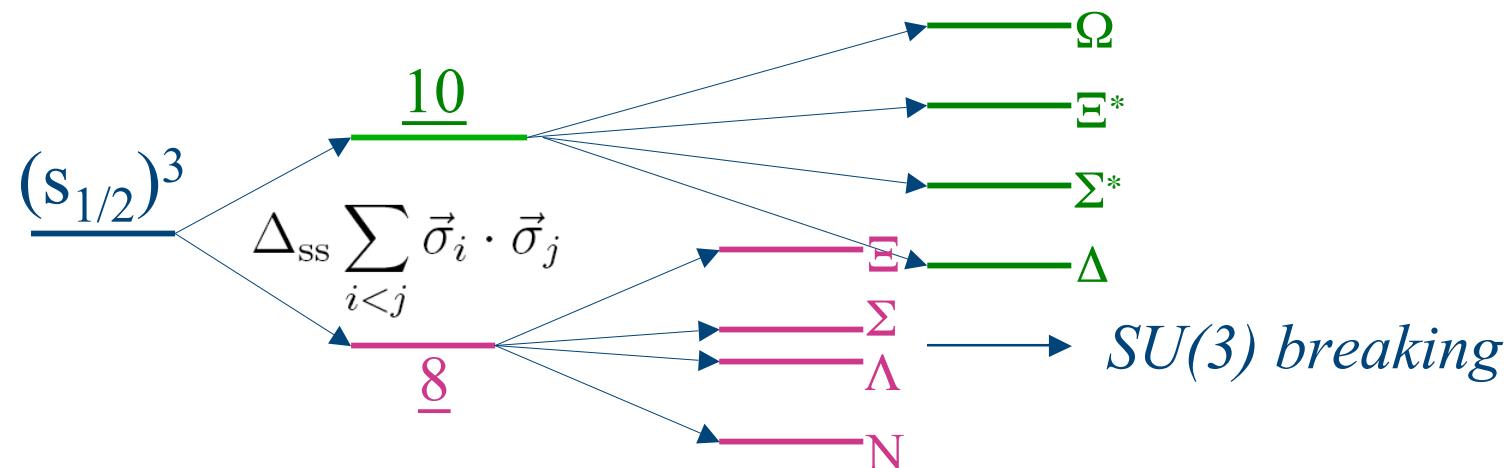
$(s_{1/2})^3$

$J = 1/2$

$J = 3/2$

8

10



HF interaction in Baryon

N- Δ mass splitting (300 MeV) $\leftrightarrow \Delta_{ss} \sim 50$ MeV

Λ - Σ mass splitting (~ 77 MeV) from SU(3) breaking

$$\Sigma_{HF} = \boxed{\Delta_{ss}} \{ \vec{\sigma}_u \cdot \vec{\sigma}_d + \boxed{\xi} \times \vec{\sigma}_s \cdot (\vec{\sigma}_d + \vec{\sigma}_u) \}$$

$\downarrow 50 \text{ MeV}$

$$\Lambda \quad (ud)_{I=0,S=0} \text{ s} \quad 50 \text{ MeV} \times [(-3) + 0 * \xi]$$

$$\Sigma \quad (ud)_{I=1,S=1} \text{ s} \quad 50 \text{ MeV} \times [1 + (-4) * \xi]$$

ξ - factor: s-u, s-d HF interaction is weaker than u-d.

$$\text{for } \xi = 3/5 \rightarrow \Sigma - \Lambda = (8/15) \times 150 \text{ MeV} = 80 \text{ MeV}$$



Origin of HF Interaction

(1) One gluon exchange
or color-magnetic (CM) interaction

Breit-Fermi, DeRujula-Georgi-Glashow (1975)

$$\alpha_s \lambda_i \frac{\vec{\sigma}_i \cdot \vec{q}}{m_i} \frac{1}{q^2} \lambda_j \frac{\vec{\sigma}_j \cdot \vec{q}}{m_j} \simeq \frac{\alpha_s}{m_i m_j} (\lambda_i \cdot \lambda_j) (\vec{\sigma}_i \cdot \vec{\sigma}_j) \delta(\vec{r}_{ij})$$

SU(3) breaking $m_u/m_s \sim 3/5$

$$\Sigma_{\text{CM}} = -\Delta_{\text{CM}} \sum_{i < j} \xi_{ij} (\lambda_i^c \cdot \lambda_j^c) (\vec{\sigma}_i \cdot \vec{\sigma}_j)$$

N- Δ mass splitting (300 MeV) $\leftrightarrow \Delta_{\text{CM}} \sim 18.75$ MeV

for Θ^+ Carlson et al., hep-ph/0307396



Origin of HF Interaction

(2) Pseudoscalar meson exchange
or flavor dependent (FD) interaction

Glozman, Riska, Phys. Rep. 208 (1996)

$$\Sigma_{\text{FD}} = -\Delta_{\text{FD}} \sum_{i < j} \xi_{ij} (\lambda_i^f \cdot \lambda_j^f) (\vec{\sigma}_i \cdot \vec{\sigma}_j)$$

N- Δ mass splitting (300 MeV) $\leftrightarrow \Delta_{\text{FD}} \sim 30$ MeV

for Θ^+ **Stancu, Riska, hep-ph/0307010, 0402044**

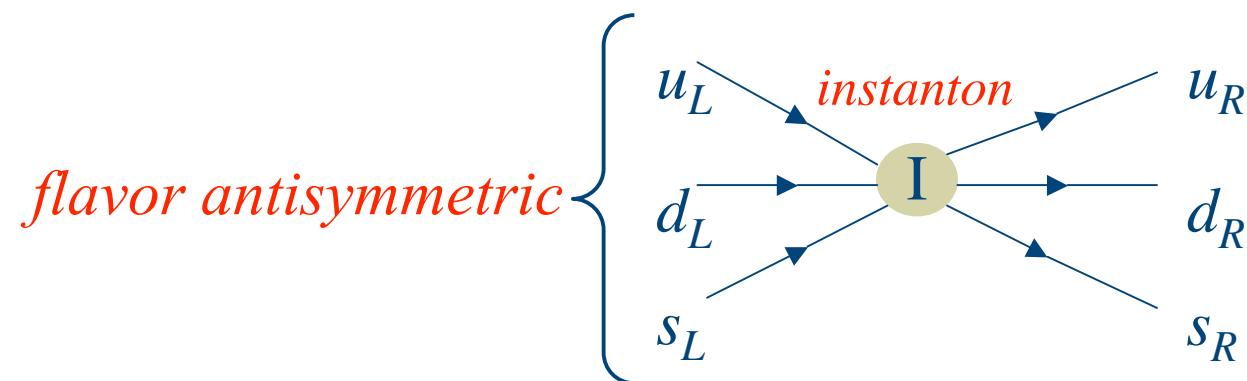
CM and FD **Jennings, Maltman, hep-ph/0308286**
Bijker et al., hep-ph/0310281



Origin of HF Interaction

(3) Instanton-induced-interaction (III)

aka Kobayashi-Maskawa-'t Hooft (KMT)
instanton-light-quark couplings



Origin of HF Interaction

(3) Instanton-induced-interaction (III)

flavor antisymmetric u-d-s 3-body repulsion

flavor antisymmetric 2-body attraction

$$V_{\text{III}}^{(3)} = V^{(3)} \sum_{(ijk)} \mathcal{A}^f \left[1 - \frac{1}{7} (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_k + \boldsymbol{\sigma}_k \cdot \boldsymbol{\sigma}_i) \right] \delta(\mathbf{r}_{ij}) \delta(\mathbf{r}_{jk})$$

$$V_{\text{III}}^{(2)} = V^{(2)} \sum_{i < j} \mathcal{A}^f \left[1 - \frac{1}{5} (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \right] \delta(\mathbf{r}_{ij})$$

$$= V_{ij}^{(2)} (2/5) (1 - \boxed{\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j}) \delta(r_{ij}) \quad \text{in the baryon}$$

proportional to $1/m_i m_j$

**Shuryak-Rosner (1989)
Takeuchi-Oka (1989)**



Origin of HF Interaction

(3) Instanton-induced-interaction (III) (2-body part)

$$\Sigma_{\text{III}} = \Delta_{\text{III}} \sum_{i < j} \mathcal{A}_{ij}^f \xi_{ij} [1 - \frac{1}{5}(\vec{\sigma}_i \cdot \vec{\sigma}_j)]$$

N- Δ mass splitting (300 MeV) $\leftrightarrow \Delta_{\text{III}} \sim 125$ MeV

It is quite likely that the realistic HF interaction is a
combination of Σ_{CM} , Σ_{FD} , and Σ_{III} .



Quark Models for Θ^+

- ◆ Crude estimates of the baryon masses
 - CM interactions

$$\Sigma_{CM} = -\Delta_{CM} \sum_{i < j} (\lambda_i \cdot \lambda_j) (\sigma_i \cdot \sigma_j) \quad \Delta_{CM} = 150/8 \text{ MeV}$$

$$M_N = 3 m_q + \langle \Sigma_{CM} \rangle_N = 360 \times 3 - 150 \approx 930 \text{ MeV}$$

$$M_\Delta = 3 m_q + \langle \Sigma_{CM} \rangle_\Delta = 360 \times 3 + 150 \approx 1230 \text{ MeV}$$

$$M_\Lambda = 3 m_q + \Delta m + \langle \Sigma_{CM} \rangle_{\Lambda\Sigma} = 360 \times 3 + 180 - 150 \approx 1110 \text{ MeV}$$

$$\Delta m = m_s - m_q \sim 540 - 360 = 180 \text{ MeV}$$



Quark Models for Θ^+

- ◆ H dibaryon Jaffe (1977)

Strangeness = -2 , $B = 2$

$\Lambda\Lambda$ threshold = 2231 MeV

$$M_H = 6 m_q + 2 \Delta m + \langle \Sigma_{CM} \rangle_H = 360 \times 6 + 2 \times 180 - 450 \approx 2070 \text{ MeV}$$

The 20-year searches for the bound state were not successful.

The 3-body (u-d-s) III gives ~ 160 MeV repulsion.

Takeuchi, Oka (1991)



Quark Models for Θ^+

- ◆ Θ^+ in Σ_{CM} ($1/2^-$: $q^4 (L=0, S=1) + \bar{s}$: $J=1/2$)
$$M_\Theta = 5 m_q + \Delta m + \langle \Sigma_{\text{CM}} \rangle_{5q} \quad (1/2^- : q^4 L=0, S=1, J=1/2)$$
$$= 360 \times 5 + 180 - 250 \approx 1730 \text{ MeV} \quad \text{exp. } 1540 \text{ MeV}$$
- ◆ ~ 200 MeV extra attraction is necessary for $\Theta^+(1/2^-)$ in Σ_{CM}
- ◆ Other interactions?
 - FD interaction slightly less repulsive.
 - III (2-body) is strongly attractive
- $$M_\Theta = 5 m_q + \Delta m + \langle \Sigma_{\text{III}} \rangle_{5q} = 360 \times 5 + 180 - 660 \approx 1320 \text{ MeV}$$
- III (3-body) $\sim +50$ MeV repulsive

Shinozaki, Takeuchi, Oka (preliminary)



Quark Models for Θ^+

Evaluation of HF interactions for $\Theta^+(1/2^-)$

<i>SU(3) limit</i>	$\langle \Sigma_{\text{CM}} \rangle$	$\langle \Sigma_{\text{FD}} \rangle$	$\langle \Sigma_{\text{III}} \rangle$ (2-body)
	$\Delta_{\text{CM}} \sim 18.7 \text{ MeV}$	$\Delta_{\text{FD}} \sim 30 \text{ MeV}$	$\Delta_{\text{III}} \sim 125 \text{ MeV}$
N	$-8\Delta_{\text{CM}}$	$-14\Delta_{\text{FD}}$	$-\frac{12}{5}\Delta_{\text{III}}$
Δ	$+8\Delta_{\text{CM}}$	$-4\Delta_{\text{FD}}$	$0\Delta_{\text{III}}$
K	$-16\Delta_{\text{CM}}$	$2\Delta_{\text{FD}}$	$-\frac{8}{5}\Delta_{\text{III}}$
$\Theta^+(1/2^-)$	$-\frac{56}{3}\Delta_{\text{CM}}$	$-\frac{28}{3}\Delta_{\text{FD}}$	$-\frac{36}{5}\Delta_{\text{III}}$
$\Theta^+ - (N + K)$	$+100 \text{ MeV}$	$+80 \text{ MeV}$	-400 MeV



Quark Models for Θ^+

- ◆ $\Theta^+ (1/2^+) \quad (1/2^+ : q^4 (L=1, S=0) + \bar{s} : J=1/2)$

$$M_\Theta = 5 m_q + \Delta m + \langle \Sigma_{CM} \rangle_{5q} + \Delta E(\Delta L=1)$$
$$= 360 \times 5 + 180 - \underline{620 + 450} \approx 1810 \text{ MeV}$$

$$M_\Theta = 5 m_q + \Delta m + \langle \Sigma_{FD} \rangle_{5q} + \Delta E(\Delta L=1)$$
$$= 360 \times 5 + 180 - \underline{660 + 450} \approx 1770 \text{ MeV}$$

- ◆ It is *unlikely* to reverse the $1/2^- \leftrightarrow 1/2^+$ ordering in conventional constituent quark model.



Quark Models for Θ^+

Evaluation of HF interactions for $\Theta^+(1/2^+)$

	$\langle \Sigma_{\text{CM}} \rangle$	$\langle \Sigma_{\text{FD}} \rangle$
<i>SU(3) limit</i>	$\Delta_{\text{CM}} \sim 18.7 \text{ MeV}$	$\Delta_{\text{FD}} \sim 30 \text{ MeV}$
$\Theta^+(1/2^+)$	$(-28 \sim -33) \Delta_{\text{CM}}$	$-22 \Delta_{\text{FD}}$
$\Theta^+ - (N + K)$	$(-75 \sim -170) \text{ MeV}$	-300 MeV
$\Theta^+(1/2^-) - \Theta^+(1/2^+)$	$175 \sim 270 \text{ MeV}$	380 MeV

depends on wave function/ range of the interactions



Quark Models for Θ^+

- ◆ Those who predict $1/2^+$ in conventional quark model

Stancu hep-ph/0402044

FD with subtract 510 MeV to fit $\Theta^+(1540)$

Jennings, Maltman hep-ph/0308286

CM and FD compared

assume $\Delta E (\Delta L=1) = 210 - 250$ MeV



Missing attraction

Exotic possibilities

◆ Diquarks

Jaffe, Wilczek, PRL 91 (2003) 232003

Karliner, Lipkin, hep-ph/0307232

ud diquarks		color	spin	
$I = 1$	uu, {ud}, dd	3*	1^+	
		6	0^+	
$I = 0$	$[ud]_0$	3*	0^+	strongly attractive ~ 420 MeV
		6	1^+	
$I = 0$	$[ud]_T$	3*	1^-	attractive in the instanton vacuum ~ 570 MeV
	tensor diquark			

Shuryak, Zahed hep-ph/0310270



Missing attraction

Jaffe, Wilczek

Phys. Rev. Lett. 91 (2003) 232003

2 diquarks + \bar{s} model

$[ud]_0 \times [ud]_0$: color antisymmetric

$L = 1 \Rightarrow 1/2^+, 3/2^+ \quad M = 840 + 560 + \Delta E(\Delta L) \sim 1400 + \Delta E(\Delta L)$

However, note $\Delta E(\Delta L=1) \sim 400 - 450$ MeV

Karliner, Lipkin

hep-ph/0307232

diquark + triquark $L = 1$

$\Delta E(\Delta L=1) \sim 209$ MeV ?

from $\Delta [D_s(0^+, 2317) - D_s(0^-, 1969)] - HF$

Shuryak, Zahed

hep-ph/0310270

$[ud]_0 (0^+, 420$ MeV) + $[ud]_T (1^-, 570$ MeV) + $\bar{s} (560$ MeV)

$L = 0 \Rightarrow 1/2^+, 3/2^+$



Diquark spectrum

- ◆ Diquark spectrum in lattice QCD (Landau gauge)

Wetzorke, Karsch, hep-lat/0008008

Diquark state	($\bar{3}0\bar{3}$)	($61\bar{3}$)	($\bar{3}16$)	(606)
$ma_{(FSC)}$: MEM result	0.60(2)	0.70(3)	0.74(9)	—
$ma_{(FSC)}$: 2-exp. fit	0.62(2)	0.73(4)	0.77(17)	0.50(15)

maximal entropy method

- ◆ Bethe-Salpeter calculation in the rainbow-ladder QCD

Maris, nucl-th/0204020

diquark mass $m(0^+) \sim 800$ MeV



Diquark spectrum

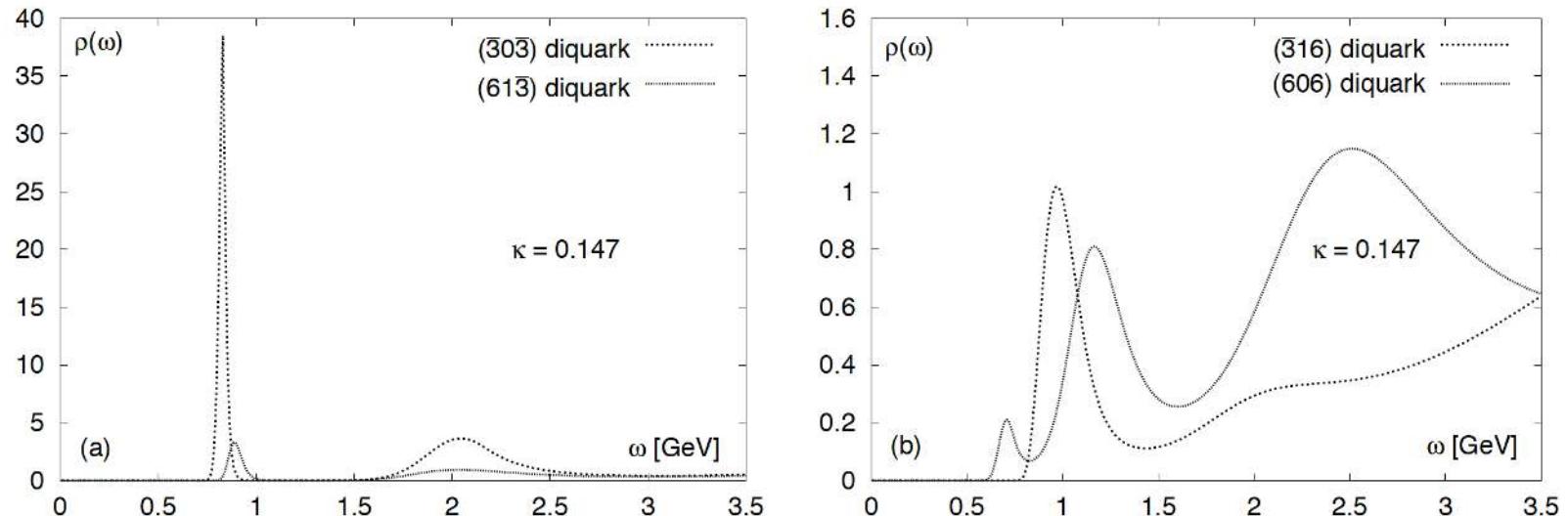


Figure 2: Spectral functions for color anti-triplet (a) and sextet diquark states (b).



Missing attraction

- ◆ Diquark in the nucleon (and the other gs baryons)
Diquark enhancement breaks SU(6) symmetry.

ex.

$$p = [ud]_0 u \quad n = [ud]_0 d$$

$$\mu_p / \mu_n = e_p / e_n = -2$$

if the quarks do not have anomalous moments.



Missing attraction

- ◆ Chiral bag model

Hosaka, Phys. Lett. B571 (2003) 55

$uudd$ ($I=0, J^\pi = 1^-$) + \bar{s} ($J^\pi = 1/2^-$)
 $\Rightarrow 1/2^+, 3/2^+$

$$K^\pi = 1^-$$

- ◆ Bag excitation energy

$E(p_{3/2}) - E(s_{1/2}) = 1.16/R \sim 230$ MeV
seems too small?

Can the bag model describe excited states?

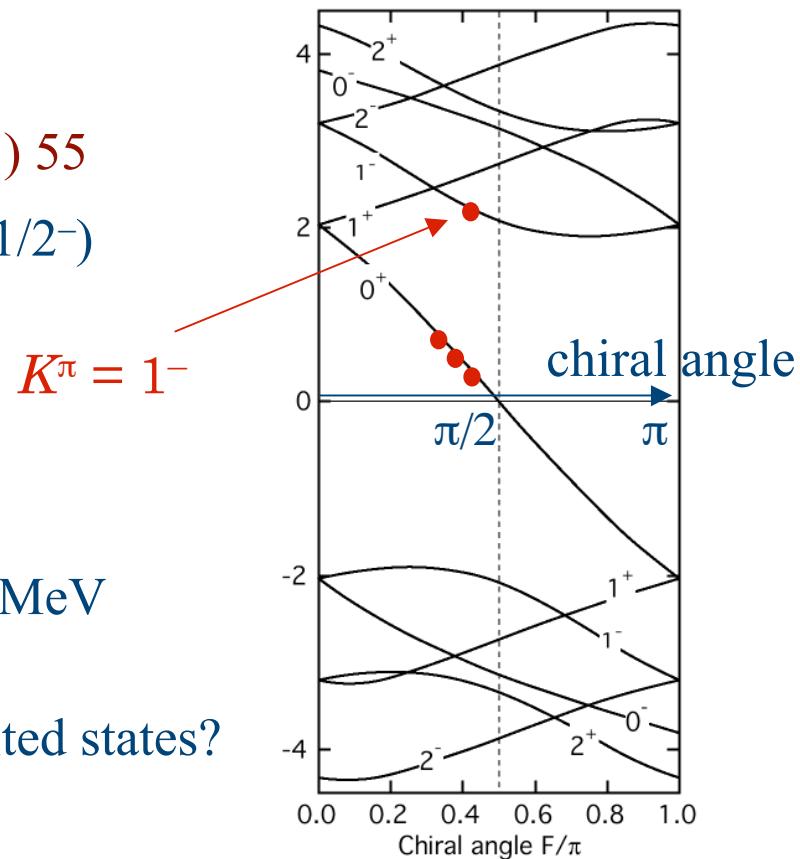


Figure 2: Eigenenergies of the hedgehog quark in the chiral bag model as functions of the chiral angle F .



QCD prediction

- ◆ QCD sum rule

Zhu, Phys. Rev. Lett. 91 (2003) 232002

Matheus et al., Phys. Lett. B578 (2004) 323

No parity projection was made.

Only the chiral even terms are considered.

Sugiyama, Doi, Oka, Phys. Lett. B581 (2004) 167
determine mass and parity

- ◆ Lattice QCD

Csikor, Fodor, Katz, Kovacs, JHEP 0311 (2003) 070

Sasaki, hep-lat/0310014 *Sasaki's talk*

Kentucky group, Lee et al., (Lattice03, HYP2003) *Mathur's talk*



QCD Sum Rule for Θ^+

J. Sugiyama, T. Doi, M. O. PLB581 (2004) 167

- ◆ Assume $J=1/2, I=0$
- ◆ 5-quark local operator for $J=1/2$ baryon

$$J_{\Theta^+}(x) = \epsilon^{abc} \epsilon^{def} \epsilon^{cfg} \{u_a^T(x) C d_b(x)\} \{u_d^T(x) C \gamma_5 d_e(x)\} C \bar{s}_g^T(x)$$

	0^- diquark	0^+ diquark	\bar{s}
color	3_c^*	✗ 3_c^*	✗ 3_c^*
flavor	3_f^*	✗ 3_f^*	✗ 3_f^*



QCD Sum Rule for Θ^+

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- ◆ Parity projection
- ◆ Retarded Green's function at rest, $\mathbf{q} = 0$.

$$\Pi(q^0) = \int d^4 q e^{iq \cdot x} i \langle 0 | \theta(x^0) J(x) \bar{J}(0) | 0 \rangle |_{\vec{q}=0}$$

$$-\frac{1}{\pi} \text{Im} \Pi(q^0) = \boxed{A(q^0)} \gamma^0 + \boxed{B(q^0)}$$

$$A(q^0) = \frac{1}{2} (\rho^+(q^0) + \rho^-(q^0))$$

$$B(q^0) = \frac{1}{2} (\rho^+(q^0) - \rho^-(q^0))$$



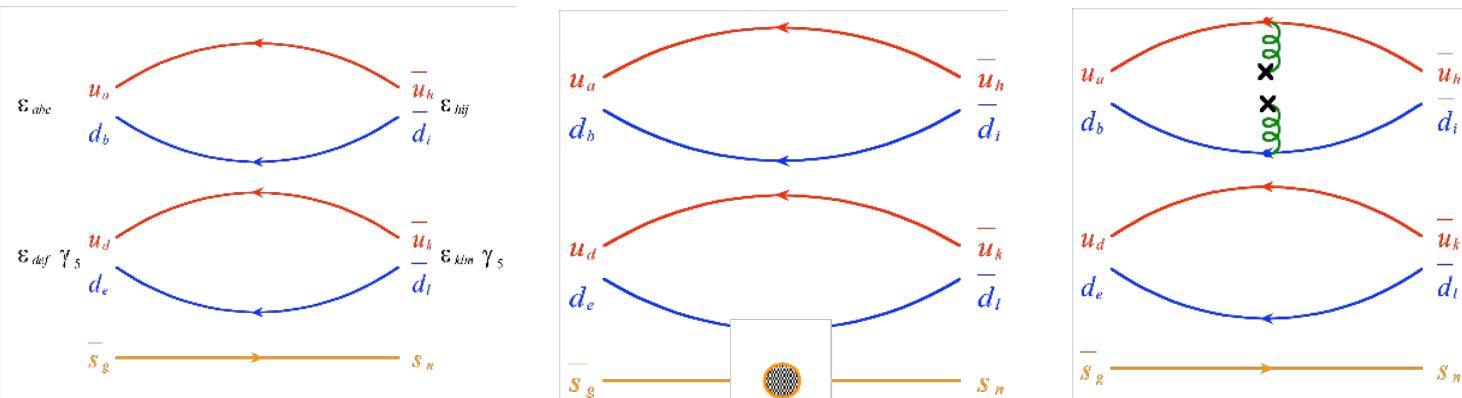
QCD Sum Rule for Θ^+

$$-\frac{1}{\pi} \text{Im}\Pi(q^0) = A(q^0)\gamma^0 + B(q^0)$$

$$A_{\text{OPE}}(q_0) = \frac{q_0^{11}}{5! 5! 2^{10} 7 \pi^8} + \frac{q_0^7}{3! 5! 2^8 \pi^6} m_s \langle \bar{s}s \rangle + \frac{q_0^7}{5! 3! 2^{10} \pi^6} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle - \frac{q_0^5}{4! 3! 2^9 \pi^6} m_s \langle \bar{s}g_s \sigma \cdot G s \rangle$$

chiral odd

$$B_{\text{OPE}}(q_0) = \frac{q_0^{10} m_s}{5! 5! 2^{10} \pi^8} - \frac{q_0^8}{4! 5! 2^7 \pi^6} \langle \bar{s}s \rangle + \frac{q_0^6}{3! 4! 2^9 \pi^6} \langle \bar{s}g_s \sigma \cdot G s \rangle$$



QCD Sum Rule for Θ^+

- ◆ Spectral function
- ◆ assume the continuum part is equal to the OPE

$$\rho_{\text{ph}}^\pm(q_0) = |\lambda_\pm^2| \delta(q_0 - m_\pm) + \theta(q_0 - \sqrt{s_{\text{th}}}) \rho_{\text{OPE}}^\pm(q_0)$$

- ◆ suppress $q_0 > M$ contribution by a weight function

$$W(q_0) = \exp\left(-\frac{q_0^2}{M^2}\right)$$

$$\int \rho_{\text{ph}}^\pm(q_0) W(q_0) dq_0 = \int \rho_{\text{OPE}}^\pm(q_0) W(q_0) dq_0$$



QCD Sum Rule for Θ^+

- ◆ Sum rule *The chiral odd terms distinguish parity.*

$$|\lambda_{\pm}|^2 e^{-\frac{m_{\pm}^2}{M^2}} = \frac{1}{3!4!2^7\pi^6} \left[\frac{1}{5600\pi^2} I_{11}(M, s_{\text{th}}) \boxplus \frac{1}{800\pi^2} I_{10}(M, s_{\text{th}}) m_s \right. \\ \boxplus \frac{1}{20} I_8(M, s_{\text{th}}) \langle \bar{s}s \rangle + \frac{1}{10} I_7(M, s_{\text{th}}) m_s \langle \bar{s}s \rangle \\ + \frac{1}{40} I_7(M, s_{\text{th}}) \langle \frac{\alpha_s}{\pi} G^2 \rangle \boxplus \frac{1}{4} I_6(M, s_{\text{th}}) \langle \bar{s}g_s \sigma \cdot G s \rangle \\ \left. - \frac{1}{4} I_5(M, s_{\text{th}}) m_s \langle \bar{s}g_s \sigma \cdot G s \rangle \right],$$

$$I_n(M, s_{\text{th}}) \equiv \int_0^{\sqrt{s_{\text{th}}}} dq_0 q_0^n e^{-\frac{q_0^2}{M^2}}$$

- ◆ Differentiate wrt $1/M^2$



QCD Sum Rule for Θ^+

- ◆ QCD parameters: condensates, mass, ...
determined so as to reproduce the masses
of the octet baryons

$$\begin{aligned}\langle \bar{q}q \rangle &= (-0.23[\text{GeV}])^3 \\ \langle \bar{s}s \rangle &= 0.8 \times \langle \bar{q}q \rangle \\ \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle &= (0.33[\text{GeV}])^4 \\ \langle \bar{s}g\sigma G s \rangle &= m_0^2 \langle \bar{s}s \rangle \\ m_0^2 &= 0.8[\text{GeV}^2] \\ m_s &= 0.12[\text{GeV}]\end{aligned}$$

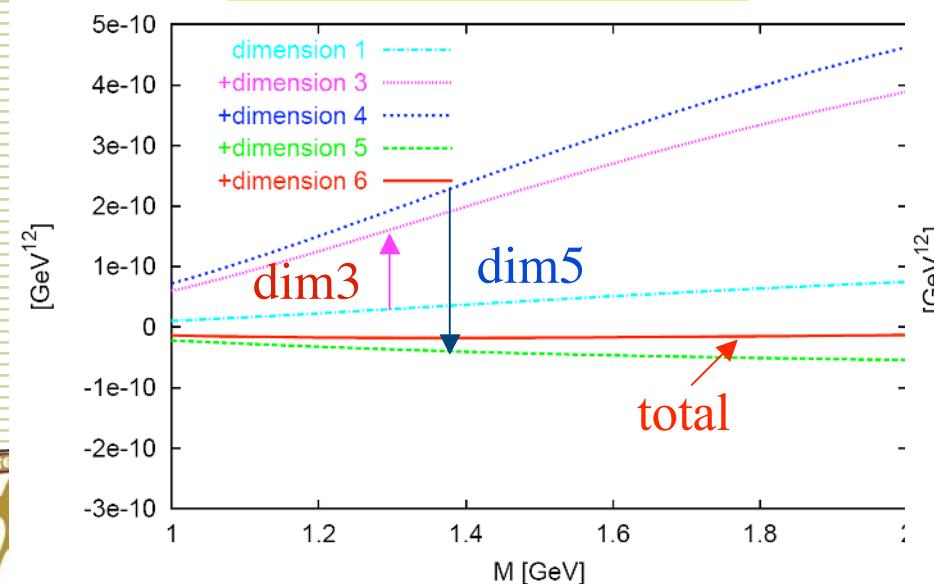


QCD Sum Rule for Θ^+

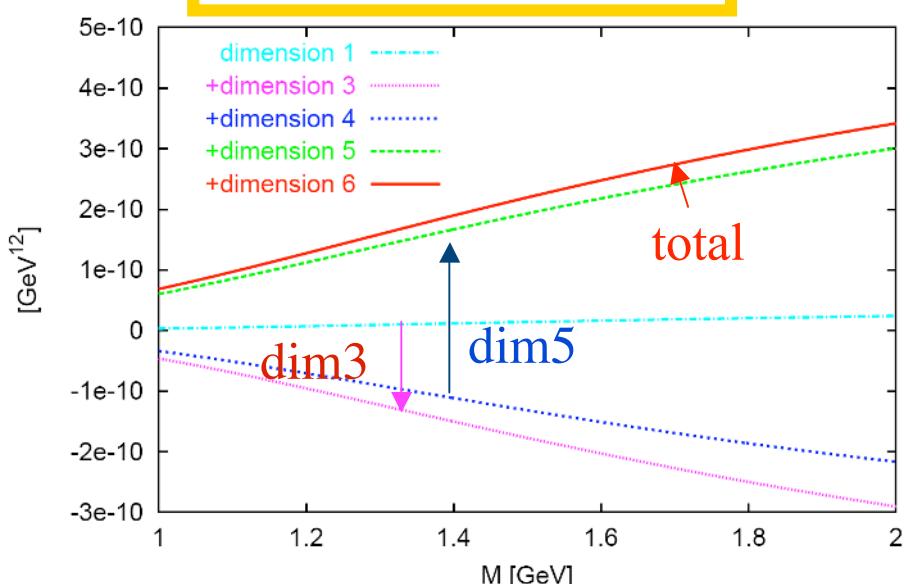
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- ◆ positivity of $|\lambda_{\pm}|^2 e^{-\frac{m_{\pm}^2}{M^2}}$

positive parity

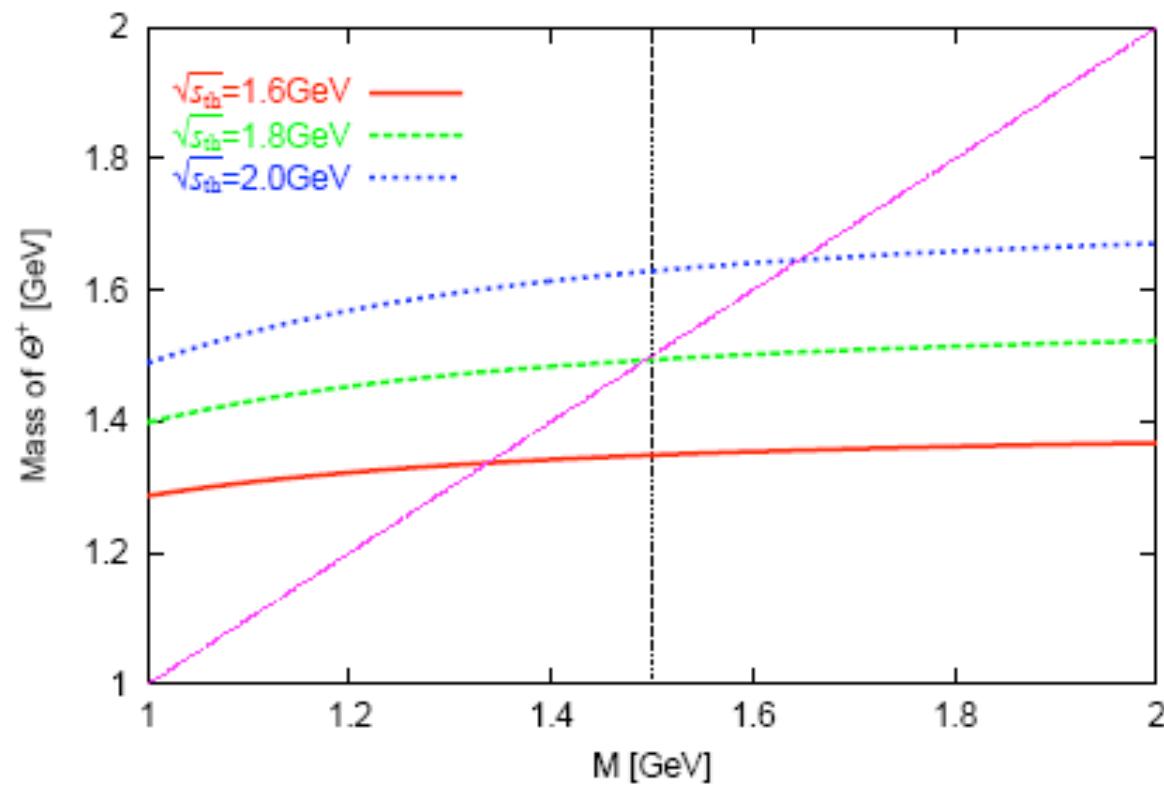


negative parity



QCD Sum Rule for Θ^+

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QCD Sum Rule for Θ^+

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◆ Conclusion

- Parity splitting caused by chiral symmetry breaking by m_s and $\langle\bar{q}q\rangle$ and $\langle\bar{q}\sigma Gq\rangle$ condensates
- Parity determined by the positivity of the correlation function $\Rightarrow 1/2^-$
- Mass $1/2^- \Theta^+ \text{ baryon} \approx 1.3 - 1.7 \text{ GeV}$



Lattice QCD for Θ^+

- ◆ Lattice QCD calculations

- F. Csikor, et al., JHEP 0311 (2003) 070
- S. Sasaki, hep-lat/0310014
- Kentucky group

Each uses a **different** operator for Θ^+ .

All agree that the ground state has

$J=1/2, I=0$, **negative parity**

and the positive parity state is far high above.



Conclusion

- ◆ QCD predicts a (maybe) $1/2^-, I=0, \Theta^+$.
It may not be easy to distinguish Θ^+ from the KN threshold.
- ◆ Many models suggest positive parity baryons.
It seems unlikely to have $1/2^-$ higher than $1/2^+$ in the quark model.
- ◆ Mass prediction
The masses are overestimated by ≥ 100 MeV- 200 MeV.
The discrepancy is larger for $1/2^+$.
Ideas of diquark correlations need careful confirmation.
- ◆ Width
Arndt et al. nucl-th/0308012
 $\Gamma < 1$ MeV in order to be consistent with KN PSA.



Width

- ◆ How natural is the narrow width?

$$\mathcal{L}_{\text{int}}^{(-)} = g^- \bar{\psi}_N \psi_\Theta \phi_K \quad \Gamma(1/2^-) = \frac{(g^-)^2}{4\pi} \frac{M_N + E_N}{M_\Theta} p$$
$$\mathcal{L}_{\text{int}}^{(+)} = g^+ \bar{\psi}_N \gamma^5 \psi_\Theta \phi_K \quad \Gamma(1/2^+) = \frac{(g^+)^2}{4\pi} \frac{M_N - E_N}{M_\Theta} p$$
$$p = 269 \text{ MeV}/c$$
$$\frac{\Gamma(1/2^-)}{\Gamma(1/2^+)} = 51 \left(\frac{g^-}{g^+} \right)^2$$

phase space difference for $L=0/1$

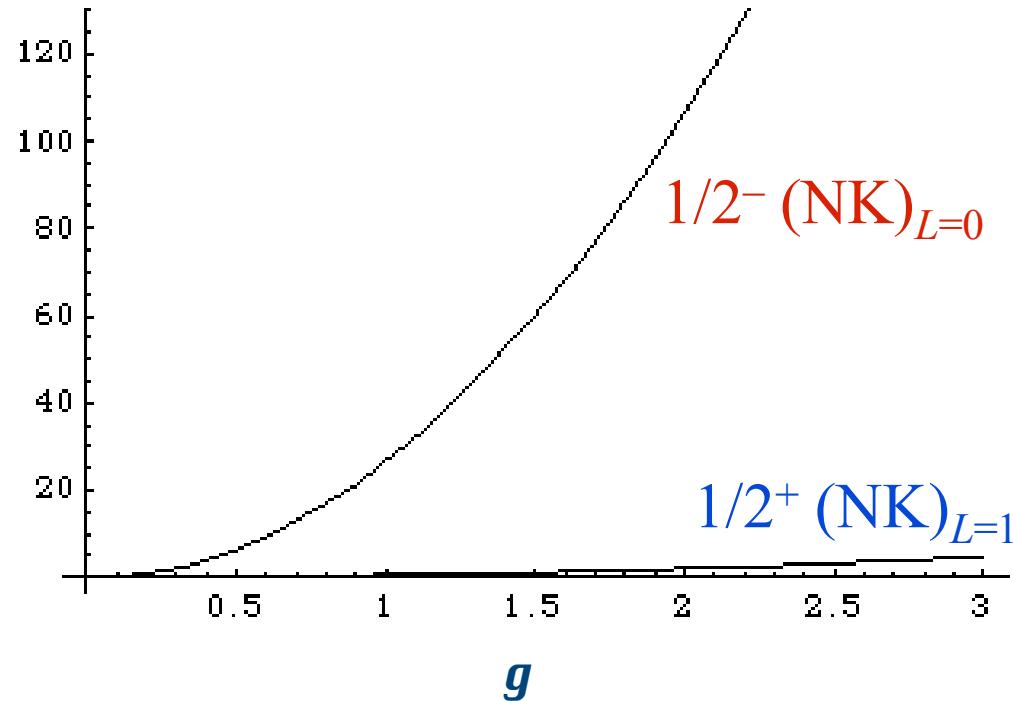




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Width

Γ (MeV)



KN phase shifts

K^+ -nucleon scattering and exotic $S=+1$ Baryons

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$\Gamma \leq 1 \text{ MeV} \Rightarrow g^+ = 1.38 \text{ for } 1/2^+$

$g^- = 0.19 \text{ for } 1/2^-$

A mechanism of strong suppression of the decay
is necessary.

S wave resonance might be very exotic.



Conclusion

- ◆ We need more experimental data
 - spin and parity
 - width
 - 10* members in order to confirm "pentaquark" structure.
 - excited states ex. $J=3/2$ states, $I=1$ states?
- ◆ and more theoretical ideas
 - How to suppress the width?
 - Is this related to other narrow hadron states? ex. D_s^* , X
Does the heavy quark play a key role, or a spectator?
 - more exotic states? ex. molecular states

