JHEP HYPER VERSION

Received: *December 16, 1998,* Accepted: *January 11, 1999*

Theoretical update of pseudoscalar meson distribution amplitudes of higher twist: the nonsinglet case

Patricia Ball

Heisenberg fellow
CERN–TH, CH–1211 Geneva 23, Switzerland
E-mail: Patricia.Ball@cern.ch

Abstract: We discuss the two and three particle light-cone distribution amplitudes (DAs) of pseudoscalar nonsinglet mesons of twist 3 and 4. Using nonlocal operator identities and conformal expansion, we derive closed expressions for several DAs. We also include meson-mass corrections which prove to be dominant in the twist 4 DAs of K and η mesons. Explicit parametrizations for the DAs of π, K and η mesons are given, with the numerical input parameters determined from QCD sum rules.

Keywords: Nonperturbative effects, QCD.

JHEP01(1999)010

Contents

1. Introduction **1**

2. Definition of distribution amplitudes **2**

3. Conformal partial wave expansion and equations of motion **7**

4. Meson-mass corrections **9**

5. Twist 3 distribution amplitudes **11**

6. Twist 4 distribution amplitudes **13**

7. Summary and conclusions **17**

1. Introduction

Meson distribution amplitudes (DAs) describe the momentum fraction distributions of partons in a meson, in a particular Fock state, with a fixed number of constituents. In the standard treatment of exclusive processes in QCD [1, 2], cross sections are expanded in inverse powers of the momentum transfer; the size of these power-suppressed corrections, ordered by increasing twist, is determined by the convolution of a perturbative hard scattering amplitude with a soft nonperturbative DA of given twist. The leading twist DA ϕ, which describes the momentum distribution of the valence quarks in the meson, is related to the meson's Bethe–Salpeter wave function ϕ_{BS} by

$$\phi(x) \sim \int^{|k_\perp|<\mu} d^2k_\perp\, \phi_{BS}(x, k_\perp)\,.$$

Here μ denotes the separation scale between perturbative and nonperturbative regime. The study of these leading twist 2 DAs has attracted much attention in the literature, in particular for the case of the π [3, 4, 5], but only a few investigations are devoted to higher twist distributions, which determine the preasymptotic behaviour of hard exclusive processes. Higher twist DAs originate from three different sources and describe either contributions of "bad" components in the wave function and in particular of components with "wrong" spin projection or contributions of transverse motion of quarks (antiquarks) in the leading twist components or contributions of higher Fock states with additional gluons and/or quark-antiquark pairs.

DAs of the π of twist 3 and 4 have been studied in [6] in the chiral limit, based on the techniques of nonlocal operator product expansion and conformal expansion. In refs. [7, 8, 9], vector meson DAs of twist 3 and 4 have been studied, also including corrections in the meson-mass. In this paper we extend the analysis of [6] to include also terms in the meson-mass in twist 3 and 4 DAs of pseudoscalar octet mesons. As discussed in [9], the structure of these mass corrections is more complicated than for deep-inelastic lepton-hadron scattering, where the corrections, being induced by kinematics, do not involve new information on dynamics and can be absorbed into a redefinition of the scaling variable, known as Nachtmann scaling [10]. The situation with exclusive decays is different, as matrix elements of operators containing total derivatives, specifically

$$\partial^2 O^{(2)}_{\mu_1\mu_2\dots\mu_n} \qquad \text{and} \qquad \partial_{\mu_1} O^{(2)}_{\mu_1\mu_2\dots\mu_n}\,,$$

where $O^{(2)}$ is a leading twist operator, vanish for forward-scattering, but do contribute to exclusive processes. Contributions of the first type can be taken into account consistently for all moments of DAs, while contributions of the second type are more complicated and can be unravelled only order by order in the conformal expansion. Numerically, as expected, these mass terms turn out to be small for the π, but are dominant for K and the octet DAs of η. We shall not discuss the octet DAs of the η' in this paper. The results are of direct relevance for the discussion of, for instance, meson transition form factors, $\gamma\gamma^* \to \eta$, and also for B meson decays into light mesons, see e.g. [11].

2. Definition of distribution amplitudes

Amplitudes of light-cone dominated processes involving pseodoscalar mesons can be expressed in terms of matrix elements of gauge invariant nonlocal operators sandwiched between the vacuum and the meson state, e.g. a matrix element over a two particle operator,

$$\Big\langle 0\,|\bar{u}(x)\Gamma[x,-x]d(-x)|\,\pi^-(P)\Big\rangle\,, \tag{2.1}$$

where Γ is a generic Dirac matrix structure and we use the notation $[x,y]$ for the path-ordered gauge factor along the straight line connecting the points x and y:

$$[x,y] = \mathrm{P}\exp\left[ig\!\int_0^1\! dt\,(x-y)_\mu A_\mu(tx+(1-t)y)\right]. \tag{2.2}$$

For notational convenience, we refer explicitly to π^- mesons. For other (nonsinglet) mesons, one has to use appropriate SU(3) currents, e.g. $1/\sqrt{2}\,\langle 0|\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d|\pi^0\rangle$ etc.

As mentioned in the introduction, we are in particular interested in meson-mass corrections. In contrast to vector mesons, whose mass is of order $\Lambda_{\rm QCD}$ and nonvanishing also in the chiral limit, pseudoscalar meson-masses scale linearly with the sum of quark masses. For consistency, we will thus keep all such terms in the analysis of the equations of motion, but neglect terms in the difference of quark masses. We thus also neglect contributions in the DAs of K mesons that are antisymmetric under the exchange of strange and nonstrange quark.

The asymptotic expansion of exclusive amplitudes in powers of large momentum transfer corresponds to the expansion of amplitudes like (2.1) in powers of the deviation from the light-cone $x^2 = 0$. As always in quantum field theory, such an expansion generates divergences and has to be understood as an operator product expansion in terms of renormalized nonlocal operators on the light-cone, whose matrix elements define meson DAs of increasing twist. To leading logarithmic accuracy, the coefficient functions are just taken at tree-level and the distributions have to be evaluated at the scale $\mu^2 \sim x^{-2}$. In this section we present the necessary expansions and introduce a complete set of meson DAs to twist 4 accuracy. This set is, in fact, overcomplete, and different distributions are related to one another via the QCD equations of motion, as detailed in secs 5 and 6.

To facilitate the discussion of matrix elements on the light-cone, it is convenient to introduce light-like vectors p and z such that

$$p_\mu = P_\mu - \frac{1}{2} z_\mu \frac{m^2}{pz}\,, \tag{2.3}$$

where P_μ is the meson momentum, $P^2 = m^2$. We also need the projector onto the directions orthogonal to p and z:

$$g^\perp_{\mu\nu} = g_{\mu\nu} - \frac{1}{pz}(p_\mu z_\nu + p_\nu z_\mu)\,, \tag{2.4}$$

and will use the notations

$$a. \equiv a_\mu z^\mu\,, \qquad a_* \equiv a_\mu p^\mu/(pz)\,, \tag{2.5}$$

for an arbitrary Lorentz vector a_μ.

We use the standard Bjorken–Drell convention [12] for the metric and the Dirac matrices; in particular $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and the Levi–Civita tensor $\epsilon_{\mu\nu\lambda\sigma}$ is defined as the totally antisymmetric tensor with $\epsilon_{0123} = 1$. The covariant derivative is defined as $D_\mu = \partial_\mu - igA_\mu$. The dual gluon field strength tensor is defined as $\widetilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$.

We start with the two particle DAs of the π meson. For the axial vector operator, the light-cone expansion to twist 4 accuracy reads:

$$\begin{aligned}\left\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\pi^-(P)\right\rangle &= if_\pi P_\mu \int_0^1 du\, e^{i\xi Px}\left[\phi_\pi(u) + \frac{1}{4}m_\pi^2 x^2 \mathbb{A}(u)\right] + \\ &\quad + \frac{i}{2} f_\pi m_\pi^2 \frac{1}{Px} x_\mu \int_0^1 du\, e^{i\xi Px}\, \mathbb{B}(u)\,.\end{aligned} \tag{2.6}$$

ϕ_π is the leading twist 2 DA, $\mathbb{A}$ and $\mathbb{B}$ contain contributions from operators of twist 2, 3 and 4. For brevity, here and below we do not show gauge factors between the quark and the antiquark fields; we also use the short-hand notation

$$\xi = 2u - 1 \,.$$

The decay constant f_π is defined, as usual, as

$$\langle 0|\bar{u}(0)\gamma_\mu\gamma_5 d(0)|\pi^-(P)\rangle = i f_\pi P_\mu \,. \tag{2.7}$$

Numerically, one has $f_\pi = 131\,\mathrm{MeV}$ and $f_K = 160\,\mathrm{MeV}$ [13]. For η, the situation is more complicated due to the mixing with η', and differing results for the coupling to the octet current, f_η^8, are available in the literature. We quote, for instance, $f_\eta^8 \approx 130\,\mathrm{MeV}$ [14] and, from a more recent analysis, $f_\eta^8 = 159\,\mathrm{MeV}$ [15]. In the numerical analysis we will use the ilustrative value $f_\eta^8 = 130\,\mathrm{MeV}$. As we shall see, the DAs themselves do not depend critically on the decay constants, although the matrix element (2.6) does.

The Lorentz invariant amplitude $\mathbb{B}$ can be interpreted in terms of meson DAs, defined in terms of nonlocal operators at strictly light-like separations, which can most conveniently be written using light-cone variables. For the axial vector current, the two particle DAs of the π meson are defined as

$$\begin{aligned}\Big\langle 0|\bar{u}(z)\gamma_\mu\gamma_5 d(-z)|\pi^-(P)\Big\rangle &= i f_\pi p_\mu \int_0^1 du\, e^{i\xi pz}\,\phi_\pi(u) + \\ &\quad + \frac{i}{2} f_\pi m^2 \frac{1}{pz} z_\mu \int_0^1 du\, e^{i\xi pz} g_\pi(u) \,.\end{aligned} \tag{2.8}$$

Comparing the above with (2.6), one finds

$$\mathbb{B}(u) = g_\pi(u) - \phi_\pi(u) \,. \tag{2.9}$$

The relation of these DAs to those defined by Braun and Filyanov, ref. [6], is given by

$$\frac{d}{du} g_2^{BF}(u) = -\frac{1}{2}\lim_{m_\pi^2\to 0} m_\pi^2 \mathbb{B}(u)\,, \qquad g_1^{BF}(u) - \int_0^u dv g_2^{BF}(v) = \frac{1}{16}\lim_{m_\pi^2\to 0} m_\pi^2 \mathbb{A}(u)\,. \tag{2.10}$$

In the local limit $x_\mu \to 0$, (2.6) yields the normalization conditions:

$$\begin{aligned}&\int_0^1 du\,\phi_\pi(u) = 1\,,\\ &\int_0^1 du\,\mathbb{B}(u) = 0 \qquad \Longrightarrow \qquad \int_0^1 du\, g_\pi(u) = 1\,.\end{aligned}$$

Two more matrix elements define DAs of twist 3 [6]:

$$\Big\langle 0|\bar{u}(x)i\gamma_5 d(-x)|\pi^-(P)\Big\rangle = \frac{f_\pi m_\pi^2}{m_u + m_d}\int_0^1 du\, e^{i\xi Px}\,\phi_p(u), \tag{2.11}$$

$$\begin{aligned}\Big\langle 0|\bar{u}(x)\sigma_{\alpha\beta}\gamma_5 d(-x)|\pi^-(P)\Big\rangle &= -\frac{i}{3}\frac{f_\pi m_\pi^2}{m_u+m_d}\left\{1 - \left(\frac{m_u+m_d}{m_\pi}\right)^2\right\}\times\\ &\quad \times(P_\alpha x_\beta - P_\beta x_\alpha)\int_0^1 du\, e^{i\xi Px}\,\phi_\sigma(u).\end{aligned} \tag{2.12}$$

Also these two DAs are normalized to unity:

$$\int_0^1 du\, \phi_{(p,\sigma)}(u) = 1\,.$$

The normalization factor in (2.12) differs from the one obtained in [6] by a term of $O(m_u + m_d) \sim O(m_\pi^2)$, which is tiny for the π, but amounts to 10% for the K. This may be of particular relevance for calculations of the $B \to K$ decay form factor in the framework of QCD sum rules on the light-cone, e.g. [11], to which the twist 3 DAs give a sizeable contribution.

At this point we would like to comment on the numerical values to be used for the normalization factors in (2.11) and (2.12). Evidently, it is difficult to give precise numbers as long as the quark masses are not more accurately known. To circumvent this problem, we invoke chiral perturbation theory (see e.g. [16] for a nice introduction), which relates meson to quark masses in the following way: define the constant B_0 via the (nonstrange) quark condensate:

$$\langle 0|\bar{q}q|0\rangle = -\frac{f_\pi^2}{2}\, B_0\,,$$

at the scale $\mu \approx 1\,\mathrm{GeV}$. Then the meson-masses are given by

$$\begin{aligned} m_\pi^2 &= (m_u + m_d) B_0\,,\\ m_K^2 &= (m_{u,d} + m_s) B_0\,,\\ m_{\eta_8}^2 &= \frac{2}{3}\left\{\frac{1}{2}\,(m_u + m_d) + 2m_s\right\} B_0\,, \end{aligned} \tag{2.13}$$

where we neglect small corrections in $(m_d - m_u)^2$. With the standard value of the quark condensate, $\langle 0|\bar{q}q|0\rangle(1\,\mathrm{GeV}) = -(0.24 \pm 0.01)\,\mathrm{GeV}^3$, one finds $B_0 = (1.6 \pm 0.2)\,\mathrm{GeV}$. Thus we have

$$\frac{m_\pi^2 f_\pi^2}{m_u + m_d} = f_\pi^2 B_0 = (0.027 \pm 0.003)\,\mathrm{GeV}^3\,.$$

The situation is slightly different for the K and η (which we consider as a pure octet state in this section). Proceeding like with the π, one finds (letting $m_{u,d} = 0$):

$$\begin{aligned} \frac{m_K^2 f_K^2}{m_s} &= f_K^2 B_0 &&= (0.041 \pm 0.005)\,\mathrm{GeV}^3,\\ \frac{m_\eta^2 (f_\eta^8)^2}{m_s} &= \frac{4}{3}\,(f_\eta^8)^2 B_0 &&= (0.045 \pm 0.006)\,\mathrm{GeV}^3, \end{aligned} \tag{2.14}$$

where one might worry, however, that the constant B_0 be affected by SU(3) violation. Using the actual values for the meson masses and $m_s(1\,\mathrm{GeV}) = 150\,\mathrm{MeV}$, one finds

$$\frac{m_K^2 f_K^2}{m_s} = 0.042\,\mathrm{GeV}^3\,, \qquad \frac{m_\eta^2 (f_\eta^8)^2}{m_s} = 0.042\,\mathrm{GeV}^3\,,$$

which is in good agreement with the results from chiral perturbation theory.

Let us now define the three particle DAs. To twist 3 accuracy, there is only one:

$$\langle 0|\bar{u}(z)\sigma_{\mu\nu}\gamma_5 gG_{\alpha\beta}(vz)d(-z)|\pi^-(P)\rangle = i\,\frac{f_\pi m_\pi^2}{m_u+m_d}\Big(p_\alpha p_\mu g^\perp_{\nu\beta} - p_\alpha p_\nu g^\perp_{\mu\beta} - \\ - p_\beta p_\mu g^\perp_{\nu\alpha} + p_\beta p_\nu g^\perp_{\alpha\mu}\Big)\,\mathcal{T}(v,pz)+\cdots \quad (2.15)$$

where the ellipses stand for Lorentz structures of twist 5 and higher and where we used the following short-hand notation for the integral defining the three particle DA:

$$\mathcal{T}(v,pz) = \int \mathcal{D}\underline{\alpha}\, e^{-ipz(\alpha_u-\alpha_d+v\alpha_g)}\mathcal{T}(\alpha_d,\alpha_u,\alpha_g)\,. \quad (2.16)$$

Here $\underline{\alpha}$ is the set of three momentum fractions α_d (d quark), α_u (u quark) and α_g (gluon). The integration measure is defined as

$$\int \mathcal{D}\underline{\alpha} = \int_0^1 d\alpha_d d\alpha_u d\alpha_g \delta(1-\alpha_u-\alpha_d-\alpha_g)\,.$$

There are also four three particle DAs of twist 4, defined as

$$\Big\langle 0|\bar{u}(z)\gamma_\mu\gamma_5 gG_{\alpha\beta}(vz)d(-z)|\pi^-(P)\Big\rangle = p_\mu(p_\alpha z_\beta - p_\beta z_\alpha)\,\frac{1}{pz}\,f_\pi m_\pi^2\mathcal{A}_\parallel(v,pz)+ \\ +(p_\beta g^\perp_{\alpha\mu} - p_\alpha g^\perp_{\beta\mu})f_\pi m_\pi^2\mathcal{A}_\perp(v,pz)\,, \quad (2.17)$$

$$\Big\langle 0|\bar{u}(z)\gamma_\mu ig\widetilde{G}_{\alpha\beta}(vz)d(-z)|\pi^-(P)\Big\rangle = p_\mu(p_\alpha z_\beta - p_\beta z_\alpha)\,\frac{1}{pz}\,f_\pi m_\pi^2\mathcal{V}_\parallel(v,pz)+ \\ +(p_\beta g^\perp_{\alpha\mu} - p_\alpha g^\perp_{\beta\mu})f_\pi m_\pi^2\mathcal{V}_\perp(v,pz)\,. \quad (2.18)$$

A short synopsis of the various light-cone projections of the three-particle matrix elements and their relation to DAs is given in table 1.

For completeness, let us mention that also four particle twist 4 DAs exist, corresponding to contributions of Fock states with two gluons or an additional $q\bar{q}$ pair. Such distributions will not be considered in this paper for two reasons: first, it is well known [17] that four particle twist 4 operators do not allow the factorization of vacuum condensates such as $\langle\bar{\psi}\psi\rangle$, $\langle G^2\rangle$. Because of this, their matrix elements cannot be estimated reliably by existing methods (e.g. QCD sum rules), although they are generally expected to be small. Second, and more importantly, the four

Twist	$(\mu\nu\alpha\beta)$	$\bar{\psi}\sigma_{\mu\nu}\gamma_5\widetilde{G}_{\alpha\beta}\psi$	$(\mu\alpha\beta)$	$\bar{\psi}\gamma_\mu\gamma_5 G_{\alpha\beta}\psi$	$\bar{\psi}\gamma_\mu\widetilde{G}_{\alpha\beta}\psi$
3	$\cdot\perp\cdot\perp$	$\mathcal{T}$			
4			$\cdot\cdot *$	$\mathcal{A}_\parallel$	$\mathcal{V}_\parallel$
			$\perp\perp\cdot$	$\mathcal{A}_\perp$	$\mathcal{V}_\perp$

Table 1: Identification of three-particle DAs with projections onto different light-cone components of nonlocal operators. For example, $\perp\perp\cdot$ refers to $\bar{\psi}\gamma_\perp\gamma_5 G_{\perp\cdot}\psi$.

particle distributions decouple from the QCD equations of motion in the two lowest conformal partial waves. To this accuracy, therefore, it is consistent to put them to zero. Vice versa, nonvanishing four particle distributions necessitate the inclusion of higher conformal spin corrections to distributions with less particles, which are beyond the approximation adopted in this paper.

3. Conformal partial wave expansion and equations of motion

The aim of this paper is to express the DAs defined in the previous section in a model-independent way by a minimal number of nonperturbative parameters. The one key ingredient in solving this task is the use of the QCD equations of motion which will allow us to reveal interrelations between the different DAs of a given twist. Nonlocal operators on or near the light-cone can conveniently be treated in the framework of the string-operator technique developped by Balitskii and Braun [18]. In the present context, we need the following nonlocal operator identities [9]:

$$\begin{aligned}
\frac{\partial}{\partial x_\mu}\,\bar u(x)\gamma_\mu\gamma_5 d(-x) &= -i\int_{-1}^{1} dv\, v\bar u(x)x_\alpha g G_{\alpha\mu}(vx)\gamma_\mu\gamma_5 d(-x) + \\
&\quad +(m_u-m_d)\bar u(x)i\gamma_5 d(-x)\,, && (3.1)\\
\partial_\mu\{\bar u(x)\gamma_\mu\gamma_5 d(-x)\} &= -i\int_{-1}^{1} dv\, \bar u(x)x_\alpha g G_{\alpha\mu}(vx)\gamma_\mu\gamma_5 d(-x) + \\
&\quad +(m_u+m_d)\bar u(x)i\gamma_5 d(-x)\,, && (3.2)\\
\partial_\mu \bar u(x)\sigma_{\mu\nu}\gamma_5 d(-x) &= -i\frac{\partial}{\partial x_\nu}\,\bar u(x)\gamma_5 d(-x) + \int_{-1}^{1} dv\, v\bar u(x)x_\rho g G_{\rho\nu}(vx)\gamma_5 d(-x) - \\
&\quad -i\int_{-1}^{1} dv\, \bar u(x)x_\rho g G_{\rho\mu}(vx)\sigma_{\mu\nu}\gamma_5 d(-x) + \\
&\quad +(m_d-m_u)\bar u(x)\gamma_\nu\gamma_5 d(-x)\,, && (3.3)\\
\frac{\partial}{\partial x_\mu}\,\bar u(x)\sigma_{\mu\nu}\gamma_5 d(-x) &= -i\partial_\nu \bar u(x)\gamma_5 d(-x) + \int_{-1}^{1} dv\, \bar u(x)x_\rho g G_{\rho\nu}(vx)\gamma_5 d(-x) - \\
&\quad -i\int_{-1}^{1} dv\, v\bar u(x)x_\rho g G_{\rho\mu}(vx)\sigma_{\mu\nu}\gamma_5 d(-x) - \\
&\quad -(m_u+m_d)\bar u(x)\gamma_\nu\gamma_5 d(-x)\,. && (3.4)
\end{aligned}$$

Here ∂_μ is the total derivative defined as

$$\partial_\mu\left\{\bar u(x)\Gamma d(-x)\right\} \equiv \frac{\partial}{\partial y_\mu}\left\{\bar u(x+y)[x+y,-x+y]\Gamma d(-x+y)\right\}\Big|_{y\to 0}\,.$$

By taking matrix elements of the above relations between the vacuum und the π^- meson state, one obtains exact integral representations for those DAs that are not dynamically independent.

The other key ingredient in our approach is the use of conformal expansion [19, 6] which, analogously to partial wave decomposition in quantum mechanics, allows one to separate transverse and longitudinal variables in the wave function. The dependence on transverse coordinates is represented as scale-dependence of the relevant operators and is governed by renormalization-group equations, the dependence on the longitudinal momentum fraction is described in terms of irreducible representations of the corresponding symmetry group, the collinear conformal group SL(2,$\mathbb{R}$). The conformal partial wave expansion is explicitly consistent with the equations of motion since the latter are not renormalized. The expansion thus makes maximum use of the symmetry of the theory in order to simplify the dynamics.

To construct the conformal expansion for an arbitrary multi-particle distribution, one first has to decompose each constituent field into components with fixed Lorentz spin projection onto the light-cone. Each such component has conformal spin

$$j = \frac{1}{2}(l+s)\,,$$

where l is the canonical dimension and s the (Lorentz) spin projection. In particular, $l = 3/2$ for quarks and $l = 2$ for gluons. The quark field is decomposed as $\psi_+ \equiv (1/2)\not{z}\not{p}\psi$ and $\psi_- = (1/2)\not{p}\not{z}\psi$ with spin projections $s = +1/2$ and $s = -1/2$, respectively. For the gluon field strength there are three possibilities: $G_{\cdot\perp}$ corresponds to $s = +1$, $G_{*\perp}$ to $s = -1$ and both $G_{\perp\perp}$ and $G_{\cdot *}$ correspond to $s = 0$.

Multi-particle states built of fields with definite Lorentz spin projection can be expanded in irreducible unitary representations of SL(2,$\mathbb{R}$) with increasing conformal spin. The explicit expression for the DA of a m-particle state with the lowest possible conformal spin $j = j_1 + \ldots + j_m$, the so-called asymptotic DA, is

$$\phi_{as}(\alpha_1, \alpha_2, \ldots, \alpha_m) = \frac{\Gamma(2j_1 + \cdots + 2j_m)}{\Gamma(2j_1)\cdots\Gamma(2j_m)}\, \alpha_1^{2j_1-1}\alpha_2^{2j_2-1}\cdots\alpha_m^{2j_m-1}\,. \qquad (3.5)$$

Here α_k are the corresponding momentum fractions. This state is nondegenerate and cannot mix with other states because of conformal symmetry. Multi-particle irreducible representations with higher spin $j+n, n = 1, 2, \ldots$, are given by polynomials of m variables (with the constraint $\sum_{k=1}^m \alpha_k = 1$), which are orthogonal over the weight-function (3.5).

In particular, for the leading twist 2 DA ϕ_π defined in (2.6), the expansion goes in Gegenbauer polynomials:

$$\phi_\pi(u, \mu^2) = 6u(1-u)\left(1 + \sum_{n=1}^{\infty} a_{2n}\left(\mu^2\right) C_{2n}^{3/2}(2u-1)\right). \qquad (3.6)$$

To leading logarithmic accuracy, the (nonperturbative) Gegenbauer moments a_n renormalize multiplicatively with

$$a_n(Q^2) = L^{\gamma_n/b}\, a_n\left(\mu^2\right),$$

where $L \equiv \alpha_s(Q^2)/\alpha_s(\mu^2)$, $b = (11N_c - 2N_f)/3$, and the anomalous dimension γ_n is given by

$$\gamma_n = 4C_F\left(\psi(n+2) + \gamma_E - \frac{3}{4} - \frac{1}{2(n+1)(n+2)}\right).$$

In this paper, we work to next-to-leading order in conformal spin and thus truncate[1] the above expansion of ϕ_π after the term in $n = 1$. For the π, the corresponding Gegenbauer moment was determined in e.g. [4] from QCD sum rules, for K, we use the value determined in [11]:

$$a_2^\pi(1\,\text{GeV}) = 0.44\,, \qquad a_2^K(1\,\text{GeV}) = 0.2\,, \qquad a_2^\eta(1\,\text{GeV}) = 0.2\,. \tag{3.7}$$

The value for η is new and follows from an analysis of the QCD sum rule in [4] by fixing the continuum threshold s_0 to reproduce $f_\eta^8 = 130\,\text{MeV}$.

4. Meson-mass corrections

The structure of meson-mass corrections in inclusive processes is in general more complicated than that of target-mass corrections in deep inelastic scattering, which can be resummed using the Nachtmann variable [10]. The terms entering the Nachtmann variable are just the subtracted traces of the leading twist forward-scattering matrix element, which can also for exclusive processes be summed to all orders. Let us illustrate this point with the aid of the two-point correlation function of scalar fields:

$$\langle 0|\phi(x)\phi(-x)|P\rangle = \int_0^1 du\, e^{i\xi Px}\left[\psi(u) + \frac{1}{4}\,x^2m^2\psi_2(u) + O(x^4)\right].$$

Here ψ is the leading twist DA; ψ_2 receives contributions from both the subtraction of traces in the leading twist operator, also dubbed "kinematical" corrections, and from intrinsic "dynamical" higher twist corrections. The subtraction of traces can be done on the operator level by making use of the condition [18]

$$\frac{\partial^2}{\partial x_\alpha \partial x_\alpha}\,[\phi(x)\phi(-x)]_{\text{l.t.}} = 0\,, \tag{4.1}$$

which translates into the condition that all local operators arising in the Taylor expansion be traceless. A formal solution is [18]

$$[\phi(x)\phi(-x)]_{\text{l.t.}} = \phi(x)\phi(-x) + \\ + \sum_{n=1}^{\infty}\int_0^1 dt\, t\left(-\frac{1}{4}\,x^2\right)^n [t(1-t)]^{n-1}\left[\frac{\partial^2}{t^2\partial x_\alpha\partial x_\alpha}\right]^n \phi(tx)\phi(-tx)\,.$$

[1]Note that a thorough discussion of the shape of the π DA of leading twist necessitates the inclusion of higher terms in the conformal expansion. In this paper, however, we concentrate on higher twist DAs which constitute *corrections* to the leading twist DAs that are suppressed by powers of the characteristic momentum transfer in hard reactions, so we feel justified in neglecting higher order conformal corrections to these corrections.

JHEP01(1999)010

To order x^2, one thus has

$$\phi(x)\phi(-x) = [\phi(x)\phi(-x)]_{\rm l.t.} - \frac{1}{4}\,x^2\int_0^1 dt\,t\,\partial^2[\phi(tx)\phi(-tx)]_{\rm l.t.} + \\ +\text{interaction terms } + O(x^4)\,. \tag{4.2}$$

Note that $\partial^2[\ldots]_{\rm l.t.}$ is a higher twist operator. Taking a forward-scattering matrix element, $\langle P|\ldots|P\rangle$, the second term on the right-hand side vanishes, and all mass corrections arise from subtracting traces in the leading twist matrix element. This is the source of the Nachtmann corrections. In the following we will calculate the corresponding matrix element for exclusive processes,

$$\langle 0\,|[\phi(x)\phi(-x)]_{\rm l.t.}|\,P\rangle\,, \tag{4.3}$$

exactly, i.e. summing up all terms in x^2.

It is clear that (4.3) can only depend on the leading twist DA $\psi(u)$. Writing (4.3) as a Taylor-series over the moments of ψ, we can subtract the traces explicitly for each term in the expansion, which yields ($P^2 = m^2$):

$$\langle 0\,|[\phi(x)\phi(-x)]_{\rm l.t.}|\,P\rangle = \sum_{n=0}^{\infty}\frac{i^n}{n!}\,\langle\langle O_n\rangle\rangle\left(\frac{1}{4}\,m^2x^2\right)^{n/2} U_n(Px/\sqrt{m^2x^2})\,. \tag{4.4}$$

Here U_n are the Chebyshev polynomials

$$U_n(x) = \sum_{j=0}^{[\frac{n}{2}]}(-1)^j\begin{pmatrix} n-j\\ j\end{pmatrix}(2x)^{n-2j}\,,$$

and $\langle\langle O_n\rangle\rangle$ are the moments of ψ:

$$\langle\langle O_n\rangle\rangle = \int_0^1 du\,\xi^n\,\psi(u)\,.$$

Using the generating function of U_n, it proves possible to sum (4.4) explicitly (see also [20]):

$$\langle 0|[\phi(x)\phi(-x)]_{\rm l.t.}|P\rangle = \int_0^1 du\,\psi(u)\frac{1}{\sqrt{(Px)^2-m^2x^2}}\times \\ \times\frac{d}{du}\left[e^{i\xi Px/2}\sin\left(\xi\sqrt{(Px)^2-m^2x^2}/2\right)\right]. \tag{4.5}$$

In the spirit of Nachtmann, we would like to absorb all terms in m^2 into a new scaling variable $Px\to(Px)'$. Although we did not succeed in finding such a variable, the above expression can be simplified considerably by introducing

$$2(Px)' \equiv Px + \sqrt{(Px)^2-m^2x^2}\,,$$

so that

$$\langle 0|[\phi(x)\phi(-x)]_{\rm l.t.}|P\rangle \;=\int_0^1 du\,\frac{\psi(u)}{1-\frac{m^2x^2}{4(Px)'^2}}\left(\exp\{i\xi(Px)'\}-\right.$$

$$\left.-\frac{m^2x^2}{4(Px)'^2}\exp\left\{i\xi(Px)'\,\frac{m^2x^2}{4(Px)'^2}\right\}\right). \tag{4.6}$$

Expanding to $O(x^2)$, this can be written as

$$\langle 0|[\phi(x)\phi(-x)]_{\rm l.t.}|P\rangle \;=\int_0^1 dt\int_0^1 du\,\left[e^{i\xi Px}+\frac{1}{4}\,m^2x^2t\,\xi^2e^{i\xi Pxt}\right]\psi(u)\,,$$

and combining with (4.2), we get

$$\langle 0|\phi(x)\phi(-x)]|P\rangle \;=\int_0^1 dt\int_0^1 du\,\left[e^{i\xi Px}+\frac{1}{4}\,m^2x^2t(1+\xi^2)e^{i\xi Pxt}\right]\psi(u)\,.$$

This means that both sources of mass corrections, the subtraction of traces in the leading twist matrix element and the higher twist operator containing total derivatives, act in the same direction and thus enlarge the mass correction terms. We will observe the same effect, enlarged mass corrections, also in QCD.

5. Twist 3 distribution amplitudes

The twist 3 DAs of the π have already been studied in [6]. Here we extend this study by including terms in $\rho_\pi^2\equiv(m_u+m_d)^2/m_\pi^2\sim O(m_\pi^2)$.

To next-to-leading order in conformal spin, the only three particle DA $\mathcal{T}$ gets expanded as

$$\mathcal{T}(\underline{\alpha})=360\eta_3\alpha_u\alpha_d\alpha_g^2\left\{1+\omega_3\,\frac{1}{2}\,(7\alpha_g-3)\right\}. \tag{5.1}$$

η_3 is defined as

$$\langle 0|\bar u\sigma_{\mu\nu}\gamma_5 gG_{\alpha\beta}d|\pi^-\rangle=if_\pi\eta_3\,\frac{m_\pi^2}{m_u+m_d}(P_\alpha P_\mu g_{\nu\beta}-P_\alpha P_\nu g_{\mu\beta}-P_\beta P_\mu g_{\nu\alpha}+P_\beta P_\nu g_{\alpha\mu})\,, \tag{5.2}$$

and ω_3 is defined as

$$\langle 0|\bar u\sigma_{\mu\xi}\gamma_5[iD_\mu,gG_{\alpha\xi}]d-\frac{3}{7}\,i\partial_\beta\bar u\sigma_{\mu\xi}\gamma_5 gG_{\alpha\xi}d|\pi^-\rangle=i\,\frac{f_\pi m^2}{m_u+m_d}\,2P_\alpha P_\beta P_\mu\,\frac{3}{28}\,\eta_3\omega_3+$$

$$+O(\text{higher twist})\,. \tag{5.3}$$

In the notations of [6]:

$$\eta_3\equiv R=\frac{f_{3\pi}}{f_\pi}\,\frac{m_u+m_d}{m_\pi^2}\,,\qquad \omega_3\equiv\omega_{10}\,.$$

These parameters are scale-dependent with ($C_A=N_c$)

$$\eta_3(Q^2)=L^{\gamma_3^\eta/b}\eta_3(\mu^2)\,,\qquad \gamma_3^\eta=\frac{16}{3}\,C_F+C_A\,,$$

$$\omega_3(Q^2)=L^{\gamma_3^\omega/b}\omega_3(\mu^2)\,,\qquad \gamma_3^\omega=-\frac{25}{6}\,C_F+\frac{7}{3}\,C_A\,.$$

Numerical values are obtained from QCD sum rules [6, 21] and collected in table 2.

	π	K	η
a_2	0.44	0.2	0.2
η_3	0.015	0.015	0.013
ω_3	-3	-3	-3
μ^2 [GeV]^2	0.0077	0.096	0.12

Table 2: Input parameters for twist 3 DAs, calculated from QCD sum rules. The accuracy is about 30%. Renormalization scale is 1 GeV.

The two particle DAs $\phi_{p,\sigma}$ are determined by $\mathcal{T}$ and $\rho_\pi^2\phi_\pi$. As an analysis of the matrix elements of the exact operator relations in sec. 3 leads to integro-differential equations that cannot be solved in a closed form, we prefer to perform an analysis of moments:

$$\begin{aligned} M_n^p &= \delta_{n0} + \frac{n-1}{n+1}\, M_{n-2}^p + 2(n-1)M_{n-2}^{\mathcal{T}_1} + \frac{2(n-1)(n-2)}{n+1}\, M_{n-3}^{\mathcal{T}_2} - \rho_\pi^2\, \frac{n-1}{n+1}\, M_{n-2}^\phi, \\ M_n^\sigma &= \delta_{n0} + \frac{n-1}{n+3}\, M_{n-2}^\sigma + \frac{6(n-1)}{n+3}\, M_{n-2}^{\mathcal{T}_1} + \frac{6n}{n+3}\, M_{n-1}^{\mathcal{T}_2} - \rho_\pi^2\, \frac{3}{n+3}\, M_n^\phi. \end{aligned} \tag{5.4}$$

Here we use the notation $M_n^P = \int_0^1 du\, \xi^n \phi_P(u)$ and the functions

$$\phi_{\mathcal{T}_1} = \int_0^u d\alpha_d \int_0^{\bar{u}} d\alpha_u\, \frac{2}{\alpha_g}\, \mathcal{T}(\underline{\alpha}), \qquad \phi_{\mathcal{T}_2} = \int_0^u d\alpha_d \int_0^{\bar{u}} d\alpha_u\, \frac{2}{\alpha_g^2}\, (\alpha_d - \alpha_u - \xi)\, \mathcal{T}(\underline{\alpha}).$$

Except for the new terms in ρ_π^2, the relations for moments agree with those obtained in [6].

Conformal expansion imposes that ϕ_p gets expanded in Gegenbauer polynomials $C_n^{1/2}$ and ϕ_σ in $C_n^{3/2}$ [6]. From the recursion relations for moments we find:

$$\begin{aligned} \phi_p(u) &= 1 + \left(30\eta_3 - \frac{5}{2}\,\rho_\pi^2\right) C_2^{1/2}(\xi) + \left(-3\eta_3\omega_3 - \frac{27}{20}\,\rho_\pi^2 - \frac{81}{10}\,\rho_\pi^2 a_2\right) C_4^{1/2}(\xi), \\ \phi_\sigma(u) &= 6u(1-u)\left\{1 + \left(5\eta_3 - \frac{1}{2}\,\eta_3\omega_3 - \frac{7}{20}\,\rho_\pi^2 - \frac{3}{5}\,\rho_\pi^2 a_2\right) C_2^{3/2}(\xi)\right\}. \end{aligned} \tag{5.5}$$

In fig. 1 we plot the two two particle DAs of twist 3 for π, K and η mesons. Evidently, the effect of mass corrections is not negligible and for ϕ_p even modifies the

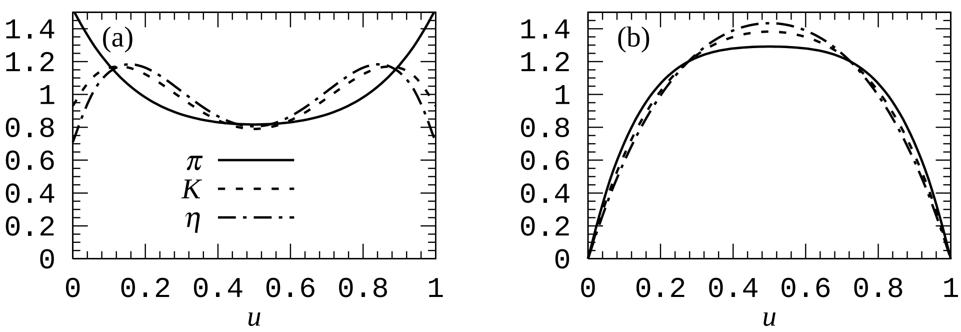

Figure 1: The two particle DAs of twist 3: $\phi_p(u)$ (a) and $\phi_\sigma(u)$ (b).

JHEP01(1999)010

shape of the DA near the endpoints, which is due to the dependence of the coefficient of $C_4^{1/2}$ on ρ_π^2. We would like to recall, however, that the above parametrizations are to be understood in the sense of (mathematical) distributions rather than as models that are valid point by point, and that they are always intended to be convoluted with smooth perturbative scattering amplitudes, which in particular will smooth out the effect of neglected higher order terms in the conformal expansion.

6. Twist 4 distribution amplitudes

In this section we repeat the analysis of twist 4 DAs performed in [6] in a more systematic way and extend it by including mass correction terms.

Due to G-parity, in the chiral limit, the DAs $\mathcal{A}_\parallel$ and $\mathcal{A}_\perp$ are antisymmetric under the exchange of α_d and α_u, whereas $\mathcal{V}_\parallel$ and $\mathcal{V}_\perp$ are symmetric; contributions of "wrong" G-parity give rise to asymmetric contributions to the two particle DAs of K and are neglected in the following. The distributions $\mathcal{A}_\parallel$ and $\mathcal{V}_\parallel$ correspond to the light-cone projection $\gamma.G_{.*}$ (see Table 1) and have the conformal expansion

$$\begin{aligned}\mathcal{V}_\parallel(\underline{\alpha}) &= 120\alpha_u\alpha_d\alpha_g(v_{00} + v_{10}(3\alpha_g - 1) + \cdots)\,,\\ \mathcal{A}_\parallel(\underline{\alpha}) &= 120\alpha_u\alpha_d\alpha_g(0 + a_{10}(\alpha_d - \alpha_u) + \cdots)\,,\end{aligned} \tag{6.1}$$

respectively. Note that the leading spin contribution to $\mathcal{A}_\parallel$ vanishes because of G-parity (for massless quarks).

The DAs $\mathcal{V}_\perp$ and $\mathcal{A}_\perp$, on the other hand, correspond to the projection $\gamma_\perp G_{.\perp}$ and thus do not describe states with a definite projection of the quark spins onto the light-ray z_μ. We separate the different quark spin projections with the aid of the auxiliary amplitudes $\mathcal{H}^{\uparrow\downarrow}$ and $\mathcal{H}^{\downarrow\uparrow}$ defined as

$$\begin{aligned}\left\langle 0 \left| \bar{u}(z) ig\widetilde{G}_{\alpha\beta}(vz)\gamma.\gamma_\mu\gamma_* d(-z) \right| \pi^- \right\rangle &= f_\pi m_\pi^2 \left(p_\beta g^\perp_{\alpha\mu} - p_\alpha g^\perp_{\beta\mu}\right) \mathcal{H}^{\uparrow\downarrow}(v, pz)\,,\\ \left\langle 0 \left| \bar{u}(z) ig\widetilde{G}_{\alpha\beta}(vz)\gamma_*\gamma_\mu\gamma. d(-z) \right| \pi^- \right\rangle &= f_\pi m_\pi^2 \left(p_\beta g^\perp_{\alpha\mu} - p_\alpha g^\perp_{\beta\mu}\right) \mathcal{H}^{\downarrow\uparrow}(v, pz)\,.\end{aligned} \tag{6.2}$$

The original distributions $\mathcal{A}_\perp$ and $\mathcal{V}_\perp$ are then given by

$$\begin{aligned}\mathcal{V}_\perp(\underline{\alpha}) &= -\frac{1}{2}\left(\mathcal{H}^{\uparrow\downarrow}(\underline{\alpha}) + \mathcal{H}^{\downarrow\uparrow}(\underline{\alpha})\right),\\ \mathcal{A}_\perp(\underline{\alpha}) &= \frac{1}{2}\left(\mathcal{H}^{\uparrow\downarrow}(\underline{\alpha}) - \mathcal{H}^{\downarrow\uparrow}(\underline{\alpha})\right).\end{aligned} \tag{6.3}$$

$\mathcal{H}^{\uparrow\downarrow}$ and $\mathcal{H}^{\downarrow\uparrow}$ have a simple expansion in terms of Appell polynomials, to wit:

$$\begin{aligned}\mathcal{H}^{\uparrow\downarrow}(\underline{\alpha}) &= 60\alpha_u\alpha_g^2\left[h_{00} + h_{01}(\alpha_g - 3\alpha_d) + h_{10}\left(\alpha_g - \frac{3}{2}\alpha_u\right)\right],\\ \mathcal{H}^{\downarrow\uparrow}(\underline{\alpha}) &= 60\alpha_d\alpha_g^2\left[h_{00} + h_{01}(\alpha_g - 3\alpha_u) + h_{10}\left(\alpha_g - \frac{3}{2}\alpha_d\right)\right],\end{aligned} \tag{6.4}$$

where we have taken into account the symmetry properties, i.e.

$$\mathcal{H}^{\uparrow\downarrow}(\alpha_d,\alpha_u) \equiv \mathcal{H}^{\downarrow\uparrow}(\alpha_u,\alpha_d)\,.$$

From (6.3), the following relations can be derived immediately:

$$\begin{aligned} \mathcal{V}_\perp(\underline{\alpha}) &= -30\alpha_g^2\Big[h_{00}(1-\alpha_g)+h_{01}\Big[\alpha_g(1-\alpha_g)-6\alpha_u\alpha_d\Big] \\ &\qquad +h_{10}\Big[\alpha_g(1-\alpha_g)-\frac{3}{2}(\alpha_u^2+\alpha_d^2)\Big]\Big]\,, \\ \mathcal{A}_\perp(\underline{\alpha}) &= 30\alpha_g^2(\alpha_u-\alpha_d)\Big[h_{00}+h_{01}\alpha_g+\frac{1}{2}\,h_{10}(5\alpha_g-3)\Big]\,. \end{aligned} \tag{6.5}$$

The DAs $\mathcal{V}_{\perp,\parallel}$ and $\mathcal{A}_{\perp,\parallel}$ depend, to next-to-leading accuracy in the conformal spin, on a total of six parameters: v_{00} and h_{00} of leading conformal spin and v_{10}, a_{10}, h_{10} and h_{01} of NLO conformal spin. Our next task is to relate these parameters to independent local matrix elements. Defining

$$\left\langle 0\left|\bar{u}\gamma_\xi ig\widetilde{G}_{\xi\alpha}d\right|\pi^-\right\rangle = f_\pi m_\pi^2\eta_4 P_\alpha, \tag{6.6}$$

which is equivalent to

$$\left\langle 0\left|\bar{u}\gamma_\alpha ig\widetilde{G}_{\mu\nu}d\right|\pi^-\right\rangle = -\frac{1}{3}\,f_\pi m_\pi^2\eta_4(P_\mu g_{\nu\alpha}-P_\nu g_{\mu\alpha})\,,$$

it follows

$$h_{00} = v_{00} = -\frac{1}{3}\,\eta_4\,. \tag{6.7}$$

η_4 is scale-dependent with

$$\eta_4(Q^2) = L^{\gamma_4^\eta/b}\,\eta_4\left(\mu^2\right), \qquad \gamma_4^\eta = \frac{8}{3}\,C_F\,.$$

To NLO in conformal spin, beyond the matrix elements already defined above, we need only one more matrix element of a conformal quark-gluon operator:

$$\begin{aligned}\left\langle 0\left|\bar{u}\left[iD_\mu, ig\widetilde{G}_{\nu\xi}\right]\gamma_\xi d-\frac{4}{9}\,i\partial_\mu\bar{u}ig\widetilde{G}_{\nu\xi}\gamma_\xi d\right|\pi^-\right\rangle &= f_\pi m_\pi^2\eta_4\omega_4\left(P_\mu P_\nu-\frac{1}{4}\,m_\pi^2 g_{\mu\nu}\right)+ \\ &\quad +O(\text{twist }5).\end{aligned} \tag{6.8}$$

The scale-dependence of ω_4 is given by

$$\omega_4(Q^2) = L^{\gamma_4^\omega/b}\,\omega_4\left(\mu^2\right), \qquad \gamma_4^\omega = \frac{10}{3}\,C_A-\frac{8}{3}\,C_F\,.$$

Numerical values for η_4 and ω_4 were calculated from QCD sum rules [22, 6] and are collected in table 3. In the notation of ref. [6], $\delta^2 \equiv m_\pi^2\eta_4$, $\epsilon \equiv 21/8\,\omega_4$.

The procedure how to relate v_{10}, a_{10}, h_{10} and h_{01} to local matrix elements is described in detail in ref. [9], so that we mention only the essentials. Three of the four necessary relations follow from an analysis of various light-cone projections of the matrix elements of the operators

$$O^{(1)}_{\alpha\beta\mu\nu} = -\bar{u}\left(i \overleftarrow{D}_\beta\, g\widetilde{G}_{\mu\nu} + g\widetilde{G}_{\mu\nu} i \overrightarrow{D}_\beta\right)\gamma_\alpha d\,,$$
$$O^{(2)}_{\alpha\beta\mu\nu} = \bar{u}\left(-i \overleftarrow{D}_\beta\, gG_{\mu\nu} + gG_{\mu\nu} i \overrightarrow{D}_\beta\right)\gamma_\alpha\gamma_5 d\,.$$

	π	K	η
a_2	0.44	0.2	0.2
η_3	0.015	0.015	0.013
ω_3	-3	-3	-3
η_4	10	0.6	0.5
ω_4	0.2	0.2	0.2

Table 3: Input parameters for twist 4 DAs, calculated from QCD sum rules. The accuracy is about 30%. Renormalization scale is 1 GeV.

The fourth relation can be derived from the operator identity

$$\begin{aligned}\frac{4}{5}\partial_\mu E_{\mu\alpha\beta} &= -12i\bar{u}\gamma_\rho\gamma_5\left\{G_{\rho\beta}\overrightarrow{D}_\alpha - \overleftarrow{D}_\alpha G_{\rho\beta} + (\alpha\leftrightarrow\beta)\right\}d - \\ &\quad -4\partial_\rho\bar{u}\left(\gamma_\beta\widetilde{G}_{\alpha\rho} + \gamma_\alpha\widetilde{G}_{\beta\rho}\right)d - \frac{8}{3}\,\partial_\beta\bar{u}\gamma_\sigma\widetilde{G}_{\sigma\alpha}d - \\ &\quad -\frac{8}{3}\,\partial_\alpha\bar{u}\gamma_\sigma\widetilde{G}_{\sigma\beta}d + \frac{28}{3}\,g_{\alpha\beta}\partial_\rho\bar{u}\gamma_\sigma\gamma_5\widetilde{G}_{\sigma\rho}d\,,\end{aligned}$$

where

$$E_{\mu\alpha\beta} = \left[\frac{15}{2}\bar{u}\gamma_\mu\gamma_5\overleftrightarrow{D}_\alpha\overleftrightarrow{D}_\beta\, d - \frac{3}{2}\partial_\alpha\partial_\beta\bar{u}\gamma_\mu\gamma_5 d - \text{traces}\right]_{\text{symmetrized}}$$

is a leading twist 2 conformal operator. The matrix element of this operator is proportional to the Gegenbauer moment a_2 of the twist 2 DA and brings in a dynamical mass correction in the twist 4 DAs. This is precisely the effect mentioned in the introduction: total derivatives of twist 2 operators enter higher twist DAs.

The final results for the NLO parameters read:

$$\begin{aligned}a_{10} &= \frac{21}{8}\,\eta_4\omega_4 - \frac{9}{20}\,a_2\,,\\ v_{10} &= \frac{21}{8}\,\eta_4\omega_4\,,\\ h_{01} &= \frac{7}{4}\,\eta_4\omega_4 - \frac{3}{20}\,a_2\,,\\ h_{10} &= \frac{7}{2}\,\eta_4\omega_4 + \frac{3}{20}\,a_2\,.\end{aligned}\tag{6.9}$$

For $a_2\to 0$, these results agree with those obtained in [6].

We are now in the position to derive expressions also for the remaining two DAs of twist 4, g_π and $\mathbb{A}$. From the operator relations (3.1) and (3.2), one obtains

$$g_\pi(u) = 2\phi_p(u) - \phi_\pi(u) + \frac{d}{du}\int_0^u d\alpha_d\int_0^{\bar{u}} d\alpha_u\,\frac{2}{\alpha_g}\left(\mathcal{A}_\parallel(\underline{\alpha}) - 2\mathcal{A}_\perp(\underline{\alpha})\right),\tag{6.10}$$

$$\begin{aligned}\mathbb{A}(u) &= 12\int_0^u dv\int_0^v dw\,(g_\pi(w) - \phi_\pi(w)) - 2\int_0^u dv(2v-1)(\phi_\pi(v) + g_\pi(v)) + \\ &\quad + \int_0^u d\alpha_d\int_0^{\bar{u}} d\alpha_u\,\frac{4}{\alpha_g^2}\,(\alpha_d - \alpha_u - \xi)\left(2\mathcal{A}_\perp(\underline{\alpha}) - \mathcal{A}_\parallel(\underline{\alpha})\right).\end{aligned}\tag{6.11}$$

g_π corresponds to a definite quark spin projection and thus has a simple expansion in Gegenbauer polynomials $C_n^{1/2}$:

$$g_\pi(u) = \sum_{i=0}^{\infty} g_{2i} C_{2i}^{1/2}(\xi)\,. \tag{6.12}$$

From (6.10), one finds:

$$\begin{aligned} g_0 &= 1\,, \\ g_2 &= 1 + \frac{18}{7}\,a_2 + 60\eta_3 + \frac{20}{3}\,\eta_4\,, \\ g_4 &= -\frac{9}{28}\,a_2 - 6\eta_3\omega_3\,, \end{aligned}$$

where we neglect terms of $O(m_u + m_d)$ induced by ϕ_p, as g_π itself enters the matrix element of the axialvector current already as $O(m_\pi^2) = O(m_u + m_d)$, cf.~(2.8).

The expansion of $\mathbb{A}$ is not that straightforward and involves logarithms:

$$\begin{aligned} \mathbb{A}(u) = 6u\bar{u}\Bigg\{ &\frac{16}{15} + \frac{24}{35}\,a_2 + 20\eta_3 + \frac{20}{9}\,\eta_4 + \left(-\frac{1}{15} + \frac{1}{16} - \frac{7}{27}\,\eta_3\omega_3 - \frac{10}{27}\,\eta_4\right) C_2^{3/2}(\xi) + \\ &+ \left(-\frac{11}{210}\,a_2 - \frac{4}{135}\,\eta_3\omega_3\right) C_4^{3/2}(\xi)\Bigg\} + \left(-\frac{18}{5}\,a_2 + 21\eta_4\omega_4\right) \times \\ &\times \Big\{2u^3(10 - 15u + 6u^2)\ln u + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln\bar{u} + u\bar{u}(2 + 13u\bar{u})\Big\}. \end{aligned}$$

In fig. 2 we plot $m_P^2 g_P$ and $m_P^2\mathbb{A}$ for the mesons $P = \pi, K, \eta$. Whereas the DA $m_P^2 g_P$ is not too different for the three different mesons, the impact of meson-mass corrections on $m_P^2\mathbb{A}$ is more noteworthy. For the area under the curve, i.e. the overall normalization $N_P = \int_0^1 du\, m_P^2 \mathbb{A}(u)$, we find:

$$N_\pi = 0.47\,, \qquad N_K = 0.70\,, \qquad N_\eta = 0.77\,.$$

The result for N_η is essentially independent of the precise value of f_η^8. The impact of meson-mass corrections is thus rather profond for the DA $\mathbb{A}$. Likewise the change in normalization of ϕ_σ, this shift in $\mathbb{A}$ may have a noticeable effect on the $B \to K$ form factors calculated from QCD sum rules on the light-cone.

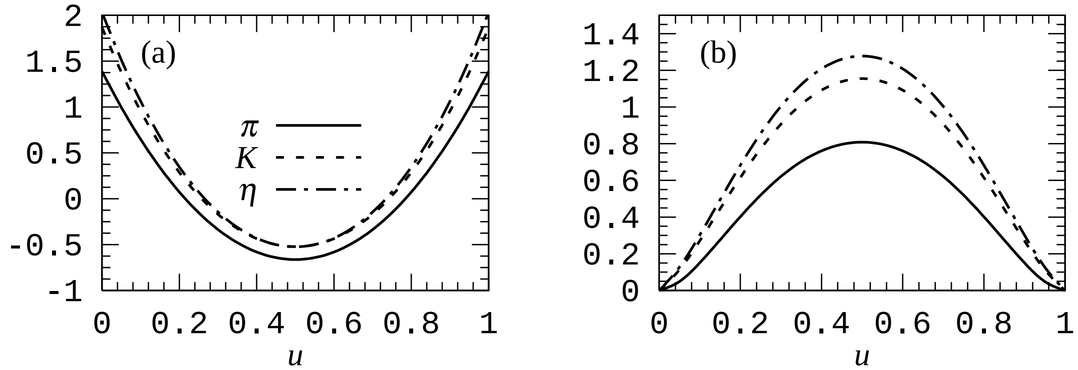

Figure 2: The two particle DAs of twist 4: $g_P(u)$ (a) and $\mathbb{A}$ (b) for $P = \pi$, K, η.

7. Summary and conclusions

In this paper we have studied the twist 3 and 4 two and three particle DAs of pseudoscalar nonsinglet mesons in QCD and expressed them in a model-independent way by a minimal number of nonperturbative parameters. The work presented here is an extension of the paper [6] on π DAs and is in particular concerned with corrections in the meson mass. The one ingredient in our approach is the use of the QCD equations of motion, which allow one to express dynamically dependent DAs in terms of independent ones. The other ingredient is conformal expansion which makes it possible to separate transverse and longitudinal variables in the wave functions, the former ones being governed by renormalization group equations, the latter ones being described in terms of irreducible representations of the corresponding symmetry group.

The analysis of twist 4 DAs is complicated by the fact that the twist 4 terms are of different origin: there are, first, "intrinsic" twist 4 corrections from matrix elements of twist 4 operators. There are, second, admixtures of matrix elements of twist 3 operators, as the counting of twist in terms of "good" and "bad" projections on light-cone coordinates does not exactly match the definition of twist as "dimension minus spin" of an operator. There are, third, meson-mass corrections, which one may term kinematical corrections, that come, on the one hand, from the subtraction of traces in the leading twist operators and, on the other hand, from higher twist operators containing total derivatives of twist 2 operators. Meson-mass corrections of the first kind are formally analogous to Nachtmann corrections in inclusive processes, while the contribution of operators with total derivatives is a specific new feature in exclusive processes, which makes the structure of these corrections much more complex.

Our final results are collected in secs. 5 and 6. In contrast to the case of vector mesons whose mass is finite also in the chiral limit, pseudoscalar octet mesons bring in the complication that their mass squared depends linearly on the quark masses, so that for consistency one also has to take into account terms of $O((m_u+m_d)^2/m_\pi^2) \sim O(m_\pi^2)$. The effect of meson-mass corrections is noticeable for twist 3 DAs and in particular for the twist 4 DA $\mathbb{A}$, whose normalization increases by $\sim 50\%$, when comparing the π with the K.

We hope that our results will contribute to a better understanding of SU(3) breaking effects in hard exclusive processes and in particular to the investigation of B and B_s decay form factors into π, K and η mesons from QCD sum rules on the light-cone.

Acknowledgments

The author is supported by DFG through a Heisenberg fellowship.

References

[1] S.J. Brodsky and G.P. Lepage, in: *Perturbative Quantum Chromodynamics*, A.H. Müller ed., p. 93, World Scientific, Singapore 1989.

[2] V.L. Chernyak and A.R. Zhitnitsky, *JETP Lett.* **25** (1977) 510; *Yad. Fiz.* **31** (1980) 1053;
A.V. Efremov and A.V. Radyushkin, *Phys. Lett.* **B 94** (1980) 245; *Teor. Mat. Fiz.* **42** (1980) 147;
G.P. Lepage and S.J. Brodsky, *Phys. Lett.* **B 87** (1979) 359; *Phys. Rev.* **D 22** (1980) 2157;
V.L. Chernyak, V.G. Serbo and A.R. Zhitnitsky, *JETP Lett.* **26** (1977) 594; *Sov. J. Nucl. Phys.* **31** (1980) 552.

[3] V.L. Chernyak and A.R. Zhitnitsky, *Phys. Rep.* **112** (1984) 173.

[4] V.M. Braun and I.E. Filyanov, *Z. Physik* **C 44** (1989) 157.

[5] For instance:
S.V. Mikhailov and A.V. Radyushkin, *Phys. Rev.* **D 45** (1992) 1754;
I. Halperin, *Z. Physik* **C 56** (1992) 615;
S.S. Agaev, *Int. J. Mod. Phys.* **A 8** (1993) 2605;
T. Huang, B.Q. Ma and Q.X. Shen, *Phys. Rev.* **D 49** (1994) 1490 [hep-ph/9402285];
V.M. Belyaev and M.B. Johnson, *Phys. Lett.* **B 423** (1998) 379 [hep-ph/9707329];
V.Yu. Petrov *et al.*, *Pion and photon light-cone wave functions from the instanton vacuum*, hep-ph/9807229.

[6] V.M. Braun and I.E. Filyanov, *Z. Physik* **C 48** (1990) 239.

[7] P. Ball *et al.*, *Nucl. Phys.* **B 529** (1998) 323 [hep-ph/9802299].

[8] P. Ball and V.M. Braun, *Handbook of higher twist distribution amplitudes of vector mesons in QCD*, hep-ph/9808229, to appear in the Proceedings of the 3rd Workshop on *Continuous Advances in QCD*, Minneapolis (MN), USA, April 1998.

[9] P. Ball and V.M. Braun, *Higher twist distribution amplitudes of vector mesons in QCD: twist-4 distributions and meson mass corrections*, hep-ph/9810475.

[10] O. Nachtmann, *Nucl. Phys.* **B 63** (1973) 237.

[11] P. Ball, *J. High Energy Phys.* **09** (1998) 005 [hep-ph/9802394].

[12] J.D. Bjorken and S.D. Drell, *Relativistic Quantum Fields* McGraw-Hill, New York 1965.

[13] Particle Data Group (C. Caso et al.), *Eur. Phys. J.* **C 3** (1998) 1.

[14] P. Ball, J.M. Frere and M. Tytgat, *Phys. Lett.* **B 365** (1996) 367 [hep-ph/9508359].

[15] T. Feldmann, P. Kroll and B. Stech, *Phys. Rev.* **D 58** (1998) 114006 [hep-ph/9802409].

[16] A. Pich, *Probing the standard model of particle interactions*, talk given at Les Houches Summer School in Theoretical Physics, Session 68: Les Houches, France, 28 Jul – 5 Sep 1997, hep-ph/9806303.

[17] E.V. Shuryak and A.I. Vainshtein, *Nucl. Phys.* **B 199** (1982) 451; *Nucl. Phys.* **B 201** (1982) 141.

[18] I.I. Balitskii and V.M. Braun, *Nucl. Phys.* **B 311** (1989) 541.

[19] S.J. Brodsky *et al.*, *Phys. Lett.* **B 91** (1980) 239; *Phys. Rev.* **D 33** (1986) 1881; *Phys. Lett.* **B 167** (1986) 347;
Yu.M. Makeenko, *Sov. J. Nucl. Phys.* **33** (1981) 440, *Yad. Fiz.* **33** (1981) 842;
Th. Ohrndorf, *Nucl. Phys.* **B 198** (1982) 26;
D. Müller, *Phys. Rev.* **D 51** (1995) 3855 [hep-ph/9411338]; *Phys. Rev.* **D 58** (1998) 054005 [hep-ph/9704406];
A. Belitsky and D. Müller, *Phys. Lett.* **B 417** (1998) 129 [hep-ph/9709379]; *Nucl. Phys.* **B 537** (1999) 397 [hep-ph/9804379].

[20] I.I. Balitskii and V.M. Braun, *Nucl. Phys.* **B 361** (1991) 93.

[21] A.R. Zhitnitsky, I.R. Zhitnitsky and V.L. Chernyak, *Sov. J. Nucl. Phys.* **41** (1985) 284, *Yad. Fiz.* **41** (1985) 445.

[22] V.A. Novikov *et al.*, *Nucl. Phys.* **B 237** (1984) 525.