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THEORIES EQUIVALENT TO SPECIAL RELATIVITY

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INTRODUCTION

The purpose of this paper is twofold: (1) to discuss the basis of the Lorentz transformations showing that the invariance of the velocity of light has in them a role even more important than usually believed, and (2) to find the complete set of theories empirically equivalent to the special theory of relativity (STR) under the assumption that the one-way velocity of light is not measurable.

In particular it will be shown that any modification of the coefficients of the Lorentz transformations, however small, gives rise to an ether theory, in the sense that the modified theory necessarily predicts the existence of a privileged frame that in principle can be detected experimentally. Therefore all the theories equivalent to STR but based on different transformation laws, must necessarily negate the validity of the relativity principle. We will come thus to the surprising conclusion that if the one-way velocity of light is not measurable, the content of the relativity principle is entirely conventional, since it can be affirmed or negated without any practical change in the predictive power of the theory.

SPACE-TIME TRANSFORMATIONS

The task of the present section is to study once more an old problem, how to set up the most general transformation laws of Cartesian coordinates and of time between two different inertial systems S and S'. We will show that these transformations must be linear if the following conditions hold:

"Empty" space is homogeneous, that is, it has the same properties in all points. Also time is homogeneous, that is, all properties of space remain the same with passing time.

We suppose that there is at least one inertial reference frame in which Maxwell's equations are valid, and that it coincides with S ("stationary system"). With respect to S the velocity of light is c in all directions, a well known consequence of Maxwell's equations. Therefore in S clocks can be synchronized by using Einstein's procedure.

Let us consider the most general form of space-time transformations:

$$\begin{aligned} x' &= f(x, y, z, t) \\ y' &= g(x, y, z, t) \\ z' &= h(x, y, z, t) \\ t' &= e(x, y, z, t) \end{aligned} \tag{1}$$

where f, g, h, e are four functions of the space-time coordinates of the system S. Particularly interesting is the function e giving the time t' of S'. There is of course a considerable arbitrariness in the operative definition of simultaneity for two clocks placed in different points of a moving inertial system [1], and one can therefore define a "time" which is not the same in the two inertial systems S(x, y, z, t) and S'(x', y', z', t') considered in (1), and such that the "delay" t' - t (which a priori can be positive, zero, or negative) depends not only on the velocity of S' with respect to S, but also on the considered geometrical point. In other words a clock W' of S' can be retarded with respect to the clock W of S passing near it by a quantity depending not only on t, but also on the coordinates x', y', z' of the point where W' is placed. One can write therefore:

$$t' = \varepsilon(x', y', z', t) \tag{2}$$

where ε can be called "synchronisation function" and informs about how t' depends on t and on the point of S' where the clock W' is placed. Given a function of several variables it is in general enough that some of them are changed in order to obtain a different value of the function. Therefore, if we consider two events simultaneous in S (same t), but taking place in different points of space, Eq. (2) implies that they will in general not be simultaneous in t'. This is the relativity of simultaneity, of course. There is a considerable arbitrariness in the choice of ε which is largely conventional since it depends on the procedures used for synchronising clocks in S' [2]. The function ε is however not totally conventional, since its dependence on t gives rise to well known phenomena, e.g. to the positive result found by Hafele-Keating [3]. In any case ε is equivalent to e, since if one substitutes in (2) the first three Eq.s (1) one obtains a function depending on x, y, z, t, just like e. The simplest synchronisation in S' is of course the one implying no dependence of time t' on the space variables [4], but such a choice in not compatible with the relativity principle. Nevertheless it retains a great physical interest, as we will see.

From space-time homogeneity it follows that the variation of position generated by the addition in S of a rod of length Δx parallel to the x axis has the same effect in S' on the co-ordinates x', y', z' and on the time t', whichever be the point x, y, z and the time t where the rod is added. The functions (1) are very general and a priori it is of course not necessary that the considered Cartesian coordinates of S and of S' be orthogonal. One can write:

$$\begin{aligned} x' + \Delta x' &= f(x + \Delta x, y, z, t) \\ y' + \Delta y' &= g(x + \Delta x, y, z, t) \end{aligned} \tag{3}$$

$$z' + \Delta z' = h(x + \Delta x, y, z, t)$$
$$t' + \Delta t' = e(x + \Delta x, y, z, t)$$

There is naturally an effect of Δx also on t' since we saw that by changing the position at fixed t in general the time t' changes. In agreement with the previous considerations we assume that all the variations of the primed variables in the left-hand sides of (3) are independent of x, y, z, t but depend only on Δx. By subtracting (1) from the previous relations one has:

$$\Delta x' = f(x + \Delta x, y, z, t) - f(x, y, z, t) \qquad (4)$$

and so on. The left-hand side, and therefore also the right-hand side, must be independent of x, y, z, t. This will remain true if everything is divided by Δx and the limit $\Delta x \to 0$ is considered. It follows that

$$a_1 \equiv \frac{\partial f}{\partial x} \quad ; \quad b_1 \equiv \frac{\partial g}{\partial x} \quad ; \quad c_1 \equiv \frac{\partial h}{\partial x} \quad ; \quad d_1 \equiv \frac{\partial e}{\partial x}$$

will be independent of x, y, z, t, and therefore constant.

We can make analogous considerations with rods of length Δy and Δz, respectively parallel to the axes y and z, and with a time interval Δt. In the latter case we consider a fixed point x, y, z of S and in it an increase Δt of time t, and assume that the latter gives rise to variations of t' and of x', y', z' which are independent both of x, y, z and of the time t at which Δt is assumed to start. From these new conditions one obtains that also

$$a_2 \equiv \frac{\partial f}{\partial y} \quad ; \quad a_3 \equiv \frac{\partial f}{\partial z} \quad ; \quad a_4 \equiv \frac{\partial f}{\partial t}$$

must be constant (that is, independent of x, y, z, t). Analogous constants can obviously be found from the functions g, h, e, but we do not write them all down. As far as the function f is concerned one obtains by integration:

$$\frac{\partial f}{\partial x} = a_1 \quad \Rightarrow \quad x' = a_1 x + \alpha_1(yzt)$$

$$\frac{\partial f}{\partial y} = a_2 \quad \Rightarrow \quad x' = a_2 y + \alpha_2(xzt)$$

$$\frac{\partial f}{\partial z} = a_3 \quad \Rightarrow \quad x' = a_3 z + \alpha_3(xyt)$$

$$\frac{\partial f}{\partial t} = a_4 \quad \Rightarrow \quad x' = a_4 t + \alpha_4(xyz)$$

where the first result is obtained by integrating over x, the second one over y, and so on. The functions α_1, α_2, α_3, and α_4 are integration "constants": that is, constant with respect to the integration variable, but of course dependent on the other three variables. The previous four relations must be simultaneously valid, but <u>can be compatible with one another only if x' is linear in all the space-time variables of S</u>. Analogous conclusions can obviously be obtained for y', z', t' and one can write:

$$
\begin{aligned}
x' &= a_1 x + a_2 y + a_3 z + a_4 t + a_5 \\
y' &= b_1 x + b_2 y + b_3 z + b_4 t + b_5 \\
z' &= c_1 x + c_2 y + c_3 z + c_4 t + c_5 \\
t' &= d_1 x + d_2 y + d_3 z + d_4 t + d_5
\end{aligned}
\qquad (5)
$$

The transformation laws (5) will be our starting point in the coming sections and will be used , in particular, for evaluating the velocity of light in theories more general than the STR.

SIMPLIFIED CHOICE OF AXES

We have so obtained a very general set of transformations containing twenty coefficients, which are constant with respect to x, y, z, t, but can a priori depend on the particular system S' considered, and especially on its velocity v relative to the stationary system S. It is of course possible to reduce strongly the number of free coefficients by considering that Cartesian coordinates are perfectly arbitrary, and thus to be chosen on the basis of convenience criteria. In particular one can choose the axes in S and in S' in such a way that the straight line joining their origins be parallel to their relative velocity. This implies that there is a time at which the two origins coincide, and this is assumed to happen at time zero both in S and in S'. In other words we write:

$$[x = y = z = t = 0] \quad \Rightarrow \quad [x' = y' = z' = t' = 0]$$

where "⇒" is the symbol of implication. This condition used in (5) gives:

$$a_5 = b_5 = c_5 = d_5 = 0 \qquad (6)$$

Let us next assume that the plane (x, y) coincides with the plane (x', y') for all times t. One must then have for all x, y, t:

$$[z = 0] \quad \Rightarrow \quad [z' = 0]$$

From the third Eq. (5) one gets immediately:

$$c_1 = c_2 = c_4 = 0 \qquad (7)$$

Now we assume that also the plane (x, z) coincides with the plane (x', z') at all times t. One must then have for all x, z, t:

$$[y = 0] \quad \Rightarrow \quad [y' = 0]$$

From the second Eq. (5) one obtains:

$$b_1 = b_3 = b_4 = 0 \tag{8}$$

We saw that our Cartesian coordinates are not necessarily orthogonal: therefore the coincidence of two planes of coordinates has in general no implication for the third plane. We assume however that at time $t = 0$ also the plane (y, z) coincides with the plane (y', z'): this is like saying that the two systems of coordinates overlap exactly at time zero. Therefore:

$$[t = x = 0] \quad \Rightarrow \quad [x' = 0]$$

From the first Eq. (5) one has:

$$a_2 = a_3 = 0 \tag{9}$$

We consider now the condition arising from velocity: let the origin of S' (equation $x' = 0$) be seen from S to move with velocity **v** parallel to the x axis, that is with equation $x = vt$. In other words:

$$[x = vt] \quad \Rightarrow \quad [x' = 0]$$

From the first Eq. (5) one has:

$$a_4 = -a_1 v \tag{10}$$

We can now rewrite the transformation laws (5) by keeping into account (6), (7), (8), (9), and (10). They become:

$$\begin{aligned} x' &= a_1(x - v\,t) \\ y' &= b_2\,y \\ z' &= c_3\,z \\ t' &= d_1\,x + d_2\,y + d_3\,z + d_4\,t \end{aligned} \tag{11}$$

These are the most general transformation laws for two systems of Cartesian coordinates, in general non orthogonal, perfectly overlapping at times $t = t' = 0$ and with the velocity **v** of S' parallel to the axes x and x'.

If we specify <u>at this point</u> that the Cartesian coordinates are orthogonal, we can also assume a complete equivalence of the axes y and z. In fact no physical phenomenon can distinguish them, if space is isotropic. Therefore:

$$b_2 = c_3 \quad ; \quad d_2 = d_3 \tag{12}$$

It is now necessary to invert the system (11), of course after keeping into account (12). It is a simple matter to obtain:

$$x = \frac{1}{a_1}\,x' + v\,Q$$

$$y = \frac{1}{b_2}\,y' \tag{13}$$

$$z = \frac{1}{b_2} z'$$

$$t = Q$$

where

$$Q = \frac{1}{R} \left[t' - \frac{d_1}{a_1} x' - \frac{d_2}{b_2} (y' + z') \right] \tag{14}$$

with

$$R = d_4 + d_1 v \tag{15}$$

These results are not valid if $a_1, b_2, R = 0$, cases devoid of any physical interest, since they respectively imply that x' vanishes for all x, that y' vanishes for all y, and that t' does not depend on t in the x'y'z' space, as one can see from (11). Eq.s (11)-(15) will allow us to calculate the velocity of light in S'. The same velocity in S is c by hypothesis. By assuming that c holds also in S' it is of course possible to move towards the Lorentz transformations. The generality of our results will make it easy to study the empirical consequences of theories equivalent to SRT, but not based on the relativity principle.

THE ONE-WAY VELOCITY OF LIGHT

In the inertial system S consider two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ and suppose that a light signal leaves P_1 at time t_1 and arrives in P_2 at time t_2. Since in S Maxwell's equations have an unlimited validity one must have:

$$c^2 \Delta t^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 \tag{16}$$

where

$$\Delta x = x_2 - x_1 \;;\; \Delta y = y_2 - y_1 \;;\; \Delta z = z_2 - z_1 \;;\; \Delta t = t_2 - t_1$$

The events "departure of the light signal from P_1 at time t_1" and "arrival of the light signal in P_2 at time t_2" will be described in S' respectively as "departure of the light signal from P_1' at time t_1' " and "arrival of the light signal in P_2' at time t_2' ", where primed and unprimed variables are related via (13) as follows

$$x_1 = \frac{1}{a_1} x_1' + v Q_1 \quad ; \quad x_2 = \frac{1}{a_1} x_2' + v Q_2$$

$$y_1 = \frac{1}{b_2} y_1' \quad ; \quad y_2 = \frac{1}{b_2} y_2'$$

$$z_1 = \frac{1}{b_2} z_1' \quad ; \quad z_2 = \frac{1}{b_2} z_2'$$

$$t_1 = Q_1 \quad ; \quad t_2 = Q_2$$

Here Q_1 and Q_2 have expressions similar to (14) with the space-time variables bearing the index "1" and "2", respectively. By subtracting from one another the relations of every line one gets equations strictly similar to (13), but written in terms of space-time intervals:

$$\begin{aligned} \Delta x &= \frac{1}{a_1}\Delta x' + v\,\Delta Q \\ \Delta y &= \frac{1}{b_2}\Delta y' \\ \Delta z &= \frac{1}{b_2}\Delta z' \\ \Delta t &= \Delta Q \end{aligned} \tag{17}$$

where

$$\Delta x' = x_2' - x_1' \; ; \; \Delta y' = y_2' - y_1' \; ; \; \Delta z' = z_2' - z_1' \; ; \; \Delta t' = t_2' - t_1'$$

and

$$\Delta Q = \frac{1}{R}\left[\Delta t' - \frac{d_1}{a_1}\Delta x' - \frac{d_2}{b_2}(\Delta y' + \Delta z')\right] \tag{18}$$

with R given by (15) as before. From (16) it follows:

$$c^2\,\Delta Q^2 = \left[\frac{1}{a_1}\Delta x' + v\,\Delta Q\right]^2 + \left(\frac{1}{b_2}\right)^2(\Delta y'^2 + \Delta z'^2) \tag{19}$$

which can be solved as a second degree equation in ΔQ and gives:

$$\Delta Q = \frac{\beta \Delta x'}{c a_1(1-\beta^2)} \pm \frac{1}{c}\left[\frac{\Delta x'^2}{a_1^2(1-\beta^2)^2} + \frac{\Delta y'^2 + \Delta z'^2}{b_2^2(1-\beta^2)}\right]^{1/2} \tag{20}$$

where, as usual, $\beta = v/c$. By inserting (18) in (20) one can obtain $\Delta t'$, the time of propagation in S' of the light signal from $P_1'(x_1', y_1', z_1')$ to $P_2(x_2', y_2', z_2')$. The result is

$$\Delta t' = \frac{d_1}{a_1}\Delta x' + \frac{d_2}{b_2}(\Delta y' + \Delta z') + \frac{R\,\beta\Delta x'}{c a_1(1-\beta^2)} \pm \frac{R}{c}\left[\frac{\Delta x'^2}{a_1^2(1-\beta^2)^2} + \frac{\Delta y'^2 + \Delta z'^2}{b_2^2(1-\beta^2)}\right]^{1/2} \tag{21}$$

Of course $\Delta t'$ is positive also in the case $\Delta x' = 0$ and $\Delta y' + \Delta z' = 0$ (but with $\Delta y'^2 + \Delta z'^2 \neq 0$). Therefore the minus sign in (21) must be discarded. Consider now in S' a suitable system of polar coordinates with centre on the straight line joining P_1' and P_2', which are of course <u>fixed points</u> of S':

$$\Delta x' = \Delta r' \cos\theta' \; ; \; \Delta y' = \Delta r' \sin\theta' \cos\varphi' \; ; \; \Delta z' = \Delta r' \sin\theta' \sin\varphi' \tag{22}$$

By inserting (22) in (21) one obtains:

$$\frac{\Delta t'}{\Delta r'} = \left[\frac{d_1}{a_1} + \frac{R\beta}{ca_1(1-\beta^2)} \right] \cos\theta' + \frac{d_2}{b_2} \sin\theta'(\sin\varphi' + \cos\varphi') +$$

$$+ \frac{R}{c} \left[\frac{\cos^2\theta'}{a_1{}^2(1-\beta^2)^2} + \frac{\sin^2\theta'}{b_2{}^2(1-\beta^2)} \right]^{1/2} \qquad (23)$$

This ratio between time and distance represents of course the inverse velocity of the light signal with respect to S'. Its dependence on θ' and φ' implies that in general it will not equal c^{-1}. The task of the following section is to study the case in which (23) is not only independent of the angles, but also exactly equal to the known value of the inverse velocity of light. These requirements are of course consequences of the relativity principle. In the last section we will however also consider points of view not compatible with relativity.

INVARIANCE OF THE VELOCITY OF LIGHT

The condition of isotropy of the velocity of light in S' can easily be seen from (23) to be equivalent to the following three requirements:

1) Independence of the azimuthal angle φ':

$$d_2 = 0 \qquad (24)$$

2) Disappearance of the term linear in $\cos\theta'$:

$$d_1 = -\frac{\beta}{c} d_4 \qquad (25)$$

3) Disappearance of the residual dependence on θ':

$$b_2 = a_1\sqrt{1-\beta^2} \qquad (26)$$

The three previous conditions are clearly sufficient for the isotropy of the velocity of light in S', but they are also necessary as can easily be shown by requiring that the derivatives of (23) vanish for all angles. From (24)-(26) one easily obtains:

$$\frac{\Delta t'}{\Delta r'} = \frac{R}{ca_1(1-\beta^2)} \qquad (27)$$

Eq. (27) implies that the velocity of light is isotropic, but it does not yet establish that it equals c. This can be imposed as a new condition and one then obtains:

$$d_4 = a_1 \qquad (28)$$

From (25) and (28) one also gets d_1 in terms of a_1. One should notice the great theoretical power of Einstein's condition on the velocity of light, that has given four relations for the five coefficients entering in (11) [after taking into account

(12)]. The most general transformation laws between inertial systems leaving the velocity of light isotropic and equal to c are thus:

$$x' = a_1 (x - \beta ct)$$

$$y' = a_1 \sqrt{1-\beta^2}\, y$$

$$z' = a_1 \sqrt{1-\beta^2}\, z \tag{29}$$

$$t' = a_1 (t - \beta x/c)$$

where only one undetermined coefficient is left, namely a_1. Nothing more can be said if only the invariance of the velocity of light is assumed, but a_1 can be fixed by using the relativity principle in a different way.

LAST CONSEQUENCE OF RELATIVITY

The simplest way to determine a_1 is to observe that by inverting the system (29) one has

$$y = \frac{1}{a_1 \sqrt{1 - \beta^2}}\, y'$$

The only difference between the previous relation and the second of (29) is in the factor multiplying the space variable, which is here inverted. Such a coefficient gives the "contraction" of a rod put on the y axis of S and seen from S', while its inverse gives the "contraction" of the same rod put on the y' axis of S' and seen from S. Obviously, if in one case there is a real contraction (i.e., if the coefficient is less than unity), in the other case there is an expansion, and vice versa. The principle of relativity requires however that S and S' should observe the same effect, and the only possibility to achieve this is clearly to require that the said coefficient has value unity, i.e. that

$$a_1 = \frac{1}{\sqrt{1-\beta^2}} \tag{30}$$

Obviously (29) and (30) together are equivalent to the Lorentz transformations. Initially it had been assumed that the velocity of light was c only in S, but later this requirement has been extended also to S' and a complete symmetry between the two systems has been introduced. Given the arbitrariness of S and S' we can conclude that the Lorentz transformations hold for any pair of inertial systems. The particular system S from which our reasoning started was called "stationary", but in the relativistic line of thought it loses at the end every peculiarity and becomes any one of the infinitely many equivalent inertial systems that can be conceived.

A different form of the transformation of time can be obtained by substituting the first Eq. (29) into the fourth one, and by using (30). One gets so:

$$t' = \sqrt{1-\beta^2}\, t - \frac{\beta}{c} x' \tag{31}$$

from which one sees that the "delay" of the time t' of S' with respect to the time t of S has a double origin since it arises both from the factor multiplying t and from the presence of the term proportional to x'. In order to obtain the velocity c for the light signals in all inertial frames a very particular clock synchronisation was needed, which generated the precise dependence (31) of time on space. Recent research has led to the conclusion that such a synchronization is basically conventional and does not necessarily reflect an objective property of physical reality[5]. Einstein himself was aware of this aspect of clock synchronization[6].

THE TWO-WAY VELOCITY OF LIGHT

It is important to stress that Eq.s (24)-(30) are all necessary consequences of the relativity principle. Their eventual violation implies thus that relativity itself does not hold as a description of nature. It can therefore be said that the STR is "unstable", in the sense that any shift, however small, of any one of the five coefficients a_1, b_2, d_4, d_1, and d_2 away from their relativistic values [given respectively by (30), (26), (28), (25), and (24)] implies necessarily the existence of a privileged system[7]. In other words, either Lorentz has given mankind a final truth with his transformations, or some kind of ether shall have to be accepted in the future[8]. After all Einstein modified his negative opinion about ether and after 1916 reverted to acceptance of this conception[9], and today there are even proposals of detecting it with suitable new experiments[10].

The problem is not only that experiments can never check a mathematical expression with infinite precision, so that it is always possible to conceive small deviations of the coefficients from their relativistic values; the problem is also that arguments have recently been advanced[11] in favour of the thesis that the one-way velocity of light is measurable neither directly nor indirectly, with the consequence that d_1 and d_2 are essentially arbitrary and only dependent on the synchronization procedure chosen in the moving reference frame.

In fact we have seen that the left-hand side of (23) is the inverse <u>one-way</u> velocity of light in the direction specified by θ' and φ'. In practically all the performed measurements only the two-way velocity of light has been measured in a system S' instantaneously at rest with respect to the Earth, for example by sending a light pulse from a point P_1' to a point P_2' in which a mirror was placed that reflected the light back to P_1'. In such experiments the velocity of light was measured as the ratio between twice the distance $\Delta r'$ from P_1' to P_2' and the time necessary for the $P_1'P_2'P_1'$ round trip. Introducing the symbols F and B to specify the forward (from P_1' to P_2') and backward (from P_2' to P_1') trip, respectively, one must have for the time intervals:

$$\Delta t'_{FB} = \Delta t'_F + \Delta t'_B$$

which can also be written in terms of velocities:

$$\frac{2\Delta r'}{c'_{FB}(\theta')} = \frac{\Delta r'}{c'_F(\theta',\varphi')} + \frac{\Delta r'}{c'_B(\theta',\varphi')} \qquad (32)$$

where c' indicates the velocity of light in S'. By eliminating $\Delta r'$ from (32) we see that <u>the inverse two-way velocity of light is the average between the inverse forward and backward one-way velocities.</u> Given the obvious fact that

$$c'_B(\theta',\varphi') = c'_F(\pi-\theta',\varphi'+\pi)$$

one has from (23)

$$\frac{1}{c'_{FB}(\theta')} = \frac{R}{c}\left[\frac{\cos^2\theta'}{a_1^2(1-\beta^2)^2} + \frac{\sin^2\theta'}{b_2^2(1-\beta^2)}\right]^{1/2} \qquad (33)$$

which justifies our notation $c'_{FB}(\theta')$, since all dependence on φ' has disappeared. As one can see the terms proportional to d_1 and d_2 are indeed absent in (33).

If we now impose only the condition that in S' the two-way velocity of light equals c, from (33) we get the results:

$$b_2 = a_1\sqrt{1-\beta^2} \quad ; \qquad R = a_1(1-\beta^2)$$

Because of (15) the last equation is equivalent to:

$$d_4 = a_1(1-\beta^2) - d_1 v$$

Therefore the most general transformation laws giving a two-way velocity of light equal to c in all inertial frames are

$$\begin{aligned} x' &= a_1(x - \beta ct) \\ y' &= a_1\sqrt{1-\beta^2}\; y \\ z' &= a_1\sqrt{1-\beta^2}\; z \\ t' &= a_1(1-\beta^2)\,t + d_1(x - \beta ct) + d_2(y+z) \end{aligned} \qquad (34)$$

These transformations represent a good approximation to reality, since the two-way velocity of light is known with a precision better than 10^{-11}. It is possible to introduce in (34) some further information, by considering that time dilation is also a well established phenomenon[12], even though the numerical precision is not as good as that concerning the two-way velocity of light. A clock at rest in the origin of S' satisfies $y = z = 0$ and $x = \beta ct$ and the last Eq. (34) gives the well known time-dilation effect only if a_1 satisfies (30). This can be inserted in (34) and the first three equations become identical with the corresponding ones of the Lorentz transformations, but the last one becomes:

$$t' = \sqrt{1-\beta^2}\; t + d_1(x - \beta ct) + d_2(y+z)$$

At this point length contraction by the usual factor $\sqrt{1-\beta^2}$ is a consequence of the transformations (34). As is well known this phenomenon has never been directly verified[13], in spite of opposite claims[14]. Length contraction has been shown to find a natural explanation in the frame of prerelativistic physics, due to deformation of the electromagnetic field of moving charges[15]. There is of course a considerable freedom left in the transformations (34), but nevertheless not enough to accept easily the idea that the Galilei transformations should be preferred[16]. This can still be done only by invoking the strange idea that Earth is at rest in ether. In all cases (34) represent the complete set of theories equivalent to special relativity: if d_1 and d_2 are varied, different theories are obtained which are all equivalent to STR as far as the explanation of experimental results is concerned. In all cases but that of STR such theories negate the relativity principle which becomes thus a disposable convention. In the case $d_1 = d_2 = 0$, corresponding to the so-called absolute synchronization[1], the Tangherlini transformations[4] are obtained, which are of course incompatible with the relativity principle, but are nevertheless particularly simple and elegant.

If the one-way velocity of light should turn out to be measurable, contrary to expectations, the previous results would anyway imply that today there is still a large set of theories logically possible, because compatible with empirical evidence, and that only future experiments will choose the right one.

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