

Theory and Design of Digital Communication Systems

Providing the underlying principles of digital communication and the design techniques of real-world systems, this textbook prepares senior undergraduate and graduate students for the engineering practices required in industry. Covering the core concepts, including link analysis, modulation, demodulation, spread spectrum, equalization, channel fading effects, and channel coding, it provides step-by-step mathematical derivations to aid understanding of background material. In addition to describing the basic theory, the principles of system and subsystem design are introduced, enabling students to visualize the intricate connections between subsystems and understand how each aspect of the design supports the overall goal of achieving reliable communications. Throughout the book, theories are linked to practical applications with over 250 real-world examples, whilst 370 varied homework problems in three levels of difficulty enhance and extend the text material. With this textbook, students can understand how digital communication systems operate in the real world, learn how to design subsystems, and evaluate end-to-end performance with ease and confidence.

Tri T. Ha is a Professor in the Department of Electrical and Computer Engineering at the Naval Postgraduate School (NPS), Monterey, California, a position he has held since 1989. Prior to joining NPS he worked at Fairchild Industries and GTE, and was an Associate Professor at Virginia Tech for four years. He is an IEEE Fellow who has written two previous textbooks, and his current research interests are in wireless communications and cyber warfare.

Cambridge University Press

978-0-521-76174-1 - Theory and Design of Digital Communication Systems

Tri T. Ha

Frontmatter

[More information](#)

Theory and Design of Digital Communication Systems

TRI T. HA

Naval Postgraduate School, Monterey, California



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
978-0-521-76174-1 - Theory and Design of Digital Communication Systems
Tri T. Ha
Frontmatter
[More information](#)

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521761741

© Cambridge University Press 2011

This publication is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without the written
permission of Cambridge University Press.

First published 2011

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

Library of Congress Cataloging in Publication data

Ha, Tri T., 1949–
Theory and design of digital communication systems / Tri T. Ha.

p. cm.
ISBN 978-0-521-76174-1 (hardback)

1. Digital communications. I. Title.
TK5103.7.H35 2011

384–dc22
2010024374

ISBN 978-0-521-76174-1 Hardback

Additional resources for this publication at www.cambridge.org/9780521761741

Cambridge University Press has no responsibility for the persistence or
accuracy of URLs for external or third-party internet websites referred to
in this publication, and does not guarantee that any content on such
websites is, or will remain, accurate or appropriate.

Cambridge University Press

978-0-521-76174-1 - Theory and Design of Digital Communication Systems

Tri T. Ha

Frontmatter

[More information](#)

World peace must develop from inner peace. Peace is not the absence of violence. Peace is the manifestation of human compassion.

14th Dalai Lama of Tibet (Inscription on United States Congressional Medal)

Contents

<i>Preface</i>	<i>page</i> xvii
<i>Acknowledgements</i>	xix
<i>List of symbols</i>	xx
<i>List of abbreviations</i>	xxx
1 Introduction	1
1.1 Brief overview	1
1.2 Scope	3
1.3 Summary	8
Bibliography	9
2 Deterministic signal analysis	10
Introduction	10
2.1 General description of deterministic signals	10
Continuous-time signals	11
Discrete-time signals	11
Periodic and aperiodic signals	12
Analog and digital signals	13
2.2 Power and energy	16
2.3 Orthogonal signals	21
2.4 Signal space	27
Gram–Schmidt procedure	27
Signal vectors	28
2.5 Linear time-invariant systems	34
2.6 Convolution	35
2.7 Fourier series of continuous-time periodic signals	37
Parseval relation for periodic signals	39
Unit step and unit impulse functions	39
Power spectral density	41
Bandwidth	41
Frequency shifting property	42
Response of an LTI system to a periodic signal	45
2.8 Fourier transform of continuous-time signals	47
Frequency shifting property	50
Parseval relation	52
Fourier transform of periodic signals	53

Response of LTI systems	53
Tables of Fourier properties and Fourier transform pairs	56
2.9 Autocorrelation	56
Autocorrelation and energy spectral density	57
Autocorrelation and power spectral density	59
Output autocorrelation and power spectral density	61
2.10 Sampling	62
Sampling theorem	62
Impulse-train sampling	63
Reconstruction with an ideal lowpass filter	64
2.11 Bandpass signals	68
Representations	68
Response of an LTI bandpass system	70
2.12 Summary	72
Problems	73
Further reading	77
Bibliography	77
3 Random signal analysis	78
Introduction	78
3.1 Review of probability theory	78
Total probability theorem	79
Bayes theorem	79
Independence	79
Union bound	79
3.2 Random variables	79
Bayes theorem and total probability revisited	81
3.3 Random processes	82
Autocorrelation and autocovariance	82
Types of random processes	85
Power spectral density	92
3.4 Gaussian process	100
Linear transformation	100
Sampling	102
Sufficient statistics for signal processing in white Gaussian noise	103
Karhunen–Loeve expansion	105
Whitening filter	106
The central limit theorem	108
3.5 Gaussian-derived processes	108
Rayleigh and Rice processes	108
Squared envelope	111
Sum of squared Gaussian processes	112
Nakagami-m density function	113
Log-normal density function	114

3.6 Summary	117
Problems	118
Further reading	121
Bibliography	121
4 Information theory and channel coding	122
Introduction	122
4.1 Entropy of a discrete source	122
4.2 Source coding	124
Huffman codes	126
Shannon noiseless source coding theorem (first theorem)	127
4.3 Discrete channel	128
Mutual information	129
Channel capacity	131
Shannon noisy channel coding theorem (main theorem)	131
4.4 Gaussian channel	134
Differential entropy	136
Mutual information and channel capacity	138
Bandlimited Gaussian channel capacity	144
Gaussian channel with discrete inputs	144
4.5 Channel coding with block codes	147
4.6 Low-density parity-check codes (LDPC)	152
Tanner graph	153
Message passing algorithm (MPA)	154
4.7 Channel coding with convolutional codes	160
Hard decoding–BSC	163
Soft decoding–DMC	165
Soft decoding–Gaussian input vector	167
4.8 Summary	169
Problems	170
Further reading	175
Bibliography	175
5 Communication link analysis	177
Introduction	177
5.1 Basic wireless communication link	177
Thermal noise	178
Effective noise temperature	181
Receiver noise model	187
System signal-to-noise ratio	190
5.2 Cellular communication link	191
Frequency spectrum	192
Major cellular standards	192
Cell connection	195

Path loss: two-ray ground reflection model	196
Hata model	199
Modified Hata model	202
Reciprocity and the reverse link	203
5.3 Co-channel interference in a narrowband cellular system	203
Combined signal-to-interference and noise ratio	206
Sectoring	208
Microcell-zoning	210
5.4 CDMA cellular link analysis	211
Forward link	211
Reverse link	214
5.5 Satellite communication link	215
5.6 Summary	219
Problems	220
Further reading	224
Bibliography	224
6 Modulation	225
Introduction	225
6.1 Review of double sideband-amplitude modulation (DSB-AM)	225
6.2 Digital modulation	229
6.3 Phase shift keying (PSK)	231
6.4 Differential phase shift keying (DPSK)	234
6.5 Amplitude shift keying (ASK)	236
6.6 Frequency shift keying (FSK)	238
Power spectral density	240
6.7 Minimum shift keying (MSK)	242
Power spectral density	243
Modulator	247
6.8 Gaussian minimum shift keying (GMSK)	249
6.9 The need for M-ary modulation	251
6.10 M-ary amplitude shift keying (MASK)	252
Signal space	252
Power spectral density	253
Modulator	255
6.11 M-ary phase shift keying (MPSK)	255
Signal space	256
Power spectral density	258
Modulator	260
Offset quadrature phase shift keying (OQPSK)	260
6.12 Differential M-ary phase shift keying (DMPSK)	262
Alternative differential encoding for DQPSK	264
Direct symbol mapping	265
Modulator	265

6.13 $\pi/4$ -shifted differential quadrature phase shift keying ($\pi/4$ -DQPSK)	266
Signal space	267
Direct symbol mapping	268
6.14 M-ary quadrature amplitude modulation (MQAM)	270
Signal space	270
Power spectral density	271
Differential MQAM (DMQAM)	274
6.15 Code shift keying (CSK)	275
Power spectral density	276
Modulator	276
6.16 M-ary frequency shift keying (MFSK)	277
Power spectral density	278
Modulator	279
6.17 Continuous phase modulation (CPM)	280
Power spectral density	281
Modulator	282
6.18 Orthogonal frequency division multiplexing (OFDM)	282
Practical baseband implementation	283
Cyclic prefix	287
6.19 Trellis coded modulation (TCM)	290
Ungerboeck TCM	291
Pragmatic TCM	295
6.20 Summary	298
Problems	299
Further reading	302
Bibliography	302
7 Demodulation	305
Introduction	305
7.1 The matched filter	306
Time domain interpretation	306
Frequency domain interpretation	310
Output signal and noise waveforms	310
Decision variable	311
Summary	312
7.2 The correlator	312
7.3 The matched filter–envelope detector (noncoherent matched filter)	313
Output signal-to-noise ratio	317
Decision variable	318
Summary	318
7.4 The quadrature correlator–square law detector (noncoherent correlator)	318
7.5 The threshold detector	320
Optimum threshold	320
Maximum likelihood criterion	321

7.6 The maximum detector	322
Gaussian decision variables	323
Rice and Rayleigh decision variables	323
7.7 Binary demodulation	325
Coherent PSK	326
Coherent DPSK	328
Direct detection ASK	329
Coherent FSK	331
Coherent MSK and precoded MSK and GMSK	332
Noncoherent FSK and MSK	337
Noncoherent DPSK	339
Performance summary of binary modulation techniques	345
7.8 Minimum Euclidean distance detector	345
Symbol error probability	348
7.9 M-ary maximum detector	350
Gaussian decision variables	351
Rice and Rayleigh decision variables	353
7.10 M-ary demodulation	355
Coherent L -path demodulator: a signal space approach	355
Coherent M -path demodulator	358
Noncoherent M -path demodulator	360
7.11 Coherent MASK	362
7.12 Coherent MPSK	366
7.13 Coherent DMPSK	371
7.14 Noncoherent DMPSK	375
7.15 Noncoherent $\pi/4$ -DQPSK	379
7.16 Coherent MQAM and DMQAM	380
7.17 Coherent CSK and MFSK	383
7.18 Noncoherent CSK and MFSK	384
7.19 Coherent CPM with sequence detection	386
7.20 Coherent CPM with symbol-by-symbol detection	395
7.21 Noncoherent CPM	397
7.22 Performance summary of M-ary modulation techniques	401
7.23 OFDM demodulation	402
Timing synchronization	403
Carrier phase synchronization	409
7.24 Binary demodulation with convolutional codes	410
Hard decoding	411
Soft decoding—Gaussian input vector	412
Soft decoding— χ^2 input vector	414
7.25 TCM demodulation and decoding	416
7.26 Summary	422
Appendix 7A: The Q-function	422

Problems	423
Further reading	430
Bibliography	430
8 Spread spectrum	432
Introduction	432
8.1 Direct sequence modulation	432
Orthogonal covering	436
IS-95 forward link	440
Code division multiple access (CDMA)	443
IS-95 reverse link	444
8.2 Direct sequence demodulation	446
Quadrature orthogonal covering demodulation	450
Noncoherent demodulation of DS-CSK	454
CDMA: performance evaluation	457
8.3 Frequency hop spread spectrum	460
Partial-band jamming	461
Multi-tone jamming	462
Follower jamming	464
8.4 Summary	466
Problems	466
Further reading	471
Bibliography	471
9 Intersymbol interference and equalization	473
Introduction	473
9.1 Intersymbol interference	473
Nyquist criterion for zero ISI	475
9.2 Optimum demodulator for bandlimited channel	477
Condition for maximum SNR_0	480
Condition for zero ISI	480
Solution for $ H_T(f) $ and $ H_R(f) $	480
9.3 Zero-forcing linear equalizer (ZF-LE)	482
Summary	493
9.4 Mean-square error linear equalizer (MSE-LE)	494
9.5 Zero-forcing decision-feedback equalizer (ZF-DFE)	501
9.6 Mean-square error decision-feedback equalizer (MSE-DFE)	506
9.7 Maximum likelihood sequence detection	509
9.8 Fractionally spaced equalizer (FSE)	516
9.9 Summary	517
Problems	518
Further reading	520
Bibliography	520

10 Fading channels	522
Introduction	522
10.1 Physical characterization of fading channels	524
Time-varying effect	524
Space-varying effect	526
Summary of fading characteristics	527
10.2 Mathematical representation of fading channels	528
Channel impulse response	528
Multipath autocorrelation and Doppler profiles	530
Clarke–Doppler power spectrum	532
Generalized Doppler power spectrum	534
10.3 Coherent demodulation	536
Equivalent complex-valued demodulator	536
Rayleigh	539
Rice	540
Nakagami-m	541
Effect of channel tap error	543
10.4 Pilot symbol-aided decision-feedback demodulation	547
Differential and double-differential decision-feedback algorithms	549
10.5 OFDM	552
Low mobility	556
High mobility	557
10.6 Noncoherent demodulation	558
Rayleigh	558
Rice	558
Nakagami-m	559
Doppler tracking of MFSK	559
Doppler tracking of CSK	562
10.7 Pilot tone-aided demodulation of orthogonal covering signal	566
Complex spreading and despreading	566
Doppler analysis	567
10.8 Noncoherent demodulation of offset quadrature DS-CSK	572
10.9 Time diversity	574
Level crossing rate	575
Average fade duration	577
10.10 Maximal ratio combining (MRC)	578
Rayleigh	579
Rice	581
Nakagami-m	581
10.11 Selection combining (SC)	583
10.12 Equal gain combining (EGC)	585
Coherent EGC	585
Noncoherent EGC	586

10.13	Frequency diversity	587
	Fast frequency hop (FFH)	588
	OFDM subcarrier combining	588
	Rake receiver	590
10.14	Receive antenna diversity (SIMO)	595
	Mobile station antennas	596
	Base station antennas	597
	Performance	598
10.15	Transmit antenna diversity (MISO)	600
	Space-time coding	600
	Alamouti code	602
	Rate $\frac{1}{2}$ space-time code	603
	Sufficient statistics	604
	Rate $\frac{3}{4}$ space-time code	604
10.16	Transmit–receive antenna diversity (MIMO)	605
10.17	Channel capacity	608
	Slow fading	609
	Slow fading–receive antenna diversity	610
	Slow fading–transmit antenna diversity	611
	Slow fading–transmit and receive antenna diversity	612
	Slow fading–OFDM	612
	Fast fading	615
10.18	Summary	616
	Appendix 10A: Complex-valued demodulators	617
	Appendix 10B: Bit error probabilities	617
	Rayleigh	618
	Nakagami-m	618
	Rayleigh–diversity: χ^2 -density function with $2L$ degrees of freedom	619
	Problems	620
	Further reading	627
	Bibliography	627
	<i>Index</i>	629

Preface

This book was written with two goals in mind: to provide the underlying principles of digital communication and to study design techniques integrated with real world systems. The ultimate aim of a communication system is to provide reliable transmission of information to the user(s). This fundamental foundation was established in 1948 by Claude Shannon, the founding father of information theory, and led eventually to the development of modern digital communication. Analog communication is near extinction or at the very gate of it. The full spectrum dominance of digital communication has arrived and new frontiers are being established every decade; from cellular systems to wireless LAN and MAN, the bit rates are being pushed ever higher for ubiquitous mobile applications.

Knowing the limit of digital transmission is vital to the design of future communication systems, particularly mobile wireless systems, where both spectrum and power are precious resources, and design techniques can be used to manipulate these two main resources to fit real world applications. No single technique can cover all the requirements of a modern communication system, which makes it necessary for students to understand the intricate web between subsystems, each designed to support others to achieve the common goal of reliable communication.

The book contains more than 250 examples to help students achieve a firmer understanding of the subject. The problems at the end of each chapter follow closely the order of the sections. They are designed for three levels: level one covers the straightforward application of equations; level two requires patience and deep thinking; whilst level three requires some research of the literature to assist in finding a solution. A solutions manual for the instructor accompanies the book.

The book was written for both senior undergraduate and graduate students studying communications at universities and colleges. The entire book is suitable for two-semester courses in digital communications. The first course is typically a one-semester senior course in digital communication, which may be taken by students new to studying communications (the conventional wisdom is that students should learn analog communication before learning digital communications) or after completing an introductory course in communication systems (one that is heavy in analog communication systems such as AM and FM). The second course is a one-semester course for graduate students who already have a firm background in random variables and processes. The practical material included in this book (much of it focused on commercial and military systems) will be helpful for practitioners and professionals in the digital communication field.

As in the learning of any subject, some prerequisites are required for the reading of this book. A first course in probability theory is necessary and exposures to random processes

would be helpful. Readers should also be familiar with linear system analysis. A knowledge of analog communication is helpful but not required. For readers who do not have the patience to go through all the design techniques but would appreciate the beauty of the underlying principles, we recommend our favorite book, *Principles of Digital Communication*, authored by the legendary Robert G. Gallager.

Acknowledgements

I would like to express my thanks to Dr. Phil Meyler of Cambridge University Press for his enthusiastic support and for suggesting the title of the book to closely reflect its coverage. I would like to thank the anonymous reviewers for their valuable comments that helped in improving the book. I am grateful for the support of my friends Nathan Beltz, Professor Vicente Garcia, Professor Jeff Knorr, Professor Frank Kragh, Donna Miller, Rita Painter, Professor Clark Robertson, Professor Wei Su, Dr. Jan Tighe, and Dr. Charlie Victory. I also would like to thank my students over the years for their valuable suggestions to improve the original notes. I am grateful to my parents for providing me the opportunity to go to the United States for higher education, and to Thuy, Khanh, Tuan, Huong and Dennis, Thu, and Hien for their help and support over the years. This book is dedicated to the people who work tirelessly for world peace.

Symbols

A	amplitude, smallest signal amplitude in MQAM, azimuth angle
\mathbf{A}	matrix \mathbf{A}
$\ \mathbf{A}\ ^2$	squared Frobenius norm of matrix \mathbf{A}
\mathbf{A}^*	conjugate and transpose of a matrix \mathbf{A}
A_d	number of paths of Hamming weight d that merge with the all-zero paths
A_e	effective aperture area of the receiver antenna
$A \cap B, AB$	intersection of set A and set B , A and B
$A \cup B, A + B$	union of set A and set B , A or B
a	Gaussian filter parameter
$a(h_R)$	correction factor in Hata model
a_f	frequency sensitivity
a_k	Fourier series coefficients
B	bandwidth
B_d	information weight (number of information bit errors) of all paths of Hamming weight d
$B_{d_{free}}$	information weight (number of information bit errors) of all paths of Euclidean distance d_{free} of TCM
C	channel capacity in coded bits/input symbol, correlation
\mathbf{C}	channel capacity in coded bits/second
C_o	outage capacity
$c(\mathbf{D}_K)$	correlation metric in MLSD
C_N	correlation metric in CPM
$C_X(t_1, t_2)$	autocovariance of the random process $x(t)$
$C(z)$	transfer function of causal ZF-LE, z -transform of the sequence $c(k)$
$Cov(N_j N_k)$	covariance of two noise samples
\mathbf{c}	code word, PN sequence
c_i	a coded bit, a differentially coded bit
$c(t)$	PN function of N chips
$\mathbf{c}(t)$	complex PN function
$c_I(t)$	PN function of I-channel
$c_Q(t)$	PN function of Q-channel
$c(\mathbf{x} \mathbf{H}_i)$	metric for CPM demodulation
D_i	i th symbol in the symbol stream

d	distance
\mathbf{d}	data sequence
dB	decibel
d_{free}	free distance of a convolutional code
\mathbf{d}_{free}	Euclidean free distance of TCM
$d_i, d(i)$	normalized bit or symbol amplitude, $d_i \in \{-1, +1\}$, $d_i \in \{0, 1\}$
$\{d_i\}, \{d(i)\}$	data sequence
d_{min}	minimum Hamming distance, minimum Euclidean distance
$d(u,v)$	Hamming distance between two code words
$d(\mathbf{x}, \mathbf{s}_i)$	Euclidean distance between two vectors
E	energy (with or without a subscript), smallest symbol energy in MQAM or MASK, electric-field wave, elevation angle
E_b	bit energy
\mathcal{E}_b	diversity bit energy
$E(d,t)$	free space E -field at distance d from transmitter and time t
E_h	hop energy
$EIRP$	effective isotropic radiated power
E_0	free space E -field at distance d_0 from transmitter
E_s	symbol energy
\mathcal{E}_s	diversity symbol energy
$\mathbf{E}(X)$	expected value (mean value) of X
$\mathbf{E}(X^2)$	mean-square value of X^2
e	2.718
\mathbf{e}	error word
$e(k)$	error sequence of MSE-LE
$e(t)$	error process
F	noise figure
$F\{x(t)\}$	Fourier transform of $x(t)$
$F(z)$	minimum-phase transfer function, transfer function of a synthetic channel, z -transform of the sequence $f(k)$
$F^*(1/z^*)$	maximum-phase function
$F^{-1}\{X(f)\}$	inverse Fourier transform of $X(f)$
${}_2F_1$	Gauss hypergeometric function
$F_{ h ^2}^{(-1)}(p_o)SNR$	outage signal-to-noise ratio
$F_X(x)$	distribution function of X
$F_{XY}(x,y)$	joint distribution function of X and Y
$f_{ h }(x)$	density function of the channel tap magnitude $ h $
$f_{XY}(x,y)$	joint density function of X and Y
$f_{X Y}(x y)$	conditional density function of X given Y
f	frequency, Doppler shift

f_c	carrier frequency
f_D	Doppler spread
f_j	instantaneous carrier frequency of FH
$f_{L,i}$	instantaneous local carrier frequency of a frequency synthesizer
f_m	maximum Doppler shift
f_s	sampling frequency
$f_X(x)$	density function of X
$f_X(\mathbf{x})$	density function of vector \mathbf{X}
G	amplifier gain, gain of a two-port network
\mathbf{G}	generator matrix, space-time block code matrix
\mathbf{G}	TCM asymptotic coding gain
G_{DC}	gain of a downconverter
G_{LNA}	gain of a low-noise amplifier
G_R	receiver antenna gain
G_R/T_s	antenna gain-to-noise temperature ratio of the earth station
G_S/T_{sat}	antenna gain-to-noise temperature ratio of the satellite
G_T	transmitter antenna gain
g	parity
$g(t)$	pulse
$g_n(t)$	orthonormal pulse shapes in OFDM
$g(x)$	code generator polynomial
\mathbf{H}	Hadamard matrix (with or without a subscript), parity check matrix, channel tap matrix
\mathbf{H}	source entropy in bits/second
H_i	hypothesis
$H(f)$	transfer function or frequency response
$H_{FE}(f)$	transfer function of the front-end filter of the receiver
$ H(f) $	magnitude response (amplitude response)
$H_T(f)$	transfer function of transmit filter
$H_R(f)$	transfer function of receive filter
$H(k)$	N -point DFT of the sequence $h(n)$
$H(X)$	entropy of the discrete random variable (discrete source) X
h	Planck constant, digital modulation index, complex channel tap
\mathbf{h}	row vector of a Hadamard matrix
$\hat{\mathbf{h}}$	MMSE of the vector \mathbf{h}
$h_i, h(i)$	i th channel tap
$\tilde{h}(i)$	complex channel tap
$h_L(t)$	complex envelope of the impulse response $h(t)$
$h(n)$	sequence used in OFDM
h_R	receive antenna height

h_T	transmit antenna height
$h(t)$	impulse response
$h(\mathbf{X})$	differential entropy of a continuous random variable X
$h(X Y)$	conditional differential entropy of a continuous random variable X
$h(\mathbf{X})$	differential entropy of a continuous n -dimensional random vector \mathbf{X}
$h(x)$	monic binary irreducible primitive polynomial, PN code polynomial
I	interference power, photodiode current
I_m	interchannel interference
I	MUI variable
$I(X)$	self-information of the discrete random variable (discrete source) X
$I(u_i, v_j)$	pair-wise mutual information
$I(U, V)$	mutual information
$I_0(\bullet)$	modified Bessel function of the first kind of zero order
$I_n(\bullet)$	modified Bessel function of the first kind of n th order
J	jamming variable
$J_0, J_0(\bullet)$	jamming spectral density, Bessel function of zero order
K	number of simultaneous users in CDMA, Kelvin
\mathbf{K}	covariance matrix
k	integer, Boltzmann constant, number of information bits in a block code, number of inputs of a convolutional encoder
k_0	free space wave number
k/n	code rate
L_C	path loss
L	diversity order, loss of a two-port network
$L(f)$	transfer function of an equalizer
L_r	receive antenna diversity
L_s	number of symbol times for m transmitted symbols in transmit diversity
L_{TL}	loss of a transmission line
L_t	transmit antenna diversity
L_{dB}	mean value in decibels of the log-normal density variable
$L(\lambda, P_1, \dots, P_n)$	Lagrangian
l	length of a code word
\bar{l}	average length of a code word
\ln	natural logarithm
$\ln \Lambda(X y)$	conditional ln-likelihood ratio
\log	base-10 logarithm
\log_2	base-2 logarithm
M	number of distinct M-ary symbols
$\mathbf{M}(\mathbf{r} \mathbf{c})$	path metric in Viterbi algorithm or log-likelihood function
m	mean value of a random variable, Nakagami-m parameter

$m_A(X^{-1})$	arithmetic mean of $1/X(f)$
$m_G(X)$	geometric mean of $X(f)$
m_n	metric for OFDM timing synchronization
$m(t)$	message signal
$\mathbf{m}_{ij}^{(l)}$	message sent by the bit node i to check node j at the l th iteration
$\hat{\mathbf{m}}_{ji}^{(l)}$	message sent by the check node j to bit node i at the l th iteration
m_I^2	sum of the squares of the means
$m_X(t)$	mean value of the random process $x(t)$
N	noise variable, available noise power, number of OFDM subcarriers, period of a PN sequence
\mathbf{N}	noise vector, complex noise
\mathcal{N}	noise variable at detector input
\mathbf{N}	noise variable
\mathfrak{N}	complex noise variable, complex noise vector
$N(f)$	power spectral density of the equivalent lowpass noise $n(t)$
N_D	number of branches in a frequency bin determinant
N_H	number of hop bins
N_i	system noise power
N_n	average number of nearest neighbors of a signal vector
$N(0, \sigma^2)$	Gaussian random variable with zero mean and variance σ^2
$\mathbf{N}(0, \sigma^2)$	Gaussian vector with iid components of zero mean and variance σ^2
N_k	noise sample
$N_0/2$	power spectral density of noise
\mathcal{N}_V	average number of level crossings
n	code word length, path loss exponent
n_I, N_I	in-phase noise variables
n_Q, N_Q	quadrature noise variables
(n, k)	block code of k information bits and code word length n
$n(t)$	noise
$n_L(t)$	complex envelope of bandpass noise
$n_0(t)$	output noise of a matched filter
P	power
$P_{c,h}$	probability of correctly identifying the frequency bin
$P(d)$	pair-wise error probability ($\Pr(\mathbf{c} \rightarrow \mathbf{c}')$)
P_e	error probability (bit, symbol, code word)
$P(f)$	energy spectrum, Fourier transform of pulse shape $p(t)$
P_j	power of a jamming signal
P_p	peak power
$\Pr(A)$	probability of A
$\Pr(A, B)$	joint probability of A and B

$\Pr(A B)$	conditional probability of A given B
$\Pr(\mathbf{c} \rightarrow \mathbf{c}')$	pair-wise error probability
$\Pr(\mathbf{c} \mathbf{r})$	a posteriori probability
$\Pr(\mathbf{r} \mathbf{c})$	likelihood of the transmitted code vector \mathbf{c}
P_T	transmit power
$P(z)$	linear predictor in MSE-DFE
p	crossover probability of a BSC, probability of a binary symbol
p_{out}	outage probability
$p_{UV}(u_i, v_j)$	joint distribution of u_i, v_j
$p(v_j u_i)$	transition probability of a discrete channel
$p_X(x_i)$	distribution of the discrete random variable $X, i = 1, 2, \dots, n$
$p(t)$	pulse shape
$\mathcal{Q}(a, b)$	Marcum \mathcal{Q} -function
$Q(x)$	Gaussian integral Q -function of argument x
$Q(z)$	transfer function of the composite channel in suboptimum MSE-LE
q	optimum number of jamming tones
R	resistance, Rayleigh random variable, source rate in symbols/second
R_b	bit rate
R_c	chip rate
\mathcal{R}	responsivity
R_e	Earth's radius (6378 km)
$R_h(\tau, \mathbf{t}')$	multipath autocorrelation profile
$R_{h_i}(\mathbf{t}')$	i th path autocorrelation
$R(i - j)$	autocorrelation of the data sequence $\{d_i\}$
R_s	symbol rate
$R(t)$	envelope of a bandpass process
\mathcal{R}_V	level crossing rate
R_w	Walsh chip rate
$R_x(\tau)$	autocorrelation of WSS random process $x(t)$
$R_X(t_1, t_2)$	autocorrelation of the random process $x(t)$
r	value assumed by a random variable R , code rate, spectral efficiency
\mathbf{r}	received word
r^2	signal-to-intertone interference ratio
r_e	extinction ratio
S	input variable of a Gaussian channel
\mathbf{S}	input vector of a Gaussian channel
\mathcal{S}	sample space
$S(f)$	power spectral density (with or without a subscript)
$S_h(\tau, \mathbf{f}')$	multipath Doppler profile
$S_{h_i}(\mathbf{f}')$	i th path Doppler power spectrum

\mathbf{s}	syndrome vector, orthogonal covering symbol
$\text{sgn}(x)$	signum function
$\text{sign}[x]$	sign of x
$s(t)$	digital signal
$\{s_i(t)\}$	set of M digital signals, $i = 1, 2, \dots, M$
$\{\mathbf{s}_i\}$	set of M signal vectors in the signal space, $i = 1, 2, \dots, M$
$\ \mathbf{s}_i\ $	norm of signal vector \mathbf{s}_i
s_{ik}	coefficients of the Gram–Schmidt orthogonal expansion, I – Q values of a two-dimensional signal vector
$s_L(n)$	time samples of an OFDM signal
$s_L(t)$	complex envelope a bandpass signal $s(t)$
$s_0(t)$	output signal of a matched filter
s_p	pilot symbol
T	time interval, period, sampling period
\mathbf{T}	phase error rotation matrix
T_A	antenna noise temperature
T_b	bit time
T_c	chip time, channel coherence time
T_d	time delay, multipath delay spread
T_{DC}	effective noise temperature of a downconverter
T_{DM}	effective noise temperature of a demodulator
T_e	effective noise temperature
T_h	hop time
T_{LNA}	effective noise temperature of a low-noise amplifier
T_n	physical temperature of the resistor
T_0	reference temperature, time interval, period
T_p	pulse width
$\text{Tr}(\mathbf{A})$	trace of matrix \mathbf{A}
T_s	symbol time, system noise temperature
t	time, error-correcting capability of a block code
U	set of M input symbols of a discrete channel
$\bigcup_{i=1}^M A_i$	union of A_i set
$u(t)$	unit step function
\mathbf{u}	message vector
V	voltage, set of Q output symbols of a discrete channel
$\text{Var}(X)$	variance of X
V_{rms}	root mean-square voltage
V_T	threshold voltage
v	radial velocity
$v(t)$	voltage signal

W	bandwidth, watt
$\{w_n(t)\}$	set of Walsh functions, $n = 1, 2, \dots, M$
X	random variable, discrete source, output variable of a Gaussian channel or a matched filter
\mathbf{X}	random vector, output vector of a Gaussian channel
\bar{X}	mean value (expected value) of X
$\overline{X^2}$	mean-square value of X^2
$X(e^{j2\pi f'})$	discrete-time Fourier transform of the sequence $x(k)$
$ X(f)^2 $	energy spectral density of the energy signal $x(t)$
$X(f)$	Fourier transform of $x(t)$, folded spectrum
$X(k)$	N -point DFT of the sequence $x(n)$
X^n	n th extension of the discrete source X
$X_T(f)$	Fourier transform of $x_T(t)$, $2T$ -truncation of $x(t)$, $-T \leq t \leq T$
$X(z)$	z -transform of the sequence $x(k)$, transfer function of the composite channel in optimum MSE-LE
x	value assumed by a random variable X
\mathbf{x}	value assumed by a random vector \mathbf{X}
$[x]$	integer part of x
$x(n)$	discrete-time signal, sequence used in OFDM
$x(t)$	continuous-time signal (with or without a subscript)
$x_I(t)$	in-phase component of the bandpass signal $x(t)$
$\{x_k(t)\}$	set of L orthonormal basis functions, $k = 1, 2, \dots, L$
$\{\mathbf{x}_k\}$	set of L orthonormal basis vectors, $k = 1, 2, \dots, L$
$x_L(t)$	complex envelope (equivalent lowpass signal) of the bandpass signal $x(t)$
$x_p(t)$	periodic signal
$x_Q(t)$	quadrature component of the bandpass signal $x(t)$
$x_s(t)$	sampled function
$x_T(t)$	$2T$ -truncation of $x(t)$, $-T \leq t \leq T$
$Y(k)$	N -point DFT of the sequence $y(n)$
$y(n)$	sequence
$y(t)$	continuous-time function
\mathbf{Z}	pre-mapped vector at the input of the combiner
$Z(k)$	frequency samples of an OFDM signal (the L - Q values of symbols of OFDM subcarriers)
Z_0	amplifier transimpedance
$z_k(t)$	complex envelope of the k th OFDM subcarrier
$*$	linear convolution
\otimes	circular convolution
$()^*$	complex conjugate
α	arbitrary constant

α_n	normalized signal amplitude in MQAM
α_p	complex Doppler factor
$ \alpha_p $	Doppler loss factor
β	proportionality constant, roll-off factor of a raised-cosine filter
γ	threshold
$\gamma_{k,m}$	complex Doppler loss factor
γ_n	MSK data stream
Γ	ground reflection coefficient, gamma function
ΔF	frequency offset in OFDM
Δf	peak frequency deviation
$\Delta \varepsilon_l$	differential Doppler phase error
$\Delta \hat{\varepsilon}_{l-1}$	post-estimated differential Doppler phase error
$\Delta \varepsilon_l - \Delta \hat{\varepsilon}_{l-1}$	double-differential Doppler phase error
δ	jamming pulse duty cycle, fraction of FH bandwidth being jammed, fraction of a hop being jammed
δ_{ij}	0 for $i \neq j$ and 1 for $i = j$
$\delta(t)$	unit impulse function
ε	phase error
ε_l	Doppler phase error
θ	phase
θ_k	azimuth angle of the k th wave
θ_L	Earth station longitude
θ_ℓ	Earth station latitude
θ_S	GEO satellite longitude
$\theta(t)$	phase function
λ	wavelength, Lagrange multiplier
μ	conditional mean value
$\Lambda(X y)$	conditional likelihood ratio
Π	product, fractional coverage area
ρ	spatial correlation coefficient
$\rho_X(\boldsymbol{\tau})$	normalized autocovariance of the random process $x(t)$
σ^2	variance of noise
σ_{dB}	standard deviation of the log-normal density variable in decibels
σ_X^2	variance of the random variable X
σ_s^2	power of the diffuse paths
τ	time delay variable, average fade duration
$\tau_i(t)$	path delay
φ	phase state in CPM
φ_k	polar angle of the k th wave
ϕ	impossible event, null set, phase in MFSK and CPM, phase shift

$\Phi_0(f)$	power spectral density of the equalizer output noise
$\{\phi_k(t)\}$	set of orthonormal eigenfunctions of the noise autocorrelation
χ	voice activity factor or data duty cycle
χ^2	chi-square
ψ	angle of mobile direction with respect to the x -axis, Doppler phase error
$\Psi(f)$	power spectral density of sampled noise
$\Psi_0(f)$	power spectral density of output noise of ZF-LE
$\Psi(t)$	phase of a bandpass process
Ω	mean-square value of the envelope of the Nakagami- m process

Abbreviations

2G	second generation
3G	third generation
A/D	analog/digital conversion
AGN	additive Gaussian noise
AMPS	advanced mobile phone system
APD	avalanche photodiode
ASK	amplitude shift keying
AWGN	additive white Gaussian noise
BCH	Bose–Chaudhuri–Hocquenghem code
BEC	binary erasure channel
BPA	belief propagation algorithm
BSC	binary symmetric channel
CDM	code division multiplexing
CDMA	code division multiple access
CDMA 2000	3G CDMA
CP	cyclic prefix
CPM	continuous phase modulation
CP-MFSK	continuous phase M-ary frequency shift keying
CRC	cyclic redundancy check
CSI	channel side information
CSIR	channel side information at the receiver
CSK	code shift keying
D/A	digital/analog conversion
DD-DF	double-differential decision-feedback algorithm
D-DF	differential decision-feedback algorithm
DEMUX	demultiplexer
DFS	decision-feedback selection
DFT	discrete Fourier transform
DMC	discrete memoryless channel
DMPSK	differential M-ary phase shift keying
DMQAM	differential quadrature amplitude modulation
DPSK	differential phase shift keying
DQPSK	differential quadrature phase shift keying

DSB-AM	double sideband–amplitude modulation
DS	direct sequence
DS-CSK	direct sequence–code shift keying
DS-PSK	direct sequence–phase shift keying
DS-SS	direct sequence spread spectrum
DTFT	discrete-time Fourier transform
EGC	equal gain combining
EIRP	effective isotropic radiated power
ESN	electronic serial number
ETACS	extended total access cellular system
FCC	Federal Communications Commission
FDM	frequency division multiplexing
FDMA	frequency division multiple access
FFH	fast frequency hop
FFT	fast Fourier transform
FH	frequency hop
FIR	finite impulse response
FM	frequency modulation
FSE	fractionally spaced equalizer
FSK	frequency shift keying (binary frequency shift keying)
fT	frequency–time product
GEO	geostationary orbit
GMSK	Gaussian minimum shift keying
GPS	global positioning system
GSM	global system for mobile communication
$ h ^2 SNR$	instantaneous SNR
ICI	intercarrier interference
ICI	interchannel interference
IDFT	inverse discrete Fourier transform
IEEE	Institute of Electrical and Electronics Engineers
IFFT	inverse fast Fourier transform
iid	independent and identically distributed
IIR	infinite impulse response
IPI	intrapath interference
IS	interim standard
ISI	intersymbol interference
<i>ISI</i>	intersample interference
JDC	Japanese digital cellular system
JTACS	Japanese total access communication system
LDPC	low-density parity-check code

LFSR	linear feedback shift-register
LLR	ln-likelihood ratio
LR	likelihood ratio
L-REC	rectangular pulse of duration L symbols
L-RC	raised cosine pulse shape of duration L symbols
LSB	lower sideband
LTI	linear time-invariant
MAP	maximum a posteriori
MASK	M-ary amplitude shift keying
MFSK	M-ary frequency shift keying
MIMO	multiple-input multiple-output
MIN	mobile identification number
MIP	multipath intensity profile
MISO	multiple-input single-output
ML	maximum likelihood
MLSD	maximum likelihood sequence detection
MMSE	minimum mean-square error
MPA	message passing algorithm
MPSK	M-ary phase shift keying
MQAM	quadrature amplitude modulation
MRC	maximal ratio combining
MSC	mobile switching center
MSE-DFE	mean-square error decision-feedback equalizer
MSE-LE	mean-square error linear equalizer
MSK	minimum shift keying
MUI	multi-user interference
MUX	multiplexer
NAMPS	narrowband advanced mobile phone system
NRZ	non-return-to-zero
NTACS	narrowband total access communication systems
OFDM	orthogonal frequency division multiplexing
OOK	on-off keying
OQPSK	offset quadrature phase shift keying
PCS	personal communication system
PD	pin photodiode
PDC	Pacific (or personal) digital cellular system
PDF	probability distribution function
pdf	probability density function
$\pi/4$ -DQPSK	$\pi/4$ shift differential quadrature phase shift keying
PLL	phase-locked loop

PN	pseudo-noise
PSK	phase shift keying (binary phase shift keying)
PSTN	public switched telephone network
QPSK	quadrature phase shift keying
RS	Reed–Solomon code
SC	selection combining
SCM	station class mark
SFH	slow frequency hop
SIMO	single-input multiple-output
$SINR$	signal-to-interference and noise ratio
$SINR$	path signal-to-interference-and-noise ratio
$SINR_0$	output signal-to-interference plus noise ratio
SIR	signal-to-interference ratio
SIR_i	input signal-to-interference ratio
SIR_0	output signal-to-interference ratio
$SJNR_0$	output signal-to-jamming-plus-noise ratio
SJR_i	input signal-to-jamming ratio
$SJR_{i,p}$	input signal-to-pulse jamming ratio
SJR_0	output signal-to-jamming ratio
$SJR_{0,p}$	output signal-to-pulse jamming ratio
SNR	signal-to-noise ratio
SNR	diversity symbol signal-to-noise ratio
SNR_0	output signal-to-noise ratio
SPA	sum product algorithm
TCM	trellis coded modulation
TDMA	time division multiple access
TIA/EIA	Telecommunication Industry Association/Electronic Industry Association
USB	upper sideband
USDC	US digital cellular
VCO	voltage-controlled oscillator
WCDMA	wideband CDMA (3G CDMA)
WLAN	wireless local area network
WMAN	wireless metropolitan area network
WSCS	wide-sense cyclostationary
WSS	wide-sense stationary
ZF-DFE	zero-forcing decision-feedback equalizer
ZF-LE	zero-forcing linear equalizer