JOURNAL OF RESEARCH of the National Bureau of Standards
Vol. 89, No. 1, January-February 1984

Theory of Acoustic Emission From Phase Transformations

J. A. Simmons and H. N. G. Wadley
National Bureau of Standards, Washington, DC 20234

Accepted: December 1, 1983

A theoretical framework is developed within which it is possible to predict the dynamic elastic displacement field (acoustic emission) for a phase transformation in which there is a change of both crystal structure (elastic constants) and shape (density). An integral equation is presented for the acoustic emission displacement field due to formation of inhomogeneous inclusions. This integral equation is solved by expressing the source in multipolar form and using the Eshelby equivalent inclusion method to estimate the dynamic multipolar coefficients. Expressions for the source of elastic radiation are explicitly calculated for small isotropic spherical and ellipsoidal inclusions embedded in an isotropic matrix. These expressions are used for qualitative interpretation of recent experiments on martensitic transformations in steels and for identifying the information that may be deduced about transformation dynamics from quantitative measurements of acoustic emission.

Key words: acoustic emission; martensitic phase changes; twinning.

1. Introduction

Acoustic emission (AE) is the term used for the elastic waves generated by abrupt localized changes of stress in a solid [1][1]. The waves propagate from the source of stress change to cause transient (nano-millisecond) surface displacements of a sample. These transient displacements may be detected with ultasonic transducers and are known as acoustic emission. Acoustic emission is then a method for observing rapid dynamic material processes with elastic waves. The slower, quasi-static changes of stress are not usually considered sources of acoustic emission even though their surface displacements are incorporated (as a limiting case) in theoretical formulations of acoustic emission [2]. These static displacements, normally measured with extensometers, are the basis of routine mechanical property measurements.

Acoustic emission has begun to be extensively explored as a tool for the investigation of the micromechanisms of deformation and fracture during mechanical testing [3]. It has also found increasing application as a nondestructive evaluation (NDE) technique for detecting and locating flaws in mechanical structures that are subjected to stress and the premature failure of which would have catastrophic consequences [4]. More recently, it is being considered a candidate technique for in-situ monitoring of materials processing because acoustic emission signals are emitted through some of the mechanisms by which a material responds to process variables [5]. These mechanisms may include both benign processes (e.g., phase transformations) and malevolent processes (e.g., cracking).

About the Authors, Paper: J. A. Simmons is a research mathematician and H. N. G. Wadley a metallurgist in the NBS Center for Materials Science. The work on which they report was supported by the Defense Advanced Research Projects Agency under DARPA Order No. 4275.

[1] Figures in brackets indicate the references at the end of this paper.

It has been speculated that the measurement of acoustic emission from benign mechanisms during materials processing could provide much needed in situ information about materials processing. There is current interest in using this information in tandem with recently developed process models to develop improved feedback-controlled systems for materials processing. (The malevolent mechanisms of acoustic emission, e.g., cracking, have already received and continue to attract attention as potential quality control indicators [6]).

As an example of the possible use of AE for phase transformation monitoring, consider some system where above a temperature T_1 phase A is the stable phase and below T_1 phase B is stable. Then, for $T<T_1$ material composed of the A-phase may lower its free energy by undergoing a phase change to B. Usually, the new phase has a different crystal structure so that there are changes of elastic modulus and density as well as a shape change associated with the rearrangement of atoms in the transforming volume. These changes may generate acoustic emission or internal stresses which give rise to local plastic flow and subsequent acoustic emission. If a transducer is used to detect the acoustic emission from such phase transformations, useful information may be obtained about the temperature, pressure, etc. at which the phase change occurred [7]. Furthermore, the dynamics and crystallography of the phase change are also contained—convolved with the sample and instrumentation impulse response—in the signal. The use of appropriate analysis methods may enable the measurement of hitherto unobserved aspects of phase changes. Such measurements would, because of the passive nature of this monitoring technique, emanate from phase changes unmodified by our attempts to observe them.

The majority of phase transformations occurs at a rate controlled by diffusion. This, unfortunately, is sufficiently slow (compared with the time for elastic waves emitted by the transformation to communicate with the sample boundaries) that no detectable acoustic emission is observed. Thus, diffusion-controlled phase changes, while often resulting in significant stress changes, cause mainly quasi-static surface displacement and no direct acoustic emission (as is usually the case with bainite and pearlite formation during cooling of low alloy steels [8]). In these cases acoustic emission is not a viable candidate for microstructure control during processing.

There is, however, an important class of phase transformations for which atomic diffusion is not rate controlling. These include the martensitic transformations in which the change of crystal structure is accommodated by a so-called "diffusionless" shear transformation. Diffusion, if it occurs, is over a very short range; of the order of the lattice parameter. The velocity at which the transformation may propagate varies enormously from one alloy to the next, but in some systems velocities of ~ 1000 ms^{-1} have been reported [9]. This implies that the formation of a typical 30 μm length of martensite in some alloys is formed in as little as 30 ns. In this time, elastic waves only propagate ~ 0.1 mm and transient sample displacements are observed [8,10] as the sample returns to mechanical equilibrium.

Despite the reporting by several workers [8,10] of intense acoustic emission during some martensitic transformations, effects of micromechanism (transformation velocity, volume, etc.) upon acoustic emission have not been studied. Even during the simpler processes of deformation twinning, there have only been a few tentative correlations made between micromechanism and acoustic emission signal [11,12]. A part of the problem has been the absence of a rigorous theory relating the properties of the dynamic elastic wavefield (acoustic emission radiation) to the dynamics and crystallography of atomic motions during phase transformation or twin growth. It is our purpose here to begin to apply recently developed elastodynamic techniques to the prediction of acoustic emission signals from phase changes and twinning.

2. Theoretical Framework

Consider an idealized transformation to consist of a small region of phase A (with density ρ and elastic moduli C) undergoing a change of crystal structure to form a region of phase B with a density $\rho+\Delta\rho$ and elastic moduli $C+\Delta C$. We assume that if the region B could be cut out of the

matrix, its shape would be determined by a linear transformation β^* applied to the original region of phase A. In elastodynamics, a transformation involving both a change of moduli and shape is referred to as an inhomogeneous inclusion. The calculation of the dynamic elastic wavefield for the inhomogeneity problem is complicated by:

- Coupling of the wavefields from density and modulus changes.
- Internal reflection and mode conversion of elastic waves at the inhomogeneity boundary.
- Doppler effects for high transformation velocity ($\gtrsim$20% speed of sound).

We find the acoustic emission from inhomogeneous inclusions by recourse to certain simplifying assumptions. We make the assumption that the inhomogeneity is small in comparison with the wavelengths of interest.[2] Thus, reverberations within the inhomogeneity are at frequencies above those of interest. It is also assumed that the linear velocity at which the transformation progresses through the austenite is $\lesssim$20% of the shear wave speed so that a sub-sonic approach may be used. Complications, such as transformation stress induced plastic deformation, twinning of martensite, autocatalytic phenomena, and polycrystalline anisotropy of the matrix are, for the present, put aside.

The theoretical framework we use is based upon the equivalent inclusion problem studied by Eshelby [13] and applied first to acoustic emission by Simmons and Clough [14]. As our starting point we use eq (A32) from ref. [14] to express the farfield elastic displacement field for an ellipsoidal inhomogeneity undergoing a self-similar (constant aspect ratio) change of shape. The ellipsoid volume is $V_\Omega(t)$ where $\Omega(r)$ denotes the region transformed (Ω has the value one inside the inhomogeneity and zero elsewhere), as shown in figure 1.

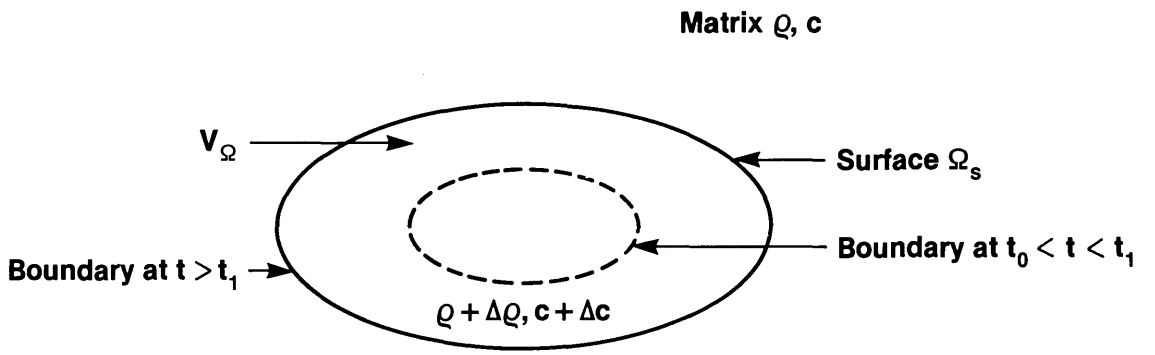

Figure 1–An ellipsoidal inhomogeneity undergoing a self-similar change of shape.

An elegant and simple way to consider the phase transformation problem is to generalize the stress and strain tensors to contain both space and time coordinates. We thus have four dimensional stress and strain tensors:

$$[\sigma_{ij}] = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & -\rho v_1 \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & -\rho v_2 \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & -\rho v_3 \end{bmatrix} \quad \text{and} \quad [u_{i,j}] = \begin{bmatrix} u_{1,1} & u_{1,2} & u_{1,3} & v_1 \\ u_{2,1} & u_{2,2} & u_{2,3} & v_2 \\ u_{3,1} & u_{3,2} & u_{3,3} & v_3 \end{bmatrix}$$

[2] We believe this to be reasonable. For example, if a region 10 μm in dimension transforms, its fundamental resonance frequency will be ~300 MHz. The upper frequency of acoustic emission measurement is normally $\lesssim$10 MHz.

where v_i is the velocity in direction x_i and ρv_i is dynamic momentum. A hat ($\hat{}$) above a subscript indicates the subscript runs from 1 to 4 where 4 represents the time dimension. The four dimensional stiffness is denoted in the matrix by:

$$\sigma_{ij} = C_{ijkl}\, u_{k,l} \qquad \text{for } i, j, k, l = 1, 2, \text{ and } 3$$

and

$$C_{i\hat{j}k4} = C_{i4k\hat{j}} = -\rho\, \delta_{4\hat{j}}\, \delta_{ik}\,.$$

For the inhomogeneity it is denoted by:

$$\sigma_{ij} = (C+\Delta C)_{ijkl}\, u_{k,l} \qquad \text{for } i, j, k, l = 1, 2, \text{ and } 3$$

and

$$(C+\Delta C)_{i\hat{j}k4} = (C+\Delta C)_{i4k\hat{j}} = -(\rho+\Delta\rho)\, \delta_{4\hat{j}}\, \delta_{ik}$$

We use the fact that the difference in moduli (ΔC) between the inhomogeneity and surrounding matrix is constant over $\Omega(t)$ to write eq (A32) in the form:

$$\begin{aligned} u_m(\vec{r},t) = \quad & -\int\int G_{mi,\hat{j}'}(\vec{r},\vec{r}',t-\tau)\Delta C_{ijkl} u_{k,\hat{l}'}(\vec{r}',\tau)\Omega(\vec{r}',\tau)\, d\vec{r}'d\tau && \text{Term 1} \\ & +\int\int G_{mi,4}(\vec{r},\vec{r}',t-\tau)\Delta\rho u_{k,4}(\vec{r}',\tau)\, d\,\vec{r}'d\tau && \text{Term 2} \\ & +\int\int G_{m\,i,\hat{j}'}(\vec{r},\vec{r}',t-\tau)(C+\Delta C)_{i\hat{j}k\hat{l}}\, \beta^*_{k\hat{l}}\,(\vec{r},\tau)\Omega(\vec{r}',\tau)\, d\,\vec{r}'d\tau && \text{Term 3} \\ & -\int\int G_{m\,i,\hat{j}'}(\vec{r},\vec{r}',t-\tau)\Delta C_{i\hat{j}k\hat{l}}\, \beta^\circ_{k\hat{l}}\,(\vec{r}',\tau)d\,\vec{r}'d\tau && \text{Term 4} \end{aligned} \qquad (1)$$

where:
$u_m(\vec{r},t)$ is the displacement at $\vec{r}$ as a function of time in direction x_m (valid both inside and outside the inhomogeneity). $G_{mi}(\vec{r},\vec{r}',t)$ is the dynamic elastic Green's tensor representing the displacement at $\vec{r}$ in direction x_m as a function of time (t) due to the application of a force impulse in direction x_i at ($\vec{r}'$,0). The subscript, j, denotes partial differentiation with respect to the x_j coordinate. $\vec{\beta}^\circ$ is the pre-existing elastic distortion and β^* the stress-free strain for the transformation; in the phase change problem, the term β^*_{14} (the "plastic velocity" component which contributes to shape change emission but not that due to momentum) is taken to be zero. The new elastic distortion is $\vec{\beta}^\circ+\vec{\beta}^T-\vec{\beta}^*$ where $\vec{\beta}^T$ is the total distortion (elastic and plastic).

In eq (1), the acoustic emission is given by the change of stress:

$$\begin{aligned} \Delta\sigma(\vec{r},t) &= (\mathbf{C}+\Delta\mathbf{C})(\vec{\beta}^\circ+\vec{\beta}^T-\vec{\beta}^*)-\mathbf{C}\,\vec{\beta}^\circ \\ &= \begin{cases} (\mathbf{C}+\Delta\mathbf{C})(\vec{\beta}^T-\vec{\beta}^*)+\Delta\mathbf{C}\,\beta^\circ & \text{for } \Omega(\vec{r},t)=1 \\ \mathbf{C}(\vec{\beta}^T-\vec{\beta}^*) & \text{for } \Omega(\vec{r},t)=0 \end{cases} \end{aligned} \qquad (2)$$

It can be seen in eq (1) that the acoustic emission arises from changes associated with the stress-

It can be seen in eq (1) that the acoustic emission arises from changes associated with the stress-free strain (term 3), and the interaction of the change in modulus with the pre-existing strain (term 4). These sources act upon the modulus changes (static and dynamic) to create further changes (terms 1 and 2). It should be clear that eq (1) is very general and describes both the acoustic emission of the phase transformation and the scattering of elastic waves ($\vec{\beta}^{\circ}$ now time varying) from inhomogeneities.

A difficulty with eq (1) is that the Green's tensor depends upon $\vec{r}-\vec{r}'$. Thus, a different Green's tensor must be used between each source point and the receiver. To overcome this problem we approximate the solution to eq (1)—for inhomogeneous transformations of fixed magnitude in the presence of a relatively constant applied stress—by the use of multipolar expansions. These simplify the dependence of the Green's tensor on $\vec{r}-\vec{r}'$. If the source is small in size ($\lesssim$20% of the shortest wavelength of interest) there is only a small error associated with using a multipolar expansion obtained by representing the Green's tensor in a Taylor's series about $\vec{r}_0'$, the centroid position. In this exposition we retain only the first term in the expansion, but higher order terms can easily be incorporated. Equation (1) then becomes:

$$
\begin{aligned}
u_m(\vec{r},t) = & -\Delta C_{ijkl}\int_{t_0}^{t_1} G_{mi,j'}(\vec{r},\vec{r}_0',t-\tau)\left[\int_{\Omega} u_{k,l'}(\vec{r}',\tau)d\vec{r}'\right]d\tau \\
& +\Delta\rho\int_{t_0}^{t_1} G_{mi,j'}(\vec{r},\vec{r}_0',t-\tau)\left[\int_{\Omega} u_{k,4}(\vec{r}',\tau)d\vec{r}'\right]d\tau \\
& +\int_{t_0}^{t_1} G_{mi,j'}(\vec{r},\vec{r}_0',t-\tau)\left[(C+\Delta C)_{ijkl}\int_{\Omega}\beta^{*}_{k,l}(\vec{r}',\tau)d\vec{r}'-\Delta C_{ijkl}\int_{\Omega}\beta^{\circ}_{kl}(\vec{r}',\tau)d\vec{r}'\right]d\tau
\end{aligned}
\tag{3}
$$

In terms of the Heaviside Green's tensor G^H (displacement at $\vec{r},t$ due to a stepfunction in force at $\vec{r}',0$) we can express eq (3) as:

$$
\begin{aligned}
u_m(\vec{r},t) = & -\int_{t_0}^{t_1} G^H_{mi,j'}(\vec{r},\vec{r}_0',t-\tau)\left[\frac{d}{dt}\Delta C_{ijkl}\int_{\Omega} u_{k,l}(\vec{r}',\tau)d\vec{r}'\right]d\tau \\
& +\int_{t_0}^{t_1} G^H_{mi,4}(\vec{r},\vec{r}_0',t-\tau)\left[\frac{d}{dt}\Delta\rho\int_{\Omega} u_{k,4}(\vec{r}',\tau)d\vec{r}'\right]d\tau \\
& +\int_{t_0}^{t_1} G^H_{mi,j}(\vec{r},\vec{r}_0',t-\tau)\frac{d}{dt}\left[(C+\Delta C)_{ijkl}\int_{\Omega}\beta^{*}_{kl}(\vec{r}',\tau)d\vec{r}'-\Delta C_{ijkl}\int_{\Omega}\beta^{\circ}_{kl}(\vec{r}',\tau)d\vec{r}'\right]d\tau
\end{aligned}
\tag{4}
$$

Equations (3) and (4) have the physical interpretation that the acoustic emission at $\vec{r},t$ is obtained from a dynamic multipolar source (in our truncated expansion considered dipolar) at the inhomogeneous inclusion. When an inhomogeneity is present, the magnitude of the source has a "feedback" component on the right hand side of the equation. It is this feedback component that complicates the inhomogeneity problem.

The solution to this integral equation is still not possible unless recourse is made to a final simplifying approximation. The one commonly used in scattering problems, the Born approximation, consists of replacing $\vec{u}$ on the right hand side of eq (1) or eq (4) with the values of $\vec{u}$ obtained from eq (1) without terms 1 and 2 (the homogeneous problem). We feel this weak scattering approximation is less appropriate here because of the large differences in modulus that may occur between the inhomogeneity and matrix.

Once it is recognized that the source appears to be a force multipole located at the centroid of the inclusion, we can use information about the static case and the assumption of ellipsoidal shape, which has not yet been needed to approximate the value of the source strength. To understand how we apply such a quasi-static approximation, consider a point outside but near the ellipsoid Ω. Suppose we were to stop the growth of the inhomogeneity at some time t^*. Then, after a short time, the longitudinal and transverse wavefronts generated by the dipole before t^* would pass through the point and from then on the point would only experience the static displacement associated with the presence of a static multipolar force combination.[3] Thus, the multipolar density $\vec{M}(t,t^*)$ describing the dynamic motion must be consistent with that of the static case, $\vec{M}^\infty(t^*)$, i.e.:

$$\lim_{t \to t_1} \vec{M}(t,t^*) = \vec{M}^\infty(t^*)$$

where it is assumed that t_1 is sufficiently greater than t^* that the process "comes to rest." The physical distinction between $\vec{M}(t,t^*)$ and $\vec{M}^\infty(t^*)$ arises from the feedback effects of the growth dynamics and multiple reflections within the ellipsoid. We shall ignore these dynamic feedback effects and correct only for those feedback effects produced by the static component of the dipole field.[4]

To obtain the static correction, i.e. the value of $\vec{M}^\infty(t^*)$, we know that $\vec{\beta}^*$ has a fixed value throughout $\Omega(t)$. We assume $\vec{\beta}^\circ$ to be fixed and constant in the region of $\Omega(t)$ and recall that $\vec{G}^\infty(\vec{r}\,,\vec{r}_0') = \lim_{t \to \infty} \vec{G}^H(\vec{r},\vec{r}_0',t)$. Then, we replace $\vec{G}^H$ by $\vec{G}^\infty$ in eq (4) and integrate the source terms with respect to time to give:

$$u_m^\infty(\vec{r},t) = G_{mi,j}^\infty(\vec{r},\vec{r}_0')\left[(C+\Delta C)_{ijkl}\beta_{kl}^* V_\Omega(t^*) - \Delta C_{ijkl}(\beta_{kl}^\circ V_\Omega(t^*) + \int_\Omega u_{k,l}^\infty(\vec{r}',t^*))\right] d\vec{r}' \tag{5}$$

The solution to this problem is well known from the equivalent inclusion method of Eshelby [13]. In fact, for an ellipsoidal region, $u_{k,l}^\infty$ is constant over the ellipsoidal region if $\vec{\beta}^*$ and $\vec{\beta}^\circ$ are also constant (it is also true that it is a polynomial in $\vec{r}'$, if the strains are polynomials in $\vec{r}'$). The effective dipole density associated with the inclusion can then be easily derived.

Using a six-dimensional vector terminology (such as the Voight convention) where vectors are symmetric 3×3 matrices, one can easily show that (now replacing t^* by t):

$$\overline{\Delta\sigma}(t) = [\mathbf{I} + \Delta\mathbf{C}\,\tilde{\mathbf{D}}]^{-1}[(\mathbf{C}+\Delta\mathbf{C})\vec{\beta}^* - \Delta\mathbf{C}\,\vec{\beta}^\circ] V_\Omega(t) \tag{6a}$$

and

$$\overline{\Delta\sigma}_{i4}(t) = 0 \quad ,^5 \tag{6b}$$

---

[3] If the body in which this occurs has external boundaries (either free surfaces or regions of different ρ) then wavefronts are reflected from the boundaries and will pass through both our chosen point and the surface of the ellipsoid. We ignore the effect of these reflected wavefronts on the acoustic emission from the inhomogeneity.

[4] In ref. [14] a slight extension of this assumption, called the retarded density approximation was developed. In that assumption, the expanding ellipsoid was broken into two regions, an inner region, in which the full static feedback correction is applied, and the outer "shell" region of the ellipsoid, in which no feedback correction is applied.

[5] We have already assumed $\beta_{k4}^\circ = 0$. The only term that might then contribute to $\Delta\sigma_{i4}$ would arise from the term $\Delta\rho\int_\Omega u_{k4}(\vec{r}'\tau)d\vec{r}'$ which occurs in eq (3). Here, we have ignored this term, which arises from momentum effects associated to density changes in the inclusion. An alternate approach, analogous to the Born approximation, would be to modify the value of $\vec{u}$ as calculated from eq (6) by including its own "homogeneous" dynamic density contribution.

where

$$u_m(\vec{r},t) = \int_{t_0}^{t_1} G_{mi,j}(\vec{r},\vec{r}'_0,t-\tau)\overline{\Delta\sigma}_{ij}(\tau)d\tau \equiv \int_{t_0}^{t_1} G^H_{mi,j}(\vec{r},\vec{r}'_0,t-\tau)\overline{\Delta\dot{\sigma}}_{ij}(\tau)d\tau \tag{7a}$$

$$\overline{D}_{ijkl} = \tfrac{1}{2}(D_{ijkl}+D_{ijlk}) \tag{7b}$$

$$D_{ijkl} = \frac{a_1a_2a_3}{8\pi} \int_{|\xi|=1} [\tilde{G}_{ik}(\xi)\xi_j\,\xi_l+\tilde{G}_{jk}(\xi)\xi_i\,\xi_l][a_n^2\xi_n^2]^{-3/2}ds \tag{7c}$$

$$\tilde{G}_{lk}(\xi) = [C_{ijkl}\xi_j\xi_l]^{-1} \tag{7d}$$

For an elastically isotropic spherical inclusion [15]:

$$D_{ijkl} = \frac{1}{6\mu}\left[\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}-\frac{1}{5(1-\nu)}(\delta_{ij}\delta_{kl}+\delta_{lk}\delta_{jl}+\delta_{il}\delta_{jk})\right] \tag{7e}$$

and for the disc shaped anisotropic inhomogeneity with disc unit normal $\vec{\nu}$ [16]:

$$D_{ijkl} = \tfrac{1}{2}[\nu_i\nu_l\tilde{G}_{jk}(\vec{\nu})+\nu_j\nu_l\tilde{G}_{ik}(\vec{\nu})] \tag{7f}$$

For an isotropic matrix, eq (7f) becomes:

$$\tilde{D}_{ijkl} = \tfrac{1}{4}\left[\frac{\nu_i\nu_l}{\mu\delta_{jk}+(\lambda+\mu)\nu_j\nu_k}+\frac{\nu_i\nu_l}{\mu\delta_{ik}+(\lambda+\mu)\nu_i\nu_k}+\frac{\nu_i\nu_k}{\mu\delta_{jl}+(\lambda+\mu)\nu_j\nu_l}+\frac{\nu_k\nu_j}{\mu\delta_{il}+(\lambda+\mu)\nu_i\nu_l}\right] \tag{8a}$$

so that, for instance, if $\nu_i=\delta_{i3}$

$$\overline{D}_{ijkl} = \frac{\delta_{i3}\delta_{j3}\delta_{k3}\delta_{l3}}{\lambda+2\mu}$$

and $[\mathbf{I}+\Delta\mathbf{C}\,\overline{\mathbf{D}}]^{-1}$ arises from the inverse of the Voight matrix:

$$\frac{1}{\lambda+2\mu}\begin{bmatrix} \lambda+2\mu & 0 & \delta C_{1133} & 0 & 0 & 0 \\ 0 & \lambda+2\mu & \delta C_{2233} & 0 & 0 & 0 \\ 0 & 0 & \lambda+2\mu+\Delta C_{3333} & 0 & 0 & 0 \\ 0 & 0 & \delta C_{1233} & \lambda+2\mu & 0 & 0 \\ 0 & 0 & \delta C_{1333} & 0 & \lambda+2\mu & 0 \\ 0 & 0 & \delta C_{2333} & 0 & 0 & \lambda+2\mu \end{bmatrix} \tag{8b}$$

3. Discussion

The theoretical framework outlined above has several consequences for those interested in studying the dynamics of twinning and martensitic phase changes. The above model shows that the acoustic emission signal contains information about six properties of a martensitic transformation (or twin):

1. Volume of region transformed (of martensitic lath)
2. Dilatational strain
3. Shear/rotational strain
4. Habit plane
5. Internal stress magnitude (through its interaction with $\Delta\mathbf{C}$)
6. Duration of the reaction

In fact, from eq (6)

$$\overline{\Delta\sigma}(t)=[\mathbf{I}+\Delta\mathbf{C}\ \overline{\mathbf{D}}]^{-1}\ [(\mathbf{C}+\Delta\mathbf{C})\vec{\beta}^{*}-\Delta\mathbf{C}\ \vec{\beta}^{\circ})]V_{\Omega}(t).$$

Ignoring directionality and concentrating upon the magnitude of the stress components of a dipolar source, we see that acoustic emission is proportional to the volume of material that transforms and is linearly related to the transformation strain and pre-existing (residual) stress. If $\Delta\mathbf{C}$ is sufficiently small we can ignore the terms in $\Delta\mathbf{C}$ leaving the simple relation for acoustic emission in a homogeneous medium:

$$\overline{\Delta\sigma}(t)\simeq\mathbf{C}\ \vec{\beta}^{*}V_{\Omega}(t). \tag{9}$$

Returning to the example in the introduction, we can now enumerate some potential applications for acoustic emission during the phase transformation: 1) If the transformation is accompanied by cracking one should find it possible to distinguish $\overline{\Delta\sigma}$ signatures of cracking from those of the transformation itself. 2) It should also be possible for one to distinguish between different morphologies of martensitic (lath, plate or needle) based upon their different V_{Ω} distributions. 3) If one monitors a local area in the material one could observe the evolution of residual stress. 4) Under "ideal measurement conditions" one can directly deduce the shape change tensor and habit plane dynamically and perhaps gain a deeper understanding of autocatalytic phenomena in which secondary martenstic transformations (with possibly different habit planes) are stimulated by the first transformation.

Equation (9) can be used to deduce the smallest volume of martensitically transformed material detectable by acoustic emission. It is known that the smallest displacements detectable by an acoustic emission transducer is $\sim 10^{-14}$ m. This corresponds to a dipole of 3×10^{-8} Nm strength with 30 ns risetime buried 25 mm below the receiver [2]. Using values of 200 GNm^{-2} and 0.2 for C and β^{*} gives a minimum detectable volume of 1 μm^3.

We can use the above results to comment on the work of Speich and Schwoeble [8] who monitored the acoustic emission of SAE 4300 series steels with systematically varied carbon content during transformation to martensite, as shown in figure 2. They demonstrated that acoustic emission was able to accurately determine the martensitic start (Ms) temperature of the steel. In addition, their data shows a distinct correlation between carbon concentration and acoustic emission per unit volume of sample for which they did not account.

From eq (6) we can speculate that the cause of the correlation could be due either to the increase of transformation strain (β^{*}) or an increase of individual martensitic nucleations associated with a change of martensite morphology with increasing carbon content. This may be further compounded by a consistent change of bulk residual stress with increasing carbon content or a change in reaction time whose accompanying frequency shift could affect instrument

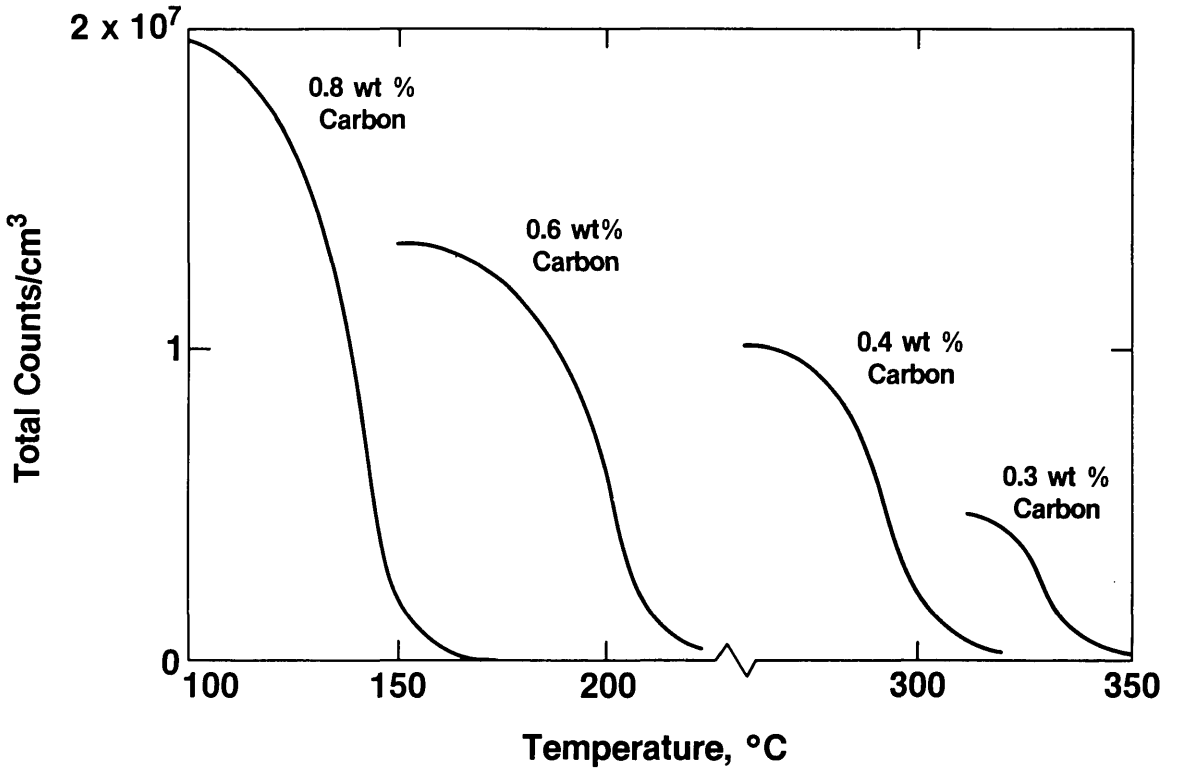

Figure 2–Acoustic emission per unit volume as a function of temperature measured during the cooling of low alloy steels of variable carbon content [8].

sensitivity. Change in lath morphology producing more (but smaller) emissions with increasing carbon content seems the most likely, but detailed metallographic studies are required.

4. Summary

An elastodynamic formulism has been used to obtain a solution for the acoustic emission from dynamic phase transformations where there is a change in the new phase (inclusion) of both shape and elastic constants. Explicit solutions for small ellipsoidal inclusions with anisotropic elastic constants are given for an isotropic matrix. This framework is used to explain how acoustic emission could be used for monitoring martensitic phase changes.

We thank J. W. Cahn and J. W. Christian for valuable discussions.

References

[1] Wadley, H. N. G.; Scruby, C. B.; Speake, J. Int. Metals Rev. **3**: 41; 1980.
[2] Scruby, C. B.; Wadley, H. N. G.; Hill, J. J. J. Phys. D: Appl. Phys. **16**: 1069; 1983.
[3] Rusbridge, K. L.; Scruby, C. B.; Wadley, H. N. G. Mat. Sci. and Eng. **59**: 151; 1983.
[4] Scruby, C. B.; Wadley, H. N. G. Prog. Nucl. Eng. **11**(3): 275; 1983.

[5] Wadley, H. N. G.; Mehrabian, R. Proc. Symp. Nondestructive Methods for Material Property Determination, Hershey, PA, April 6-8, 1983.
[6] Vahaviolos, S. J.; Carlos, N. F.; Siykovje, S. J.; Ternowschk, S. J. Mat. Eval. **39**: 1057; 1981.
[7] Canelli, G.; Cantelli, R. *Advances in Acoustic Emission* Proc. Int. Conf. on Acoustic Emission, Anaheim, CA; 1979. H. L. Dunegan and W. F. Hartman, eds., Dunhart, 1981.
[8] Speich, G. R.; Schwoeble, A. J. ASTM STR-571: 40; 1975.
[9] Fine, M.; Meshi, M.; Wayman, C. Martensitic Transformation, Academic Press, New York; 1978.
[10] Baram, J.; Rosen, M. Acta Met. **30**: 655; 1982.
[11] van Doren, S. L.; Pond, R. B.; Green, R. E. J. Appl. Phys. **47**(10): 4343; 1976.
[12] Tanaka, H.; Horivchi, R. Scripta Met. **9**: 777; 1975.
[13] Eshelby, J. D. J. Mech. Phys. Solids **17**: 177; 1969.
[14] Simmons, J. A.; Clough, R. B. *Dislocation Modelling of Physical Systems*, Proc. Int. Conf., Gainesville, FL, June 1980, Pergammon Press, 1981.
[15] Kneer, G. Phys. Stat. Sol. **9**: 825; 1965.
[16] Kinoshita, N.; Mura, T. Phys. Stat. Sol. **5**(a): 759; 1971.