SLAC-PUB-692
November 1969
(TH)

A THEORY OF DEEP INELASTIC LEPTON-NUCLEON SCATTERING AND LEPTON PAIR ANNIHILATION PROCESSES* IV

DEEP INELASTIC NEUTRINO SCATTERING

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ABSTRACT

This is the last in a series of four papers devoted to a theoretical study based on canonical field theory of the deep inelastic lepton processes. In the present paper we present the detailed calculations leading to the limiting behavior--or the "parton model"--for deep inelastic neutrino scattering, i.e.

$$\nu + p \rightarrow e^- + \text{"anything"}$$

$$\bar{\nu} + p \rightarrow e^+ + \text{"anything"}$$

where "anything" refers to all possible hadrons. In particular we show that the structure functions depend only on the ratio of energy to momentum transfer $2M\nu/q^2$ as conjectured by Bjorken on general grounds. Experimental implications including sum rules and the relation of ν and $\bar{\nu}$ cross sections to each other as well as to deep inelastic electron scattering cross sections are derived and discussed.

(Submitted to Physical Review)

* Work supported by the U.S. Atomic Energy Commission

I. Introduction

In this fourth and final article of a series of papers[1] on lepton-hadron interactions we study neutrino and antineutrino scatterings in the deep inelastic region.

The smallness of the fine structure constant for lepton electromagnetic interactions and of the Fermi coupling constant for their weak interactions permits the lowest order perturbation expansion in these parameters. We assume the weak currents of the leptons to be well-described by the universal V-A theory. The conserved vector current hypothesis of Feynman and Gell-Mann[2] and the Cabibbo theory of the weak currents for the hadrons[3] are also generally accepted as working assumptions.

Apart from the question of whether the weak interaction is really of current-current type or is mediated by intermediate vector bosons, neutrinos as well as antineutrinos, like electrons and muons in electromagnetic interactions, also probe the structure of hadrons via scatterings from hadron targets. The parton model derived in previous papers of this series[1] for deep inelastic electron scattering can be generalized to a form appropriate for neutrino and antineutrino scattering. Accomplishing this generalization is the task of the present paper.[4] An important and characteristic aspect of the parton model lies in the fact that in an infinite momentum frame of the hadron target the currents (electromagnetic and weak) have point-like, incoherent interactions with long-lived and almost free constituents (partons) of the hadrons in the deep inelastic region. The point vertices of the electromagnetic or weak currents can easily be isolated and identified. The structure functions revealed by these probes thus become directly related to the structure of the hadrons -- more precisely, they record the longitudinal momentum distributions of the hadron's constituents which interact with these currents. Electron scattering and neutrino as well as antineutrino scattering are therefore intimately related through the dynamical connection between electromagnetic and weak currents. Such quantitative connections and their general experimental implications are derived and discussed in the following along with the extension of the parton model to parity violating weak currents.

A brief description of this work and summary of our main results were presented in the first paper of this series.[1]

II. Deep Inelastic Neutrino and Antineutrino Scattering and Derivation of the Parton Model

The kinematics for inelastic neutrino or antineutrino scattering from a nucleon target

(i) $\nu_\ell + P \rightarrow \ell +$ "anything"; $\ell = e$ or μ,

(ii) $\bar{\nu}_\ell + P \rightarrow \bar{\ell} +$ "anything"

are identical with the inelastic electron-nucleon scattering when the lepton rest masses are neglected, an approximation we shall make in this paper. The differential cross section for neutrino scattering in the rest system of the nucleon target is given by[5]

$$\frac{d^2\sigma^\nu}{d\cos\theta\, d\epsilon'} = \frac{G^2}{\pi}(\epsilon')^2\left[W'_2(q^2,\nu)\cos^2\frac{\theta}{2} + 2W'_1(q^2,\nu)\sin^2\frac{\theta}{2} + W'_3(q^2,\nu)\,\frac{\epsilon+\epsilon'}{M}\,\sin^2\frac{\theta}{2}\right] \tag{1}$$

where ϵ is the incident neutrino energy and ϵ', θ are the energy and angle of the outgoing lepton, G is the Fermi coupling constant ($G \approx 10^{-5}/M^2$). The invariants q^2 and ν are respectively the invariant momentum transfer and energy transfer in the laboratory frame to the nucleon. The structure functions W'_1, W'_2 and W'_3 are defined as

$$W'_{\mu\nu} = 4\pi^2 \frac{E_p}{M} \sum_n <P|J_\mu{}^c(0)|n><n|J_\nu{}^{c\dagger}(0)|P> (2\pi)^4\delta^4 (q + P - P_n)$$
$$= -g_{\mu\nu} W'_1(q^2, \nu) + \frac{1}{M^2} P_\mu P_\nu W'_2(q^2, \nu) + \frac{i\epsilon_{\mu\nu\lambda\kappa} P^\lambda q^\kappa}{2M^2} W'_3(q^2, \nu) + \cdots \tag{2}$$

where P_μ and q_μ are respectively the four momentum vectors of the nucleon and of the momentum transfer; an average over the nucleon spin is understood. The dots in (2) denote additional terms proportional to q_μ or q_ν which therefore do not contribute to the inelastic scattering cross section because the lepton current is conserved in the zero lepton mass approximation. The third structure function W'_3 appears as a result of parity nonconservation in weak interactions; $J_\mu{}^c(x)$ is the Cabibbo current describing the hadronic weak interactions.

For antineutrino scattering the expressions corresponding to (1) and (2) are

$$\frac{d^2\sigma^{\bar\nu}}{d\cos\theta\, d\epsilon'} = \frac{G^2}{\pi} (\epsilon')^2 \left[W''_2(q^2, \nu)\cos^2\frac{\theta}{2} + 2W''_1(q^2, \nu)\sin^2\frac{\theta}{2} - W''_3(q^2, \nu)\frac{\epsilon + \epsilon'}{M}\sin^2\frac{\theta}{2}\right] \tag{3}$$

and

$$W''_{\mu\nu} = 4\pi^2 \frac{E_p}{M} \sum_n <P|J_\mu{}^{c\dagger}(0)|n><n|J_\nu{}^c(0)|P> (2\pi)^4\delta^4 (q + P - P_n)$$
$$= -g_{\mu\nu} W''_1(q^2, \nu) + \frac{1}{M^2} P_\mu P_\nu W''_2(q^2, \nu) + i\frac{\epsilon_{\mu\nu\lambda\kappa} P^\lambda q^\kappa}{2M^2} W''_3(q^2, \nu) + \cdots \tag{4}$$

with an obvious interpretation for the notation. The isotopic property of the Cabbibo current leads to simple relations between neutrino and antineutrino scattering on protons and neutrons:

$$W'_{\mu\nu}(\nu p) = W''_{\mu\nu}(\bar\nu n)$$
$$W'_{\mu\nu}(\nu n) = W''_{\mu\nu}(\bar\nu p). \tag{5}$$

In the following we shall concentrate our attention on neutrino scattering. The results obtained can be translated immediately into those for antineutrino scattering.

As in the study of inelastic electron scattering we work in the infinite momentum center of mass frame of the initial neutrino (antineutrino) and nucleon where

$$q^o = \frac{2M\nu - Q^2}{4P}, \quad q_3 = \frac{-2M\nu - Q^2}{4P}, \quad |q_\perp| = \sqrt{Q^2} + 0\left(\frac{1}{P^2}\right) \qquad (6)$$
$$Q^2 \equiv |q^2|$$

with the nucleon momentum $\underset{\sim}{P}$ along the 3 axis. We undress the Cabibbo current operator $J_\mu{}^c$ in terms of the corresponding bare or free current operator $j_\mu{}^c$ by the U transformation

$$J_\mu{}^c(x) = U^{-1}(t)\, j_\mu{}^c(x) U(t) \qquad (7)$$

with

$$U(t) = \left(e^{-i\int_{-\infty}^{t} dt\, H_I(t)}\right)_+ . \qquad (8)$$

In our field theoretic model H_I and $J_\mu{}^c$ are respectively

$$H_I(t) = ig\int d^3x\, \overline{\psi}\gamma_5 \underset{\sim}{\tau}\psi \cdot \underset{\sim}{\pi} \qquad (9)$$

and

$$J_\mu{}^c = \overline{\psi}_n \gamma_\mu (1 - i\gamma_5)\psi_p - \sqrt{2}\, i\, \pi^o \overleftrightarrow{\partial}_\mu \pi^+ . \qquad (10)$$

The pion contribution to $J_\mu{}^c$ is the consequence of the conserved vector current hypothesis of Feynman and Gell-Mann.[2] Here we neglect strange particles as well as strangeness changing weak currents and set the Cabibbo angle $\theta_c \approx 0$. Refinements to include such effects can be made but the corrections are expected to be negligible in the present context since $\theta_c \approx 0$ empirically.

As can be readily verified with the explicit representation of the Dirac matrices given in Paper II, the time and third component along P_μ of the Cabbibo weak current (10) are "good components" in the same sense as are the corresponding components of the electromagnetic current discussed in II. By restricting j_μ^c to the good components ($\mu = 0$ or 3) the formal derivation of the parton model given in Paper II for inelastic electron scattering can be immediately adapted to the present case to obtain, in the Bjorken limit (Lim_{bj}) of large Q^2 and $M\nu$ with the ratio $w \equiv 2M\nu/Q^2$ fixed:

$$\mathrm{Lim}_{bj} W'_{\mu\nu} = 4\pi^2 \frac{E_p}{M} \int (dx)\, e^{iqx} \langle UP | j_\mu^{\,c}(x) j_\nu^{\,c\dagger}(0) | UP \rangle \tag{11}$$

for μ, $\nu = 0$ or 3.

There is a minor complication, however. Eq. (11) for μ, $\nu = 0$ or 3 sheds no light on the structure function W'_3, since the antisymmetric pseudotensor $\epsilon_{\mu\nu\lambda\kappa} P^\lambda q^\kappa \equiv 0$ for $\mu, \nu = 0$ or 3 as a result of the fact that P_μ has no transverse components ($P_1 = P_2 = 0$). Thus, to obtain information about W'_3 we <u>must</u> extend the parton model to transverse components of j_μ^c ($\mu = 1, 2$). Nevertheless, a minimum and sufficient extension for our purpose is to consider only the case in which one of the two currents in (2) is a transverse component and the other a good component. Then $\epsilon_{0132} P_3 q_2 \neq 0$, for instance, and W'_3 can be identified. Notice that these non-diagonal combinations of currents do not contribute to W'_1 or W'_2, since both $g_{\mu\nu}$ and $P_\mu P_\nu$ vanish identically for these tensor indices.

To extend the parton model to transverse components of weak currents in the limited manner described above let us recall first the main conclusion reached in the derivation of (11) for good components of the weak current. The physical picture of a scattering process as described by (11) in the infinite momentum frame (6) may be summarized as follows (see Fig. 1). Each term in the infinite series of the old-fashioned time-ordered perturbation expansion of $|UP>$ represents a multi-particle state with all constituents moving forward along the infinite initial nucleon momentum $\underset{\sim}{P}$. The weak current scatters one of the constituents and imparts to it a very large transverse momentum $q_\perp$ in the Bjorken limit. This scattered constituent emits and reabsorbs pions and nucleon-antinucleon pairs; they form a group of particles moving close to each other which we call (B) in Fig. 1. The unscattered constituents of $|UP>$ also emit and reabsorb pions and nucleon-antinucleon pairs. They form a second group of particles moving close to each other along the direction of P which we call (A) in Fig. 1. In the Bjorken limit of large Q^2 and $M\nu$, there is no interaction nor interference between the two groups of particles; and in addition, the energy differences between $|UP>$ and $|P>$ as well as between $|Un>$ and $|n>$ can be neglected. Eq. (11) then follows from these simplifications in the Bjorken limit and from the unitarity of the U matrix.

The whole discussion can be applied with one modification to the case where only one transverse component of $J_\mu^{\ c}$ appears in (2). For definiteness we will assume that $J_\mu^{\ c}(x)$ is a transverse, or bad, component ($\mu = 1$ or 2) and $J_\nu^{\ c}(0)$ a good component ($\nu = 0$ or 3) of the weak current. In the infinite momentum frame (6) the vertex of a weak transverse current can to leading order in P create or absorb a nucleon-antinucleon pair with one member of the pair necessarily having a negative longitudinal momentum. In contrast to a good current this leads to a vertex of order P. Also we need consider only the nucleon current in the present discussion, since the transverse component of the pion current will be proportional not to P which is purely longitudinal, but to a bounded, finite transverse momentum vector other q, which does not contribute due to lepton current conservation. The particle with negative longitudinal momentum must be annihilated or change the direction of its longitudinal momentum at the next strong vertex if it is created by the weak vertex. If it is absorbed by the weak vertex, it must come from the strong vertex immediately preceding the weak vertex as was discussed in detail in Paper II on the basis of the familiar energy denominator arguments. These two kinds of weak vertices that create or absorb a nucleon pair and which involve a particle with negative longitudinal momentum will be simply called Z type weak vertices. A typical example is given in Fig. 2. In the Bjorken limit there can be no interaction nor interference between the two groups of particles (A) and (B) as discussed in Paper II because the transverse momentum cutoff at every strong vertex prevents any overlap between them as $q_{\perp} \to \infty$. Moreover, to form the tensor $W'_{\mu\nu}$ the two amplitudes $<P|J_{\mu}^{c}|n>$ and $<n|J_{\nu}^{c\dagger}|P>$ can combine to give a nonvanishing contribution in the Bjorken limit only if the two amplitudes produce two identical groups of particles (A) and (B). The presence of a particle with negative longitudinal momentum at a weak vertex does not alter these conclusions. Several examples with a Z type weak vertex contributing to $W'_{\mu\nu}$ are illustrated in Fig. 3. It is clear from momentum conservation at each vertex and the examples in Fig. 3 that if a Z type weak vertex creates a particle with negative longitudinal momentum, this particle must have a momentum $\overline{\underset{\sim}{P}}_n = -\underset{\sim}{P}_n$ where $\underset{\sim}{P}_n$ is the momentum of the particle in the other half of the diagram which is to interact with a good component of the weak current. If the Z type weak vertex absorbs a particle with negative longitudinal momentum, this particle must have a momentum $\overline{P}'_n = -P'_n$, where P'_n is the final momentum of the particle immediately after being scattered from a good current in the other half of the diagram. Since the overall energy conserving delta function which appears in (2) depends only on the energies of the final real particles which have the identical momenta regardless of the presence or absence of a Z type weak vertex, we can, by the same arguments given in Paper II, replace it by the energy conserving delta function across the weak vertex of the good current. As an example, consider diagram (c") of Fig. 3. In terms of the momentum labels given there and the parametrization

$$\begin{aligned} &\underset{\sim}{P}_1 = \eta_1 \underset{\sim}{P} + \underset{\sim}{k}_{1\perp}, \quad \underset{\sim}{k}_1 = (1-\eta_1)\underset{\sim}{P} - \underset{\sim}{k}_{1\perp}, \quad \underset{\sim}{k}_{1\perp} \cdot \underset{\sim}{P} = 0, \quad 0<\eta_1<1 \\ &\underset{\sim}{P}'_1 = \underset{\sim}{P}_1 + \underset{\sim}{q} = \overline{\underset{\sim}{P}'_1} \\ &\underset{\sim}{P}_2 = \eta_2 \underset{\sim}{P}'_1 + \underset{\sim}{k}_{2\perp}, \quad \underset{\sim}{k}_2 = (1-\eta_2)\underset{\sim}{P}'_1 - \underset{\sim}{k}_{2\perp}, \quad \underset{\sim}{k}_{2\perp} \cdot \underset{\sim}{P}'_1 = 0, \quad 0<\eta_2<1 \end{aligned} \tag{12}$$

we obtain, in the Bjorken limit,

$$\begin{aligned} q^o + E_p - E_2 - \omega_1 - \omega'_2 &= q^o + (P + \frac{M^2}{2P}) - \left(\eta_2 P'_1 + \frac{k^2_{2\perp} + M^2}{2\eta_2 P'_1}\right) - \left((1-\eta_2)P'_1 + \frac{k^2_{2\perp} + \mu^2}{2(1-\eta_2)P'_1}\right) \\ &\quad - \left((1-\eta_1)P + \frac{k^2_{1\perp} + \mu^2}{2(1-\eta_1)P}\right) \\ &= q^o + \eta_1 P - P'_1 + \frac{1}{P}\, 0\,(1) \\ &= q^o + E_1 - E'_1 + \frac{1}{P}\, 0\,(1) \\ &= \frac{1}{2\eta_1 P}(2M\nu\eta_1 - Q^2) + \frac{1}{P}\, 0\,(1) \end{aligned}$$

and therefore

$$\delta(q^o + E_p - E_2 - \omega_1 - \omega'_2) = \delta(q^o + E_1 - E'_1) \tag{13}$$

which verifies our statement.

We will now show that diagrams with a Z type weak vertex do not contribute to the structure functions W'_1, W'_2 and W'_3 in the Bjorken limit.

Our argument makes use of relativistic covariance and the fact that the final result will be in the form of the covariant tensor (2). There is no need to consider the Z type weak vertices for the good components of the current as was analyzed in Paper II. As discussed earlier we need consider only the case in which one of the two currents in (2) is a transverse component -- i.e. W_{01} for example which contributes only to W'_3. To leading order in $P \to \infty$ we are looking for potential contributions proportional to $\epsilon_{01\mu\nu} P^{\mu} q^{\nu} \sim P q_{\perp}$. The question therefore is: can there be any contributions proportional to a transverse component of the momentum transfer, $q_{\perp}$, and simultaneously proportional to P arising from a Z graph? The graphs without a Z occurring at the weak vertex will in general give rise to a contribution of this form with the P coming from the good component and the q from the bad or transverse current of the weak current. However, if a Z is introduced at the vertex of the bad component of the weak current this vertex must provide both a q and a P factor. The P factor is needed to overcome the additional factor of $\frac{1}{P^2}$ from the energy denominator connecting two adjacent Z vertices which multiples the extra factor of P appearing at a Z vertex involving the strong interaction γ_5. However, the following table of vertices shows that the transverse component of a weak Z vertex will give either a factor of q or P but not both:

$$
\begin{aligned}
\bar{u}_{P_n} \gamma_3 u_{P_n+q} &= \frac{1}{2M} U_1^* (2P_n + q_3 + \sigma_3 \sigma \cdot q_\perp) U_2 \\
\bar{u}_{P_n} \gamma^o u_{P_n+q} &= \frac{1}{2M} U_1^* (2P_n + q^o + \sigma_3 \sigma \cdot q_\perp - \frac{Q^2}{2P_n}) U_2 \\
\bar{u}_{P_n} \gamma_3 \gamma_5 u_{P_n+q} &= \frac{1}{2M} U_1^* (2P_n \sigma_3 + q_3 \sigma_3 + \sigma \cdot q_\perp) U_2 \\
\bar{u}_{P_n} \gamma^o \gamma_5 u_{P_n+q} &= \frac{1}{2M} U_1^* (2P_n \sigma_3 + q^o \sigma_3 + \sigma \cdot q_\perp - \sigma_3 \frac{Q^2}{2P_n}) U_2 \\
\bar{u}_{P_n} \gamma_\perp u_{P_n+q} &= \frac{1}{2M} U_1^* [(\sigma_\perp \sigma \cdot q_\perp - q_\perp) + q_\perp - \sigma_\perp \sigma_3 \frac{Q^2}{2P_n}] U_2 \\
\bar{u}_{P_n} \gamma_\perp \gamma_5 u_{P_n+q} &= \frac{1}{2M} U_1^* [\sigma_3 (\sigma_\perp \sigma \cdot q_\perp - q_\perp) + \sigma_3 q_\perp + \sigma_\perp \frac{Q^2}{2P_n}] U_2
\end{aligned}
\tag{14}
$$

$$\bar{u}_{P_n}\gamma_\perp V_{-P_n-q} = \frac{1}{2M} U_1^*[2P_n\sigma_\perp - \sigma_3\sigma_\perp(\sigma_\perp \cdot q_\perp)] U_2$$

$$\bar{u}_{P_n}\gamma_\perp\gamma_5 V_{-P_n-q} = \frac{1}{2M} U_1^* [2P_n\sigma_3\sigma_\perp - \sigma_\perp(\sigma_\perp \cdot q_\perp)] U_2$$

Furthermore, in addition to the two weak vertices, q_μ appears only in the <u>invariant</u> structures q^2 and $M\nu$ in the energy conserving delta function in the Bjorken limit, as illustrated by the simple example (13). As a result, there is no way to introduce vectorial dependence on q_μ other than from the two weak vertices. Consequently, these diagrams do not contribute at all to the structure function W'_3 since there is no novanishing contribution from these diagrams in the Bjorken limit which is of the form $Pq_\perp$, and the tensor $\epsilon_{\mu\nu\lambda\kappa}P^\lambda q^\kappa \propto Pq_\perp$ for $\mu = 1$, $\nu = 0$, contributes only to terms of order $\frac{1}{P}(Pq_\perp)$.

We will now show by explicit calculation how the relativistic construction of $W'_{\mu\nu}$ can be carried out without ambiguity when (11) is extended to include transverse components of the weak current. Since Z type weak vertices are omitted the evaluation of (11) involves the calculation of the diagonal matrix elements

$$\int (dx) e^{iqx} \langle k_n | j_\mu^c(x) j_\nu^{c\dagger}(0) | k_n \rangle = \frac{1}{4\pi^2}\frac{2}{2\omega_n}(2k_{n\mu} + q_\mu)(2k_{n\nu} + q_\nu)\delta(q^2 + 2k_n \cdot q) \tag{15}$$

for a pion current contribution, and

$$\begin{aligned}\int (dx) e^{iqx} &\langle P_{ns} | j_\mu^c(x) j_\nu^{c\dagger}(0) | P_{ns'} \rangle \\ &= \frac{1}{4\pi^2}\cdot\frac{2M}{2E_n}\bar{u}_{P_n}(s)\gamma_\mu(1-\gamma_5)[M+\gamma(P_n+q)]\gamma_\nu(1-\gamma_5)u_{P_n}(s')\delta(q^2+2P_n\cdot q)\end{aligned} \tag{16}$$

for a nucleon contribution. In (15) and (16), k_n and P_n are the momenta, respectively, born by the pion and the nucleon constituent to be scattered by the current, and s, s' are the spins of the nucleon constituent.

To simplyfy (16) we notice that

$$\begin{aligned} &\bar{u}_{P_n}(s)\gamma_\mu(1-\gamma_5)[M+\gamma(P_n+q)]\gamma_\nu(1-\gamma_5)u_{P_n}(s') \\ &= 2\bar{u}_{P_n}(s)\gamma_\mu\gamma(P_n+q)\gamma_\nu(1-\gamma_5)u_{P_n}(s') \\ &= 2\bar{u}_{P_n}(s)\left\{\frac{4P_{n\mu}P_{n\nu}}{2M} + g_{\mu\nu}\frac{-2P_n(q+P_n)}{2M} - \gamma(P_n+q)\, i\sigma_{\mu\nu}(\gamma_5)\right. \\ &\quad \left. -2P_{n\mu}\gamma_\nu(\gamma_5) + g_{\mu\nu}\,\gamma(P_n+q)(\gamma_5) + \gamma(P_n+q)\, i\sigma_{\mu\nu}\right\} u_{P_n}(s') \end{aligned} \tag{17}$$

where use has been made of the Dirac equation, and of the identity

$$\gamma_\mu\gamma_\nu = +g_{\mu\nu} - i\sigma_{\mu\nu} \tag{18}$$

with $\sigma_{\mu\nu} \equiv \frac{i}{2}[\gamma_\mu, \gamma_\nu]$; terms proportional to q_μ or q_ν are neglected. The last three terms in (17) cannot contribute to the final answer, as we now show. Aside from the weak vertices q_μ enters the matrix element only in the scalar forms q^2 and $q \cdot P = M\nu$. For example the delta functions in (15) and (16) depend only on $Q^2 - 2M\nu\eta$, where η is the fraction of longitudinal momentum born by the constituent on which the bare current lands. Thus, the $P_{n\mu}\gamma_\nu(\gamma_5)$ term in (17) must eventually appear in the final answer as a pseudotensor constructed entirely from P_μ alone; this is obviously impossible. Similarly, $\gamma(P_n + q)(\gamma_5)$ in (17) must appear as a pseudoscalar constructed from P_λ and q_κ; this too cannot occur. Finally, $\gamma(P_n + q)\sigma_{\mu\nu}$ must appear as a second rank antisymmetrical tensor in P_μ and q_ν, that is as $P_\mu q_\nu - P_\nu q_\mu$. This form, however, is absent from our final answer by lepton current conservation. In any event its coefficient must vanish in the final answer, as time reversal invariance and parity conservation in strong interactions forbid the appearance of such a tensor $(P_\mu q_\nu - P_\nu q_\mu)$ in $W'_{\mu\nu}$.

To proceed further, we simplify the third term in (17) with the aid of the identity

$$\sigma_{\mu\nu}\gamma_5 = -\frac{i}{2}\epsilon_{\mu\nu\lambda\kappa}\sigma^{\lambda\kappa} = \frac{1}{2}\epsilon_{\mu\nu\lambda\kappa}\gamma^{\lambda}\gamma^{\kappa} \tag{19}$$

where $\epsilon_{\mu\nu\lambda\kappa}$ is the totally antisymmetrical tensor with the normalization $\epsilon^{0123} = 1$. Thus, when sandwiched between two Dirac spinors of same momentum, we have

$$\gamma(P_n + q)\, i\sigma_{\mu\nu}\ \ \gamma_5 \rightarrow \frac{1}{2M}[2P_n\cdot(P_n + q) + \gamma(P_n + q)(M - \gamma P_n)]\ \ \frac{i}{2}\ \epsilon_{\mu\nu\lambda\kappa}\gamma^{\lambda}\gamma^{\kappa}. \tag{20}$$

The first term cannot contribute to the final answer since $2P_n\cdot(P_n + q) = 2M^2 + Q^2$ by the delta function in (16), and $\epsilon_{\mu\nu\lambda\kappa}\gamma^{\lambda}\gamma^{\kappa}$ must appear as a pseudotensor constructed from P_μ alone which is impossible. Commuting $M - \gamma P_n$ through $\gamma^{\lambda}\gamma^{\kappa}$ to operate on the Dirac spinor, we obtain

$$\begin{aligned}\gamma(P_n + q)\, i\sigma_{\mu\nu}(\gamma_5) &\rightarrow \frac{1}{2M}(2i)\gamma(P_n + q)\,\epsilon_{\mu\nu\lambda\kappa}P_n^{\lambda}\gamma^{\kappa} \\ &= \frac{1}{2M}(2i)\left[-\epsilon_{\mu\nu\lambda\kappa}P_n^{\lambda}q^{\kappa} + \epsilon_{\mu\nu\lambda\kappa}P_n^{\lambda}(P_n + q)_{\tau}\ \sigma^{\tau\kappa}\right]\end{aligned} \tag{21}$$

The second step follows from the use of identity (18). Again, the last term cannot contribute to the final answer, as there is no way to construct from P_μ alone a second and third rank tensor antisymmetrical in $\mu\nu$ corresponding to $\epsilon_{\mu\nu\lambda\kappa}P_n^{\lambda}P_{n\tau}\sigma^{\tau\kappa}$ and $\epsilon_{\mu\nu\lambda\kappa}P_n^{\lambda}\sigma^{\tau\kappa}$, respectively. Collecting, we get the final answer,

$$\begin{aligned}\int (dx)\, e^{+iqx} < P_n s|j_\mu^{c}(x) j_\nu^{c\dagger}(0)|P_n s'> &= \delta_{ss'}\,\delta(q^2 + 2P_n\cdot q) \times \\ &\frac{1}{4\pi^2}\frac{1}{2E_n}\cdot 2[4P_{n\mu}P_{n\nu} - Q^2 g_{\mu\nu} + 2i\epsilon_{\mu\nu\lambda\kappa}P_n^{\lambda}q^{\kappa}]\end{aligned} \tag{22}$$

If the current lands on an antinucleon instead of a nucleon, the corresponding result is

$$\int (dx)\, e^{+iqx} <\overline{P}_n s | j_\mu^{\,c}(x)\, j_\nu^{\,c\dagger}(0) | \overline{P}_n s'> = \delta_{ss'}\, \delta(q^2 + 2\overline{P}_n q) \times$$
$$\frac{1}{4\pi^2}\, \frac{1}{2\overline{E}_n}\, 2[4\overline{P}_{n\mu}\overline{P}_{n\nu} - Q^2 g_{\mu\nu} + 2i\,\epsilon_{\mu\nu\lambda\kappa}\overline{P}_n^{\;\lambda} q^\kappa] \tag{23}$$

The sign difference between the last terms of (22) and (23) is due to the fact that a (V–A) coupling for particles corresponds to a (V + A) coupling for antiparticles, and vice versa.

In the infinite momentum frame (6) and in the Bjorken limit an approximation consistent with relativistic covariance is to make the substitution

$$k_{n\mu} = \eta P_\mu, \quad P_{n\mu} = \eta P_\mu, \quad \overline{P}_{n\mu} = \eta P_\mu \tag{24}$$

in (15), (22) and (23) with η the fraction of the longitudinal momentum born by the constituent on which the current lands. Following a procedure employed in Paper II we expand $|UP>$ in a complete set of multiparticle states

$$|UP> = \sum_n a_n |n>, \qquad \sum_n |a_n|^2 = 1 \tag{25}$$

Eqs. (11), (15), (22), (23), (24) and (25) combine to give

$$W'_{\mu\nu} = \frac{2}{w} \sum_n |a_n|^2 <n| \frac{P_\mu P_\nu}{M^2} \left[\frac{1}{\nu} \sum_i (\lambda_{n,i}^{\,F})^2\, \delta(\eta_{n,i} - \frac{1}{w}) + \frac{1}{\nu} \sum_j (\lambda_{n,j}^{\,B})^2 \delta(\eta_{n,j} - \frac{1}{w})\right]$$
$$- g_{\mu\nu}\, \frac{w}{2M} \sum_i (\lambda_{n,i}^{\,F})^2\, \delta(\eta_{n,i} - \frac{1}{w}) \tag{26}$$
$$+ i\, \frac{\epsilon_{\mu\nu\lambda\kappa} P^\lambda q^\kappa}{2M^2}\, \frac{1}{\nu}\, w \sum_i [\pm (\lambda_{n,i}^{\,F})^2]\, \delta(\eta_{n,i} - \frac{1}{w})\, |n>$$

where $\eta_{n,i}$ or $\eta_{n,j}$ have the same meaning as η in (24); $\lambda_{n,i}^{\,F}$ ($\lambda_{n,j}^{\,B}$) is the "weak charge" of the i^{th} (j^{th}) Fermion (Boson) constituent in the state $|n>$ of (25). In the last term of (26) the upper (lower) sign applies to a nucleon (antinucleon). For neutrino scattering we have

$$\nu \text{ scattering: } \lambda^F_{n,i} = 1 \quad \text{for neutron, antiproton,} \qquad (27)$$
$$= 0 \quad \text{for proton, antineutron:}$$

$$\lambda^B_{n,j} = 1 \quad \text{for } \pi^o, \ \pi^- \qquad (28)$$
$$= 0 \quad \text{for } \pi^+.$$

Eq. (26) is expressed explicitly in terms of the invariants Q^2 and $M\nu$ with the possible tensors properly extracted. It thus satisfies manifest relativistic covariance, and the results for W'_1 and W'_2 are precisely those expected from employing only good currents. Obviously (26) can now apply to all components of the weak current. We conclude therefore that the procedure leading to (26) is self consistent and justifies our omission of diagrams with any Z type weak vertex.

III. Results and Predictions

We now turn to some important theoretical and experimental implications of the results we have just obtained. Many but not all of these results were presented in Paper I.

It follows from (26) that W'_1, $\nu W'_2$ and $\nu W'_3$ are universal functions of w in the Bjorken limit of large Q^2 and $M\nu$. We define

$$\text{Lim}_{bj}\, MW'_1(q^2, \nu) = F'_1(w),$$
$$\text{Lim}_{bj}\, \nu W'_2(q^2, \nu) = F'_2(w), \qquad (29)$$
$$\text{Lim}_{bj}\, \nu W'_3(q^2, \nu) = F'_3(w).$$

These structure functions can be immediately identified from (26):

$$F'_\alpha(w) = F'_\alpha(w)^N + F'_\alpha(w)^{\overline{N}} + F'_\alpha(w)^\pi, \ \alpha = 1, 2, 3 \qquad (30)$$

$$F'_2(w) = \frac{2}{w} \sum_n \ |a_n|^2 \langle n| \sum_i \lambda^2_{n,i}\, \delta(\eta_{n,i} - \frac{1}{w})|n\rangle \qquad (31)$$

In (31) the summation over i extends over all Fermion and Boson constituents in state $|n\rangle$. The superscripts N, $\overline{N}$ and π on $F'_\alpha(w)$ denote the contributions from nucleon, antinucleon and pion lines on which the current lands, respectively. The other structure functions are also determined:

$$\begin{aligned} F'_1(w)^\pi &= 0, & F'_3(w)^\pi &= 0 \\ F'_1(w)^N &= \frac{w}{2} F'_2(w)^N, & F'_3(w)^N &= w\, F'_2(w)^N \\ F'_1(w)^{\overline{N}} &= \frac{w}{2} F'_2(w)^{\overline{N}}, & F'_3(w)^{\overline{N}} &= -wF'_2(w)^{\overline{N}}. \end{aligned} \tag{32}$$

Completely analogous results hold for antineutrino scattering structure functions. One may define $F''_{1,2,3}(w)$, for example, by replacing $W'_{1,2,3}$ in (29) by the corresponding $W''_{1,2,3}$. If one introduces the corresponding $F''_\alpha(w)$ for $F'_\alpha(w)$ in (30, (31) and (32), one obtains the analogous results for deep inelastic antineutrino scattering. The "weak charges" for antineutrino scattering are

$$\bar{\nu} \text{ scattering:} \quad \begin{aligned} \lambda^F_{n,i} &= 1 \quad \text{for proton, antineutron} \\ &= 0 \quad \text{for antiproton, neutron} \end{aligned} \tag{33}$$

$$\begin{aligned} \lambda^B_{n,j} &= 1 \quad \text{for } \pi^o,\ \pi^+ \\ &= 0 \quad \text{for } \pi^- . \end{aligned} \tag{34}$$

Eq. (26) or (31) leads to a sum rule for neutrino-proton scattering.

$$\int_1^\infty \frac{dw}{w} F'_2(w)^{(\nu p)} = 2 \sum_n |a_n|^2 \left(\sum_i \lambda^2_{n,i}\right) \tag{35}$$

In our model this becomes

$$\int_1^\infty \frac{dw}{w} F'_2(w)^{(\nu p)} = 2 \sum_n \left(n_{\bar{p}} + n_{\pi^-} + n_n + n_{\pi^o}\right) |a_n|^2 \tag{36}$$

where n_n, $n_{\bar{p}}$, n_{π^-} and n_{π^0} denote respectively the number of neutrons, antiprotons, π^-'s and π^0's in the state $|n\rangle$ that appears in the expansion of $|UP\rangle$.

A similar sum rule holds for antineutrino-proton scattering

$$\int_1^\infty \frac{dw}{w} F''_2(w)^{(\bar{\nu}p)} = 2 \sum_n (n_p + n_{\pi^+} + n_{\bar{n}} + n_{\pi^0}) |a_n|^2. \tag{37}$$

Therefore

$$\int_1^\infty \frac{dw}{w} [F''_2(w)^{(\bar{\nu}p)} - F'_2(w)^{(\nu p)}] = 2 \sum (n_p + n_{\pi^+} + n_{\bar{n}} - n_n - n_{\bar{p}} - n_{\pi^-}) |a_n|^2 \tag{38}$$

Electric charge conservation imposes the condition

$$(n_p + n_{\pi^+}) - (n_{\bar{p}} + n_{\pi^-}) = 1 \tag{39}$$

and the baryon number conservation requires

$$(n_p + n_n) - (n_{\bar{p}} + n_{\bar{n}}) = 1 \tag{40}$$

These two conditions together fix

$$n_{\bar{n}} - n_n = n_{\pi^-} - n_{\pi^+} \tag{41}$$

and (38) becomes

$$\int_1^\infty \frac{dw}{w} [F''_2(w)^{(\bar{\nu}p)} - F'_2(w)^{(\nu p)}] = 2 \sum_n (n_p - n_{\bar{p}}) |a_n|^2 \tag{42}$$

This is our analog of Adler's sum rule[6] for neutrino-proton and antineutrino-proton scattering. Adler's sum rule is valid for any fixed values of Q^2 and in particular yields

$$\int_1^\infty \frac{dw}{w} [F''_2(w)^{(\bar{\nu}p)} - F'_2(w)^{(\nu p)}] = 2 \tag{43}$$

in the Bjorken limit. The reason that (42) does not satisfy the Adler relation is because the weak current (10) in our model does not have an axial vector partner for the vector pion current and hence violates the Gell-Mann commutator relations for the axial and vector charges that are used to derive (42). Eq. (43) would be satisfied if there were only π^o's and protons in our model, since in (42) we would then have $n_p - n_{\bar{p}} = 1$ by electric charge and baryon conservation, there being no neutron. The normalization condition for the a_n's then yields (43).

As another more realistic model of a theory of strong interactions which obeys chiral symmetry and the Gell-Mann algebra of charges we consider the σ model[7] which contains an even parity spinless isoscalar meson σ in addition to the odd parity spinless isovector pion. The chiral invariant interaction Hamiltonian is given by

$$H_I = -g \int d^3x \, \bar{\psi} \, (\sigma - i\gamma_5 \underset{\sim}{\tau} \cdot \underset{\sim}{\pi}) \psi \tag{44}$$

and the weak current has the chiral structure

$$J_\mu^{\,c} = \bar{\psi}_n \gamma_\mu (1 - \gamma_5) \psi_p - \sqrt{2}\, i \, (\pi^o \overleftrightarrow{\partial}_\mu \pi^+ - \sigma \overleftrightarrow{\partial}_\mu \pi^+) \tag{45}$$

This is a particular example satisfying the $SU_2 \times SU_2$ symmetry group and can also be generalized to $SU_3 \times SU_3$. A parton model can be derived and discussed for the interaction (44) and the weak current (45). In this chiral model the weak charges for nucleons and antinucleons remain the same as in the earlier pion model without the σ, but the weak charges for mesons are different. We have for neutrino scattering

$$\begin{aligned} \nu \text{ scattering: } (\lambda^B_{n,j})^2 &= 1 \quad \text{for } \sigma, \pi^o \\ &= 2 \quad \text{for } \pi^- \\ &= 0 \quad \text{for } \pi^+ \end{aligned} \tag{46}$$

and for antineutrino scattering

$$\bar{\nu} \text{ scattering: } (\lambda^{B}_{n,j})^2 = 1 \text{ for } \sigma,\ \pi^{o}$$
$$= 2 \text{ for } \pi^{+} \qquad (47)$$
$$= 0 \text{ for } \pi^{-}.$$

The sum rules similar to (36) and (37) become

$$\int_1^\infty \frac{dw}{w} F'_2(w)^{(\nu p)} = 2 \sum_n (n_{\bar{p}} + n_n + n_{\pi^o} + n_\sigma + 2n_{\pi^-}) |a_n|^2 \qquad (48)$$

$$\int_1^\infty \frac{dw}{w} F''_2(w)^{(\bar{\nu} p)} = 2 \sum_n (n_p + n_{\bar{n}} + n_{\pi^o} + n_\sigma + 2n_{\pi^+}) |a_n|^2 \qquad (49)$$

and therefore

$$\int_1^\infty \frac{dw}{w} [F''_2(w)^{(\bar{\nu} p)} - F'_2(w)^{(\nu p)}] = 2 \sum_n (n_p + 2n_{\pi^+} + n_{\bar{n}} - n_{\bar{p}} - 2n_{\pi^-} - n_n) |a_n|^2. \qquad (50)$$

The two conditions (39) and (40), and hence (41), are also true in this model. Consequently

$$\int_1^\infty \frac{dw}{w} [F''_2(w)^{(\bar{\nu} p)} - F'_2(w)^{(\nu p)}] = 2 \sum_n (n_p + n_{\pi^+} - n_{\bar{p}} - n_{\pi^-}) |a_n|^2$$
$$= 2 \sum_n |a_n|^2 \qquad (51)$$
$$= 2$$

by charge conservation. The Adler sum rules (43) are therefore satisfied in this model. Similarly we can derive from (26) or (31) and (32) the following results for $F'_3(w)$ and $F''_3(w)$:

$$\int_1^\infty \frac{dw}{w^2} F'_3(w)^{(\nu p)} = 2 \sum_n (n_n - n_{\overline{p}}) |a_n|^2 \tag{52a}$$

$$\int_1^\infty \frac{dw}{w^2} F''_3(w)^{(\overline{\nu} p)} = 2 \sum_n (n_p - n_{\overline{n}}) |a_n|^2 \tag{52b}$$

which hold for both the original pion triplet model and the pion triplet plus the σ singlet model since only the nucleon weak current contributes to F'_3 and F''_3. Adding (52a) and (52b) we get

$$\begin{aligned}\int_1^\infty \frac{dw}{w^2} [F'_3(w)^{(\nu p)} + F''_3(w)^{(\overline{\nu} p)}] &= 2\sum_n (n_p + n_n - n_{\overline{p}} - n_{\overline{n}}) |a_n|^2 \\ &= 2\end{aligned} \tag{53}$$

by the baryon conservation. Sum rules for $F'_3(w)$ and $F''_3(w)$ similar to (53) are discussed for different models by Gross and Llewellyn Smith.[8]

In the Bjorken limit the differential cross sections (1) and (3) can be expressed in terms of the structure functions $F'_{1,2,3}(w)$ for neutrino scattering and $F''_{1,2,3}(w)$ for antineutrino scattering:

$$\begin{aligned}\frac{d^2\sigma^\nu}{d\epsilon' d\cos\theta} = \frac{G^2}{\pi}\epsilon'\left(\frac{\epsilon'}{\nu}\right)\Bigg[& F'_2(w)^N \left\{\cos^2\frac{\theta}{2} + (w + \frac{2\epsilon - \nu}{\nu} w)\frac{\nu}{M}\sin^2\frac{\theta}{2}\right\} \\ & + F'_2(w)^\pi \cos^2\frac{\theta}{2} \\ & + F'_2(w)^{\overline{N}} \left\{\cos^2\frac{\theta}{2} + (w - \frac{2\epsilon - \nu}{\nu} w)\frac{\nu}{M}\sin^2\frac{\theta}{2}\right\}\Bigg]\end{aligned} \tag{54a}$$

and

$$\frac{d^2\sigma^{\bar{\nu}}}{d\epsilon' d\cos\theta} = \frac{G^2}{\pi}\epsilon'\left(\frac{\epsilon'}{\nu}\right)\left[F''_2(w)^N\left\{\cos^2\frac{\theta}{2} + (w - \frac{2\epsilon-\nu}{\nu}w)\frac{\nu}{M}\sin^2\frac{\theta}{2}\right\}\right.$$

$$+ F''_2(w)^\pi \cos^2\frac{\theta}{2} \tag{54b}$$

$$\left. + F''_2(w)^{\overline{N}}\left\{\cos^2\frac{\theta}{2} + (w + \frac{2\epsilon-\nu}{\nu}w)\frac{\nu}{M}\sin^2\frac{\theta}{2}\right\}\right]$$

Inserting the variables $Q^2 = 4\epsilon^2(1-y)\sin^2\frac{\theta}{2}$, $y = \frac{\nu}{\epsilon}$ and $d\epsilon' d\cos\theta = \frac{My}{1-y}\,dy\,d(\frac{1}{w})$, we find

$$\frac{d^2\sigma^{\nu}}{d(\frac{1}{w})dy} = \frac{G^2}{\pi}(M\epsilon)[F'_2(w)^N + F'_2(w)^\pi(1-y) + F'_2(w)^{\overline{N}}(1-y)^2] \tag{55a}$$

$$(0<y<1)$$

$$\frac{d^2\sigma^{\bar{\nu}}}{d(\frac{1}{w})dy} = \frac{G^2}{\pi}(M\epsilon)[F''_2(w)^{\overline{N}} + F''_2(w)^\pi(1-y) + F''_2(w)^N(1-y)^2] \tag{55b}$$

$$(0<y<1)$$

If the integration over the inelasticity y is performed, we get

$$\frac{d\sigma^{\nu}}{d(\frac{1}{w})} = \frac{G^2}{\pi}(M\epsilon)[F'_2(w)^N + \frac{1}{2}F'_2(w)^\pi + \frac{1}{3}F'_2(w)^{\overline{N}}] \tag{56a}$$

and

$$\frac{d\sigma^{\bar{\nu}}}{d(\frac{1}{w})} = \frac{G^2}{\pi}(M\epsilon)[F''_2(w)^{\overline{N}} + \frac{1}{2}F''_2(w)^\pi + \frac{1}{3}F''_2(w)^N] \tag{56b}$$

The difference in the neutrino and antineutrino cross sections (56a) and (56b) arises from the interchange of roles played by particles and antiparticles in the two processes. Eq. (56) predicts that the total cross sections grow linearly with the incident energy of the neutrino or antineutrino in the high energy region. This linear rise of total neutrino and antineutrino cross sections with energies is a striking prediction of this limiting behavior as discussed by Bjorken[4] and is independent of any dynamical details. It is consistent with the recently reported CERN data[9] in which $\epsilon_{max} \approx 10\mathrm{GeV}$. The linear rise with energy of the total cross sections would presumably be cut off at approximately the mass of the intermediate vector bosons if weak interactions were indeed mediated by such particles.

To help understand the difference of the neutrino and antineutrino scattering cross sections we will separate the individual contributions of left-handed, right-handed and longitudinally polarized currents. They are identified by projecting the currents onto the three polarization vectors denoted by ϵ_+^{μ}, ϵ_-^{μ} and $\epsilon_{\parallel}^{\mu}$ corresponding to a right-handed, left-handed and longitudinally polarized vector current, respectively (or in a model of weak interactions mediated by a W vector meson, of a boson of the same polarization). These vectors are given by

$$\epsilon_+^{\mu} = \frac{1}{\sqrt{2}}(0, 1, i, 0), \quad \epsilon_-^{\mu} = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \quad \epsilon_{\parallel}^{\mu} = \frac{1}{\sqrt{Q^2}}(q_3, 0, 0, q^0) \tag{57}$$

in the laboratory system where

$$q^{\mu} = (q^0, o, o, q_3) \tag{58}$$

As neutrino and antineutrino scattering are very similar, for definiteness we shall consider neutrino scattering only and define

$$\frac{1}{M}F'_+ = 4\pi^2 \frac{E_p}{M} \sum_n <P|\epsilon_+^{\mu *} J_{\mu}^{c}(0)|n><n|\epsilon_+^{\nu} J_{\nu}^{c\dagger}(0)|P>(2\pi)^4 \delta^4(q + P - P_n) \tag{59}$$

and analogously F'_- and $F'_{\parallel}$. These quantities, F'_+, F'_-, and $F'_{\parallel}$, measure the relative importance of the three polarization states of the current. Using (2) and (57), we obtain from (59)

$$
\begin{aligned}
F'_+ &= MW'_1 - \tfrac{1}{2}\sqrt{Q^2+\nu^2}\, W'_3 \\
F'_- &= MW'_1 + \tfrac{1}{2}\sqrt{Q^2+\nu^2}\, W'_3 \\
F'_\parallel &= -MW'_1 + \left(1+\frac{\nu^2}{Q^2}\right) MW'_2 .
\end{aligned}
\tag{60}
$$

In the Bjorken limit these relations reduce to

$$
\begin{aligned}
F'_+(w) &= F'_1(w) - \tfrac{1}{2} F'_3(w) \\
F'_-(w) &= F'_1(w) + \tfrac{1}{2} F'_3(w) \\
F'_\parallel(w) &= \tfrac{w}{2} F'_2(w) - F'_1(w).
\end{aligned}
\tag{61}
$$

We conclude from (61) and (32) that

$$
\begin{aligned}
F'_+(w)^N = 0, \qquad & F'_-(w)^{\overline{N}} = 0 \\
& F'_+(w)^\pi = F'_-(w)^\pi = 0
\end{aligned}
\tag{62}
$$

Eq. (62) shows that in the Bjorken limit independent of strong interaction dynamics a right-handed (left-handed) current or W boson cannot interact with a nucleon (an antinucleon); a right-handed or left-handed polarized current, or W boson, cannot interact with a skinless pion current. A simple explanation why $F'_+(w)^N = 0$ has been given in Paper I. It is a consequence of the basic assumption of our formalism that all the internal momenta of the nucleon's structure are small in the rest system of the nucleon in comparison with the asymptotically large Q^2 and $|\underset{\sim}{q}| = \sqrt{\nu^2 + Q^2} \approx \nu$ delivered by the current from the lepton line. Therefore in the Bjorken limit the current as viewed from the laboratory frame enters an assemblage of "slow" constituents of the nucleon and the one on which it lands recoils ultrarelativistically with $\underset{\sim}{q}$ leaving the others behind. If the constituent on which the bare current lands is a nucleon, by (10) that nucleon emerges with left-handed helicity -- a state which could not be created by a right-handed polarized W Boson. Thus right-handed W Bosons are absent from our model when the interaction is on the nucleon line. Similar consideration explains why $F'_{-}(w)^{\overline{N}} = 0$.

We recall that the behavior of the structure functions for deep inelastic electron scattering near the threshold $w \sim 1$ may be obtained from crossing properties of field theory and positivity of the physical cross section in the crossed channel of annihilation. Analogous considerations show that near $w \gtrsim 1$

$$F'_2(w) = (w - 1)^{2n+1}, \qquad n = 0, 1, 2, \cdots \tag{63}$$

if the nucleon current dominates; and

$$F'_2(w) = (w - 1)^{2n}, \qquad n = 0, 1, 2, \cdots \tag{64}$$

if the pion current dominates. One may also study the process of neutrino-positron (or antineutrino-electron) annihilation into an antinucleon with fixed momentum, plus anything; in complete analogy to the electron-positron annihilation process studied in Paper III. However, the feasibility of such experiments is so remote that we shall not consider them here.

Aside from the general implications of the parton model discussed above, other quantitative predictions can be made on the basis of specific models.

In the field theory model of Ref. 1 the nucleon current was found to be dominant in the very inelastic region with $w >> 1$ -- i.e. to leading order in $\ell n w > 1$, order by order of the interaction, the current landed on the nucleon line. We find in this region therefore that the neutrino cross section is given by

$$\frac{d^2\sigma^\nu}{d(\frac{1}{w})dy} = \frac{G^2}{\pi} (M\epsilon) F'_2(w)^N, \qquad w >> 1 \tag{65}$$

and the antineutrino cross section is given by

$$\frac{d^2\sigma^{\bar\nu}}{d(\frac{1}{w})dy} = \frac{G^2}{\pi}(M\epsilon)\, F''_2(w)^N (1-y)^2, \qquad w>>1 \tag{66}$$

In this kinematic region the dominant family of graphs according to our model is as illustrated in Fig. 4 and we can use simple charge symmetry to identify the neutrino reactions (via a W^+) on protons with antineutrinos (via a W^-) on neutrons and vice versa as given by (5). In particular[10]

$$\begin{aligned}\frac{d\sigma^{\nu p}}{d(\frac{1}{w})} &= 3\,\frac{d\sigma^{\bar\nu n}}{d(\frac{1}{w})}\\ & & w>>1\\ \frac{d\sigma^{\nu n}}{d(\frac{1}{w})} &= 3\,\frac{d\sigma^{\bar\nu p}}{d(\frac{1}{w})}\end{aligned} \tag{67}$$

and

$$\frac{d\sigma^{\nu p}}{d(\frac{1}{w})} + \frac{d\sigma^{\nu n}}{d(\frac{1}{w})} = 3\left[\frac{d\sigma^{\bar\nu p}}{d(\frac{1}{w})} + \frac{d\sigma^{\bar\nu n}}{d(\frac{1}{w})}\right], \qquad w>>1. \tag{68}$$

Another consequence of the ladder graphs is that the cross sections on neutrons and protons are equal as shown for inelastic electron scattering in Paper II. In fact, a similar calculation gives

$$\begin{aligned} &F'_2(w)^{\nu p} = F'_2(w)^{\nu n} = F''_2(w)^{\bar\nu p} = F''_2(w)^{\bar\nu n} = 2F_2(w)^{ep} = c\xi w^{\xi - 1},\\ & & w>>1\\ &F'_2(w)^{\nu p} - F'_2(w)^{\nu n} = -[F''_2(w)^{\bar\nu p} - F''_2(w)^{\bar\nu n}] = +\frac{2}{3}\, c\xi w^{-(\frac{1}{3}\xi+1)},\end{aligned} \tag{69}$$

where

$$\xi = \frac{3}{4\pi}\left(\frac{g^2}{4\pi}\right)\log\left[1 + \frac{k_{\perp\,max}^2}{M^2}\right]$$

and c is an unknown scale factor. All the same parameters ξ and c appear also in the structure functions of deep inelastic electron-proton scattering in the large $w>>1$ region. A factor of 2 in relations (69) between the neutrino and electron structure functions arises from the fact that $(1-\gamma_5)^2 = 2(1-\gamma_5)$. According to (69) we can rewrite (68) as

$$\frac{d\sigma^{\nu p}}{d(\frac{1}{w})} = \frac{d\sigma^{\nu n}}{d(\frac{1}{w})} = 3\,\frac{d\sigma^{\bar{\nu} p}}{d(\frac{1}{w})} = 3\,\frac{d\sigma^{\bar{\nu} n}}{d(\frac{1}{w})}, \qquad w>>1 \tag{70}$$

Eq. (68) or (70) tells us that the ratio of the limiting cross sections for large w is 3 to 1 for neutrinos relative to antineutrinos.

This ratio of 3 to 1 in the large w very inelastic region is the most striking prediction from our field theoretic basis for deriving the Bjorken limit. It presents a clear experimental challenge. For inelastic electron scattering Harari[11] has discussed the interpretation of the inelastic structure functions in terms of the contribution of the pomeron to the forward virtual compton cross section. The mechanism in our model does not correspond to this physical picture as discussed in Paper II.[12]

This model can also be used to compute the ratio of neutrino to electron scattering as a check against recent data reported at the 1969 CERN Weak Interaction conference.[9] In the large $w>>1$ region we predict from (65), (66), and (69) that

$$\frac{d^2\sigma^{\nu p}}{d(\frac{1}{w})dy} = \frac{d^2\sigma^{\nu n}}{d(\frac{1}{w})dy} = \frac{G^2}{\pi}\,(2M\epsilon)\,F_2(w)^{ep}, \qquad w>>1 \tag{71}$$

$$\frac{d^2\sigma^{\bar{\nu} p}}{d(\frac{1}{w})dy} = \frac{d^2\sigma^{\bar{\nu} n}}{d(\frac{1}{w})dy} = \frac{G^2}{\pi}\,(2M\epsilon)\,F_2(w)^{ep}(1-y)^2, \qquad w>>1 \tag{72}$$

Since the observed behavior of $F_2(w)$ in the electron scattering experiments[13] weights the large w region relatively heavily and falls off for $w \lesssim 3$ we can make an approximate prediction for the neutrino cross section in (71) by applying our result that the nucleon current dominates throughout the entire w interval in (71). Then as observed by Bjorken and Paschos[14] experimentally

$$\int_0^1 d\left(\frac{1}{w}\right) F_2(w)^{ep} \approx 0.18 \tag{73}$$

and by (71) and (73)

$$\sigma^{\nu p} = \sigma^{\nu n} \approx \frac{G^2}{\pi} (2M\epsilon)(0.16) = 4 \times 10^{-39} \mathrm{cm}^2 \times (\epsilon/\mathrm{GeV}) \tag{74}$$

The CERN data in the energy ranges up to $\epsilon_{max} \approx 10$ GeV are represented approximately by

$$\sigma^{\nu} = 6 \times 10^{-39} \mathrm{cm}^2 \times (\epsilon/\mathrm{GeV}) \tag{75}$$

which agrees with (75) within a factor of 2.

In kinematic regions where the pion current contribution is dominant, as we have conjectured in Paper II to be the case near $w = 1$, $W'^{\pi}_3 = 0$ since there is no bare axial pion current in (10). Also $W'^{\pi}_1 = 0$ as in the electromagnetic process because the convection current of spinless pions is along P_μ in the infinite momentum frame and therefore only W'_2 in (2) is non-vanishing. By a simple isotopic consideration

$$W'^{\pi}_2(\nu p) + W'^{\pi}_2(\nu n) = 4W^{\pi}_2(ep) \tag{76}$$

and by (2) and (6)

$$\left.\frac{d^2\sigma(\nu p) + d^2\sigma(\nu n)}{d^2\sigma(ep)}\right)_{\pi} = \frac{G^2(Q^2)^2}{2\pi^2\alpha^2} \approx 10^{-7}\left(\frac{Q^2}{M^2}\right)^2$$

compared at the identical values of Q^2 and $M\nu$. If the pion current instead of the nucleon current dominates throughout the entire w interval, then by (76) the same result as (74) would be obtained for the average nucleon cross section $\frac{1}{2}(\sigma^{\nu p} + \sigma^{\nu n})$ but in this case the ν and $\bar{\nu}$ cross sections would be equal instead of in the ratio of 3 : 1 for large w.

We may remark finally about the chiral model introduced by (44) and (45). In the large $w>>1$ region this model also predicts that the nucleon current dominates and the dominant diagrams are also of the form as in Fig. 4 with the dotted lines representing a σ or π. The proton and neutron cross sections are equal and given by an expression similar to (69); also (74) is valid.

IV. Summary and Conclusion

In this final article of a series of papers on lepton-hadron interactions we have extended our parton model to deep inelastic neutrino (antineutrino) scattering under the same fundamental assumption that there exists an asymptotic region in which the momentum and energy transfers to the hadrons can be made greater than the transverse momenta of their virtual constituents or "partons" as viewed in an infinite momentum frame. The specific new theoretical problem faced in this application is that of deriving the "parton" model in the presence of the additional parity violating term in the weak (V - A) current interaction. This leads to a third structure function and forces us to consider a transverse (or "bad") current component in addition to the "good" currents which were treated.

Beyond the derivation of the scaling behavior first demonstrated by Bjorken we have constructed the Adler sum rule for a field theory satisfying chiral symmetry and the Gell-Mann algebra of charges. We have also constructed sum rules of the Gross-Llewellyn-Smith type. Finally in the kinematic region of very large energy loss characterized by $w >> 1$ we have computed ratios of neutrino and antineutrino cross sections to inelastic scattering and to each other for comparison with the experiment.

From a theoretical point of view inelastic neutrino and antineutrino scatterings contain richer information than available from inelastic electron scattering. This is because the Cabibbo currents have definite internal symmetry transformation property. Unlike the deep inelastic electron scattering which can reveal only the distribution of the charged constituents, deep inelastic neutrino and antineutrino scattering together probe the distribution of all constituents inside the protons which could also be useful for a better description of purely hadronic processes. Moreover, because of parity non-conservation in the weak interaction neutrino and antineutrino scattering also yeild information about vector current - axial current interference.

ACKNOWLEDGMENT

We thank D. Levy who participated in this work during its early stage.

FOOTNOTES AND REFERENCES

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FIGURE CAPTIONS

Fig. 1 Physical picture for deep inelastic neutrino scattering in the Bjorken limit viewed in an infinite momentum frame. The weak vertex marked "X" here corresponds to a good current.

Fig. 2 Normal and Z-type weak vertices correspond to a transverse or "bad" current.

Fig. 3 Example of contributions to $W'_{\mu\nu}$ with a Z-type weak vertex.

Fig. 4 Dominant class of diagrams contributing to $W'_{\mu\nu}$ in large w region.

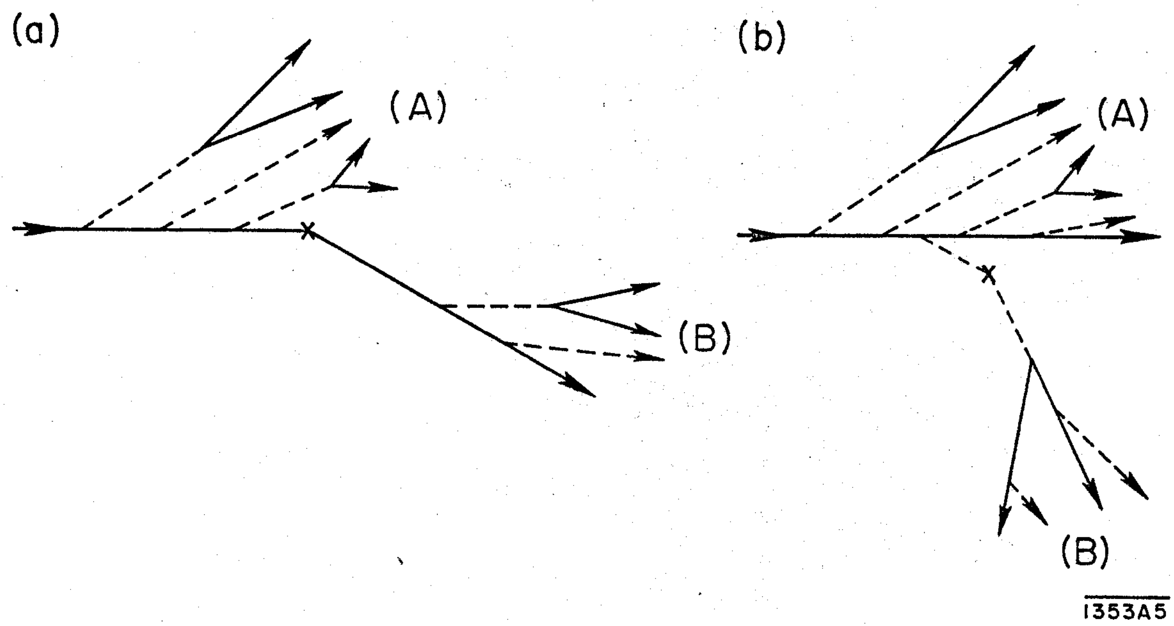

Fig. 1

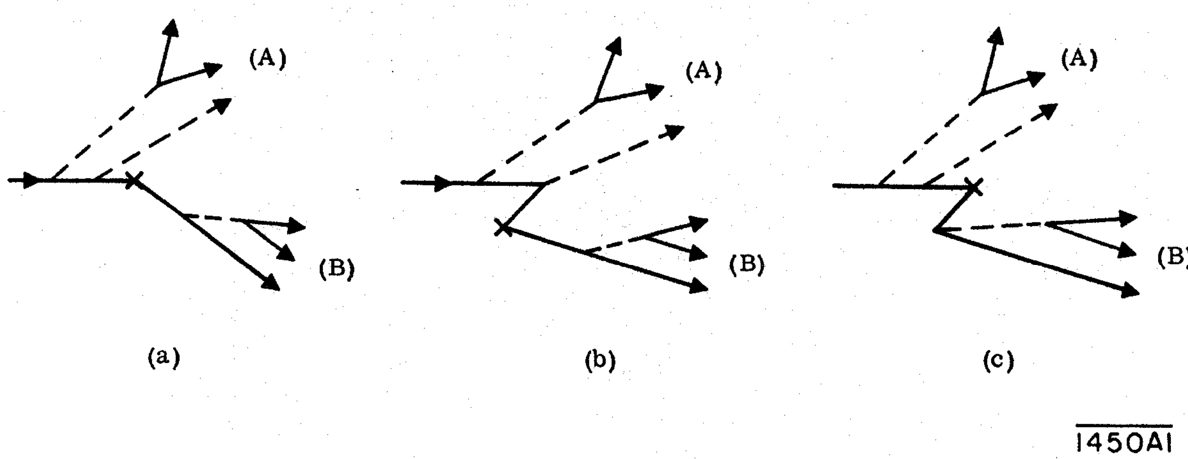

Fig. 2

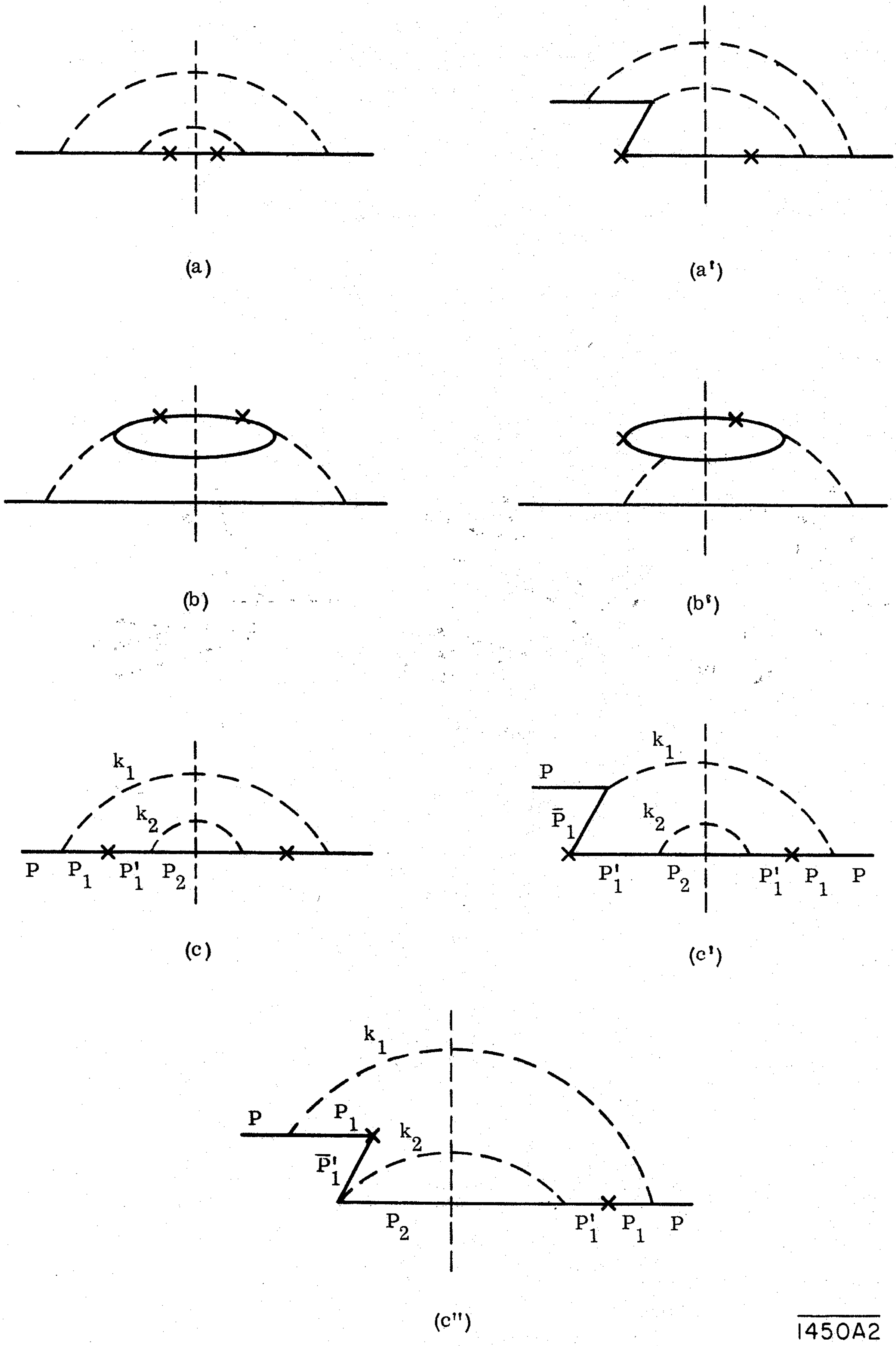
(a)
(a')
(b)
(b')
k1
k2
P
P1
P'1
P2
(c)
k1
P
P̄1
k2
P'1
P2
P'1
P1
P
(c')
k1
P
P1
k2
P̄'1
P2
P'1
P1
P
(c'')

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Fig. 3

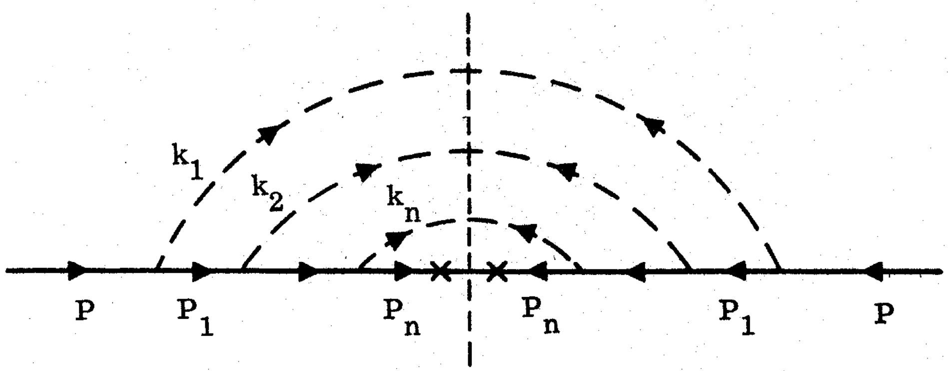

Fig. 4

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