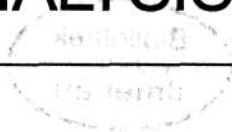

THEORY OF DISCRETE AND CONTINUOUS FOURIER ANALYSIS



H. Joseph Weaver



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