

# Theory of Electrical Breakdown in Ionic Crystals

BY H. FRÖHLICH

*Wills Physical Laboratory, University of Bristol*

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## I—INTRODUCTION

If an electrical field higher than a certain critical strength  $F$  is applied to an ionic crystal, the insulation breaks down. If the temperature is above a certain critical value  $T_0$  (usually of the order  $100^\circ\text{C}$ .),  $F$  decreases very rapidly with temperature, and the breakdown takes place some seconds after the application of the field. Wagner (cf. Semenoff and Walther 1928) has shown that in this case the breakdown is due to the Joule heat generated by ionic conduction, which causes local melting.

For temperatures less than  $T_0$ , on the other hand, the breakdown takes place in a time of the order  $10^{-8}$  sec. (Rogowsky 1928) and the variation of  $F$  with temperature is very much smaller than in the case of heat breakdown.\* The phenomenon in this case is referred to as electrical breakdown, any melting of the crystal being ruled out by the short times involved.

To explain the electrical breakdown, various theories have been proposed. The mechanical theory (cf. Semenoff and Walther 1928) assumes that the breakdown is due to mechanical rupture of the crystal, caused by the forces which the electrical field exerts on the ions. According to this theory the electrical strength, like the mechanical strength, should depend very strongly on cracks and other crystal imperfections. Experiments by v. Hippel (1932) have shown, however, that the breakdown field is almost the same for different specimens and does not depend on their source or method of preparation.

On the other hand, Joffe has assumed (cf. Semenoff and Walther 1928) that the breakdown is due to an ionization of the ions by the moving ions, carrying the current. This theory does not allow, however, for the short time in which the breakdown occurs. v. Hippel and others (v. Hippel 1935) have therefore suggested the following mechanism: At any temperature a few electrons will be in the "conduction level", i.e. free to move through the lattice. If these can gain enough energy from the field to ionize the

\* Exact measurements of  $F$  in a homogeneous field have been made only at room temperature (v. Hippel 1935).

(negative) ions of the lattice, the number of free electrons will increase very rapidly. This will then lead to the breakdown similarly as in gases. v. Hippel (1935) supports these ideas by showing that the breakdown always occurs in such directions in the lattice, along which an electron has to surmount the lowest potential walls. (For a NaCl lattice, for example, this is the case in the 110 direction.)

In order to calculate the energy gained by the electron from the field, we need to know the mean free path of the electron in the lattice. In this paper, therefore, we make a quantitative calculation of the mean free path, and hence the breakdown field, on the assumption that the phenomenon is due to electrons.\*

## 2—THE CONDITION FOR BREAKDOWN

In an ionic crystal a very few electrons will always be in the conduction level. Their number increases with increasing field strengths but for small field strengths an equilibrium number is reached (cf. below). We shall assume these electrons to behave as though free (i.e. we neglect the effect of the lattice field). We denote the kinetic energy of such an electron by  $E$ , and assume that  $E$  is of the same order of magnitude but less than the energy  $\mathcal{T}$  required to excite or ionize the ions of the crystal ( $\sim 5$  e-volts). When such an electron is deflected by a collision with the lattice vibrations the gain or loss of energy is of the order  $h\nu$  ( $\sim 0.02$  e-volt), so that the collision is nearly elastic as in the theory of metals. Under these conditions the current density per electron is given by the usual formula

$$I = \frac{e^2 \tau F}{Vm}, \quad (1)$$

where  $F$  is the field strength,  $\tau$  is the time of relaxation<sup>†</sup> and  $V$  is the volume of the crystal. Therefore, the energy  $A$ , transferred per second from the field to the electron, is given by

$$A = IFV = e^2 F^2 \tau / m. \quad (2)$$

The calculation of  $\tau$  in § 4 gives

$$A = \text{const. } F^2 E^{\frac{3}{2}}.$$

\* v. Hippel (1935) has already proposed some ideas for the interaction of an electron with an ionic crystal. The following calculations, however, lead to another conception of this interaction.

<sup>†</sup>  $\tau$  is defined as the time in which the component of the momentum of the electron in a certain direction is reduced to the  $e$ th part (cf. § 4).

The electron also transfers a certain energy  $B$  per second to the lattice. In § 5 we shall find

$$B = \text{const. } E^{-\frac{1}{2}}.$$

The rate of loss of energy is thus

$$B - A = \text{const. } E^{-\frac{1}{2}} - \text{const. } F^2 E^{\frac{3}{2}}.$$

We see, therefore, that for low energies ( $E < E'$ ) the electron loses energy and for high energies ( $E > E'$ ) it gains energy. The critical energy  $E'$ , where\*  $A = B$  depends on the field strength  $F$ :

$$E' \propto 1/F.$$

Now the condition for breakdown is that no stationary state for the electron distribution can be reached, i.e. that an electron, with energy less than  $\mathcal{T}$  (the ionization potential), is capable of gaining energy from the field but that no reverse process exists. Hence the condition for breakdown is

$$E' \leq \mathcal{T}.$$

If this is not the case, the field cannot cause any instability because it cannot produce any *more* electrons capable of ionization. Thus, the critical field strength  $F$  where the breakdown begins can be calculated from the condition

$$E' = \mathcal{T},$$

i.e.  $A(E, F) = B(E), \quad E = \mathcal{T}. \quad (3)$

From the above considerations it follows that in a field, higher than the breakdown field  $F$ , the electron has in general to suffer quite a number of collisions before reaching the energy  $\mathcal{T}$ . We should notice, however, that even in a field weaker than the breakdown field it might happen that an electron having by chance an energy nearly equal to  $\mathcal{T}$ , and moving in the direction of the field, is accelerated in the field in such a way that it reaches between *two* collisions the energy  $\mathcal{T}$ . Processes of this sort do not lead, however, to any instability. They have only the effect that the number of electrons in the conductive levels increases with increasing field strength (but there exists a stationary state) and that therefore also the conductivity increases. For field strengths near to the breakdown field this increase of the conductivity has been observed. But as v. Hippel (1935) has shown, at the breakdown field the current density increases almost discontinuously. Thus the breakdown cannot be considered as a continuous increase of the ordinary current.

\* An electron with the energy  $E'$  is in an unstable equilibrium.

### 3—INTERACTION BETWEEN THE LATTICE VIBRATIONS AND THE ELECTRON

The investigations of Born and v. Karman (Born and Göppert-Mayer 1933) on the elastic vibrations of a diatomic polar lattice show that the normal modes can be divided up into two branches, the acoustic and the optical branch. The number of normal modes in each branch is  $3N$  (i.e. the total number is  $6N$ ), where  $2N$  is the number of ions.

Each elastic deformation of the lattice is connected with a certain polarization. Each elastic wave, therefore, corresponds to a certain "polarization wave". The oscillations of the optical branch are those in which two neighbouring ions of opposite sign vibrate in opposite directions. These waves correspond therefore to long polarization waves. On the other hand, in the acoustic branch, neighbouring ions of opposite sign have almost the same displacement. These waves correspond, therefore, to polarization wave-length, nearly equal to the lattice constant.†

According to Born and v. Karman (Born and Göppert-Mayer 1933) the oscillations of the optical branch have all nearly the same frequency  $\nu$  (the Reststrahlen frequency). Since the optical branch gives much the greater polarization of the lattice, it gives a much stronger interaction with the electron. Thus it will involve only a small error if we give the Reststrahlen frequency  $\nu$  to all waves.

For a long polarization wave (wave number  $\mathbf{w}$ ) the polarization  $\mathbf{P}_w$  per unit volume may be obtained as follows: Let  $u_+$ ,  $u_-$  denote the displacements of the ions of the two signs in the neighbourhood of a given point, and let

$$u_w = u_+ + u_-.$$

Then clearly

$$P_w = eu_w/2a^3, \quad (4)$$

where  $2a^3$  is the volume of the unit cell, i.e.  $a$  the distance between neighbouring ions.

For those waves in which the polarization varies very rapidly from point to point (short polarization waves, long elastic waves) we shall adopt the same device as that used by Debye in his theory of specific heats and treat the crystal as a continuous medium. We shall treat the elastic waves as giving rise to a polarization wave

$$u_w(x, y, z) = \frac{1}{(2N)^{\frac{1}{2}}} \{b_w e^{i(\mathbf{w}, \mathbf{r})} + b_w^* e^{-i(\mathbf{w}, \mathbf{r})}\}, \quad (4a)$$

where

$$b_w = \text{const. } e^{2\pi i \nu t}.$$

† The wave number of an elastic wave differs from that of a polarization wave by a vector in the reciprocal lattice.

Since for each wave number  $w$  there are two transverse waves and one longitudinal one,  $w$  will have all values between 0 and  $w_0$ , given by

$$6N = 3 \frac{2a^3 N}{(2\pi)^3} \frac{4\pi}{3} w_0^3,$$

i.e. 
$$w_0 = \frac{(6\pi^2)^{\frac{1}{3}}}{a} \simeq 2^{\frac{1}{3}} \frac{\pi}{a}. \quad (5)$$

The value  $w = 0$  corresponds to an infinite wave length, i.e. to a constant polarization.

For our further considerations it will be necessary to know the total energy of a polarization wave. Let  $M^+$  and  $M^-$  denote the masses of the ions of the two signs. In our continuum theory, we shall have to assume the masses and charges of each sort of ion to be uniformly distributed. If there is no displacement the density of positive and negative charges is the same all over the continuum. Since the force acting on positive charges is opposite to that acting on negative charges, and since both  $u_+$  and  $u_-$  have the same dependence on time, it follows that

$$M^+ u_+ = M^- u_-.$$

Using this relation, it is possible to calculate the total energy in the same way as has been done for the vibrations of a monatomic lattice.† One thus finds that the total energy is represented by the energy of an harmonic oscillator with a frequency  $\nu$ , amplitude

$$X_w = b_w + b_w^*, \quad (6)$$

momentum 
$$Y_w = -2\pi i \nu M (b_w - b_w^*), \quad (7)$$

and mass  $M$ , where 
$$\frac{1}{M} = \frac{1}{M^+} + \frac{1}{M^-}. \quad (8)$$

In order to consider the interaction of an electron with the dielectric, it is convenient to express the total energy in a Hamiltonian form. The Hamiltonian is composed of three terms

$$H = H_{\text{osc.}} + H_{\text{el.}} + W.$$

$H_{\text{osc.}}$  is the sum of the Hamiltonians of all the oscillators‡

$$H_{\text{osc.}} = \sum_w \left( \frac{Y_w^2}{2M} + 2\pi^2 \nu^2 M X_w^2 \right).$$

† (Cf. Peierls 1929.) In our case the kinetic energy is the sum of the kinetic energies of the positive and negative ions. The total energy is twice the kinetic energy as for any kind of harmonic oscillation.

‡ The interaction energy between the waves is zero, if they are assumed to be exactly harmonic (cf. Peierls 1929).

$H_{el}$  is the Hamiltonian of the electron, moving in the field of the lattice if all ions are exactly in their equilibrium positions. Since we have assumed the electron to behave like a free electron,  $H_{el}$  is given by

$$H_{el} = p^2/2m,$$

where  $p$  is the momentum of the electron.

$W$  is the interaction energy between the electron and the polarization waves. Thus

$$W = e\Sigma\phi_w,$$

where  $\phi_w$  is the potential of a single wave with wave number  $w$  at the position of the electron.  $\phi_w$  is determined by

$$\nabla^2\phi_w = 4\pi \operatorname{div} \mathbf{P}_w. \tag{9}$$

From this equation it follows that only longitudinal waves have an interaction with the electron. Using equation (4), we find for the solution of (9) according to (4a), (6) and (7)

$$\begin{aligned} \phi_w &= -\frac{4\pi ie}{2a^3w(2N)^{\frac{1}{2}}} \{b_w e^{i(\mathbf{w}, \mathbf{r})} - b_w^* e^{-i(\mathbf{w}, \mathbf{r})}\} \\ &= \frac{2\pi e}{a^3w(2N)^{\frac{1}{2}}} \left\{ X_w \sin(\mathbf{w}, \mathbf{r}) + \frac{Y_w}{2\pi\nu M} \cos(\mathbf{w}, \mathbf{r}) \right\}. \end{aligned} \tag{10}$$

From the Hamiltonian  $H$ , we may, for instance, calculate the classical polarization  $P$  of the dielectric due to an electron. We find  $P \propto e/r$  for distances  $r \gg a$ . For  $r \rightarrow 0$ ,  $P$  tends to a certain finite value in contrast to the classical picture where  $\epsilon \dagger$  is considered a constant and  $P \rightarrow \infty$  as  $r \rightarrow 0$ .

#### 4—TIME OF RELAXATION

According to the quantum theory, the momenta  $p$  and  $Y_w$  have to be replaced in the usual way by operators. The energy of each polarization wave consists then of a number of quanta  $h\nu$ .

For the calculation of the time of relaxation, we may treat  $W$  as a perturbation. Then, the zero order approximation of the wave function of the total system is given by a product

$$\Psi = \prod_w \chi(X_w, n_w) \psi(\mathbf{r}, \mathbf{k}),$$

and the total energy by

$$U = \sum_w (n_w + \frac{1}{2}) h\nu + E_k.$$

†  $\epsilon$  is the dielectric constant.

Here,  $\mathbf{k}$  is the wave number and  $\psi$  the wave function of the electron

$$\psi = e^{i(\mathbf{k}, \mathbf{r})} / V^{\frac{1}{2}},$$

$E_k$  is its energy:

$$E_k = \hbar^2 k^2 / 2m,$$

$n_w$  is the number of quanta  $h\nu$  of the oscillator  $\mathbf{w}$ , and  $\chi(X_w, n_w)$  is its wave function.

We now calculate the probability  $\Phi_w$  per second that the electron makes a transition to the state  $\mathbf{k}'$ , coupled with a transition of the oscillator  $\mathbf{w}$  from  $n_w$  to  $n'_w$ . By Dirac's perturbation theory,  $\Phi_w$  is given by†

$$\Phi_w = \frac{1}{\hbar^2} |M_{kk'w}|^2 \frac{\partial \sin^2 \xi t}{\partial t \xi^2},$$

where

$$\hbar \xi = E_{k'} - E_k + (n'_w - n_w) h\nu$$

and 
$$M_{kk'w} = \frac{1}{V} \int \chi^*(X_w, n'_w) e^{-i(\mathbf{k}', \mathbf{r})} e \phi_w \chi(X_w, n_w) e^{i(\mathbf{k}, \mathbf{r})} dX_w d'.$$

Introducing  $\phi_w$  from equation (10) and using the well-known matrix elements for an oscillator, we find that the electron may either absorb ( $\propto \Phi_w^a$ ) or emit ( $\propto \Phi_w^e$ ) a quantum  $h\nu$  (as in the theory of metals). The matrix elements can easily be worked out. In the case of emission,  $M_{kk'w}$  is different from zero only if  $\mathbf{k}'$  is determined by

$$\mathbf{k}' = \mathbf{k} - \mathbf{w}. \quad (11)$$

The matrix element is then

$$M_{kk'w} = \frac{2\pi e^2}{a^3 w (2N)^{\frac{1}{2}}} \left[ \frac{\hbar(1+n_w)}{4\pi M\nu} \right]^{\frac{1}{2}}$$

and hence

$$\Phi_w^e = \left( \frac{2\pi e^2}{a^3 w} \right)^2 \frac{1+n_w}{2Mh\nu} \frac{\partial \sin^2 \xi t}{\partial t \xi^2}. \quad (11a)$$

In the case of absorption, we find similarly

$$\mathbf{k}' = \mathbf{k} + \mathbf{w} \quad (12)$$

and

$$\Phi_w^a = \left( \frac{2\pi e^2}{a^3 w} \right)^2 \frac{n_w}{2Mh\nu} \frac{\partial \sin^2 \xi t}{\partial t \xi^2}. \quad (12a)$$

$\mathbf{k}'$ , therefore, is completely determined by conservation of momentum [(11) and (12)].  $\Phi_w$  is big only if  $\xi = 0$  (conservation of energy). From the

† By taking the average over a small energy interval, it can easily be seen that  $\Phi_w$  is independent of the time.

conservation of energy and momentum, it follows† that the angle  $\alpha$  between  $\mathbf{k}$  and  $\mathbf{w}$  is given by

$$\cos \alpha = -\frac{w}{2k} \pm \frac{2m}{\hbar^2} \frac{h\nu}{2kw} \quad (+ \text{ for absorption, } - \text{ for emission}). \quad (13)$$

For most oscillators  $w$ , the second term in (13), is small.

We can now calculate the time of relaxation  $\tau$ . Let the field  $F$  be in the  $x$  direction, so that the change of the  $x$  component  $k_x$  of the wave number of the electron by the field is

$$\left(\frac{\partial k_x}{\partial t}\right)_{\text{field}} = \frac{eF}{\hbar}.$$

Let  $\Delta k_x(\mathbf{w})$  be the mean change of  $k_x$  by one collision with the oscillator  $\mathbf{w}$ . Then,  $\tau$  is defined as (Fröhlich 1936, § 14)

$$\frac{k_x}{\tau} = -\sum_{\mathbf{w}} \Delta k_x(\mathbf{w})(\Phi_w^a + \Phi_w^e). \quad (14)$$

This definition leads immediately to equation (1), since the right-hand side the mean change of  $k_x$  per second, due to collisions, i.e.

$$\frac{k_x}{\tau} = -\left(\frac{\partial k_x}{\partial t}\right)_{\text{collision}}.$$

Equation (1) then follows from the condition for a stationary state,‡

$$\left(\frac{\partial k_x}{\partial t}\right)_{\text{field}} + \left(\frac{\partial k_x}{\partial t}\right)_{\text{collision}} = 0.$$

Let us introduce polar co-ordinates  $(w, \theta, \phi)$  in  $\mathbf{w}$  space, with the axes parallel to  $\mathbf{k}$ , i.e. with  $\theta = \alpha$ . Since the  $\Phi_w$ 's do not depend on  $\phi$ , we may take the average of  $\Delta k_x(\mathbf{w})$  in equation (14) over the azimuth  $\phi$ . The result (Fröhlich 1936, § 13)

$$\overline{\Delta k_x(\mathbf{w})} = -\frac{w^2}{2k^2} k_x.$$

Replacing the sum in (14) by an integral, we find for  $\tau$  (cf. equation (5))

$$\frac{1}{\tau} = \frac{2a^3 N}{(2\pi)^3} \int_0^{2\pi} d\phi \int_{-\pi}^{\pi} \sin \theta d\theta \int_0^{w_0} \frac{w^2}{2k^2} (\Phi_w^a + \Phi_w^e) w^2 dw.$$

The integration over  $\phi$  gives  $2\pi$ . The simplest way to carry out the integration over  $\theta$  is (cf. Bloch 1928, p. 589) to introduce  $\xi$  as variable instead of  $\theta$ .

† Cf., for example, Fröhlich (1936, § 13), considering that all oscillators have the same energy.

‡ In fact, the state is not stationary, but quasi-stationary, because of the exchange of energy of the electron with lattice and field (cf. § 2).



Using the fact that  $n_w$  is the same for all oscillators, namely,

$$n_w = \frac{1}{e^{h\nu/kT} - 1},$$

we finally obtain\* 
$$\frac{1}{\tau} = \frac{1}{\tau_0} \left( 1 + \frac{2}{e^{h\nu/kT} - 1} \right), \quad (15)$$

where 
$$\frac{1}{\tau_0} = \frac{2^{\frac{3}{2}} \pi^2}{8\sqrt{2}} \frac{e^4 \hbar}{m^{\frac{1}{2}} M a^5 \nu E^{\frac{3}{2}}}, \quad \text{if } E > \frac{2^{\frac{3}{2}} \pi^2}{8} \frac{\hbar^2}{ma^2}, \quad (16)$$

and 
$$\frac{1}{\tau_0} = \frac{e^4 m^{\frac{1}{2}}}{2^{\frac{3}{2}} \hbar M a^3 \nu E^{\frac{1}{2}}}, \quad \text{if } E < \frac{2^{\frac{3}{2}} \pi^2}{8} \frac{\hbar^2}{ma^2}. \quad (16a)$$

The time of relaxation  $\tau_0$  at the absolute zero of temperature is entirely due to the zero-point oscillations. In Bloch's electron theory of metals (Bloch 1928), owing to the Pauli principle, an electron cannot lose energy, and so there are *no* collisions of the conductive electrons with the zero point oscillations. This is not the case here. An electron, with an energy higher than the energy of the conduction electrons, suffers collisions with the lattice vibrations† even at  $T = 0$ .

#### 5—TRANSFER OF ENERGY TO THE LATTICE

At each collision, the energy  $h\nu$  is transferred to the lattice with a probability  $\Phi_w^e$ , or absorbed by the electron with a probability  $\Phi_w^a$ . The energy  $B$ , transferred per second to the lattice, therefore, is given by

$$B = h\nu \sum_w (\Phi_w^e - \Phi_w^a).$$

Since (cf. equations (11a) and (12a))

$$\Phi_w^e / \Phi_w^a = (1 + n_w) / n_w,$$

$B$  is independent of the temperature.

$B$  can be evaluated by a method similar to that used in the calculation of  $\tau$ . We thus find

$$B = \frac{h\nu 4k^2}{\tau_0 w_0^2} \int \frac{dw}{w}.$$

\* For  $2k < w_0$ ,  $w_0$  has to be replaced by  $2k$  because of momentum and energy theorem. This leads to equation (16a).

† If, however,  $E < h\nu$ , the electron cannot emit the energy  $h\nu$ , and (15) has to be replaced by

$$\frac{1}{\tau} = \frac{1}{\tau_0} \frac{1}{e^{h\nu/kT} - 1}.$$

In this case, therefore,  $1/\tau$  vanishes for  $T = 0$ .

As this integral diverges for  $w = 0$ , we must find the lower limit  $w'$  of  $w$ . This can be obtained from equation (13), using the fact that always

$$|\cos \alpha| \leq 1.$$

Since for small  $w$  the first term in equation (13) is small,  $w'$  is given by\*

$$w' = \frac{2\pi m v}{\hbar k}.$$

We then obtain 
$$B = \frac{h\nu}{\tau_0} \frac{4k^2}{w_0^2} \log \gamma = \frac{2^{\frac{1}{2}} \pi e^4 m^{\frac{1}{2}}}{M a^3 E^{\frac{1}{2}}} \log \gamma, \tag{17}$$

where 
$$\gamma = \frac{w_0}{w'} = \frac{\hbar k}{2^{\frac{3}{2}} m v a} = \frac{(2E)^{\frac{1}{2}}}{2^{\frac{3}{2}} m^{\frac{1}{2}} \nu a}.$$

6—BREAKDOWN FIELD

We calculate now the breakdown field  $F$  from equation (3). Using equations (2), (15), (16), (17) and (5), we obtain

$$eF = \frac{2^{\frac{1}{2}} \pi}{4} \frac{e^4}{a^4 \mathcal{F} M} \left( \frac{m h \log \gamma}{\nu} \right)^{\frac{1}{2}} \left( 1 + \frac{2}{e^{h\nu/kT} - 1} \right)^{\frac{1}{2}}. \tag{18}$$

As  $\tau$  is proportional to the mean free path  $l$  ( $l_0 =$  mean free path for  $T = 0$ ), we notice that

$$eF \propto \frac{1}{(l_0)^{\frac{1}{2}}}. \tag{19}$$

7—DISCUSSION

Equation (18) contains no arbitrary constants so that we may calculate theoretically the absolute value of  $F$ . The energy  $\mathcal{F}$  will be taken from the first maximum of the ultra-violet absorption band. For the alkali halides, the latter has been measured by Hilsch and Pohl (1930). If  $\lambda$  is the wave-length of this first maximum,  $\mathcal{F}$  is given by

$$\mathcal{F} = hc/\lambda.$$

We shall insert this expression into equation (18). Finally we shall express  $F$  in V/cm.,  $\lambda$  and  $a$  in Ångströms,  $h\nu$  in  $10^{-2}$  V and  $M$  (cf. equation (8)) in units of the hydrogen mass. We then obtain

$$F = 1.6(\log \gamma)^{\frac{1}{2}} \times 10^5 \frac{\lambda}{M a^4 (h\nu)^{\frac{1}{2}}} \left( 1 + \frac{2}{e^{h\nu/kT} - 1} \right)^{\frac{1}{2}} \text{ V/cm.}$$

\* As  $w' \ll w_0$ , it was correct to put  $w' = 0$  in the integral for  $1/\tau$ .

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Here,  $\lambda$  is the wave-length of the ultra-violet absorption,  $\nu$  is the Reststrahlen frequency,  $a$  is the distance between two neighbouring ions of opposite sign, and  $M$  is given by equation (8) in terms of the atomic weight  $M^+$  and  $M^-$  of the ions. For  $E = \mathcal{F}$ ,  $(\log \gamma)^{\frac{1}{2}}$  has nearly the same value for all alkali halides, namely, 2.6.

We shall now compare for the alkali halides the theoretical value of  $F$  at 300° K. with the experiments of v. Hippel (1935) carried out at room temperature. v. Hippel's values differ considerably from former measurements. For NaCl, for example, his value is  $15 \times 10^5$  V/cm., whereas Semenoff and Walther (1928) give  $5 \times 10^5$  V/cm. As mentioned in the introduction, v. Hippel has shown that the breakdown occurs always in the 110 direction whatever direction the field may have and that, therefore, the projection of the breakdown field on the 110 direction is always the same. In our model which does not account for any anisotropic effects, the breakdown occurs in the direction of the field. Our formula for  $F$ , therefore, corresponds to the experimental value of the field in the 110 direction. To obtain the value in the 100 direction ( $F_{100}$ ), for which experimental values usually are given, we have to multiply  $F$  by  $\sqrt{2}$ , as already mentioned by v. Hippel: thus  $F_{100} = F\sqrt{2}$ .

TABLE I— $F_{100}$  IN  $10^5$  V/CM.

	NaCl	NaBr	NaI	KCl	KBr	KI	RbCl	RbBr	RbI
Theory:									
$T = 0^\circ$ K.	6.9	6.1	4.9	3.8	3.0	2.5	2.7	2.0	1.4
$T = 300^\circ$ K.	10.7	10.6	9.3	6.6	5.6	5.1	4.8	4.2	3.1
Experimental:									
$T \approx 300^\circ$ K.	15	10	8	8	7	6	7	6	5
$m^*/m$	2.0	0.9	0.7	1.5	1.6	1.4	2.1	2.0	2.6

Considering the simplifications made in the theory, the agreement is satisfactory. The difference between theoretical and experimental values may be due to the "effective mass"  $m^*$  for an electron in the lattice field being generally greater than the mass of a free electron.

An important consequence of the theory is the dependence of  $F$  on temperature. For sufficiently high temperatures ( $kT > h\nu$ ),  $F$  should be proportional to  $\sqrt{T}$ , provided that the temperature  $T_0$ , where the heat breakdown begins, is not yet reached. (For  $T > T_0$ , cf. § 1.) As we have already stated, experiments in a homogeneous field have at present been carried out at room temperature only.

The increase of  $F$  with temperature is due to the decrease of the mean free path with temperature (cf. equations (19) and (15)). Apart from an

increase of temperature, there are two other means of decreasing the mean free path  $l$ . The first is to introduce foreign ions into the lattice,\* the second is to take very thin layers of thickness smaller than  $l$ . An increase of  $F$  in the first case has been found by v. Hippel (1935). Experiments on thin layers have been carried out by Joffe and co-workers (Joffe and Alexandrow 1933). They did not find an increase in  $F$  for thicknesses down to  $0.7 \times 10^{-4}$  cm. The mean free path, however, is only of the order  $10^{-5}$ – $10^{-6}$  cm., so that an increase of  $F$  is not yet to be expected.

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#### SUMMARY

The time of relaxation of an electron in an ionic lattice has been calculated. Hence the critical field  $F$  for electrical breakdown has been calculated quantitatively. Satisfactory agreement with the experiments of v. Hippel is obtained.

$F$  increases: (1) with increasing temperature (for high temperature  $F \propto \sqrt{T}$ , if  $T < T_0$ ,  $T_0$  is the temperature where heat breakdown begins); (2) if foreign ions are introduced into the lattice (experiments by v. Hippel); (3) for layers of about  $10^{-6}$  cm. thickness.

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\* As in the theory of conductivity, the presence of foreign ions always diminishes  $l$  (Nordheim 1931) and not only in the case where the ionic radius of the foreign ion is smaller than that of the original ion, as it was assumed by v. Hippel.