

Theory of Gain, Modulation Response, and Spectral Linewidth in AlGaAs Quantum Well Lasers

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Abstract—We investigate theoretically a number of important issues related to the performance of AlGaAs quantum well (QW) semiconductor lasers. These include a basic derivation of the laser gain, the linewidth enhancement factor α , and the differential gain constant in single and multiple QW structures. The results reveal the existence of gain saturation with current in structures with a small number of wells. They also point to a possible two-fold increase in modulation bandwidth and a ten-fold decrease in the spectral laser linewidth in a thin QW laser compared to a conventional double heterostructure laser.

I. INTRODUCTION

THE application of semiconductor lasers for optical communication requires low-noise characteristics and high-frequency dynamic performance. These properties have received considerable attention in conventional double heterostructure (DH) lasers [1]–[5]. Recently the detuned loading by the use of coupled-cavity DH lasers was proposed for improving these properties [6]. In this structure, the differential gain g' and the linewidth enhancement factor α which determine, respectively, the dynamic performance and noise characteristics, are varied by detuning the lasing frequency away from the loss peak. The basic quantum mechanical expressions for g' and α suggest yet another way to improve α and g' through changes in the density of states which occur in quantum well (QW) structures. Although QW lasers have proven to be attractive light sources because of their low threshold current densities and the low temperature sensitivity of the threshold current [7]–[11], little effort has been directed towards measurement or calculation of these noise characteristics and high-modulation performance [12]–[14].

In this paper, we investigate theoretically the effects of QW structure parameters such as the number and the thickness of QW's on the dynamic response and quantum noise, and consider the optimum QW structure for realizing low-noise characteristics and/or high-dynamic performance. In the analysis, we also consider the important issue of the use of multi-quantum well structures in order to avoid the phenomenon of gain saturation with current, as well as for improving the noise and dynamic performance. The effect of the structural parameters such as the thickness and the number of QW's on the threshold cur-

rent have been considered before [15], [16], although the gain saturation effect was not explicitly treated. In Section III the fundamental properties of differential gain are discussed, and then the effects of QW structure parameter on the laser dynamics are investigated. In Section IV, the linewidth enhancement factor α and the spontaneous emission factor are studied and the attainable reduction of the linewidth by the use of the optimum QW structures is considered. The results indicate that the modulation bandwidth (relaxation resonance frequency) is enhanced by a factor of ~ 2 in the multi-quantum well (MQW) laser and the laser linewidth (FM noise) is found to be significantly suppressed, by $\sim \frac{1}{10}$, in a single thin QW (SQW) laser compared to a conventional double heterostructure laser. In order to clarify achievable limits of these properties with quantum well effects, we will consider AlGaAs quantum well lasers which are almost free from nonradiative processes, such as the Auger recombination at room temperature.

II. LINEAR GAIN AND THRESHOLD CURRENT

A. Density of States

In the QW structures [7], a series of energy levels and associated subbands are formed owing to the quantization of electrons in the direction of the quantum well thickness. The density of states (per unit energy per unit area) of such confined electrons in the n th subband is given by

$$\rho_{c_n}(E) = \frac{m_c}{\pi \hbar^2} H(E - \epsilon_{c_n}) \quad (1)$$

where $H(x)$, m_c , \hbar , L_z , and ϵ_{c_n} are the Heaviside function, the effective mass of electrons, Planck's constant (\hbar) divided by 2π , the QW thickness, and the quantized level of electrons in the n th subband of the QW structures, respectively. When the barrier height is sufficiently big and the barrier thickness is sufficiently large, ϵ_{c_n} is equal to $(n\pi\hbar)^2/(2m_c L_z^2)$. On the other hand, if the barrier is thin or its barrier height is small so that coupling between adjacent wells exists, the degeneracy of the individual well quantized energy levels disappears and each single-well level splits into N different energy levels. In order to simplify the discussion in this paper, we will consider the weak coupling limit of QW structures in which energy broadening due to coupling is sufficiently small compared to the energy broadening (usually more than 5 meV) due to intraband relaxation time.

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B. Linear Gain and Threshold Current

When the recombination is dominated by band-to-band radiative process, the linear bulk gain derived under k -selection rule is given by

$$g(E) = \int A(\epsilon) \sum_{j=h,l} \sum_{n=1}^{\infty} \frac{\rho_{\text{red},n}^j(\epsilon)}{L_z} (f_c(E_{c_n}) - f_v(E_{v_n}^j)) \frac{(\hbar/\tau_{\text{in}})}{(E - \epsilon)^2 + (\hbar/\tau_{\text{in}})^2} d\epsilon. \quad (2)$$

The bulk gain is the gain exercised by an electromagnetic field if it were completely confined to the quantum well. E is the photon energy, j designates either light holes (1) or heavy holes (h), $\rho_{\text{red},n}^j$ is the reduced density of states which is defined by $\rho_{\text{red},n}^j = ((\rho_{c_n})^{-1} + (\rho_{v_n}^j)^{-1})^{-1}$, $\rho_{v_n}^j$ is the density of states of light holes ($j = 1$) or heavy holes ($j = h$), $f_c(f_v)$ is the Fermi-Dirac distribution function for electrons (holes) in the conduction band (the valence band) with the Fermi-energy $\epsilon_{F_c}(\epsilon_{F_v})$, τ_{in} is the intraband relaxation time. E_{c_n} and $E_{v_n}^j$ are equal to $(m_c \epsilon_{v_n}^j + m_v^j E + m_v^j \epsilon_{c_n}) / (m_c + m_v^j)$, and $(m_c \epsilon_{v_n}^j - m_c E + m_v^j \epsilon_{c_n}) / (m_c + m_v^j)$, respectively, where m_v^j and $\epsilon_{v_n}^j$ are the effective mass and the energy level of the n th subband of heavy holes ($j = h$) or light holes ($j = 1$). $A(\epsilon)$ is the coefficient related to the square of the dipole matrix element $|M|_{\text{ave}}^2$ as given by

$$A(\epsilon) = \frac{\pi e^2 \hbar}{m_0^2 c n_r E_g} |M|_{\text{ave}}^2 \quad (3)$$

where n_r is the refractive index of GaAs, e is the electron charge, m_0 is the mass of electrons, c is the light velocity, E_g is the band-gap, and \hbar is the Planck's constant. In quantum well structures, the gain depends on the polarization of the light. The dipole matrix element $|M|_{\text{ave}}^2$ which is different from that of the double heterostructure was discussed by Asada *et al.* [16]. For instance the dipole matrix element for TE mode due to an electron-heavy hole transition is given by

$$|M|_{\text{ave}}^{\text{TE}} = |M_0|_{\text{ave}}^2 (1 + E/(\epsilon_{c_n} - \epsilon_{v_n}^h)) \quad (4)$$

where $|M_0|_{\text{ave}}^2$ is the square of the dipole matrix with the conventional double heterostructure lasers and is approximately equal to $1.33 m_0 E_g$. The quasi-Fermi energy ϵ_{F_c} and ϵ_{F_v} are determined by both the charge neutrality condition and the condition that the modal gain $g_{\text{mod}}(E) = \Gamma g(E)$ at the photon energy E_l for laser oscillation is equal to the total losses α_{total} as follows:

$$g_{\text{mod}}(E_l) = \Gamma g(E_l) = \alpha_{\text{total}} \\ \equiv \Gamma \alpha_{\text{ac}} + (1 - \Gamma) \alpha_{\text{ex}} + L^{-1} \ln(1/R) \quad (5)$$

where α_{ac} , α_{ex} , R , and L are the loss in the active region, the loss in the cladding layer, the reflectivity, and the cavity length. The optical confinement factor Γ depends on the structure strongly. In this paper, we assume a multiquantum well (MQW) structure composed of thin GaAs well layers, Ga_{0.75}Al_{0.25}As barrier layers, and Ga_{0.75}Al_{0.25}As waveguide layers. The dimension of the

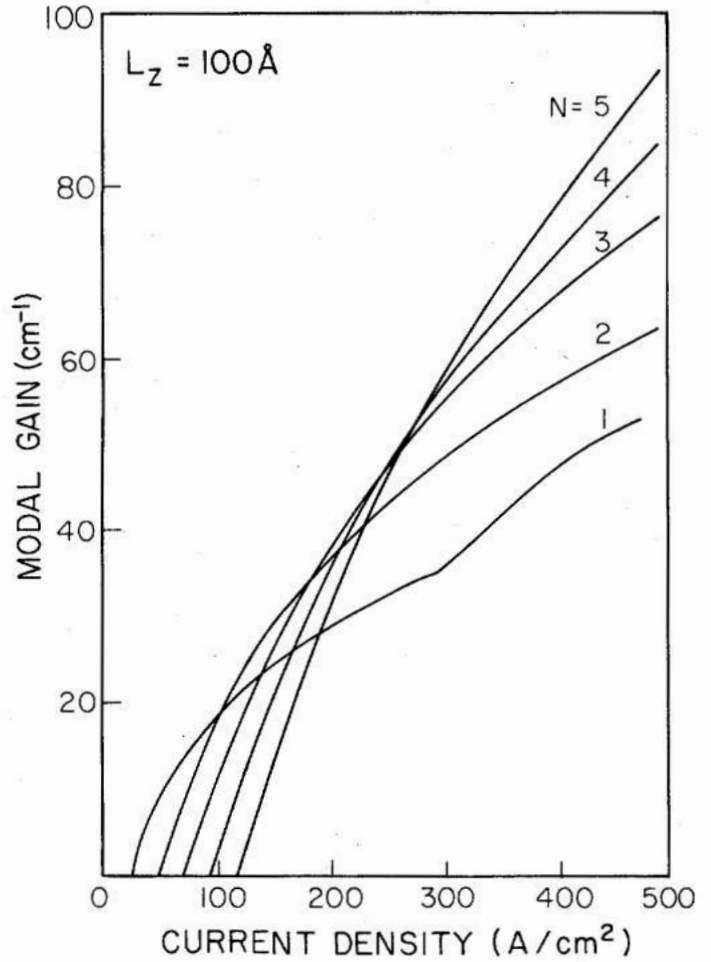


Fig. 1. The modal gain $g_{\text{mod}} (= \Gamma g)$ as a function of the injected current density J with the various number of quantum wells N . In this case, the quantum well thickness L_z is assumed to be 100 Å.

waveguide layers is determined so that the total thickness, including QW wells, barrier layers, and waveguide layers, is equal to 2000 Å. The cladding layers are a p-Ga_{0.6}Al_{0.4}As layer, and n-Ga_{0.6}Al_{0.4}As layer, respectively. In particular, when the number of QW is equal to 1, we call this structure a single QW (SQW) structure, hereafter. In these MQW structures the Γ is given approximately by [15]

$$\Gamma = 0.3 N(L_z/L_0) \quad (6)$$

where L_0 is equal to 1000 Å, N is the number of QW's.

Fig. 1 shows the maximum gain $g_{\text{mod}}(E_l)$ exercised by the laser mode as a function of the injected current density J for different number of QW's. The injected current J is given by

$$J = e L_z N \left(\iint \frac{8\pi n_r^2 E_g^2}{c^2 \hbar} A(\epsilon) \sum_{j=h,l} \sum_{n=1}^{\infty} \frac{\rho_{\text{red},n}^j(\epsilon)}{L_z} f_c(E_{c_n}) \cdot (1 - f_v(E_{v_n}^j)) \frac{(\hbar/\tau_{\text{in}})}{(E - \epsilon)^2 + (\hbar/\tau_{\text{in}})^2} d\epsilon dE \right). \quad (7)$$

The term surrounded by parenthesis is the total emission rate. In this calculation, the QW thickness L_z and τ_{in} are

assumed to be 100 \AA and 0.2 ps . The energy band-gap discontinuity ΔE_c and ΔE_v of the conduction band and the valence band at the interface of GaAs and $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ are set $0.8 \Delta E$ and $0.2 \Delta E$, respectively, where ΔE , which is the total band discontinuity, is assumed to be 333 meV . We notice a very marked flattening ("saturation") of the gain at high injected currents, especially in a single well ($N = 1$). This gain saturation is due to the step-like shape of the density of state functions, and the fact that penetration of the quasi-Fermi levels into the conduction band or valence band takes place at high current injection. The product $\rho_{\text{red}}(\epsilon)[f_c(E_c) - f_v(E_v)]$, which determines the gain, becomes a constant and no longer increases with current. The maximum gain $g_{\text{mod}}(E_l)$ available with N QW's is thus N times larger than $g_{\text{mod}}(E_l)$ with $N = 1$ since each well can now provide its saturation gain, which is equal to that of a SQW laser. We can consequently avoid the saturation effect by increasing the number of QW's although the injected current to achieve this maximum gain also increases by N times. Owing to this gain saturation effect there exists an optimum number of QW's for minimizing the threshold current for a given total loss α_{total} . From Fig. 1 we see that, for low losses, the injected threshold current is minimum in the case of $N = 1$. On the other hand, if the $\alpha_{\text{total}} > 20 \text{ cm}^{-1}$, the threshold current with $N = 1$ is larger than that of $N = 2$. At higher values of α_{total} , which call for large laser modal gain g_{mod} , a larger number of wells are needed. From Fig. 1 we see that for $\alpha_{\text{loss}} > 50 \text{ cm}^{-1}$ a five-well structure ($N = 5$) will have the lowest threshold current.

In order to clarify these situations, in Fig. 2 the threshold current as a function of the QW number with various total loss is shown. This figure indicates that when the α_{total} is low, $N = 1$ is optimum, whereas the optimum number N is larger than 1 for higher α_{loss} . Thus the gain saturation which is enhanced in a SQW structure causes the significant dependence of the optimum number N on the α_{total} for reducing threshold current.

There remains another important structural parameter, the quantum well thickness. Fig. 3 shows the threshold current change as a function of the QW thickness. In this calculation, the number of QW's with each QW thickness is optimized so that the threshold current is minimum. The results indicate that the threshold current of thinner QW lasers ($L_z = 50 \sim 100 \text{ \AA}$) is much lower than that of thicker QW lasers. We also notice that the threshold current is minimized with $L_z \sim 60 \text{ \AA}$ when α_{loss} is low ($\alpha_{\text{loss}} = 10, 30 \text{ cm}^{-1}$). This is mainly due to the fact that the current for transparency (gain is just equal to zero), which corresponds approximately to threshold current with extremely low loss, is minimized at the thickness of $L_z \sim 60 \text{ \AA}$ in the case of $N = 1$ and also due to the fact that the optimum N at each QW is 1 in the case of low α_{loss} for thin quantum well structures. Since the gain saturation is more enhanced for smaller L_z , the optimum N of thinner QW's is larger than that of thicker QW's. For $\alpha_{\text{loss}} = 50 \text{ cm}^{-1}$, $N_{\text{opt}}(L_z = 50 \text{ \AA}) = 4$, while $N_{\text{opt}}(L_z = 100 \text{ \AA}) = 2$ and $N_{\text{opt}}(L_z = 200 \text{ \AA}) = 1$.

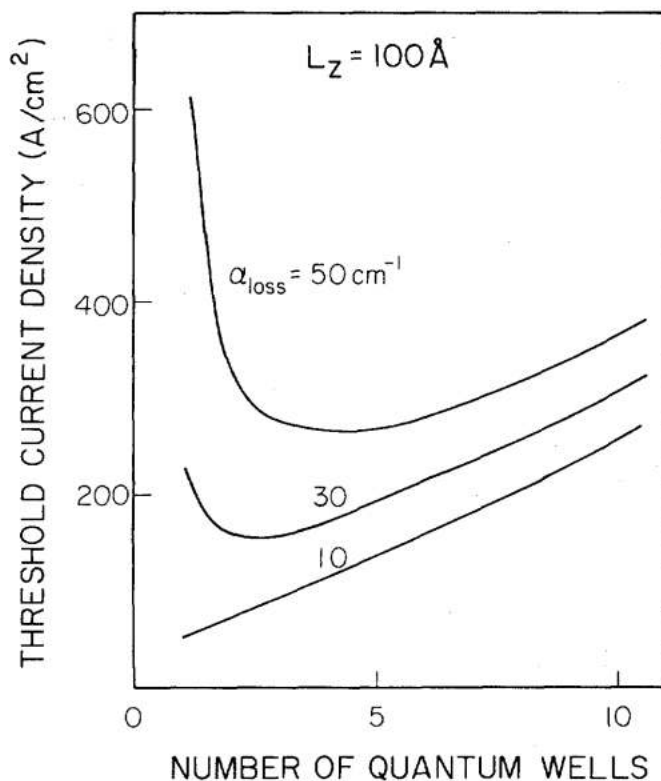


Fig. 2. The threshold current change as a function of the number N of quantum wells with the various total loss α_{loss} . In this case, the quantum well thickness L_z is assumed to be 100 \AA .

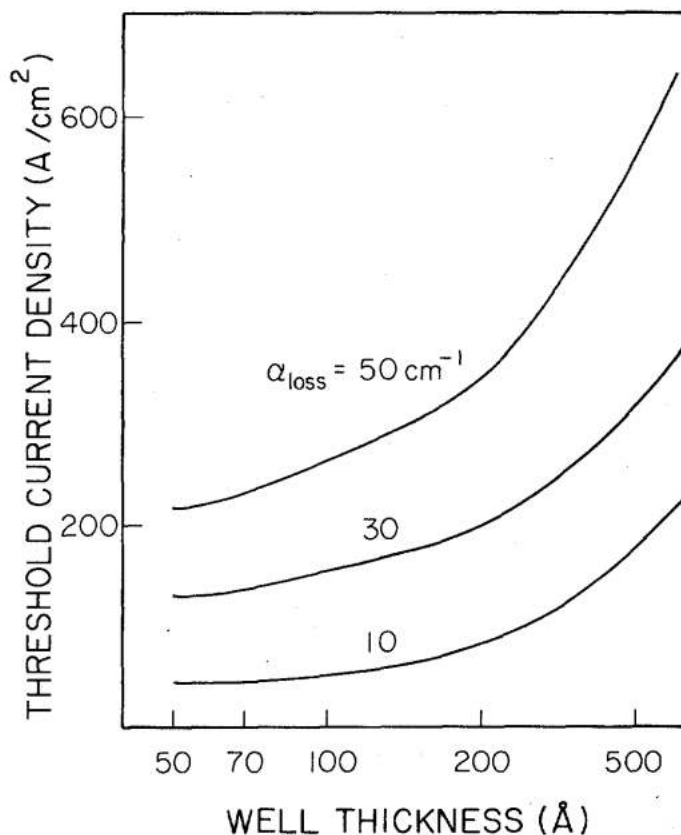


Fig. 3. The threshold current change as a function of the quantum well thickness L_z with various total loss α_{loss} . The number N of quantum wells which is optimized so that threshold current is minimum.

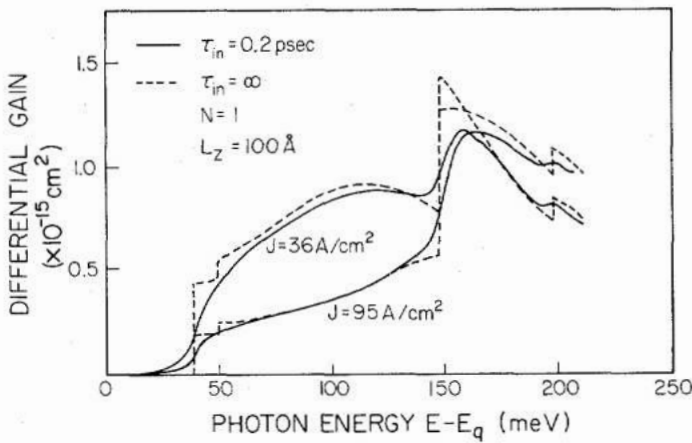


Fig. 4. The differential gain $g'(E)$ with several injected currents as a function of the photon energy subtracted the band-gap energy of GaAs ($E - E_g$). In this calculation τ_{in} is assumed to be 0.2 ps (solid curve) and ∞ s (broken curve). The number of the quantum wells is 1 and the thickness is 100 Å.

III. DIFFERENTIAL GAIN AND MODULATION BANDWIDTH

A. Differential Gain

In this section, we investigate the properties of the differential gain and calculate the modulation bandwidth of the quantum well laser. The differential gain $g'(n, E)$ at a photon energy E is defined as the derivative of the bulk gain with respect to carrier concentration, i.e., $(g'(n, E) \equiv dg(n, E)/dn)$. Since it figures in the modulation response of the semiconductor laser through the expression $f_r = \frac{1}{2} \pi \sqrt{g'(n, E)(n/c)(P_0/\tau_p)}$ (where f_r is the relaxation frequency conventionally used as a measure of the upper useful modulation frequency [1], P_0 is the photon density in the active region and τ_p is the photon lifetime in the passive cavity), $g'(n, E)$ contains the “physics” of the semiconductor laser. The derivation of f_r will be discussed in the next section. In order to emphasize the carrier dependence of the gain, we describe the gain as a function of both E and n . When we take broadening effects due to intraband relaxation into consideration the differential gain is given by

$$g'(n, E) \frac{dg(n, E)}{dn} = \int A(\epsilon) \sum_{j=h,l} \sum_{n=1}^{\infty} \frac{\rho_{red,n}^j(\epsilon)}{L_z} \cdot \left(\frac{df_c(\epsilon_{cn})}{dn} - \frac{df_v(\epsilon_{vn}^j)}{dn} \right) \cdot \frac{(\hbar/\tau_{in})}{(E - \epsilon)^2 + (\hbar/\tau_{in})^2} d\epsilon. \quad (8)$$

Fig. 4 shows the differential gain $g'(n, E)$ as a function of the excess photon energy ($E - E_g$) (E_g = band-gap energy) at two injected current densities ($J = 36, 95 \text{ A/cm}^2$) in SQW laser with a QW thickness of $L_z = 100 \text{ Å}$. The solid curve is that of $g'(n, E)$ with $\tau_{in} = 0.2$ ps and the broken line represents $g'(n, E)$ with $\tau_{in} = \infty$ (i.e., no energy broadening). We call $g'(n, E)$ without energy broadening the differential gain profile, hereafter. This differential gain profile is 0 below 39 meV which corresponds to the energy difference between the first subband of electrons and heavy holes ($\epsilon_{c1} - \epsilon_{v1}^h - E_g$). The

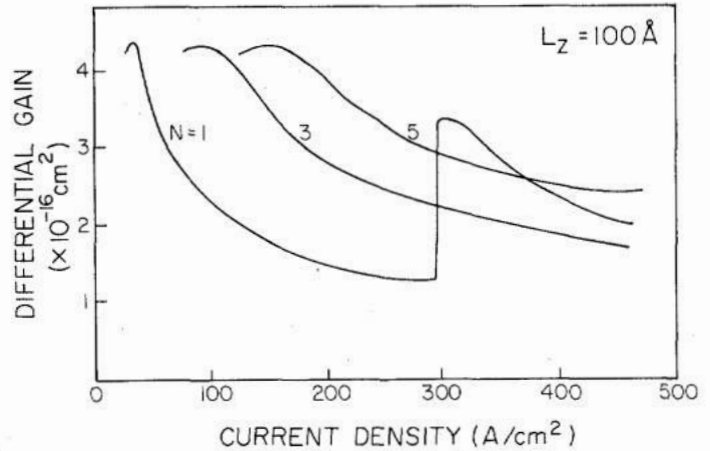


Fig. 5. The differential gain $g'(n, E_l)$, at which the gain is maximum, is shown as a function of the injected current J with various number of quantum wells. The thickness of quantum wells is 100 Å.

abrupt small change near 50 meV is due to the onset of the electron-light hole transition of the first subband. We also note other abrupt changes in the $g'(E, n)$ near 150 meV and 190 meV. These changes correspond to the electron-heavy hole, and electron-light hole transitions of the second subband, respectively. In the case of the more realistic $\tau_{in} = 0.2$ ps curves, the energy broadening of the abrupt features, by (\hbar/τ_{in}) , causes a lowering of $g'(n, E)$. In this case we can still observe dips at the onset of higher order transitions. Since the photon energy E_l that maximizes the gain, is near 40–50 meV in both cases of the injected current, this figure indicates that differential gain at E_l seems to decrease with the increase of the injected current.

In order to clarify the injected current dependence of the differential gain, we show in Fig. 5 the differential gain $g'(n, E_l)$ at maximum gain photon energy E_l as a function of the injected current in a MQW laser with various numbers of QW's. The results show that $g'(n, E_l)$ has a peaked structure, which is due to the gain saturation effects discussed in Section II. Since $g'(n, E_l)$ is a bulk parameter, it does not change with the increase of the number of QW's as long as the Fermi energy is fixed, whereas the injected current increases by a factor of N . Consequently, the $g'(E, n)$ curve is stretched in the direction of the current axis with an increase of N . Note that the second peak appears at $J \sim 300$ mA. In the current region below the second peak ($J < 300$ mA), E_l is located near the first subband. When E_l jumps from the first subband to the second subband, this second peak appears.

The differential gain also depends on the QW thickness. Fig. 6 shows the differential gain at E_l as a function of the injected current in a SQW laser with the various QW thickness (50 Å, 100 Å, and 200 Å). The results indicate two important properties of the $g'(n, E)$. First is that the thinner QW laser achieves higher differential gain. This is mainly due to the fact that the contribution of the injected current to the increment of the gain by the next higher subband is enhanced when the QW thickness is larger, and that happens at the expense of gain at E_l . Sec-

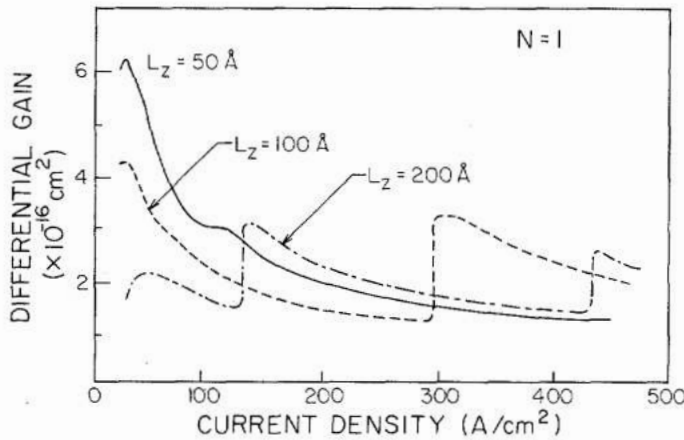


Fig. 6. The differential gain $g'(n, E_l)$, at which the gain is maximum, is shown as a function of the injected current with various quantum well thickness L_z . The number of quantum wells is equal to 1.

ond is that the $g'(n, E)$ has many peaks when $L_z = 200 \text{ \AA}$. Note that the values of the second and third peak of $g'(n, E)$ are comparable to that of the first peak. When E_l jumps from the first (second) subband to the second (third) subband, the second peak (third peak) appears. Since thicker QW's have more peaks in the density of states, the jumps appear more frequently with the increase of the L_z . In the limiting case of thick QW's, $g'(n, E)$ increases gradually with the increase of J .

B. Modulation Resonance Frequency

On the basis of the discussion concerning $g'(n, E)$ in the previous subsection, we will investigate modulation dynamics in QW lasers. The rate equation for laser dynamics can be described as follows.

$$\begin{aligned} \frac{dn}{dt} &= \frac{J(t)}{eL_z} - \frac{n_r}{c} g(n, E_l) P - \frac{n}{\tau_s} \\ \frac{dP}{dt} &= \Gamma \frac{n_r}{c} g(n, E_l) P + \beta \frac{n}{\tau_s} - \frac{P}{\tau_p} \end{aligned} \quad (9)$$

where P is the photon density (cm^{-3}), β is the spontaneous emission coefficient into the lasing mode, τ_s is the carrier lifetime, $J(t)$ (cm^{-2}) is the injected current density, n is the carrier concentration, and $g(n, E_l)$ (cm^{-1}) is the bulk gain, while $\Gamma g(n, E_l)$ is the modal gain as a function of the carrier density n (cm^{-3}) at the lasing photon energy E_l . When we discuss the carrier density in the quantum well structure we usually use the density per cm^2 , however, the proper "bookkeeping" of photons and carriers requires that n stands for carrier density per unit volume. The relaxation resonance frequency f_r is determined by a small-signal analysis of (9). The results can be simply expressed by [1]

$$f_r = \frac{1}{2\pi} \sqrt{\frac{n_r g'(n, E_l) P_o}{c} \frac{P_o}{\tau_p}}, \quad (10)$$

where P_o is the photon density in the cavity at stationary states. Note that τ_p is related to the total loss as follows.

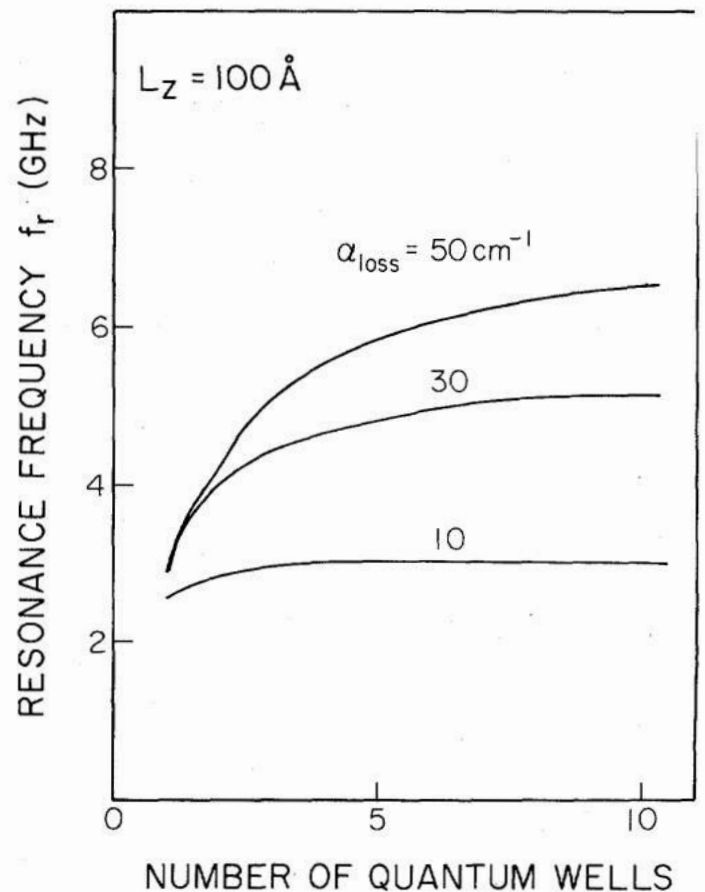


Fig. 7. The calculated results of the resonant frequency f_r as a function of N with various total loss α_{loss} , with quantum well thickness $L_z = 100 \text{ \AA}$.

$$\tau_p = \frac{n_r}{c \alpha_{\text{loss}}}. \quad (11)$$

It follows from (10) that an increase of the differential gain and of P_o , together with a decrease of τ_p , is required in order to increase the modulation bandwidth. The increase of P_o is usually limited by catastrophic mirror damage, while τ_p is determined by the total loss α_{total} . Thus P_o and τ_p depend strongly on the laser geometry. On the other hand, the differential gain $g'(n, E_l)$ depends on the properties of the QW's and on the injected current at threshold as discussed in connection with Figs. 5 and 6. Fig. 7 shows the calculated result of the resonance frequency f_r of MQW lasers with $L_z = 100 \text{ \AA}$ as a function of N with α_{loss} (photon lifetime) as a parameter taking (11) into consideration. As shown in this figure, there is the optimum N which is, usually, not equal to 1 for obtaining high f_r . This can be explained as follows. When the number of QW's is small, the threshold current J_{th} is larger than the current J_{max}^N which gives the maximum differential gain because of the gain saturation effects, leading to lower differential gain. On the other hand, the differential gain increases with the increase of N , since J_{th} approaches J_{max}^N . However, once J_{th} becomes smaller than J_{max}^N , with the further increase of N , the differential gain decreases again. Therefore there exists an optimum number N_{opt} at which $J_{\text{max}} \sim J_{\text{th}}$. Since J_{max}^N increases with the increase

of α_{loss} with the same N , the N_{opt} increases for higher α_{loss} .

In Fig. 8, the maximum resonance frequency f_r , which can be attained by optimizing N as a function of L_z with various α_{loss} , is shown by a solid curve. This result indicates that for the range of losses (α_{loss}) considered, the resonance frequency increases with the reduction of the QW thickness: the resonance frequency f_r with $L_z = 50$ Å is enhanced by 1.8 compared to the f_r of the double heterostructure laser.

IV. LASER LINEWIDTH

A. Spontaneous Emission Factor

In this section, we analyze the spectral linewidth broadening in the QW lasers and show how a substantial reduction of the linewidth can be obtained with an optimization of the quantum well structures.

The power-dependent linewidth in semiconductor lasers occurs due to the spontaneous emission that modulates the laser field both in intensity and in frequency (phase). This linewidth involves the Schawlow-Townes linewidth that is due to the instantaneous-phase fluctuation as well as to the delayed-phase fluctuation that occurs as the laser intensity returns to its steady states (amplitude phase coupling). The reduced linewidth is [3], [4]

$$\Delta\nu = \frac{v_g^2 h\nu (\Gamma g(E_l)) (L^{-1} \ln(1/R)) n_{\text{sp}}}{8\pi P_1} (1 + \alpha^2) \quad (12)$$

where P_1 is the output photon density, α is the linewidth enhancement factor, v_g is the group velocity of light in the active layer, $h\nu$ is the photon energy, and n_{sp} is defined as the spontaneous emission rate divided by total stimulated emission rate given by

$$n_{\text{sp}}(E) = \frac{\int A(\epsilon) \sum_{j=h,l} \sum_{n=1}^{\infty} \rho_{\text{red},n}^j(\epsilon) f_c (1 - f_v) \frac{(\hbar/\tau_{\text{in}})}{(E - \epsilon)^2 + (\hbar/\tau_{\text{in}})^2}}{\int A(\epsilon) \sum_{j=h,l} \sum_{n=1}^{\infty} \rho_{\text{red},n}^j(\epsilon) (f_c - f_v) \frac{(\hbar/\tau_{\text{in}})}{(E - \epsilon)^2 + (\hbar/\tau_{\text{in}})^2} d\epsilon} \quad (13)$$

If energy broadening is extremely small we can approximate n_{sp} at the photon energy E_l at which the gain is maximum ($E = E_l$) by

$$n_{\text{sp}} \approx (1 - \exp(E_l - \Delta\epsilon_F))^{-1} \quad (14)$$

where $\Delta\epsilon_F$ is the difference between the quasi-Fermi energy of electrons and that of holes ($\epsilon_{F_c} - \epsilon_{F_v}$). Equation (14) shows that n_{sp} at E_l increases substantially with the approach of $\Delta\epsilon_F$ to E_l (i.e., the total loss decreases), since E_l is fixed near $(\epsilon_{c1} - \epsilon_{v1})$ (the photon energy which corresponds to the lowest subband) in the QW lasers. Finally, n_{sp} at E_l becomes infinite when the injected current is equal to the current J_{tr} at which transparency occurs ($\Delta\epsilon_F \rightarrow E_l$). In Fig. 9, we show the calculated results of n_{sp} at E_l as a function of the injected current with various numbers of QW's. The QW thickness is assumed to be 100 Å and τ_{in} is assumed to be 0.2 ps. The observed dip corresponds to the jump of E_l from the electron-hole transition

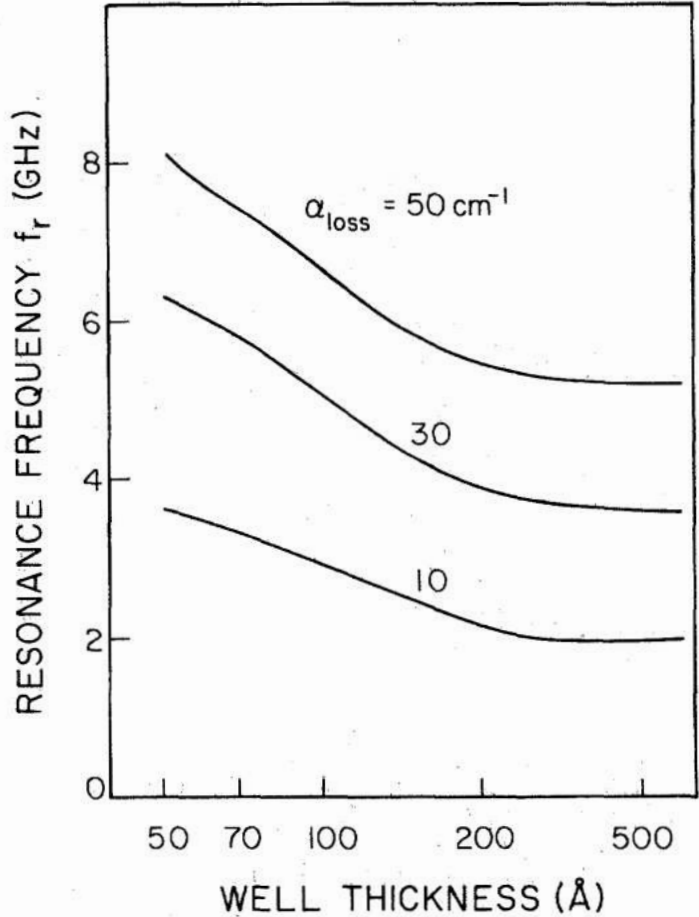


Fig. 8. The maximum resonant frequency, which is achieved by optimizing the number of quantum wells, as a function of the quantum well thickness L_z with various total loss α_{loss} .

of the first subband to the electron-hole transition of the second subband.

Fig. 10 shows the n_{sp} at E_l as a function of the injected current with various QW thicknesses. In this calculation, the number of QW's is 1 and $\tau_{\text{in}} = 0.2$ ps. We notice that the approach of n_{sp} to 1, with the increase of the injected current, occurs more rapidly when the QW thickness is smaller. We also notice that two dips appear in the case of $L_z = 200$ Å. These dips appear when E_l jumps from the electron-hole transition of the first (second) subband to the electron-hole transition of the second (third) subband.

B. Linewidth Enhancement Factor

The linewidth enhancement factor α of (12), which represents the spectral broadening due to the coupling between AM linewidth and FM noise, is given by [17]

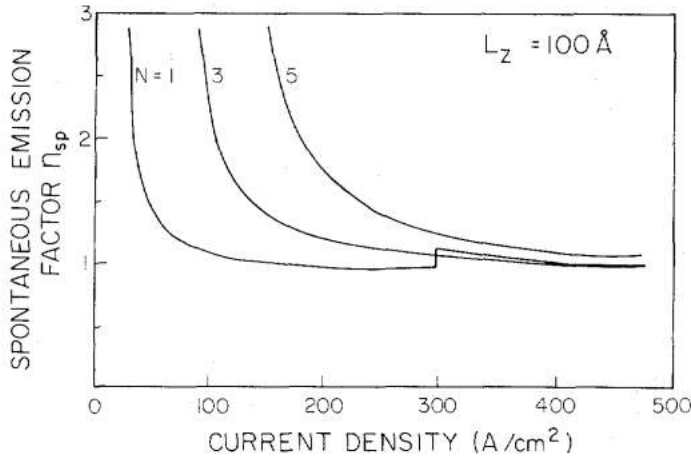


Fig. 9. The spontaneous emission factor n_{sp} as a function of the injected current, at which the gain is maximum, is shown as a function of the injected current J with various number of quantum wells. The thickness of quantum wells is 100 Å.

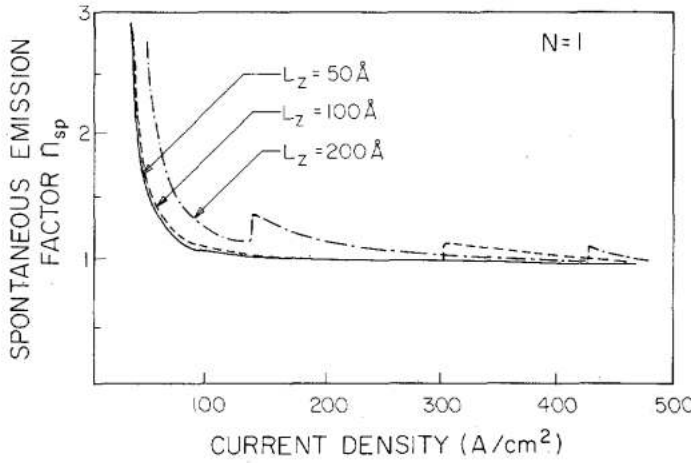


Fig. 10. The spontaneous emission factor n_{sp} as a function of the injected current, at which the gain is maximum, is shown as a function of the injected current J with various quantum well thickness. The number of quantum wells N is equal to 1.

$$\alpha = \frac{\int A(\epsilon) \sum_{j=h,l} \sum_{n=1}^{\infty} \rho_{red,n}^j(\epsilon) \left(\frac{df_c(\epsilon_{cn})}{dn} - \frac{df_v(\epsilon'_{vn})}{dn} \right) \frac{(E - \epsilon)}{(E - \epsilon)^2 + (\hbar/\tau_{in})^2} d\epsilon}{\int A(\epsilon) \sum_{j=h,l} \sum_{n=1}^{\infty} \rho_{red,n}^j(\epsilon) \left(\frac{df_c(\epsilon_{cn})}{dn} - \frac{df_v(\epsilon'_{vn})}{dn} \right) \frac{(\hbar/\tau_{in})}{(E - \epsilon)^2 + (\hbar/\tau_{in})^2} d\epsilon} \quad (15)$$

Fig. 11 shows the α parameter as a function of $(E - E_g)$ (the photon energy minus the band-gap energy) for injected currents of $J = 36 \text{ A/cm}^{-2}$, and $J = 95 \text{ A/cm}^{-2}$, respectively. We notice that α increases with the increase of the photon energy and also changes its sign from minus to plus. This is due to the change in the sign of the numerator of (15) from minus to plus, since the differential gain profile has a peaked structure as indicated in Fig. 4. Fig. 12 shows α at the maximum gain photon energy E_l as a function of the injected current for various N 's. In this calculation the QW thickness and τ_{in} are assumed to be 100 Å, and 0.2 ps, respectively. Since α is a bulk parameter, the value of α is independent of the number of quantum wells N , so that the injected current to achieve a given value of α is proportional to N , and the currents for

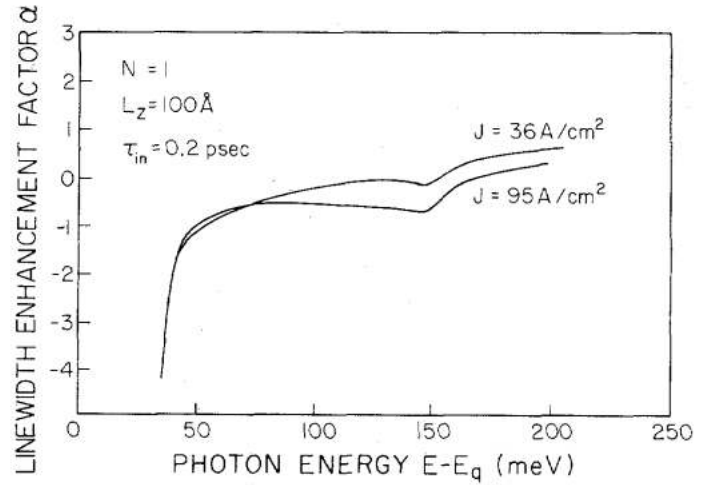


Fig. 11. The linewidth enhancement factor α with various injected currents as a function of the photon energy subtracted by the band-gap energy of GaAs ($E - E_g$).

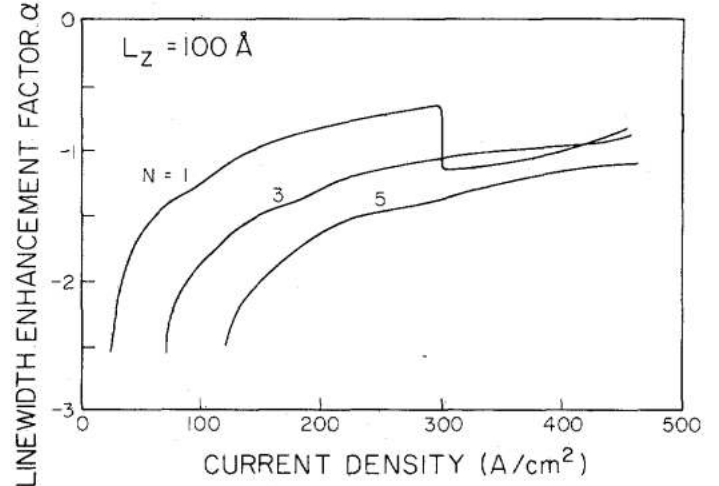


Fig. 12. The linewidth enhancement factor α as a function of the injected current at photon energy, at which the gain is maximum with various N . The thickness of quantum wells is 100 Å.

$N = 1, 3, 5$ differ only in their horizontal scale. We find that the absolute value of α ($|\alpha|$) decreases with the increases of the injected current. Therefore we can reduce the value of $|\alpha|$ by the use of high loss α_{loss} although this leads to high-threshold current.

Fig. 13 shows the α parameter as a function of the injected current with various QW thickness at E_l . The value of N is taken as unity. We notice a tendency that the smaller L_z is, the smaller α is.

C. Spectral Linewidth

Fig. 14 shows the laser spectral width $\Delta\nu$ calculated from (11) as a function of the number of QW's with various total loss ($\alpha_{total} = 10, 30, 50 \text{ cm}^{-1}$). We find that the

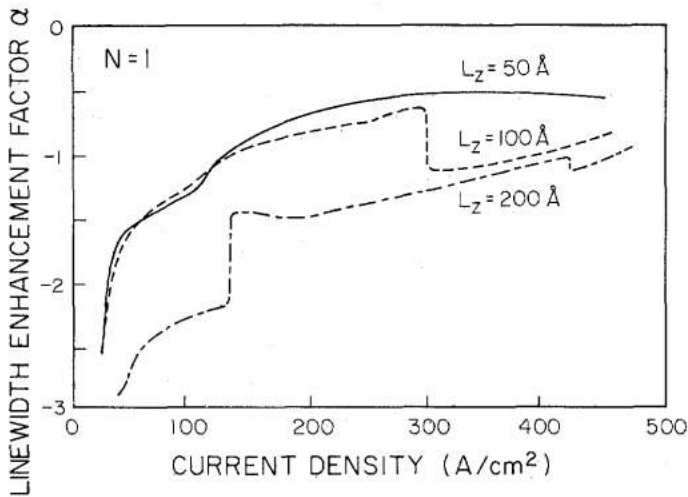


Fig. 13. The linewidth enhancement factor α as a function of the injected current with various quantum well thickness. The thickness of quantum wells is 100 Å.

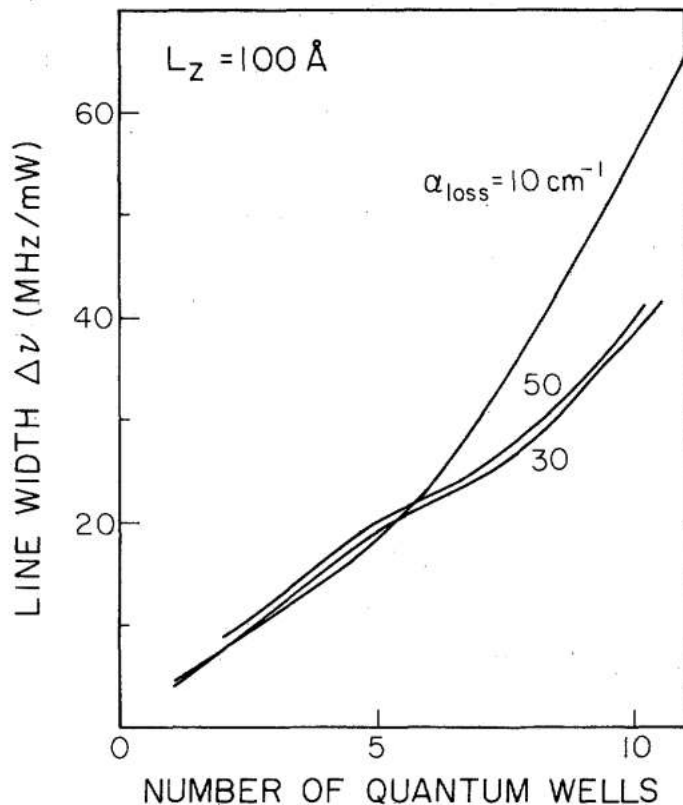


Fig. 14. The spectral linewidth $\Delta\nu$ as a function of the number of quantum wells with various total loss α_{loss} .

$\Delta\nu$ increases monotonically with the increase of the number of QW's. This is due to the fact that both α and n_{sp} are reduced with the increase of $\Delta\epsilon_F$ which is enhanced with small number of N . Therefore, in order to reduce $\Delta\nu$, a SQW laser should be used rather than a MQW laser. The substantial change in $\Delta\nu$ with the increase of N is due to the fact that the abrupt change in α occurs in the low injected-current region. Note that this change is enhanced with low α_{loss} .

The spectral width $\Delta\nu$ is also a function of the thickness L_z . Fig. 15 shows the minimum attainable $\Delta\nu$ as a function

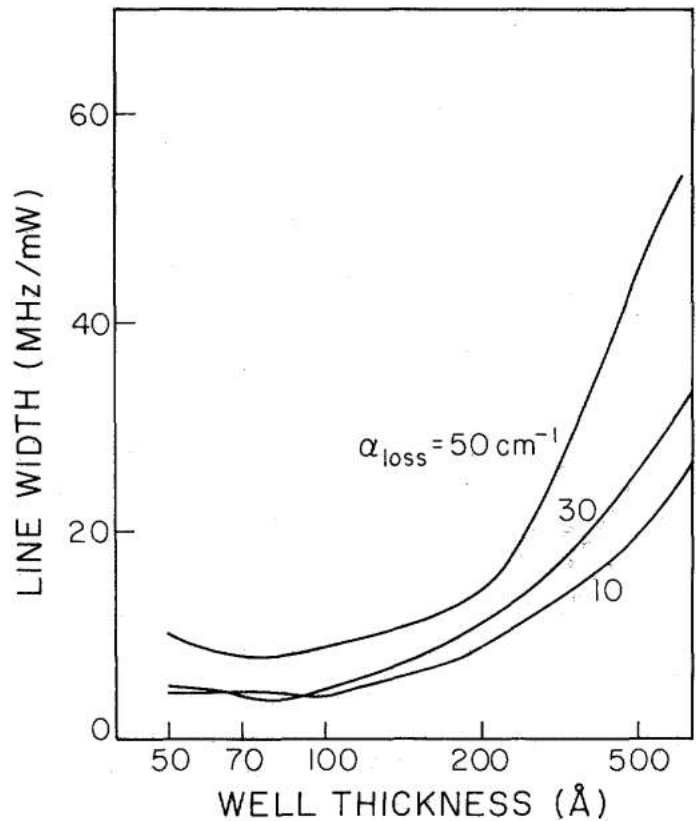


Fig. 15. The minimum spectral linewidth $\Delta\nu$ as a function of the quantum well thickness with various total loss. The number of quantum wells is optimized.

of the QW thickness with various α_{loss} . Generally speaking, we notice that $\Delta\nu$ is reduced greatly in the region of thin active layer. In the case of $\alpha_{\text{loss}} = 10 \text{ cm}^{-1}$, $\Delta\nu$ is minimum around $L_z = 80 \text{ Å}$. This is due to the fact that there is the current region in which α of $L_z = 100 \text{ Å}$ is smaller than that of $L_z = 50 \text{ Å}$, as shown in Fig. 13. Since the value $\Delta\nu$ for a DH laser ($0.1 \mu\text{m}$ active layer) is calculated to be 60 MHz/mW with $\alpha_{\text{loss}} = 30 \text{ cm}^{-1}$, the $\Delta\nu$ can be substantially reduced with a thin QW structure by factor of $\sim 1/10$ compared to DH lasers. For all L_z , the $\Delta\nu$ increases monotonically when the number of QW's increases.

V. CONCLUSIONS

We investigate theoretically the effects of quantum well (QW) structure parameters such as the number and the thickness of QW's on the gain, the dynamic response and quantum noise of QW lasers. We have considered the issues of optimum conditions for realizing low-noise characteristics and high-dynamic performance. We have found that the gain in structures with a small number of layers tends to saturate with current. The differential gain and linewidth enhancement factor of QW lasers are also discussed. The results indicate that the modulation bandwidth can be enhanced by a factor of ~ 2 in the multi-quantum well laser, and linewidth is found to be significantly suppressed by 1/10 in a single thin QW laser compared to the conventional double heterostructure laser.

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