

THEORY OF GROUP REPRESENTATIONS AND APPLICATIONS

ASIM O. BARUT

Institute for Theoretical Physics, University of Colorado,
Boulder, Colo., U.S.A.

RYSZARD RĄCZKA

Institute for Nuclear Research,
Warszawa, Polska

Second revised edition



World Scientific

Contents

PREFACE	VII
OUTLINE OF THE BOOK	XV
NOTATIONS	XIX

CHAPTER 1

LIE ALGEBRAS

§ 1. Basic Concepts and General Properties	1
§ 2. Solvable, Nilpotent, Semisimple and Simple Lie Algebras	10
§ 3. The Structure of Lie Algebras	17
§ 4. Classification of Simple, Complex Lie Algebras	20
§ 5. Classification of Simple, Real Lie Algebras	29
§ 6. The Gauss, Cartan and Iwasawa Decompositions	37
§ 7. An Application. On Unification of the Poincaré Algebra and Internal Symmetry Algebra	43
§ 8. Contraction of Lie Algebras	44
§ 9. Comments and Supplements	46
§ 10. Exercises	48

CHAPTER 2

TOPOLOGICAL GROUPS

§ 1. Topological Spaces	52
§ 2. Topological Groups	61
§ 3. The Haar Measure	67
§ 4. Comments and Supplements	70
§ 5. Exercises	71

CHAPTER 3

LIE GROUPS

§ 1. Differentiable Manifolds	75
§ 2. Lie Groups	81
§ 3. The Lie Algebra of a Lie Group	85
§ 4. The Direct and Semidirect Products	95
§ 5. Levi-Malcev Decomposition	98
§ 6. Gauss, Cartan, Iwasawa and Bruhat Global Decompositions	100
§ 7. Classification of Simple Lie Groups	106
§ 8. Structure of Compact Lie Groups	108

§ 9. Invariant Metric and Invariant Measure on Lie Groups	109
§ 10. Comments and Supplements	111
§ 11. Exercises	114

CHAPTER 4

HOMOGENEOUS AND SYMMETRIC SPACES

§ 1. Homogeneous Spaces	123
§ 2. Symmetric Spaces	124
§ 3. Invariant and Quasi-Invariant Measures on Homogeneous Spaces	128
§ 4. Comments and Supplements	132
§ 5. Exercises	132

CHAPTER 5

GROUP REPRESENTATIONS

§ 1. Basic Concepts	134
§ 2. Equivalence of Representations	139
§ 3. Irreducibility and Reducibility	141
§ 4. Cyclic Representations	145
§ 5. Tensor Product of Representations	147
§ 6. Direct Integral Decomposition of Unitary Representations	150
§ 7. Comments and Supplements	156
§ 8. Exercises	

CHAPTER 6

REPRESENTATIONS OF COMMUTATIVE GROUPS

§ 1. Irreducible Representations and Characters	159
§ 2. Stone and SNAG Theorems	160
§ 3. Comments and Supplements	163
§ 4. Exercises	164

CHAPTER 7

REPRESENTATIONS OF COMPACT GROUPS

§ 1. Basic Properties of Representations of Compact Groups	166
§ 2. Peter-Weyl and Weyl Approximation Theorems	172
§ 3. Projection Operators and Irreducible Representations	177
§ 4. Applications	179
§ 5. Representations of Finite Groups	186
§ 6. Comments and Supplements	195
§ 7. Exercises	197

CHAPTER 8

FINITE-DIMENSIONAL REPRESENTATIONS OF LIE GROUPS

§ 1. General Properties of Representations of Solvable and Semisimple Lie Groups	199
--	-----

§ 2. Induced Representations of Lie Groups	205
§ 3. The Representations of $GL(n, C)$, $GL(n, R)$, $U(p, q)$, $U(n)$, $SL(n, C)$, $SL(n, R)$, $SU(p, q)$, and $SU(n)$	213
§ 4. The Representations of the Symplectic Groups $Sp(n, C)$, $Sp(n, R)$ and $Sp(n)$	217
§ 5. The Representations of Orthogonal Groups $SO(n, C)$, $SO(p, q)$, $SO^*(n)$, and $SO(n)$	219
§ 6. The Fundamental Representations	223
§ 7. Representations of Arbitrary Lie Groups	225
§ 8. Further Results and Comments	227
§ 9. Exercises	238

CHAPTER 9

TENSOR OPERATORS, ENVELOPING ALGEBRAS AND ENVELOPING FIELDS

§ 1. The Tensor Operators	242
§ 2. The Enveloping Algebra	249
§ 3. The Invariant Operators	251
§ 4. Casimir Operators for Classical Lie Group	254
§ 5. The Enveloping Field	266
§ 6. Further Results and Comments	273
§ 7. Exercises	275

CHAPTER 10

THE EXPLICIT CONSTRUCTION OF FINITE-DIMENSIONAL IRREDUCIBLE
REPRESENTATIONS

§ 1. The Gel'fand-Zetlin Method	277
§ 2. The Tensor Method	291
§ 3. The Method of Harmonic Functions	302
§ 4. The Method of Creation and Annihilation Operators	309
§ 5. Comments and Supplements	312
§ 6. Exercises	314

CHAPTER 11

REPRESENTATION THEORY OF LIE AND ENVELOPING ALGEBRAS BY UNBOUNDED
OPERATORS: ANALYTIC VECTORS AND INTEGRABILITY

§ 1. Representations of Lie Algebras by Unbounded Operators	318
§ 2. Representations of Enveloping Algebras by Unbounded Operators	323
§ 3. Analytic Vectors and Analytic Dominance	331
§ 4. Analytic Vectors for Unitary Representations of Lie Groups	344
§ 5. Integrability of Representations of Lie Algebras	348
§ 6. FS^3 -Theory of Integrability of Lie Algebras Representations	352
§ 7. The 'Heat Equation' on a Lie Group and Analytic Vectors	358

§ 8. Algebraic Construction of Irreducible Representations	365
§ 9. Comments and Supplements	372
§ 10. Exercises	373

CHAPTER 12

QUANTUM DYNAMICAL APPLICATIONS OF LIE ALGEBRA REPRESENTATIONS

§ 1. Symmetry Algebras in Hamiltonian Formulation	378
§ 2. Dynamical Lie Algebras	382
§ 3. Exercises	386

CHAPTER 13

GROUP THEORY AND GROUP REPRESENTATIONS IN QUANTUM THEORY

§ 1. Group Representations in Physics	392
§ 2. Kinematical Postulates of Quantum Theory	394
§ 3. Symmetries of Physical Systems	406
§ 4. Dynamical Symmetries of Relativistic and Non-Relativistic Systems	412
§ 5. Comments and Supplements	417
§ 6. Exercises	418

CHAPTER 14

HARMONIC ANALYSIS ON LIE GROUPS. SPECIAL FUNCTIONS AND GROUP REPRESENTATIONS

§ 1. Harmonic Analysis on Abelian and Compact Lie Groups	421
§ 2. Harmonic Analysis on Unimodular Lie Groups	423
§ 3. Harmonic Analysis on Semidirect Product of Groups	431
§ 4. Comments and Supplements	435
§ 5. Exercises	

CHAPTER 15

HARMONIC ANALYSIS ON HOMOGENEOUS SPACES

§ 1. Invariant Operators on Homogeneous Spaces	439
§ 2. Harmonic Analysis on Homogeneous Spaces	441
§ 3. Harmonic Analysis on Symmetric Spaces Associated with Pseudo-Orthogonal Groups $SO(p, q)$	446
§ 4. Generalized Projection Operators	459
§ 5. Comments and Supplements	466
§ 6. Exercises	470

CHAPTER 16

INDUCED REPRESENTATIONS

§ 1. The Concept of Induced Representations	473
§ 2. Basic Properties of Induced Representation	487

§ 3. Systems of Imprimitivity	493
§ 4. Comments and Supplements	501
§ 5. Exercises	493

CHAPTER 17

INDUCED REPRESENTATIONS OF SEMIDIRECT PRODUCTS

§ 1. Representation Theory of Semidirect Products	503
§ 2. Induced Unitary Representations of the Poincaré Group	513
§ 3. Representation of the Extended Poincaré Group	525
§ 4. Indecomposable Representations of Poincaré Group	527
§ 5. Comments and Supplements	536
§ 6. Exercises	537

CHAPTER 18

FUNDAMENTAL THEOREMS OF INDUCED REPRESENTATIONS

§ 1. The Induction-Reduction Theorem	540
§ 2. Tensor-Product Theorem	546
§ 3. The Frobenius Reciprocity Theorem	549
§ 4. Comments and Supplements	553
§ 5. Exercises	553

CHAPTER 19

INDUCED REPRESENTATIONS OF SEMISIMPLE LIE GROUPS

§ 1. Induced Representations of Semisimple Lie Groups	555
§ 2. Properties of the Group $SL(n, C)$ and Its Subgroups	559
§ 3. The Principal Nondegenerate Series of Unitary Representations of $SL(n, C)$	560
§ 4. Principal Degenerate Series of $SL(n, C)$	567
§ 5. Supplementary Nondegenerate and Degenerate Series	570
§ 6. Comments and Supplements	577
§ 7. Exercises	578

CHAPTER 20

APPLICATIONS OF INDUCED REPRESENTATIONS

§ 1. The Relativistic Position Operator	581
§ 2. The Representations of the Heisenberg Commutation Relations	588
§ 3. Comments and Supplements	591
§ 4. Exercises	593

CHAPTER 21

GROUP REPRESENTATIONS IN RELATIVISTIC QUANTUM THEORY

§ 1. Relativistic Wave Equations and Induced Representations	596
§ 2. Finite Component Relativistic Wave Equations	601

§ 3. Infinite Component Wave Equations	609
§ 4. Group Extensions and Applications	619
§ 5. Space-Time and Internal Symmetries	626
§ 6. Comments and Supplements	630
§ 7. Exercises	636
APPENDIX A	
ALGEBRA, TOPOLOGY, MEASURE AND INTEGRATION THEORY	637
APPENDIX B	
FUNCTIONAL ANALYSIS	
§ 1. Closed, Symmetric and Self-Adjoint Operators in Hilbert Space	641
§ 2. Integration of Vector and Operator Functions	645
§ 3. Spectral Theory of Operators	649
§ 4. Functions of Self-Adjoint Operators	662
§ 5. Essentially Self-Adjoint Operators	663
BIBLIOGRAPHY	667
LIST OF IMPORTANT SYMBOLS	703
AUTHOR INDEX	706
SUBJECT INDEX	710