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Horacio E. Camblong
University of San Francisco, camblong@usfca.edu

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Theory of magnetotransport in inhomogeneous magnetic structures

Horacio E. Camblong,[a)] Shufeng Zhang, and Peter M. Levy
Department of Physics, New York University, New York, New York 10003

The origin of the giant magnetoresistance of magnetic multilayers and magnetic granular solids is investigated through a unified spin-dependent linear transport theory, in which the primary source of electrical resistivity is short-range scattering by impurities in the different magnetic or nonmagnetic regions and at the interfaces. Our theory predicts that magnetotransport in granular solids is similar to that for currents perpendicular to the plane of the layers in multilayers in that their magnetoresistance is independent of the average distance between adjacent magnetic regions.

Magnetotransport in inhomogeneous magnetic structures has been experimentally studied in both multilayers[1] and granular solids.[2] The main focus of these investigations has been the phenomenon of giant magnetoresistance, which holds promise for technological breakthroughs.

The theory of magnetotransport in multilayers with collinear magnetizations has been discussed in Refs. 3–8. Instead, magnetotransport through noncollinear magnetizations is not so well understood, either for multilayers or for granular solids.

In this paper we consider the problem of magnetotransport in inhomogeneous magnetic structures with arbitrary magnetization configurations via the introduction of a linear transport theory in the presence of effective spin-dependent fields, and we apply the formalism to three cases of experimental relevance: multilayers with currents in the plane of the layers (CIP), multilayers with currents perpendicular to the plane of the layers (CPP), and granular solids. Our model is based on scattering by impurities in the different regions of the system (layers for multilayers and granules or matrix for granular solids) as well as at the interfaces, rather than on scattering by entire regions of the inhomogeneous structure (multilayer or granular solid). We find that the characteristic exponential dependence of the magnetoresistance of CIP with respect to the thickness of the nonmagnetic spacer layer has no counterpart in either CPP or granular solids, a behavior that places them in a new class of magnetically self-averaging systems.

Our theory generalizes the real space Kubo approach of Ref. 4 and it starts from the model Hamiltonian[3]

$$H=H_0+\sum_a (v_a+j_a\hat{\mathbf{M}}_a\cdot\hat{\sigma})\,\delta(\mathbf{r}-\mathbf{r}_a), \tag{1}$$

where H_0 describes free electrons, $\hat{\sigma}$ is the one-electron spin vector operator, $\mathbf{r}_a$ is the position vector of a particular impurity, $\hat{\mathbf{M}}_a$ represents the direction of magnetization of a magnetic region, and v_a and j_a are phenomenological constants. As in Ref. 4, when the reduced Fermi wavelength $\lambda\!\!\!^{-}_F=k_F^{-1}\approx 1$ Å is much smaller than the other length scales (mean-free paths and inhomogeneity lengths), the one-particle propagator satisfies the Dyson's equation, which in real space reads

$$\left[\epsilon_F+\frac{\hbar^2}{2m}\nabla_r^2-\Sigma(\mathbf{r})\right]G^{\text{ret}}(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'), \tag{2}$$

where $\Sigma(\mathbf{r})$ is the local self-energy due to scattering in the bulk and at the interfaces. In order to analyze transport properties we look at the imaginary part

$$\Delta(\mathbf{r})=-\,\text{Im}[\Sigma(\mathbf{r})], \tag{3}$$

which we will refer to as the scattering strength.

The main difficulty posed by the existence of noncollinear magnetization is that the independence of physical properties with respect to arbitrary choices of the quantization axis requires a theory that is covariant under changes of that axis. The ensuing implication is that if the current is to be viewed as arising from different channels (two-current model), the only possible description is one in terms of the spin-dependent current densities, $\mathbf{j}_{\alpha\beta}(\mathbf{r})=\langle\hat{\mathbf{j}}_{\alpha\beta}(\mathbf{r})\rangle$, defined as the expectation values of the spinor current operators

$$\hat{\mathbf{j}}_{\alpha\beta}(\mathbf{r})=\frac{e\hbar}{mi}\,\Psi^\dagger_\alpha(r)\overleftrightarrow{\nabla}\Psi_\beta(\mathbf{r}), \tag{4}$$

where $\overleftrightarrow{\nabla}_r=(\overrightarrow{\nabla}_r-\overleftarrow{\nabla}_r)/2$ is the antisymmetric gradient operator, $\Psi_\alpha(\mathbf{r})$ is the real space one-electron field operator, and greek indicates label the two spin channels. Similarly, the spin-dependent constitutive relation

$$\mathbf{j}_{\alpha\beta}(\mathbf{r})=\sum_{\gamma,\delta}\int d^3r'\,\boldsymbol{\sigma}_{\alpha\beta,\gamma\delta}(\mathbf{r},\mathbf{r}')\cdot\mathbf{E}_{\gamma\delta}(\mathbf{r}'), \tag{5}$$

follows by absorbing the vertex corrections via a redefinition of the internal fields $\mathbf{E}_{\gamma\delta}(\mathbf{r}')$. As a result, the fourth-rank spinor two-point conductivity $\boldsymbol{\sigma}_{\alpha\beta,\gamma\delta}(\mathbf{r},\mathbf{r}')$, which contains only the contribution from the bubble diagram in the real space Kubo formula,

$$\boldsymbol{\sigma}_{\alpha\beta,\gamma\delta}(\mathbf{r},\mathbf{r}')=-\frac{2}{\pi}\frac{e^2}{\hbar}\left(\frac{\hbar^2}{2m}\right)^2 \times A_{\beta\gamma}(\mathbf{r},\mathbf{r}')\overleftrightarrow{\nabla}_{\mathbf{r}}\overleftrightarrow{\nabla}_{\mathbf{r}'}A_{\delta\alpha}(\mathbf{r}',\mathbf{r}), \tag{6}$$

[where $A_{\alpha\beta}(\mathbf{r},\mathbf{r}')=(G^{\text{ret}}_{\alpha\beta}(\mathbf{r},\mathbf{r}')-G^{\text{adv}}_{\alpha\beta}(\mathbf{r},\mathbf{r}')/2$ is the density of states function] is proportional to the bubble part of the current-current correlation of the spinor currents, i.e., $\boldsymbol{\sigma}_{\alpha\beta,\gamma\delta}(\mathbf{r},\mathbf{r}')\propto\langle[\hat{\mathbf{j}}_{\alpha\beta}(\mathbf{r}),\hat{\mathbf{j}}_{\gamma\delta}(\mathbf{r}')]\rangle$.

In the quasiclassical regime, Eq. (2) can be solved via a global WKB integration. The corresponding reduced Green's function, $\mathcal{G}(\mathbf{r},\mathbf{r}')=(\hbar^2/2m)G(\mathbf{r},\mathbf{r}')$, is

[a)]Present address: Department of Physics, University of San Francisco, San Francisco, CA 94117.

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$$g(\mathbf{r},\mathbf{r}' \approx -\frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|}\exp[ik_F|\mathbf{r}-\mathbf{r}'|]\times P_{\mathbf{r}'\to\mathbf{r}}\exp[-\tfrac{1}{2}\xi(\mathbf{r},\mathbf{r}')|\mathbf{r}-\mathbf{r}'|]. \quad (7)$$

Equation (7) has a very simple physical interpretation: it represents a propagating electron with a complex wave number $k(\mathbf{r})=[k_F^2+i2m\Delta(\mathbf{r}/\hbar^2]^{1/2}$. In Eq. (7) the two-point exponent $\xi(\mathbf{r},\mathbf{r}')$

$$\xi(\mathbf{r},\mathbf{r}')=\left(\frac{2m}{\hbar^2 k_F}\right)\frac{1}{|\mathbf{r}-\mathbf{r}'|}\int_{\Gamma[\mathbf{r},\mathbf{r}']} ds''\Delta(\mathbf{r}''), \quad (8)$$

describes the damping of the electron's wave function (loss of momentum) as it propagates along the straight path $\Gamma[\mathbf{r},\mathbf{r}']$ connecting the points $\mathbf{r}$ and $\mathbf{r}'$, and it is essentially the average scattering encountered by the electron between $\mathbf{r}$ and $\mathbf{r}'$. In the presence of spin-dependent scattering, the one-particle propagator $G^{\text{ret}}(\mathbf{r},\mathbf{r}')$, the self-energy $\Sigma(\mathbf{r})$, the scattering matrix $\Delta(\mathbf{r})$, and the two-point exponent $\xi(\mathbf{r},\mathbf{r}')$ are 2×2 spin matrices; thus, a path-ordering operator is required in Eq. (7), to reorder the noncommuting 2×2 scattering matrices in the exponential series from the point $\mathbf{r}'$ to the point $\mathbf{r}$ (from right to left). The corresponding WKB results for the two-point conductivity are [from Eq. (6)]

$$\sigma_{\alpha\beta,\gamma\delta}(\mathbf{r},\mathbf{r}')=\frac{3C_D}{4\pi}\frac{\mathbf{nn}}{|\mathbf{r}-\mathbf{r}'|^2}\{P_{\mathbf{r}'\to\mathbf{r}}\exp[-\tfrac{1}{2}\xi(\mathbf{r},\mathbf{r}')|\mathbf{r}-\mathbf{r}'|]\}_{\beta\gamma}\{P_{\mathbf{r}\to\mathbf{r}'}\exp[\tfrac{1}{2}\xi(\mathbf{r},\mathbf{r}')|\mathbf{r}-\mathbf{r}'|]\}_{\delta\alpha}, \quad (9)$$

where $C_D=e^2k_F^2/(6\pi^2\hbar)$ and $\mathbf{n}$ is a unit vector from $\mathbf{r}$ to $\mathbf{r}'$.

The computation of global or measurable transport properties requires further work, as they do not follow straightforwardly from Eq. (9). In view of this complication we will start by analyzing global properties in multilayers, and only at a later stage we will consider transport in granular solids.

For the particular case of multilayers, which are characterized by in-plane translational invariance, a Fourier transform with respect to the in-plane relative positions yields two-point conductivities $\sigma(z,z')=\sigma(k_\parallel=0;z,z')$, given by

$$\sigma(z,z')=\frac{3C_D}{2}\int_1^\infty dt\frac{1}{t}\left[\frac{1}{2}\left(1-\frac{1}{t^2}\right)\mathbf{1}_\parallel+\frac{1}{t^2}\mathbf{e}_z\mathbf{e}_z\right]\times\exp\left[-t\int_{z<}^{z>}\frac{dz''}{l(z'')}\right], \quad (10)$$

where $\mathbf{1}_\parallel$ is the unit tensor in the plane of the layers, the substitution $t=R/|z|$ has been made, and integration with respect to the in-plane azimuthal angle has rendered the tensor diagonal. Equation (10) describes electrical conduction in multilayers.[4] For the CIP case, the internal electric field induced by an external uniform field is uniform, due to the in-plane *symmetry* of the multilayers; then the global CIP conductivity can be found by integrating the two-point conductivity of Eq. (10) twice, with respect to both arguments, z and z', as has been explicitly derived in Ref. 4; the ensuing CIP conductivity and magnetoresistance exhibit a characteristic exponential dependence with respect to the thicknesses of the different layers[3,4] and the magnetoresistance vanishes exponentially in the local limit. On the other hand, for CPP the current for each spin channel is a constant[7] as seen from Eq. (14); then, we find from Eqs. (5) and (9) that the internal field $\mathbf{E}(z)$ is proportional to the local scattering rate $\Delta(z)$. Then, for the CPP geometry, the corresponding global resistivity is proportional to the average scattering encountered in each spin channel; thus, the CPP geometry for multilayers exhibits a self-averaging behavior (all transport properties are determined by the average scattering) not only in the homogeneous limit, but for all length scales, and the magnetoresistance is scale independent and does not vanish in the local limit, a result that had been predicted in Ref. 6 and was later experimentally confirmed.[9] This result leads to a current line picture, according to which transport properties are determined by the scattering sampled by current lines in the whole system, provided that all current lines be essentially equivalent.

Our analysis for multilayers suggests that there exist two categories of inhomogeneous magnetic structures (according to the behavior exhibited by their magnetoresistance): (i) magnetically self-averaging, when their magnetoresistance does not vanish exponentially and is independent of the average distance between magnetic regions (like for the CPP geometry of multilayers); (ii) magnetically non-self-averaging, when their magnetoresistance vanishes exponentially with respect to the average distance between magnetic regions (like for the CIP geometry of multilayers).

The case of granular solids requires considering limiting cases. We will first analyze the homogeneous limit, which is characterized by all mean-free paths being much larger than all inhomogeneity lengths.

In the homogeneous limit the two-point function $\xi(\mathbf{r},\mathbf{r}')$ [Eq. (8)] has a *unique limit*, as when $R=|\mathbf{r}-\mathbf{r}'|\to\infty$; in particular, this is independent of $\mathbf{r}$ and $\mathbf{r}'$, and applies to both granular solids and multilayers. For multilayers the caveat is that such a unique limit holds for almost all paths, and the exception are those paths in the plane of the layers for multilayers. For magnetic systems, the function $\xi(\mathbf{r},\mathbf{r}')$ is a spin matrix and its self-averaging limit is of the form $\bar{\xi}=\bar{\xi}_0+\bar{\boldsymbol{\xi}}\cdot\hat{\sigma}$. Thus, for both granular solids and multilayers, the "average scattering in the medium"

$$\bar{\Delta}=\frac{1}{V[\mathscr{R}]}\int_{\mathscr{R}} d^3r\Delta(\mathbf{r}) \quad (11)$$

is well defined and coincides with the asymptotic form (large R) of $\xi(\mathbf{r},\mathbf{r}')$, provided that the size of the region $\mathscr{R}$ be sufficiently large. In Eq. (11) $V[\mathscr{R}]$ is the volume of the corresponding region. The size of $\mathscr{R}$ is essentially restricted by the condition that the function $\xi(\mathbf{r},\mathbf{r}')$ become asymptotically a constant, i.e., R has is restricted by the linear dimensions L of the sample; whence, these systems have a length scale D_{sa}, such that for $D_{sa}\leq R\ll L$, and the volume integral in Eq. (11) is independent of $\mathscr{R}$. We will naturally refer to D_{sa} as the self-averaging length scale. Therefore, the homogeneous limit yields a two-point conductivity, Eq. (9) that is effectively a function of only $R=|\mathbf{r}-\mathbf{r}'|$. Then, choosing a quantization axis that diagonalizes $\bar{\Delta}$ Eq. (5) is simplified to

$$\mathbf{j}_{\alpha\beta}=\delta_{\alpha\beta}\sum_{\gamma}\left[\int d^3r'\,\boldsymbol{\sigma}_{\alpha\gamma,\gamma\alpha}(\mathbf{r},\mathbf{r}')\right]\cdot\bar{\mathbf{E}}=\delta_{\alpha\beta}(\bar{\rho}_{\alpha})^{-1}\bar{\mathbf{E}}, \tag{12}$$

where $\bar{\mathbf{E}}$ stands for the average field, and it follows that the electrical resistance in each spin channel is essentially given by the average scattering $\bar{\Delta}$ sampled by the electron in the medium, i.e.,

$$\bar{\rho}_{\alpha}=\left(\frac{2m}{\hbar^2 k_F}\right)([\bar{\Delta}^{-1}]_{\alpha\alpha})^{-1}. \tag{13}$$

This conclusion is achieved if it is assumed that the spin diffusion length is much larger than both the elastic mean-free paths and the inhomogeneity length scales, in which case the continuity equation for each spin component of the current density,

$$\nabla\cdot\mathbf{j}_{\alpha\beta}(\mathbf{r})=0, \tag{14}$$

is satisfied.

On the other hand, in the local limit, defined as the limiting scenario when all local mean-free paths are much smaller than the inhomogeneity lengths, the constitutive relation (5) becomes local; more precisely, the local conductivity becomes the produce of a one point conductivity function and the delta function $\delta(\mathbf{r}-\mathbf{r}')$. The electric field is then the product of the local resistivity and the current density, where the global resistivity is also $\bar{\rho}_{\alpha}$ along a typical current line, with the current constrained via Eq. (14).

From our analysis of limiting cases we conclude that our theory predicts that granular solids are magnetically self-averaging, due to *randomness* in the distribution of granules. In effect, the current line picture suggests that the global resistivity is proportional to the average scattering $\bar{\Delta}_{\alpha}$ in each spin channel, for all length scales, like for the CPP case. On the other hand, in the local limit for granular solids, current lines do *not* necessarily sample all the scattering in the medium but they only partially sample the scattering in the granules. Of course, the interfaces are probed regardless of the relative values of the local resistivities. This shows that the local resistivity and the current cannot be completely disentangled. In conclusion, we see that the only difference between the two limiting cases is at most the contribution from the granules. Thus, the scale dependence of the magnetoresistance will show up exponentially with respect to the size of the granules but not with respect to the average distance between adjacent granules. In this sense, granular solids are magnetically self-averaging. It should be noticed that these conclusions are based on our choice of a model in which the electrical resistivity arises from short-range impurity scattering within each region of the system rather from scattering by entire regions.

In summary, we have derived the two-point conductivity of inhomogeneous magnetic structures, as well as the global resistance and magnetoresistance of magnetically self-averaging systems, for which we have shown that they are independent of the average distance between adjacent magnetic regions.

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