Theory of quantum light emission from a strongly-coupled single quantum dot photonic-crystal cavity system

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Abstract: We present a rigorous medium-dependent theory for describing the quantum field emitted and detected from a single quantum dot exciton, strongly coupled to a planar photonic crystal nanocavity, from which the exact spectrum is derived. By using simple mode decomposition techniques, this exact spectrum is subsequently reduced to two separate user-friendly forms, in terms of the leaky cavity mode emission and the radiation mode emission. On application to study exciton-cavity coupling in the strong coupling regime, besides a pronounced modification of the usual vacuum Rabi spectral doublet, we predict several new effects associated with the leaky cavity mode emission, including the appearance of an off-resonance cavity mode and a loss-induced on-resonance spectral triplet. The cavity mode emission is shown to completely dominate the emitted spectrum, even for large cavity-exciton detunings, whereby the usual cavity-QED formulas developed for radiation-mode emission drastically fail. These predictions are in qualitative agreement with several "mystery observations" reported in recent experiments, and apply to a wide range of semiconductor cavities.

OCIS codes: (270.5580) Quantum electrodynamics; (350.4238) Nanophotonics and photonic crystals.

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1. Introduction

Since the early work of Einstein [1], the effect of the "vacuum" on physically measurable properties has been fascinating scientists, and it is now customary to define the spontaneous emission rate in a homogeneous medium as half the Einstein A coefficient. For emission into a suitably designed cavity structure, however, the concept of the photon emission having a fixed rate can break down as we enter the regime of cavity-quantum electrodynamics (cQED). Motivated in part by the prospect of creating scalable solid-state sources for quantum information science, the semiconductor community has been actively pursuing research related to cQED [2–4]. In particular, several recent breakthroughs with fabricating photonic nanostructures [5] have brought the elusive regime of single-exciton strong coupling from intellectual curiosity to experimental reality in a remarkably short time frame [6–10]. For example, Hennessy *et al.* [9] presented an experimental study of the puzzling quantum nature of a single semiconductor quantum dot (QD) strongly coupled to a planar photonic-crystal (PC) nanocavity. With the QD strategically positioned close to an electromagnetic field antinode spatial position, optical measurements clearly observe the strong-coupling regime, giving rise to the familiar anti-crossing behavior that occurs between the exciton and cavity mode. In addition to the usual cQED regimes well known from atomic optics [11], Hennessy *et al.* highlighted several apparent mysteries, perhaps *unique* to the semiconductor environment. These effects include "off-resonant excitation of the cavity mode" and a "triple peak" during the strong coupling regime. Since these observations are unusual, it has been speculated that they indicate a clear deviation from a simple artificial atom model of the QD; this view has since been echoed by additional experiments and theoretical analysis, e.g., Kaniber *et al.* [12] observe similar cavity feeding mechanisms and conclude that the coupling for large dot-cavity detunings *cannot* be explained by a simplified atomlike picture of the QD exciton.

In this work, we extend previous analytical works on single quantum dot emission into a domain more suited to the planar PC medium. We concentrate exclusively on first-order quantum correlation effects, and derive an exact spectrum that is valid for any inhomogeneous dielectric, which is then reduced to a simpler analytical form valid for planar PC cavities. Our theory is rigorous, but intuitive and clear enough to be of immediate use to experimentalists working in the field. Specifically, we recover the essential features of the experimental data, and point out the obvious failure of the commonly employed formulas for analyzing the cQED spectra in these complex semiconductor nanostructures. First, we unambiguously clarify why the off-resonant coupling is fully expected for a PC slab cavity. Second, we show how simple in-plane cavity decay can yield a spectral triplet, which is nothing more than a *which-way* interference effect, characteristic of a lossy planar PC medium. These predicted features are not related to the failure of the simple atomlike model of a QD exciton, but rather an overly simplistic medium model for the complex cavity structure. Our findings are important not just for analyzing the spectra but also for understanding higher-order correlation effects such as antibunching phenomena and anharmonic cQED. Furthermore, they can be applied to a wide range of semiconductor cavities such as micro-pillar cavities. A schematic of an example planar PC cavity, containing a single QD, is shown below in Fig. 1.

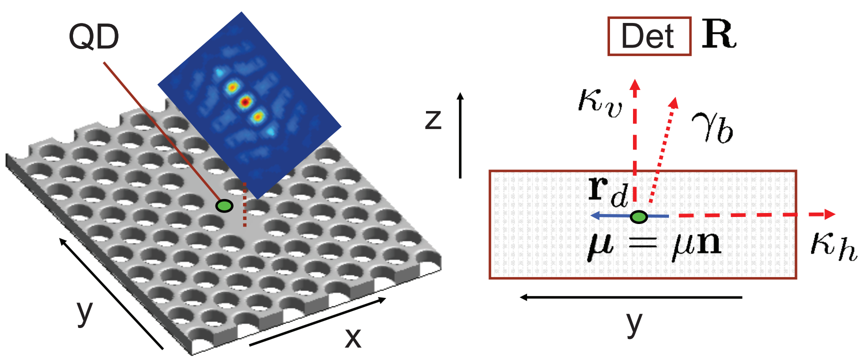

Fig. 1. Schematic of a planar semiconductor PC with an embedded QD; for a self-assembled QD, then the spatial position would be near the center of the slab to maximize coupling to the cavity mode. Also shown is a typical spatial profile of a confined cavity mode $\mathbf{f}_c(\mathbf{r},\omega_c)$, within the slab, dominated by f_c^y in this example (ω_c is the cavity resonance frequency); the effective mode volume is less than $0.1\,\mu\text{m}^3$. The right side of the figure shows a side view of the cavity, indicating the vertical background radiation-leakage (γ_b), as well as a vertical (κ_v) and horizontal (κ_h) cavity leakage. We define the cavity decay *rates* as $\Gamma_{v/h} = 2\kappa_{v/h}$, and the radiation decay rate as $\Gamma_b = 2\gamma_b$.

2. Theory

We consider a medium whose classical electromagnetic properties are described by the photon Green function [13–15], $\mathbf{G}(\mathbf{r},\mathbf{r}';\omega)$, which is the field response at $\mathbf{r}$ to a polarization dipole at $\mathbf{r}'$, as a function of frequency. The photon Green function is defined from

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Maxwell's equations, solved with a polarization dipole in a medium defined through a spatially-dependent dielectric constant, $\varepsilon(\mathbf{r})$. For a lossless inhomogeneous dielectric, it is also useful to introduce a generalized-transverse Green function, $\mathbf{K}(\mathbf{r},\mathbf{r}';\omega) = \mathbf{G}(\mathbf{r},\mathbf{r}';\omega) - \delta(\mathbf{r}-\mathbf{r}')\mathbf{I}/\varepsilon(\mathbf{r}) = \sum_{\mathbf{k}} \omega_{\mathbf{k}}^2 \mathbf{f}_{\mathbf{k}}(\mathbf{r})[\mathbf{f}_{\mathbf{k}}(\mathbf{r}')]^*/(\omega^2 - \omega_{\mathbf{k}}^2)$ [14, 15], that is defined in terms of the transverse modes of the system, $\mathbf{f}_{\mathbf{k}}$; these modes are solutions of the standard eigenvalue problem: $\left[\nabla\times\nabla\times - \omega^2/c^2\,\varepsilon(\mathbf{r})\right]\mathbf{f}_{\mathbf{k}}(\mathbf{r}) = 0$.

To describe the quantum mechanics of light-matter coupling, we adopt a canonical Hamiltonian approach introduced by Wubs *et al.* [15] and use a "two-level atom" model for the QD. The resulting Hamiltonian of the system includes one QD exciton, a sum over the light modes, and the coupling between the exciton and the light through the electric-dipole approximation:

$$H = \hbar\omega_x \hat{\sigma}^+\hat{\sigma}^- + \sum_\lambda \hbar\omega_\lambda \hat{a}_\lambda^\dagger \hat{a}_\lambda - i\hbar\sum_\lambda (\hat{\sigma}^- + \hat{\sigma}^+)(g_\lambda \hat{a}_\lambda - g_\lambda^* \hat{a}_\lambda^\dagger), \tag{1}$$

from which the time-dependent *operator equations* are derived for $\dot{\hat{a}}_\lambda, \dot{\hat{a}}_\lambda^\dagger, \dot{\hat{\sigma}}^+, \dot{\hat{\sigma}}^-$ and $\dot{\hat{\sigma}}_z$, using the Heisenberg equations of motion. We follow the usual convension, where the operators $\hat{a}_\lambda$ represent the photon modes, while the operators $\hat{\sigma}^+, \hat{\sigma}^-$ and $\hat{\sigma}_z$ are the Pauli operators of the electron-hole pair (or exciton). We consider a QD located at position $\mathbf{r}_d$, with the detected field at $\mathbf{R}$, and no initial excitation field in the spectral region of interest; this latter assumption is not a model restriction, but is similar to how the semiconductor cavity medium is typically excited, namely through incoherent loading of a higher-lying exciton state. We also assume a single exciton dipole moment $\boldsymbol{\mu} = \mathbf{n}\mu$, aligned along $\mathbf{n}$. After carrying out a Laplace transform of the operator equations [15], and using $\hat{\mathbf{E}}(\mathbf{r},t) = i\sum_\lambda [\hbar\omega_\lambda/2\varepsilon_0]^{1/2}\hat{a}_\lambda(t)\mathbf{f}_\lambda(\mathbf{r}) + H.c.$ and $g_\lambda = [\omega_\lambda/2\hbar\varepsilon_0]^{1/2}\boldsymbol{\mu}\cdot\mathbf{f}_\lambda(\mathbf{r}_d)$, one can derive the quantized electric-field operator,

$$\hat{\mathbf{E}}(\mathbf{R},\omega) = -\frac{1}{\varepsilon_0}\mathbf{K}(\mathbf{R},\mathbf{r}_d;\omega)\cdot\boldsymbol{\mu}[\hat{\sigma}^-(\omega) + \hat{\sigma}^+(\omega)], \tag{2}$$

which is *exact*. The optical spectrum is calculated from a double time integration over the first-order quantum correlation function:

$$S(\mathbf{R},\omega) = \int_0^\infty dt_2 \int_0^\infty dt_1 e^{-i\omega(t_2-t_1)} \left\langle \hat{\mathbf{E}}^{(-)}(\mathbf{R},t_2)\,\hat{\mathbf{E}}^{(+)}(\mathbf{R},t_1)\right\rangle, \tag{3}$$

where $\hat{\mathbf{E}}^{(+)}$ and $\hat{\mathbf{E}}^{(-)}$ are the positive and negative frequency components of the electric field operator. By substituting in the electric-field operator, we obtain the exact spectrum:

$$S(\mathbf{R},\omega) = \left|\frac{\mathbf{K}(\mathbf{R},\mathbf{r}_d;\omega)\cdot\boldsymbol{\mu}}{\varepsilon_0}\right|^2 \left\langle (\hat{\sigma}^-(\omega))^\dagger \hat{\sigma}^-(\omega)\right\rangle. \tag{4}$$

This analytic optical spectrum (Eq. (4)) is valid for any inhomogeneous dielectric, including waveguides and open systems, and for any number of photons in the system. Importantly, the Green function spectrum clearly highlights the essential role of light propagation, from the dot to the detector, the dynamics of which are completely contained within the propagator $\mathbf{K}(\mathbf{R},\mathbf{r}_d;\omega)$. The theory also shows that there is no need to work with a quantized cavity field operator for describing cavity mode emission from a system prepared in vacuum. Rather, any mode projection is implicitly contained within the propagator. For multiple QDs and (or) multiple excitons, an exact analytical spectral form is also possible. For example, in the presence of multiple excitons from the same QD, the formula above would contain a sum over exciton transitions (multiple Pauli operators), but the propagator remains the same.

We next make some approximations that are valid for the PC medium under investigation, and for incoherent excitation. First, we truncate the solution in the limit of one photon or one

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excitation, so that $\hat{\mathbf{E}}(\mathbf{R},\omega) = \varepsilon_0^{-1}\,\mathbf{K}(\mathbf{R},\mathbf{r}_\mathrm{d};\omega)\cdot\hat{\mathbf{d}}(\omega)/(1-\mathbf{n}\cdot\mathbf{K}(\mathbf{r}_d,\mathbf{r}_d;\omega)\cdot\mathbf{n}\,\alpha(\omega))$, where we have introduced a dipole operator term $\hat{\mathbf{d}}(\omega) = -i\boldsymbol{\mu}[\hat{\sigma}^-(t=0)/(\omega-\omega_x)+\hat{\sigma}^+(t=0)/(\omega+\omega_x)]$ and a bare polarizability amplitude $\alpha(\omega) = 2\mu^2\omega_x/[\hbar\varepsilon_0(\omega^2-\omega_x^2)]$; ω_x is the exciton resonance frequency. This is basically identical in form to the quantum electric-field operator first derived by Wubs *et al.* [15], who analyzed atoms described as harmonic oscillators embedded in an arbitrary inhomogeneous dielectric. For a planar PC medium (see Fig. 1), the medium Green function is of the form $\mathbf{K}(\mathbf{r},\mathbf{r}';\omega) = \mathbf{K}_\mathrm{cav}(\mathbf{r},\mathbf{r}';\omega)+\mathbf{K}_\mathrm{rad}(\mathbf{r},\mathbf{r}';\omega)$, where the local and nonlocal *cavity* contributions are, respectively, $\mathbf{K}_\mathrm{cav}(\mathbf{r}_d,\mathbf{r}_d;\omega) = \omega_c^2|\mathbf{f}_c(\mathbf{r}_d)|^2/(\omega^2-\omega_c^2+i\omega\Gamma_c)$ and $\mathbf{K}_\mathrm{cav}(\mathbf{R},\mathbf{r}_d;\omega) = \omega_c^2\mathbf{f}_c(\mathbf{R})\mathbf{f}_c^*(\mathbf{r}_d)/(\omega^2-\omega_c^2+i\omega\Gamma_v)$, while the radiation mode contributions represent the sum of radiation modes above the light line. In essence, we are considering a PC system that consists of a well defined cavity mode (with resonance frequency ω_c), deep inside the photonic bandgap, and a sum of radiation modes above the light line. The contribution from the radiation modes is typically significantly smaller than the resonant contribution, especially in the presence of an in-plane photonic bandgap. Another important point is that the total cavity decay rate–for example, the broadening of the local density of states (LDOS) near a cavity antinode position center of the slab–is given by the sum of two contributions, $\Gamma_c = \Gamma_v+\Gamma_h$, where Γ_v and Γ_h account for vertical and horizontal (in-plane: $x/y-$) decay loss, respectively; the horizontal decay, for example, can be caused by not having enough holes surrounding the defect cavity, or it can be caused by material losses that partly destroy the photonic bandgap. We now conveniently separate the total emitted spectrum into a background radiation-mode spectrum (emitted at a rate Γ_b) and the cavity-mode spectrum (emitted at a rate Γ_v):

$$S_\mathrm{rad}(\mathbf{R},\omega) \approx F(\mathbf{R})\,\Gamma_b\left|\frac{(\omega+\omega_x)(\omega^2-\omega_c^2+i\omega\Gamma_c)}{(\omega^2-\omega_x^2+i\omega\Gamma)(\omega^2-\omega_c^2+i\omega\Gamma_c)-4g^2\omega_c\omega}\right|^2, \tag{5}$$

$$S_\mathrm{cav}(\mathbf{R},\omega) \approx F(\mathbf{R})\,\Gamma_v\left|\frac{2g\omega_c(\omega+\omega_x)\frac{(\omega^2-\omega_c^2+i\omega\Gamma_c)}{(\omega^2-\omega_c^2+i\omega\Gamma_v)}}{(\omega^2-\omega_x^2+i\omega\Gamma)(\omega^2-\omega_c^2+i\omega\Gamma_c)-4g^2\omega_c\omega}\right|^2, \tag{6}$$

where $F(\mathbf{R})$ is a geometrical factor that depends on the detector location [16], Γ is an effective exciton decay rate, and $g = [\omega_c/2\hbar\varepsilon_0]^{1/2}\,\boldsymbol{\mu}\cdot\mathbf{f}_\mathrm{c}(\mathbf{r}_d)$ is a cavity-exciton coupling constant.

Equations (2-6) contain a rich amount of information about the coupling between a QD and a PC medium. By inspection of S_cav and S_rad, several important scaling rules and interference effects immediately become clear. First, near resonance, the ratio of the cavity mode emission and the radiation mode emission scales with $4g^2/(\Gamma_\mathrm{b}\Gamma_c)$ (assuming that $\Gamma_h \ll \Gamma_v$). Thus, for strongly coupled QDs, with g and $\Gamma_v \approx 0.1\,$meV, and $\Gamma_b \approx 0.05\,\mu$eV [9], S_cav will completely dominate the detected spectra, and by several orders of magnitude; in fact, as we will show later, this is also the case for significant exciton-cavity detunings. Second, from S_cav, the presence of in-plane decay can result in unexpected interference effects, whereby a singlet will remain in the presence of a doublet; this can lead to a spectral triplet (see Eq. (6)).

To the best of our knowledge, the above equations either extend, recover, or correct all known analytical spectral forms of single photon spectra that have been presented in the literature. For example, Carmichael *et al.* [17] and Andreani *et al.* [18] derived a rotating wave version of S_rad. Cui and Raymer [19] derived a coupled mode solution for cavity emission and radiation emission for a simple geometrical cavity system, which has been extended recently by Auffeves *et al.* [20]. Finally, our general Green function spectrum (Eq. (4)), which is exact, both corrects and extends the Green function spectrum derived by Ochiai *et al.* [21, 22].

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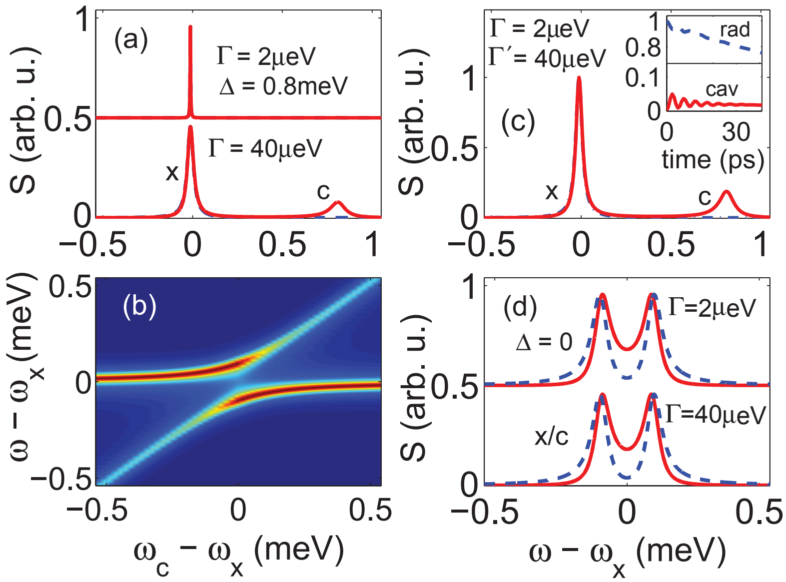

Fig. 2. Normalized light spectra from single QD photon emission in a planar PC cavity (c.f. Fig. (1)), with cavity decay rates $\Gamma_v = 0.1\,\text{meV}$ and $\Gamma_h = 0$, and exciton decay rate Γ (see text and labels in the graph). In (a) is shown the spectrum for an initially excited-exciton, with two different exciton decay rates, with a cavity mode off-resonance (from the exciton) by 0.8 meV; the blue dashed curve is the usual radiation-mode decay (S_{rad}), and the red solid curve shows the emitted spectrum from the leaky cavity mode (S_{cav}). In (b), is shown the influence of detuning as a contour plot. (c) Similar to (a) (top frame) but computed from a master equation solution with pure dephasing (Γ'); the insets display the exciton and cavity mode dynamics. In (d) is shown the on-resonance case.

3. Calculations

For representative calculations, we choose parameters similar to those measured in related experiments, and use an effective exciton broadening $\Gamma \approx 2-40\,\mu\text{eV}$, $Q_v(\omega_c/\Gamma_v) = 13\,000$ ($\Gamma_v = 0.1\,\text{meV}$), $\omega_c/2\pi \approx 317\,\text{THz}$ ($\lambda_c = 945\,\text{nm}$), and $g = 0.1\,\text{meV}$. We initially assume that $\Gamma_h = 0$. Consistent with the theory above, we consider the excitation scenario where there is no incident field, and the system evolves from an initially excited exciton; the dynamics are therefore driven by the electromagnetic vacuum. In Fig. 2(a), we show both the cavity emitted spectrum and the radiation-mode emitted spectrum, for an off-resonant cavity mode detuned by 0.8 meV; for $\Gamma = 2\,\mu\text{eV}$, we obtain essentially no visible difference between the spectral shape of the cavity emitted spectrum and the radiation-mode spectrum; however, with $\Gamma = 40\,\mu\text{eV}$, we obtain a pronounced emission peak at the cavity mode. The sensitivity to detuning in the latter case is highlighted as a contour plot in Fig. 2(b), where the cavity mode is clearly visible throughout the entire detuning range. Interestingly, the exciton-cavity coupling manifests in a cavity-emission spectral form that is simply the product of two Lorentzian lineshapes; by examining the spectral forms of S_{cav} and S_{rad} (Eqs. (5-6)), and assuming $\Gamma_c = \Gamma_v$, then $S_{\text{cav}} \propto S_{\text{rad}} L(\omega_c)$, where $L(\omega_c)$ is a Lorentzian lineshape centered at the cavity frequency; for this reason, any further detuning of the cavity mode will not influence the spectral shape at all, as the leaky cavity mode will be coupled and appear in exactly the same way, though the overall amplitude of both peaks will reduce. This feature should not be a surprise, however, as the physics of the dressed cavity-mode emission (obtaining two peaks) is identical to the coupling between two coupled cavities [12, 19, 23].

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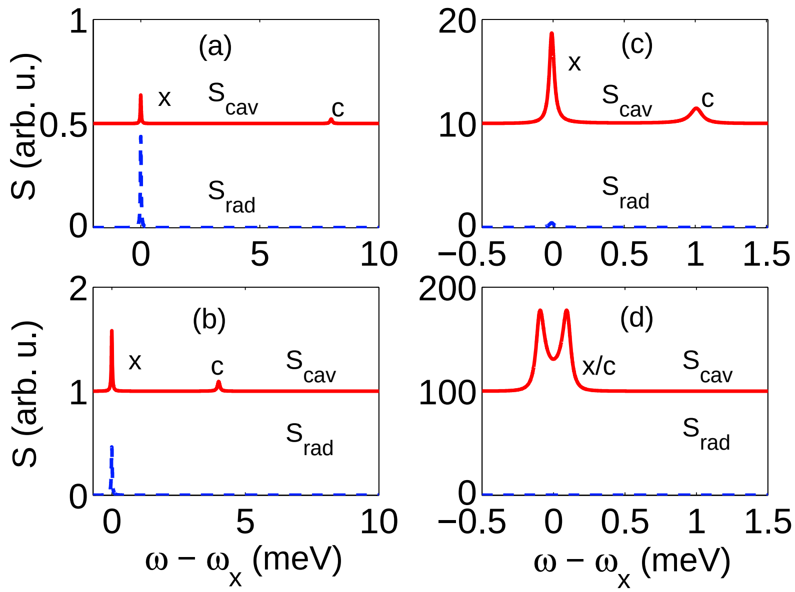

Fig. 3. Detected spectra for various cavity-exciton detunings, using the same parameters as in Fig. (2), with $\Gamma = 40\,\mu$eV. All spectra are scaled by the same constant, and the background radiation decay is $\Gamma_b = 0.05\,\mu$eV. (a) Cavity and radiation mode emission for a detuning of 8 meV. (b) Detuning of 4 meV. (c) Detuning of 1 meV showing only the cavity mode, where already $S_{\rm rad}$ plays a negligible role. (d) On-resonance case, which shows both a pronounced enhancement of the emission (Purcell effect) and the strong coupling regime.

In the calculations above, we have implicitly assumed that the effective exciton decay is "radiative," which is usually required to get analytical results for the spectra; yet this assumption is somewhat artificial, as the typical radiative decay rate for QD excitons is much smaller than the one used above, and the exciton broadening is usually dominated by phase relaxation mechanisms such a spectral diffusion and electron-phonon scattering, both of which yield "pure dephasing" [24, 25]. To demonstrate that pure dephasing leads to qualitatively the same spectral shape (as implied from Eq. (4)), and to investigate the dynamics of the cavity mode and exciton mode coupling, in Fig. 2(c) we show the computed spectra that is obtained by solving the corresponding master equation [26] of a coupled leaky cavity and exciton system [17], with a pure dephasing rate of $\Gamma' = 40\,\mu$eV; again we see a pronounced resonance at the cavity mode, when light is emitted via the cavity mode. The temporal dynamics of the two-operator one-time correlation functions, $\langle \hat{a}^\dagger(t)\hat{a}(t)\rangle$ and $\langle \hat{\sigma}^+(t)\hat{\sigma}^-(t)\rangle$, are shown as an inset; during the first few ps, already the cavity mode is efficiently excited by the exciton, which is a mechanism that also occurs for larger detunings. Finally, in the case of on-resonance excitation, shown in Fig. 2(d), we obtain the spectral doublet which is characteristic of the strong coupling regime; however, we observe a qualitatively different doublet between the cavity emitted spectrum and the radiation-mode spectrum. Although the spectral doublet appears in both cases, the cavity emission is much sharper at the wings and has a larger oscillator strength near zero detuning. For this strong coupling regime, the role of the chosen Γ is almost negligible, since the system is now dominated by enhanced radiative coupling and cavity leakage.

To clarify the relative strengths of the cavity-mode emission versus radiation-mode emission, in Fig. 3 we show the calculated spectrum–in identical normalized units–for four separate exciton-cavity detunings. With the cavity mode 8 meV away, the radiation mode spectrum dominates; but the cavity mode is still visible. For smaller detunings, both the cavity emission and

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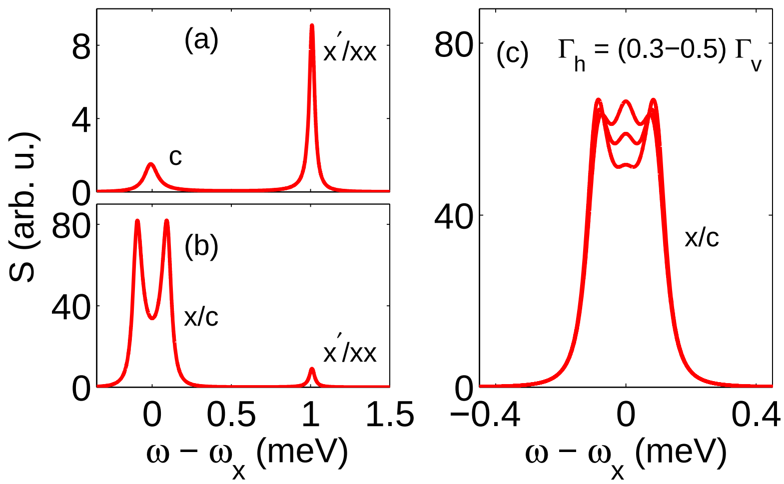

Fig. 4. (a) An example cavity-mode spectrum that results from an off-resonant excitation, from a spectrally wandered exciton or an exciton-biexciton pair. (b) The sum of the off-resonant and on-resonance spectra, showing that the total on-resonance doublet survives. (c) The simple one exciton and one cavity-mode spectrum, but in the presence of in-plane loss or decay, showing the onset of a spectral triplet.

the exciton emission increase in equal weights, eventually resulting in a pronounced Purcell effect and the strong coupling regime. For a strongly coupled cavity - PC system, evidently *only the cavity mode spectrum is needed* and this is the one that is typically observed; yet, most–if not all–semiconductor experiments to date carry out their data analysis using $S_{\rm rad}$, which can cause much confusion in interpreting the underlying physics of their data.

One might also wonder about the role of multiple excitons (spectrally off-resonance, but belonging to the same QD), which are usually assumed unimportant. To address this question, we have numerically confirmed that in the presence of several initially-excited excitons, within the spectral vicinity of the cavity mode, then a significantly larger oscillator strength appears in the cavity mode via exactly the same mechanism above; this point is consistent with the measurements of Press *et al.* [10], Hennessy *et al.* [9], and Kaniber *et al.* [12]. Thus the background excitons can certainly play a qualitatively important role, even if far off resonance. Indeed their effect would also spoil the quantum statistics of light emission from the cavity mode, which is in agreement with cavity-mode autocorrelation measurements showing little quantum correlations [9]. We will show this connection directly in a forthcoming publication, where the role of additional excitons act in concert to suppress the antibunching behavior of the cavity autocorrelation function. In addition, since part of the dressed leaky-cavity mode “lives” at the exciton resonance, extra care is needed in understanding the subtle role of spectral filtering.

Finally, we turn our attention to the strange observation of a strongly-coupled “triple peak” [9]. It has been speculated that spectral diffusion may result in a triple peak profile in the strong coupling spectrum [9], via a similar mechanism that we have explained above, namely a channeling of energy into the cavity mode for off-resonant exciton resonances; another possibility could be biexcitonic loading, where the first generated photon (which is off-resonance) feeds the cavity mode, while the second generated photon (on resonance) creates the doublet; thus, one might imagine that the statistical mixture of an off-resonant excitation and an on-resonant excitation could give rise to a triple peak. To investigate this hypothesis, in a simple qualitative way, we show in Fig. 4(a) an example of a transition that is only 1 meV away from the cavity (to look for a best case off-resonant feeding mechanism); in Fig. 4(b) is shown the sum of this contribution and the doublet contribution, which demonstrates that there is no noticeable effect on

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the doublet. We conclude that spectral wandering or biexciton loading is unlikely responsible for the observed middle peak, since the off-resonant coupling has significantly less oscillator strength than the on-resonant contribution.

We now introduce a slightly different–but more realistic–model for the PC slab cavity, that has been implicit from the beginning, and highlight some unusual differences that may occur for $S_{\rm cav}$, in a regime where Γ_h is finite, which to some degree is always the case for planar PC cavities. As shown in Fig. 4(c), a clear triple peak emerges, which becomes more pronounced with larger in-plane decay/loss; experimentally, this effect may occur as a function of pump power or through cavities that allow for in-plane decay. In the case of a pump-induced loss mechanism, nonlinear optical processes could be responsible for this loss; for example, mediated by free carrier absorption. However, we remark that such an effect would be material and PC-design specific, and this feature may or may not occur depending on the details of the fabricated structure, the pump-induced loss processes, and the pump excitation wavelength; specifically, we find a doublet if $\Gamma_h \sim< 0.25\,\Gamma_v$, else a triplet. We also point out that our model Green functions are phenomenological, but physically well motivated and confirmed by numerical calculations; in this regard, it should be noted that exact Green function calculations for planar PC media are possible [27], even with loss, and future work will explore and report on these in more detail.

Of course, one should not rule out the possible influence of spectral diffusion, nor any other well known semiconductor mechanisms such as deep-state emissions and free carrier absorption; any semiconductor QD experiment must be carefully scrutinized and analyzed. Rather, we point out that, regardless of the precise origin of the observed spectral triplet, one could deterministically engineer a situation where observations of the spectral triplet is "guaranteed," and this alone should motivate new experiments in this exciting and growing field. With properly designed high-Q cavities, and in structures where material loss is small, then a spectral doublet should be the dominant feature, which is certainly the usual case experimentally. If a triplet is obtained, then turning down the power of the pump laser, or spectrally exciting closer to the target exciton resonance could help revert back to a doublet. As mentioned above, a strongly-coupled doublet to triplet investigation could also be carried out by designing cavities with different in-plane decays. In addition, the investigation of free carrier absorption and nonlinear pumping on the Purcell effect and emitted spectrum would be an interesting experiment to try.

4. Conclusions

We have presented and exploited an exact quantized medium-dependent theory to derive "user-friendly" analytical spectra, suitable to study the quantum emission from a strongly coupled single QD - PC cavity system. Our spectral formulas clarify why the cavity-mode emission completely dominates the radiation-mode emission, which manifests in an off-resonant cavity feeding mechanism over large exciton-cavity detunings. Moreover, we have demonstrated that in-plane decay can yield a spectral triplet in the strong coupling regime, caused by a quantum interference effect between the various decay channels that a single photon can take within the medium. Both of these effects are consistent with recent experiments, and, contrary to earlier reports, can be adequately explained by simple atomlike models for the QD. These predicted effects can also be described classically using standard electromagnetic theory and an oscillator model, though naturally these would have a different physical interpretation.

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