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# THEORY OF THE DISPERSION OF MAGNETIC PERMEABILITY IN FERROMAGNETIC MATERIALS AT MICROWAVE FREQUENCIES

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

the first time in the history of the world, the people of the United States have adopted a policy of neutrality in their foreign relations. This is a remarkable achievement, and it is a great credit to the American people. It is also a wise policy, because it is based on the principle that the United States should not be drawn into any European war. The policy of neutrality has been adopted by the United States for several reasons. One reason is that the United States is a neutral power, and it does not want to be drawn into any European war. Another reason is that the United States is a neutral power, and it does not want to be drawn into any European war. The third reason is that the United States is a neutral power, and it does not want to be drawn into any European war.

President Franklin D. Roosevelt  
The White House  
Washington, D.C.  
October 1, 1939

## The Ferrimagnetic

The first class of ferrimagnetic materials is the garnet group which has a cubic crystallographic structure with a size from the a.c. permeability of the iron and manganese lattice up to frequencies near 100 cps/sec., to the hysteresis permeability of the third by Hagen and Bubens<sup>1</sup> at 20<sup>1</sup> cps/sec.

At frequencies below about 100 rad/sec the skin depth for the conduction of the alternating magnetic field in a specimen is smaller generally than the size of the elementary ferrimagnetic domains. Domain dimensions are of the order of  $10^{-7}$  to  $10^{-6}$  cm. It is plausible to expect that the theory that the initial concepts of the domain magnetism lead to is not valid at higher frequencies since the coupled magnetic field has a larger effect over the entire volume of a domain. The usual physical approach must be discarded; theory of domain dynamics applies to this situation. The theory is found to account satisfactorily the observed trends of the existing experimental data on magnetic dielectric and ferromagnetic resonance at microwave frequencies.

Ferrimagnetic materials are composed of small regions called domains.<sup>2</sup> In general the size of a domain is smaller than that of the microcrystalline structure of the material. The atomic spins in a domain are nearly all lined up in the same direction, i.e. the principal direction of the domain spin varies from one domain to the other. In the demagnetized state the domains are randomly oriented giving paramagnetic resonance.

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1. E. Hagen and J. Bubens, Ann. Phys. (19), pp. 276 (1933).
  2. For an account of domain dynamics of ferrimagnetics see F. Bitter, Introduction to Ferromagnetism, (Harrap Book Co., New York, 1937), and R. Becker and W. Döring, Resonanzparamagnetismus, (Flowers Bros., Ann Arbor, Michigan, 1945).

the first time in the history of the world, the people of the United States have been compelled to make a choice between two political parties, each of which has a distinct and well-defined platform, and each of which has a definite and well-defined object in view. The people of the United States have been compelled to make a choice between two political parties, each of which has a distinct and well-defined platform, and each of which has a definite and well-defined object in view.

With the exception of the first two, all the remaining species have been described from the Indian Peninsula.

It is also important to note that a large number of vehicles are currently being used for purposes other than those for which they were originally designed. These include examples such as the use of vehicles for the transport of dangerous goods, hazardous wastes, and the transport of radioactive materials. The use of vehicles for these purposes is responsible for significant air pollution problems and the reduction of vehicle efficiency.

Such a situation may well occur, since the pulse will be restricted at high frequencies by the presence of a resistor.

Fig. 10. Depth distribution of marks with incomplete penetration of surface layers in the direction of the maximum surface stress.

37. The author wishes the following maps are exhibited  
or located in the public library and used as illustrations by Congress and the Senate;  
and the Secretary.

On the other hand, the effects of the various factors, such as age, sex, ethnicity, education and life-style, may be more difficult to disentangle, particularly in older people.

19. *W. L. Sargent*, *U.S. Fish Commission*, *Report*, 1874, p. 10.

1. **W. Preacher**, *Journal of Nonparametric Statistics*, 1994, 4, 147-171.

2. *Leucania agorista* H. M. Smith

2. The first two years of the project will be spent in the field, collecting data and establishing the basic parameters of the system. This will involve extensive fieldwork, including surveys of the area, collection of samples, and analysis of data. The results of this work will be used to develop a detailed understanding of the system and its dynamics.

3. In the third year, the project will focus on developing a model of the system. This will involve the use of mathematical and statistical techniques to analyze the data collected in the field. The goal of this work will be to create a model that can predict the behavior of the system under different conditions.

4. The fourth year will be devoted to the validation of the model. This will involve comparing the predictions of the model with the actual data collected in the field. The goal of this work will be to ensure that the model is accurate and reliable.

5. The fifth year will be spent in the field, continuing to collect data and validate the model. This will involve further surveys of the area and collection of samples. The goal of this work will be to refine the model and improve its accuracy.

6. The sixth year will be devoted to the final analysis of the data and the development of recommendations for management. This will involve the use of the model to predict the effects of different management practices on the system. The goal of this work will be to provide recommendations for the sustainable management of the system.

7. The seventh year will be the final year of the project. This will involve the final analysis of the data and the preparation of a report. The report will summarize the findings of the project and provide recommendations for the sustainable management of the system.

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## III. MEASUREMENTAL RESULTS

A graphical review of the experimental data on the permeability of ferromagnetic materials (including alloys and ducts) at frequencies between 300 mc/sec and 10,000 mc/sec has been published by Allanson<sup>6</sup>. It is proposed here for the purpose of comparison with the theory developed below to summarize briefly the results of measurements on iron, nickel and cobalt at frequencies above 100 mc/sec, including some data not available to Allanson.

### RESULTS

Experimental values for iron and steel are plotted in Fig. 1, as determined by Arkadiev<sup>9</sup>, Hoag and Jones<sup>10</sup>, Potapenko and Sanger<sup>11</sup>, Lindman<sup>12</sup>, Hoag and Gottlieb<sup>13</sup>, Glathart<sup>14</sup>, and E. Maxwell<sup>15</sup>. The curve labeled  $\mu_R$  is drawn through the points for the experimental permeabilities deducted from resistive losses in a circuit element containing the ferromagnetic specimen, while the curve labeled  $\mu_L$  is deduced from the reactance of the circuit element. The relationship between  $\mu_L$  and  $\mu_R$  is discussed in the next section.

The measurements of Lindman have not been taken into account in drawing the  $\mu_L$  curve, since his values are far out of line with those of Hoag and Jones, Hoag and Gottlieb, Glathart, and Potapenko and Sanger. The apparent discrepancy here may be due to real differences in the dimensions of surface domains or in the electrical conductivity of the surface layer of the specimen. Maxwell has studied the effect of surface finish on microwave attenuation in wave guides and finds considerable

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6. J. T. Allanson, Journ. Inst. Elect. Eng., 92 (pt. III), 247 (1945); see also, V. Arkadiev, Jour. of Phys. USSR 2, 373 (1945).
  9. See reference 6.
  10. J. B. Hoag and H. Jones, Phys. Rev. 42, 571 (1932).
  11. C. Potapenko and R. Sanger, Naturwiss. 21, 818 (1933); Zeit. f. Phys. 104, 779 (1937).
  12. K. F. Lindman, Zeit. f. Tech. Phys. 19, 159 (1938).
  13. J. B. Hoag and N. Gottlieb, Phys. Rev. 55, 410L (1939).
  14. J. L. Glathart, Phys. Rev. 55, 833 (1939).
  15. E. Maxwell, M.I.T. R.L. Report 854 (1946), unclassified. The values of the permeabilities credited to Maxwell were calculated from his values for the attenuation of 1.25 cm radiation in rectangular wave guides.

the first time in the history of the world, the number of people in the world has exceeded the number of people who have ever lived before. This is a remarkable achievement, and it is a cause for celebration. However, it is also a reminder of the fragility of our planet and the need to take care of it. We must work together to ensure that we continue to live in harmony with the natural world and to protect the environment for future generations.

The world population is projected to reach 8 billion by 2030, and 9 billion by 2050. This is a significant increase, and it will put additional pressure on resources and the environment.

### Conclusion

The world population is a complex topic with many different perspectives. It is important to understand the various factors that contribute to population growth, and to consider the implications of such growth for the environment and for society. By working together, we can ensure that we continue to live in harmony with the natural world and to protect the environment for future generations.

Population growth is a complex issue that requires careful consideration and analysis. It is important to understand the various factors that contribute to population growth, and to consider the implications of such growth for the environment and for society.

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### III. DEFINITION OF PERMEABILITY AND ITS MEASUREMENT

There is a great deal of confusion in the literature with regard to the connection between the permeability  $\mu_p$  determined from resistive losses (as by measuring the  $Q_c$  of a cavity or the attenuation of energy along a coax line) and the permeability  $\mu_0$  determined from reactance measurements (as by measuring the resonant frequency of a cavity or the wave length of standing waves along a coax line). It does not appear to have been realized in the literature that the two sets of measurements inherently reveal different aspects of the same fundamental physical phenomena. In this section the fundamental philosophy will be developed which underlies the interpretation, in terms of permeability, of r.f. measurements in a dispersive region. The ideas were stimulated by a paper of Rayleigh<sup>19</sup> in which the concept of an out-of-phase component of magnetization is introduced in connection with hysteresis losses.

The usual definition of initial permeability as the ratio of  $B$  to  $H$  for weak fields does not correspond at high frequencies to the quantities actually observed in experiments. In a dispersive region the value of the ratio  $B/H$  may vary from point to point in the radiation field within the specimen in both amplitude and phase. Detailed knowledge of this "permeability field", exposing it could be determined, would be less useful (for most purposes) than a convenient summing up of the magnetic behavior of the material in terms of an effective permeability.

The natural and logical method of defining the effective permeability is as follows: The impedance

$$Z_{\text{calc}}(v, \omega) = R_{\text{calc}}(v, \omega) + j X_{\text{calc}}(v, \omega) \quad (1)$$

of a circuit element containing the ferromagnetic material can in principle be calculated from Maxwell's equations in the usual form, given  $\mu$  and  $\omega$ . This calculation can actually be carried out in closed form for the important experimental geometries, such as a rectangular wave guide or coaxial line resonator. Suppose that the result of a series of measurements on the circuit element gives us experimental values

19. Lord Rayleigh, Phil. Mag., 23, 225 (1887); Scientific Papers, 2, 579 (Cambridge University Press); cf. W. Arkaiew, Zeit. f. Phys., 27, 37 (1924), and F. Vinze, Ann. d. Phys., (7) 19, 148 (1934).

$$\frac{d\mu}{dx} = \frac{1}{x} \left( \frac{\partial \mu}{\partial x} + \frac{\partial \mu}{\partial p} \frac{dp}{dx} \right)$$

The differential equation (1) is a linear differential equation with constant coefficients, so it has the general solution

$$\mu(x) = C_1 e^{k_1 x} + C_2 e^{k_2 x}$$

In general  $\mu(x)$  and its derivative  $d\mu/dx$  are independent of  $p$ , and will be considered.

In 1933 Debye and Hückel<sup>1</sup> attempted to determine the ionization potential of water, and to calculate the effect of the presence of ions on the ionization potential. In their experiments performed in the aqueous acid and alkaline regions, the transactive part was measured, and in this the effect of the ions could be a serious deflection from the theoretical values of  $\mu$ . However, the equations  $F(1)$  or  $G(1)$  could not easily be solved by direct application of Kramers' theorem.

If a term  $\alpha_{\text{exp}}^2/\alpha_{\text{exp}}$  is taken away from the term  $\alpha_{\text{exp}}^2$  taken so far,

$$F(\alpha_{\text{exp}}) = \alpha_{\text{exp}}^2 - \alpha_{\text{exp}}^2 \exp(-\alpha_{\text{exp}}) \quad (1')$$

This relation determines the value of  $\mu_{\text{exp}}$  if  $\alpha_{\text{exp}}$  is known. If we take only  $\alpha_{\text{exp}}^2/\alpha_{\text{exp}}$  the permeability is taken to zero.

$$K_{\text{calc}}(\mu_{\text{exp}}) = \alpha_{\text{exp}}^2 \exp(-\alpha_{\text{exp}}) \quad (2)$$

thus determining the real part of  $\mu_{\text{exp}}$ .

There is an almost one-to-one connection between  $\mu_{\text{exp}}$  and the complex  $\mu$ . The result is that  $\mu_{\text{exp}}$  has most of the behavior of the real part of  $\mu$ , and  $\mu_{\text{exp}}$  has little of the behavior of the imaginary part of  $\mu$ .

The preceding section, which includes the determination of a透水性常数, shows that  $\alpha_{\text{exp}}$  is connected with the conductance of a certain dielectric system in the  $\sigma$ -class case. There are ten reasons for this connection, because of the effect the rapid electric field distribution has on  $\alpha_{\text{exp}}$  and as a consequence of the

space or "cavities" and because of the grain or microstructure structure of ferrimagnetic material it is possible to have a microscopic permeability which varies from point to point in the material.

In the literature  $\mu_2$  is sometimes called the "outer permeability" and may be denoted also by  $\mu_p$  or  $\mu_K$ ;  $\mu_1$  is sometimes called the "inner permeability" and may be denoted by  $\mu_i$  or  $\mu_s$ .

### IV Theory of Domain Dynamics

In the microwave region the skin depth for field penetration is comparable to or smaller than the dimensions of a domain. For iron ratio  $\mu = 100$  we have  $S \approx 1.6 \times 10^{-3}/\sqrt{f}$ , where  $S$  is the skin depth in cm and  $f$  is the frequency in mc/sec. This gives the following values:

$f$ (mc/sec)	$10^3$	$10^4$	$10^5$
$S$ (cm)	$1.6 \times 10^{-4}$	$1.6 \times 10^{-5}$	$1.6 \times 10^{-6}$

whereas domain dimensions are estimated at  $10^{-3}$  to  $10^{-4}$  cm. It is therefore necessary to reconsider the conventional application of Maxwell's equations to the skin effect problem.

In the limiting case in which the surface energy of the domain boundaries is greater than the magnetization energy, the domain wall shifts as a whole in the direction of the applied field even if, due to the skin effect, the magnetic field only penetrates a short distance into the domain. If  $H$  is constant across the domain boundary (low frequency case) the average macroscopic magnetization  $M$  for weak fields is given by

$$M = \frac{1}{2} H \quad (5)$$

by the usual definition of the initial susceptibility  $\chi$ . If now  $h$  varies across the boundary we suppose that the effective magnetization is given by the susceptibility times the average value of  $H$  across the boundary:

$$M = \frac{1}{2} \chi_{\text{eff}} \int_{-d/2}^{d/2} h(y) dy \quad (6)$$

This assumption is based on the physical concept that the boundary force  $2 M_h$  is integrated over the area of the boundary to give a force which shifts the boundary in the direction of the applied field until when this force is balanced by the force exerted on the boundary by the internal stresses in the material. Here  $M_h$  is the saturation magnetization at the boundary in the domain. The distance the wall is shifted is the macroscopic magnetization  $M$  observed in an experiment at a given frequency; the following happens: if  $h < M_h$  the magnetization

of the domain thickness, so that the applied magnetic field only penetrates a little way down the domain boundary. The force on this boundary associated with the surface value of this field's screen is correspondingly less than the force obtaining at low frequencies, so that the boundary is shifted by a reduced amount. To the field this reduced shift looks like a reduced permeability.

It is of interest to consider as a simplified model a film one domain in thickness (Fig. 3), since in this case Maxwell's equations can be solved to give what is essentially the equation of motion of a domain boundary. The longitudinal extent of a domain along the surface of the film is supposed to be small in comparison with the thickness of the film. The domains are considered for simplicity to have only "180°" walls—that is, the domains are either magnetized in the direction of the applied field or in the opposite direction. The applied field is parallel to the surface of the film and is symmetric about the central plane of the film.

From Maxwell's equations

$$\text{curl } \mathbf{H} = 4\pi G \mathbf{B}/c \quad (8)$$

$$\text{curl } \mathbf{E} = -(i_0 + 4\pi M)/c \quad (9)$$

we get

$$\text{curl curl } \mathbf{H} = -j(4\pi\omega r^2/c^2)(B_x + 4\pi M_x) \quad (10)$$

for time dependence of the form  $\exp(j\omega t)$ . By the symmetry of the problem we have

$$\text{curl curl } \mathbf{H} = -j \frac{\partial}{\partial y} (\frac{\partial^2 H_x}{\partial y^2}) \quad (11)$$

so that

$$\frac{\partial^2 H_x}{\partial y^2} = j(\epsilon\kappa G\omega/c^2)(H_x + 4\pi M_x) \quad (12)$$

Since  $4\pi M_x$  is constant with respect to  $y$  we can add this quantity to  $H_x$  on the left side without altering the value of the derivative there. For definition  $B_x = H_x + 4\pi M_x$ , giving

$$\frac{\partial^2 B_x}{\partial y^2} = j(4\pi\omega r^2/c^2)B_x \quad (13)$$

which can be compared with the usual skin effect equations

$$\partial^2 E_x / \partial y^2 = j(4\pi G\omega/c^2) E_x \quad (14)$$

For convenience in working with Eq. (13) we shall hereafter omit the subscript  $x$  on  $E_x$ ,  $H_x$ , and  $B_x$ . We write

$$p^2 = j(4\pi G\omega/c^2) = 2j/D^2 \quad (15)$$

where  $D$  is the skin depth for permeability unity. So, (13) becomes

$$\partial^2 H / \partial y^2 = p^2 H. \quad (13a)$$

A symmetric solution of this equation is

$$H = C \cosh py \quad (16)$$

where  $C$  is a constant to be determined in terms of  $H(d)$ , the magnetic field at the surface of the film. The definition of  $H$  we have  $H = B + 4\pi M$ ; or using Eq. (7),

$$H(y) = B(y) = (4\pi/2d) \int_{y-d}^d H(y') dy' ; \quad (17)$$

here  $B(y)$  is given by Eq. (16). The solution of this equation is found to be

$$B(y) = C \cosh py - (4\pi \lambda / \mu_0 pd) \sinh py \quad (18)$$

where  $\mu_0$  is defined as  $\lambda = 4\pi \lambda / \mu_0$ .

Thus the constant  $C$  is given in terms of  $H(d)$  by

$$C = H(d) \{ \cosh pd - [1 - \lambda/\mu_0] (\sinh pd/pd) \}^{-1}. \quad (19)$$

The solution of the ordinary eddy current equation (14) above for permeability  $\mu$  is

$$B(y) = M(d) \cosh py / \cosh pd + j\mu H(y) \quad (20)$$

and

$$\mu^2 = \{j(4\pi G\omega/c^2)\}^2. \quad (21)$$

Now that we have the formal solutions of both the ordinary eddy current equation and the domain eddy current equation, we can go on to calculate  $\mu$ ,  $\mu_1$ , and  $\mu_L$  by following the procedure outlined in Section VII. In calculating the impedances it is not necessary to specialize the calculation for a particular cavity or line; we can work with the intrinsic surface impedance of the film, which is defined by<sup>20</sup>

$$Z = E_{\text{tang}} / H_{\text{tang}} = E_z / H_x \quad (22)$$

From Eq. (B),

$$E_z = - (c/4\pi G) \partial h_x / \partial y \quad (23)$$

so that

$$Z = - (c/4\pi G) (\partial h_x / H_x \partial y)_{y=d} \quad (22a)$$

For the ordinary eddy current equation we have, using Eq. (20):

$$Z_{\text{ord}} = - (c/4\pi G) \frac{pd \tanh pd}{1 - (1 - 1/\mu_0)(\tanh pd/pd)} \quad (25)$$

The two expressions for  $Z$  are equal if  $g$  is chosen so that

$$qd \tanh qd = pd \frac{\tanh pd}{1 - (1 - 1/\mu_0)(\tanh pd/pd)} \quad (26)$$

that is, if the effective permeability  $\mu$  is chosen so that

$$\sqrt{\mu} \tanh pd / \sqrt{\mu} = \frac{\tanh pd}{1 - (1 - 1/\mu_0)(\tanh pd/pd)} \quad (27)$$

Hence  $p$  involves  $\sqrt{\mu}$ ; the value of  $\mu$  satisfying this equation will be complex.

Let us consider limiting cases of Eq. (27).

20. S. A. Schelkunoff, Electromagnetic Waves (Interscience, New York, 1943); J. A. Stratton, Electromagnetic Theory (McGraw-Hill, New York, 1941) p.282; J. C. Slater, Microwave Transmission (McGraw-Hill, New York, 1942) p.95.

a) Low frequency

Here  $|pd| \ll 1$ , so that we can replace  $\tanh \theta/\theta$  by unity. This gives  $\mu = \mu_0$ , the correct low frequency value.

b) Very high frequency

Here  $|pd| \gg 1$ , so that  $\tanh \theta/\theta$  approaches zero. This gives  $\mu = 1$  in agreement with the measurements of Hagen and Rubens.

Values of  $\mu$  satisfying Eq. (27) for various values of  $pd$  are given in Table 1. These values were calculated by cut-and-try methods with assistance from Kennelly's tables<sup>21</sup> and the tables prepared by Lowan, Morse, Feshbach, and Haurwitz<sup>22</sup>.

Theoretical values of  $\mu_R$  and  $\mu_I$  are also given in Table 1 according to the definitions, Eqs. (4) and (5), where we identify

$$R_{\text{exp}} \leftrightarrow R_{\text{ord}}; \quad X_{\text{exp}} \leftrightarrow X_{\text{ord}},$$

$$R_{\text{calc}} \leftrightarrow R_{\text{ord}}; \quad X_{\text{calc}} \leftrightarrow X_{\text{ord}},$$

so that  $\mu_R$  is the real number which satisfies the real part of Eq. (27).

$$\text{Re} \left\{ \sqrt{\mu_R} \tanh pd \sqrt{\mu_R} \right\} = \text{Re} \left\{ \frac{\tanh pd}{1 - (1 - 1/\mu_0)(\tanh pd/pd)} \right\} \quad (27a)$$

and  $\mu_I$  is the real number which satisfies the imaginary part:

$$\text{Im} \left\{ \sqrt{\mu_I} \tanh pd \sqrt{\mu_I} \right\} = \text{Im} \left\{ \frac{\tanh pd}{1 - (1 - 1/\mu_0)(\tanh pd/pd)} \right\}, \quad (27b)$$

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- 21. A. S. Kennelly, Tables of Complex Hyperbolic and Circular Functions (Harvard University Press, Cambridge, 1914).
  - 22. A. N. Lowan, P. M. Morse, H. Feshbach, and E. Haurwitz, Tables for Solutions of the Wave Equation for Rectangular Boundaries Having Finite Impedance, Applied Mathematics Panel Note No. 18; Section No. 5.1 - sr1046 - 2043 (June, 1945); unclassified.

Table I.  
Theoretical Permeability vs. Frequency  
Film Thickness  $2.5 \times 10^{-4}$  cm

Frequency mc/sec	Parameter $(\mu/D)^{1/2}$	Permeability $\mu$				$\mu_R$	$\mu_L$
		Amplitude	Phase	$\mu_R$	$\mu_L$		
83.	0.1	104	0°	102	100		
334.	0.2	127	- 18°	157	58		
750.	0.3	89	- 53°	160	11.5		
2080.	0.5	35	- 73°	70	1.7		
4670.	0.75	16	- 79°	32	0.5		
8200.	1.0	9	- 80°	18	0.3		
15000.	2.0	2.6	- 62°	4.5	0.2		
25000.	3.0	1.6	- 38°	2.6	0.6		
41000.	5.0	1.0	- 18°	1.7	0.9		

In Fig. 4  $\mu_R$  and  $\mu_L$  are plotted together with the smoothed experimental curves for iron taken from Fig. 1. The arbitrary constant  $k_1$ , which is the thickness of the domain film model, has been taken to be  $2.48 \times 10^{-4}$  cm; this value was chosen to make the half-value points on the experimental and theoretical curves coincide.

It is seen that the theory predicts the order of magnitude of the spacing between the  $\mu_R$  and  $\mu_L$  curves correctly. The general nature of the theoretical permeability change is in accordance with the experimental data, but the slopes of the theoretical curves are steeper than the experimental. The thickness of the film is within the limits of reasonable estimates of domain dimensions, although the thickness is somewhat on the small side.

The discrepancy in the slopes is most likely to be accounted for by local variations in domain dimensions and d.c. permeability, since these variations will act to smear out the dissipative region. The absence of the hump predicted for the  $\mu_L$  curve on the low frequency side may be due

in part to these causes and in part to the overgeneralization of the present model.

In the ordinary microwave radio range of frequencies from 3,000 to 30,000 mc/sec the permeability  $\mu$  is chiefly imaginary, according to Table I.

## 7. Ferromagnetic Resonance

Several predictions have been made that resonance effects or peaks in the permeability vs. frequency curve would be found at high frequencies; see for example the paper of Landau and Lifschitz<sup>23</sup> in which magnetic resonance is predicted in nickel at  $\sim 2500$  mc/sec.

Such effects have not been found experimentally, and it is possible to see one of the reasons why from the argument of the preceding section. The predictions have all neglected completely the effect of skin depth and eddy currents, yet in the frequency range considered we have shown that such effects are of predominant importance.

It is possible, however, that magnetic resonance effects may be detected in the magnetic oxides and sulfides of iron<sup>24</sup>. These are ferromagnetic but have low electrical conductivity, so that the skin depth will be much greater than in the ferromagnetic metals. The skin depth in magnetite ( $Fe_3O_4$ ) is  $\sim 5 \times 10^{-5}$  cm at  $10^4$  mc/sec, as compared with  $1.6 \times 10^{-5}$  cm in iron at the same frequency. The d.c. initial permeability<sup>25</sup> of magnetite is  $\sim 17$ . Measurements on films of ferromagnetic materials should also be pertinent when the film thickness is less than the calculated skin depth.

The resonance phenomena may be understood as occurring when the frequency of the applied field is equal to the Larmor frequency of the atomic spins in the internal anisotropy field. This is the field due to spin-orbit interactions and distinguishes energetically different directions of magnetization in the crystal lattice. Since the anisotropy field is of the order of 1000 gauss, the corresponding Larmor frequency is in the microwave range.

It is interesting to consider a classical model in which the atoms are replaced by non-gyroscopic bar magnets pivoted at the lattice points of the crystal. With zero applied field each magnet is attracted

23. Reference 5; see also R. Gans and K. G. Loyarte, Ann. d. Phys. [IV] 64, 209 (1921); L. Pauli, Phys. Rev. 21, 456 (1923); J. Dorfmann, Zeit. f. Phys. 17, 98 (1923); K. Kartschagin, Ann. d. Phys. [IV] 67, 325 (1922). The experiments by Kartschagin and others in which resonance phenomena were reported are now discredited.

24. The interesting possibilities of the ferromagnetic semiconductors were pointed out to the writer by Prof. A. v. Hippel, who is planning to investigate them experimentally.

25. International Critical Tables, vol. VI, p. 374

in a definite direction relative to the lattice by means of individual coiled springs representing the spin-orbit interaction, and the magnets will oscillate in an applied field with a component perpendicular to the rest position of the magnets. The sine of the angle of oscillation is proportional to the macroscopic magnetization. Resonance occurs when the applied frequency is equal to the free period of a magnet + spring unit.

The bar magnet analogy supposes that the relaxation time of the spins is sufficiently short so that gyroscopic effects may be neglected. It is not usually recognized that this assumption is being made. If this assumption is not true, the spins will precess about the field direction without lining up. It is indeed a prerequisite for any type of magnetization that the magnetic moments have time in which to line up in the instantaneous local field to which they are subjected. The calculation of the relevant relaxation time is a problem in the kinetics of thermodynamic equilibrium.

The time-dependent processes can be described by assuming the existence of a relaxation time, as was done by Gorter and Koenig for paramagnetic relaxation, and by Landau and Lifschitz for the ferromagnetic case. The quantum mechanical calculation of the relaxation times, starting from the detailed interactions of spins with the lattice, is extremely difficult and uncertain. Calculations for the paramagnetic case have been made by Waller<sup>26</sup> and others. No calculations have been carried out for the ferromagnetic case, so far as the author is aware. It seems plausible to suppose that the strong spin-dependent coupling in ferromagnetic materials will assure that the relaxation frequency will occur above the microwave range. This question should be looked into more closely.

It should be noted in passing that the collision frequency of the lattice phonons at room temperature is  $\sim 10^{12}$  collisions per second, as estimated from values of the thermal conductivity of non-metallic crystals. This figure determines an approximate upper limit to the order of magnitude of the spin relaxation frequency. The actual spin relaxation frequency may be lower depending on the strength of the coupling between the spins and the lattice. In metals the relaxation frequency of the lattice phonons is estimated to be of the order of  $10^{13} \text{ sec}^{-1}$  at room temperature, based on electrical conductivities.

26. See, for example, I. Waller, Zeit. f. Phys. 79, 370 (1932). The writer is indebted to Prof. L. Tisza for several discussions of the paramagnetic relaxation problem.

### II. Discussion

Arkadiew<sup>27</sup> first suggested that eddy current effects might be important in ferromagnetic dispersion. This approach was developed further by Becker<sup>28</sup>, who pointed out that the local microscopic eddy currents associated with the movement of domain boundaries and the rotation of domains set up a magnetic field which opposes the applied field. This back field adds a term to the equations of motion which is proportional to the velocity of boundary movement or spin rotation; that is, the eddy currents behave like a viscous force. Becker's treatment gives a good qualitative account of the damping of irreversible displacements characterizing magnetization in medium fields, at frequencies below the microwave range, although an apparent difficulty in reconciling these results with the measurements of Sixtus and Tonks on the velocity of boundary propagation has been suggested by Miss van Leeuwen<sup>29</sup>. It should be pointed out that the local eddy current effects considered by Becker have no direct connection with the use made of the eddy current equation in the present paper, according to which the incomplete penetration of the surface domains by the applied field is the major cause of dispersion.

Becker also has given a calculation for the dispersion of the initial permeability, and this calculation leads to results in some respects similar to those of the present paper. The "back field" is calculated as in the medium field strength case just mentioned. The basis of Becker's theory supposes that the skin depth is greater than the domain dimensions, so that the calculation is not applicable to the microwave range, where the skin depth is less than the domain dimensions. At  $3 \times 10^3$  mc/sec the skin depth is only  $\sim 0.1$  of the domain thickness.

The present theory probably could be improved by working with a more complicated model than that of a film one domain thick. If the film is backed on one side by a mass of ferromagnetic material the motion of the domain boundaries in the film will induce eddy currents in the backing

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27. W. Arkadiew, C. R., Acad. Sci. URSS (Doklady) 2, 204 (1935); see also reference 3.

28. See reference 4.

29. H. J. van Leeuwen, Physica 11, 30 (1944).

material. The characteristic frequency of the dispersion due to the additional damping. However, has according to the permeability calculated in this paper is expected up to multi-vines on the low frequency side of the dispersive region the eddy current damping is still strong, while at the high frequency side the failure of the applied field to penetrate the thin leads to greatly restricted domain movement. In this case the coercive field is unimportant.

It does not seem worthwhile at this time to attempt to calculate the permeability which is a elaborate model. The present model gives results in reasonable agreement with experiment, and the dispersive mechanism proposed here appears to correspond to the physics of the situation. The most important direction in which the model should be extended would seem to be in a treatment of the case in which the surface energy of the domain wall is small in comparison with the magnetization energy, so that the domain wall yields locally to the field. The dissipation would be due to the magnetic inhomogeneity of the material. The model treated in the present paper supposes that the domain wall moves rigidly under the influence of the applied field. There are reasons for believing that both cases may occur in different actual materials.

It should be pointed out that the information regarding domain behavior obtained from dispersion measurements on metals pertain only to the domains in the surface layers of the material. With this qualification, dispersion measurements may prove to be an important method for studying domain mechanics.

#### Acknowledgments

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## Appendix I

### Relation of Intrinsic Surface Impedance to Resistive Losses and Inductance of Film

It can be shown that the resistive losses in the film considered in Section IV and the contribution of the film to the inductance of a circuit element are related directly to the intrinsic surface impedance which is defined according to Eq. (22) by  $Z = E_z/H_x$ , evaluated at  $y = d$ .

The average rate of energy loss per unit area normal to the  $y$  direction is given by the average value of the Poynting vector

$$S = - \operatorname{Re}[c/4\pi) E_y(d) H_y^*(d)],$$

when it is recalled that the film has two surfaces. Now  $E_z = Z H_x$ , so that

$$S = - (c/4\pi) H_x(d) H_y^*(d) \operatorname{Re}[Z],$$

a well-known result.

The contribution of the film to the inductance of the magnetizing circuit is given by the quotient of the magnetic flux through the film by the current in the magnetizing circuit:

$$L = \operatorname{Re}[\mu \int_{-d}^d H_x dy/J],$$

Now  $J = (c/4\pi) H_x(d)$  and  $\mu H_x = + j(c/\omega)(\partial E_z / \partial y)$ , so that

$$\omega L = 8\pi \operatorname{Re}[jZ] = - 8\pi \operatorname{Im}[Z].$$

APPENDIX

- Figure 1** Convergencability requirement for  $\alpha_{\text{eff}}$
- Figure 2** Convergencability requirement for  $\alpha_{\text{eff}}$
- Figure 3** Radial of finite domain in triangle for theoretical molecular sieving permeability
- Figure 4** Comparison of smooth, experimental values for flow with theoretical calculations using  $\phi_1 = 2.5 \times 10^{-4}$  cm

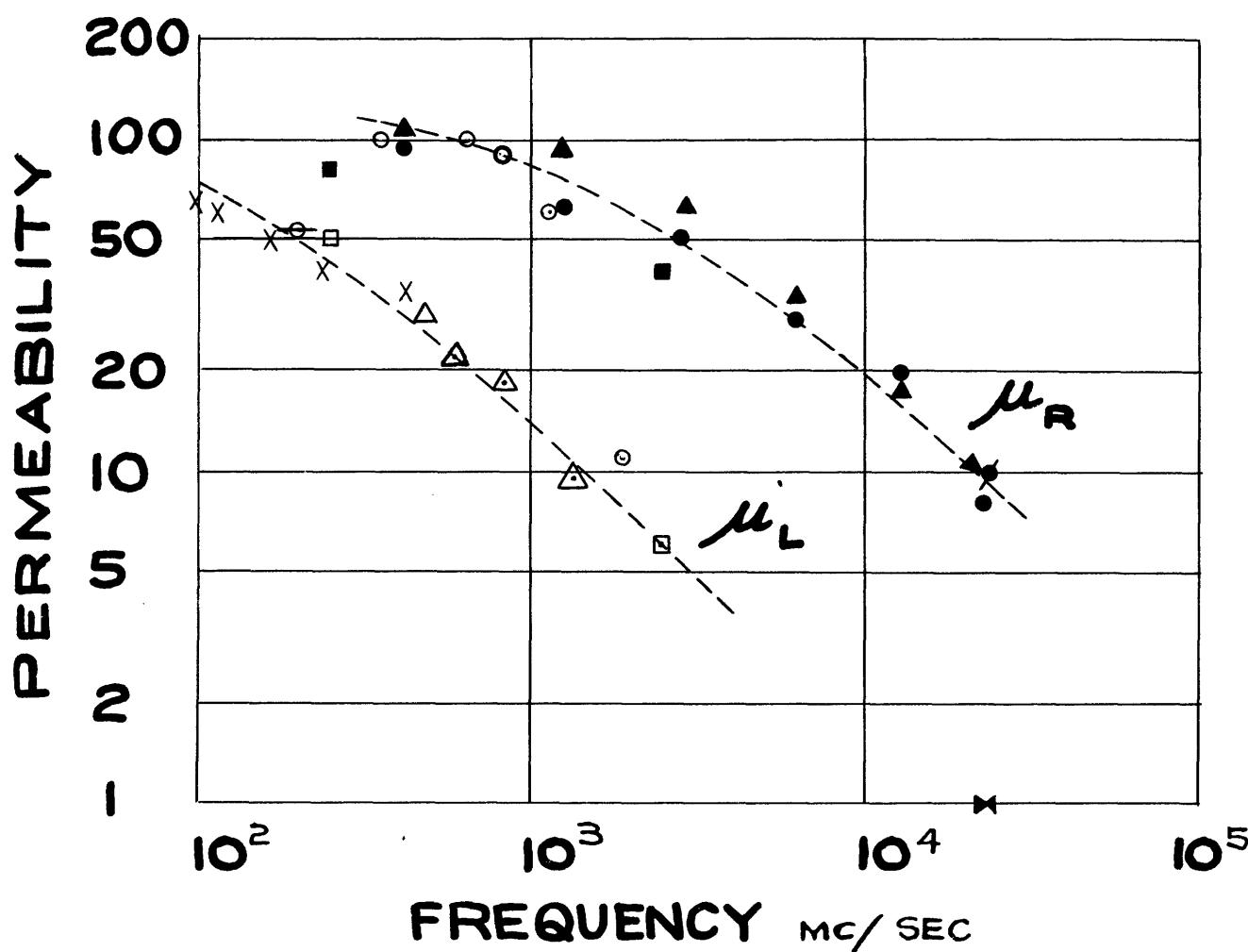
## IRON

$\mu_R$

- POTAPENKO AND SÄNGER-IRON
- ARKADIEW-ANNEALED SWEDISH IRON
- ▲ ARKADIEW - SOFT STEEL
- ◀ E. MAXWELL - ELECTROLYTIC IRON
- ◆ E. MAXWELL - COLD ROLLED STEEL

$\mu_L$

- POTAPENKO AND SÄNGER-IRON
- LINDMAN - IRON
- △ HOAG AND JONES- IRON
- × HOAG AND GOTTLIEB - IRON
- GLATHART - IRON



## NICKEL

$\mu_R$

■ ARKADIEW  
● E. MAXWELL

$\mu_L$

□ LINDMAN  
△ HOAG AND GOTTLIEB  
○ GLATHART

