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Theory of Vector Interactions in Nuclear Beta Decay

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In a previous work[1] the Siegert theorem was extended to vector interactions in nuclear beta decays on the basis of the conserved vector current hypothesis. It was already shown[1] that the experimental data of RaE are consistent with the conserved vector current hypothesis. The purpose of this note is to give the general formulae for arbitrary order of forbidden transitions, since it is highly desirable to investigate as many other non-unique forbidden beta decays as possible. Through this note the validity of non-relativistic approximation is assumed for the motion of nucleons in nuclei, so that our formalism is completely analogous to the conventional theory of nuclear radiative transitions.

§ 1. Introduction

About ten years ago several authors[2),3),4)] evaluated the ratio of the nuclear matrix elements,*)

$$B \equiv \langle\alpha\rangle/\langle r\rangle = -(\alpha Z/2R)\Lambda \tag{1}$$

where α is 1/137. According to Pursey[2)] $\Lambda \sim 2$, whereas according to Ahrens and Feenberg[3)] (for short AF) $\Lambda \sim 1$. First let us discuss the origin of their disagreement. We start from the identity,

$$[H_N, i\sum_{i=1}^{A}\tau_{\pm i}\boldsymbol{r}_i] = \sum_{i=1}^{A}\tau_{\pm i}\boldsymbol{p}_i/M + [V+H_c+H_{n-p}, i\sum_{i=1}^{A}\tau_{\pm i}\boldsymbol{r}_i], \tag{2}$$

in which τ_+ (τ_-) corresponds to the negatron (positron) emission, H_c represents the Coulomb energy, H_{n-p} the neutron-proton mass difference, and the nuclear potential V is assumed to be charge-independent for simplicity. Since $\langle p\rangle/M$ corresponds to $\langle\alpha\rangle$ in the non-relativistic approximation, we obtain from (2)

$$B = \langle\alpha\rangle/\langle ir\rangle \overset{NR}{=} \langle p\rangle/(M\langle ir\rangle) \tag{3}$$
$$= -(W_0 \mp 2.5m_e) - \langle f|[V+H_c, i\sum_{i=1}^{A}\tau_{\pm i}\boldsymbol{r}_i]|i\rangle/\langle f|i\sum_{i=1}^{A}\tau_{\pm i}\boldsymbol{r}_i|i\rangle,$$

*) The notations here follow those in the new textbook by E. J. Konopinski (to be published). The reduced matrix elements, $\langle\alpha\rangle$, $\langle\hat{r}\rangle$ and $\langle|\boldsymbol{\sigma}\times\hat{\boldsymbol{r}}|\rangle$, correspond to the conventional Konopinski-Uhlenbeck notations in the following way:

$$\langle\alpha\rangle \leftrightarrow -\int\boldsymbol{\alpha},\ \langle r\rangle = \langle\hat{r}\rangle/R \leftrightarrow \int\boldsymbol{r} \text{ and } \langle|\boldsymbol{\sigma}\times\hat{\boldsymbol{r}}|\rangle = \langle|\boldsymbol{\sigma}\times\boldsymbol{r}|\rangle/R \leftrightarrow -\int\boldsymbol{\sigma}\times\boldsymbol{r}.$$

($\hat{R}$: the nuclear radius)

where W_0 stands for energy difference between the initial and final nuclei. Equation (3) can be simplified if we assume the validity of the so-called AF approximation,

$$\langle f|[H_c, i\sum_{i=1}^{A}\tau_{\pm i}\boldsymbol{r}_i]|i\rangle = \{\langle f|H_c|f\rangle - \langle i|H_c|i\rangle\}\langle f|i\sum_{i=1}^{A}\tau_{\pm i}\boldsymbol{r}_i|i\rangle, \qquad (4a)$$

and

$$\langle f|[V, i\sum_{i=1}^{A}\tau_{\pm i}\boldsymbol{r}_i]|i\rangle = \{\langle f|V|f\rangle - \langle i|V|i\rangle\}\langle f|i\sum_{i=1}^{A}\tau_{\pm i}\boldsymbol{r}_i|i\rangle. \qquad (4b)$$

The meaning of this approximation has previously been discussed in detail.[3] Roughly speaking, this approximation is expected to be valid if the nuclear wave functions are not very different from the shell model wave functions. Equation (4b) enables us to make use of the semi-empirical formula for nuclear binding energy. Then we are led to the well-known AF formula,

$$B = \langle\alpha\rangle/\langle ir\rangle = -\left(W_0 \mp 2.5m_e \pm 1.2\frac{\alpha Z}{R} \mp 0.7\frac{\alpha Z}{R}\right), \qquad (5)$$

$$(\text{for } e^{\mp}),$$

where the Coulomb term $1.2\alpha Z/R$ was obtained by the assumption of uniform charge distribution within a sphere with radius R and $-0.7\alpha Z/R$ represents the semi-empirical estimate for the nuclear potential contributions. On the other hand, Pursey[2] assumed a specific type of exchange potentials in order to calculate the quantity $\langle f|[V, i\sum\tau_{\pm i}\boldsymbol{r}_i]|i\rangle$ and obtained a value much smaller than AF's semi-empirical estimate.

Now let us recall the Siegert theorem[5),6)] in the electromagnetic transitions of nuclei. For instance, the electric-dipole transition can be described by the operator

$$\dot{\boldsymbol{r}}_i = i[H_N, \boldsymbol{r}_i], \qquad (6)$$

which is not equal to the classical electric-dipole operator $\boldsymbol{p}_i/M$ in the presence of exchange forces. Corresponding to Eq. (2) an identity holds:

$$[H_N, i\sum_{i=1}^{A}\tau_{zi}\boldsymbol{r}_i] = \sum_{i=1}^{A}\tau_{zi}\boldsymbol{p}_i/M + [V, i\sum_{i=1}^{A}\tau_{zi}\boldsymbol{r}_i]. \qquad (7)$$

It is noticed that H_c and H_{n-p} commute with τ_{zi}. The Siegert theorem, which is a representation of charge conservation, tells us that the correct electric-dipole interaction, i.e. the left-hand side of Eq. (7), consists of the classical interaction $\sum\tau_{zi}\boldsymbol{p}_i/M$ and the term arising from non-commutability of the nuclear potential V with $\sum\tau_{zi}\boldsymbol{r}_i$. If V is a two-body potential, the term $[V, i\sum\tau_{zi}\boldsymbol{r}_i]$ represents a sort of two-body radiative interaction.

Recently, the conserved vector current (for short CVC) hypothesis in beta decay was proposed by Feynman and Gell-Mann.[7] Since the CVC hypothesis is just an analogue of charge conservation in the electromagnetic interaction, the assumption of CVC enables us to apply the Siegert theorem to beta decay with slight modifications. Namely, Eq. (7) suggests that B in Eq. (3) must be replaced by the following quantity:

$$B_{CVC} \overset{NR}{=} \{\langle p\rangle/M + \langle |[V, i\sum_{i=1}^{A}\tau_{\pm i}\boldsymbol{r}_i]|i\rangle\}/\langle ir\rangle$$

$$= -(W_0 \mp 2.5m_e) - \langle f|[H_c, i\sum_{i=1}^{A}\tau_{\pm i}\boldsymbol{r}_i]|i\rangle/\langle f|i\sum_{i=1}^{A}\tau_{\pm i}\boldsymbol{r}_i|i\rangle. \quad (8)$$

Furthermore, using the AF approximation, Eq. (4a) leads to

$$B_{CVC} \overset{AF}{=} -\left(W_0 \mp 2.5m_e \pm 1.2\frac{\alpha Z}{R}\right), \text{ (for } e^{\mp}\text{)}, \quad (9)$$

where again the factor 1.2 was obtained by assuming uniform charge distribution.

The validity of CVC can be checked by direct comparison of Eq. (9) with the quantity B obtained from the analysis of experimental data in first forbidden transitions. This method is entirely different from the weak magnetism method.[8] However, one should bear in mind that the only difference between Eqs. (9) and (5) comes from the potential contribution. Therefore, if the AF estimate, Eq. (4b), is very wrong and the true value of $\langle f|[V, i\sum\tau_{\pm i}\boldsymbol{r}_i]|i\rangle$ is negligibly small by some unknown reasons, B becomes B_{CVC} in Eq. (9).

More systematic derivation of the formula (9) and its extension to higher forbidden transitions is discussed in § 2.

§ 2. Vector interactions in general forbidden beta decays

Let us denote the leptonic field $(\bar{e}, \gamma_\mu\nu)$ or $(\bar{\nu}, \gamma_\mu e)$ by L_μ. In the presence of external L_μ, the modified Hamiltonian[6),9)] of the nuclear system is written as $H_N\{L_\mu\}$. Because the coupling with the leptonic field is very weak, we may legitimately neglect higher order terms in the expansion,

$$H_N\{L_\mu\} = H_N + H_N^{(1)}\{L_\mu\} + 1/2\, H_N^{(2)}\{L_\mu\} + \cdots, \quad (10)$$

where $H^{(1)}$ depends linearly on L_μ, and $H^{(2)}$ quadratically. Hereafter, we restrict our attention to the second term $H_N^{(1)}\{L_\mu\}$ only. The Hamiltonian H_N can be divided into two parts: charge-independent part H_I and charge-dependent part H_{II}.

$$H_N^{(1)}\{L_\mu\} = H_I^{(1)}\{L_\mu\} + H_{II}^{(1)}\{L_\mu\}, \quad (11)$$

where

$$H_I = T + V,$$

and

$$H_{II} = H_c + H_{n-p}.$$

The $H_{II}^{(1)}\{L_\mu\}$ might represent a sort of radiative correction due to Coulomb forces in nuclei and is simply omitted here in accordance with the conventional theory.

Now if we assume the validity of the CVC hypothesis, the charge-independent part $H_I\{L_\mu\}$ becomes perfectly analogous to the isovector part of electromagnetic interactions; z-component of every isovector in electromagnetic interaction corresponds to the $\pm$-component in beta decay interaction. Let us study the space part $\boldsymbol{L}$ of a four vector L_μ. Any $\boldsymbol{L}$ can be decomposed into the divergence-free $\boldsymbol{F}$ and the irrotational grad$G$.

$$H_I\{\boldsymbol{F}+\mathrm{grad}G\}=H_I\{\boldsymbol{F}\}+H_I\{\mathrm{grad}G\}$$
$$=H_I\{\boldsymbol{F}\}+e^{i\mathcal{D}}H_I\{0\}e^{-i\mathcal{D}} \tag{12}$$

where

$$\mathcal{D}=\sum_{i=1}^{A}\tau_{\pm i}C_V G(\boldsymbol{r}_i). \tag{12a}$$

The explicit form of $G(\boldsymbol{r})$ depends on the multiplicity of the leptonic field. Expanding the second term of Eq. (12) with respect to $\mathcal{D}$ and comparing with Eq. (10), we obtain

$$H_I^{(1)}\{\boldsymbol{F}+\mathrm{grad}G\}=H_I^{(1)}\{\boldsymbol{F}\}+i[\mathcal{D},H_I]$$
$$=H_I^{(1)}\{\boldsymbol{F}\}+i[\mathcal{D},H_N-H_{II}]. \tag{13}$$

Let us refer to the first term $H_I^{(1)}\{\boldsymbol{F}\}$ as magnetic interaction and the second term $i[\mathcal{D},H_I]$ as electric interaction.

2.1. *Electric interactions*

As an illustration let us assume that the leptonic field L_μ is a plane wave. Except for a normalization factor and spinor parts, $\boldsymbol{L}$ is expressed as

$$\boldsymbol{L}=\boldsymbol{u}\exp(-i\boldsymbol{k}\cdot\boldsymbol{r}). \tag{14}$$

The multipole expansion[6] is obtained by expanding the exponential factor in powers of $(\boldsymbol{k}\cdot\boldsymbol{r})$.

$$\boldsymbol{L}=\sum_{J=1}^{\infty}(-ik)^{J-1}\{\mathrm{grad}G_J+i[(\boldsymbol{u}\times\boldsymbol{k})\times\boldsymbol{r}]W_J\}, \tag{15}$$

where

$$G_J=(\boldsymbol{u}\cdot\boldsymbol{r})(\hat{\boldsymbol{k}}\cdot\boldsymbol{r})^{J-1}/J! \tag{15a}$$

and

$$W_J=J(\hat{\boldsymbol{k}}\cdot\boldsymbol{r})^{J-1}/(J+1)!, \tag{15b}$$

provided that $\hat{\boldsymbol{k}}$ represents a unit vector $\hat{\boldsymbol{k}}=\boldsymbol{k}/k$. Inserting Eq. (15) into Eq. (13), we obtain,

$$H_I^{(1)}\{\boldsymbol{L}\}=i\sum_{J=1}^{\infty}(-ik)^{J-1}\{H_I^{(1)}\{[(\boldsymbol{u}\times\boldsymbol{k})\times\boldsymbol{r}]W_J\}-[H_N-H_{II},\mathcal{D}_J]\}, \tag{16}$$

where $\mathcal{D}_J$ is referred to as the electric 2^J-pole moment operator:

$$\mathcal{D}_J=\sum_i C_V(\boldsymbol{u}\cdot\boldsymbol{r}_i)(\hat{\boldsymbol{k}}\cdot\boldsymbol{r}_i)^{J-1}\tau_{\pm i}/J! \tag{16a}$$

It is easily proved that $\mathcal{D}_1=C_V\sum_i(\boldsymbol{u}\cdot\boldsymbol{r}_i)\tau_{\pm i}$ and

$$\langle f|[H_N-H_{II}, i\mathcal{D}_1]|i\rangle/\langle f|\mathcal{D}_1|i\rangle \tag{17}$$

agrees with Eq. (8).

The other term,

$$\mathcal{M}_J\equiv-(J/(J+1)!)H_I^{(1)}\{[(\boldsymbol{u}\times\hat{\boldsymbol{k}})\times\boldsymbol{r}](\hat{\boldsymbol{k}}\cdot\boldsymbol{r})^{J-1}\}, \tag{18}$$

is referred to as the magnetic 2^J-pole moment operator, which cannot be written explicitly unless we know details of the system. Both $\mathcal{D}_J$ and $\mathcal{M}_J$ are elements of tensors of rank J, but they have opposite parities. It is also remarked that the only formal difference between CVC theory of beta decay and electromagnetic theory is the appearance of H_{II} in Eq. (16).

It is straightforward to extend the above consideration to the more realistic case including the Coulomb distortion of electron field. For that purpose it is sufficient to establish the correspondence between nuclear matrix elements in the CVC and conventional theories, because such a Coulomb effect modifies only the expansion coefficients of Eq. (15). However, as the bases of expansion it is more convenient to introduce the well-known spherical harmonics Y_{JM} and the spherical vector harmonics[10] $\boldsymbol{T}_{JM}^L$ instead of Eqs. (15a) and (15b),

$$\boldsymbol{T}_{JM}^L=\sum_{\rho=0,\pm1}\langle L, M-\rho;\ 1, \rho|J, M\rangle Y_{L,M-\rho}\,\boldsymbol{e}_\rho. \tag{19}$$

In the conventional theory*) there appear two types of reduced matrix elements,

$$\langle i^J r^J Y_J\rangle \text{ and } \langle i^L r^L \boldsymbol{\alpha}\cdot\boldsymbol{T}_J{}^L\rangle.$$

Now, noticing the identity

$$[T, \sum_i(x_i+iy_i)^J\tau_{\pm i}]=-iJ\sum_i(x_i+iy_i)^{J-1}(p_{xi}+ip_{yi})\tau_{\pm i}/M, \tag{20}$$

one can easily prove that the external leptonic field $G_J(\boldsymbol{r})=r^J Y_{JM}(\hat{\boldsymbol{r}})$ corresponds to the reduced nuclear matrix element, $\langle i^{J-1}r^{J-1}\boldsymbol{\alpha}\cdot\boldsymbol{T}_J{}^{J-1}\rangle$. According to the same prescription as Eq. (3) to Eq. (4b), we can write down the semi-empirical estimate,[4)]

$$B^{(J)}\equiv J\langle i^{J-1}r^{J-1}\,\boldsymbol{\alpha}\cdot\boldsymbol{T}_J{}^{J-1}\rangle/\langle i^J r^J Y_J\rangle$$

$$\underset{=}{AF}-\left(W_0\mp 2.5m_e\pm 0.5\frac{\alpha Z}{R}\right) \text{ for } e^{\mp}. \tag{21}$$

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*) These notations follow those of Konopinski (loc. cit.). For example, $\langle Y_0\rangle=\langle 1\rangle/\sqrt{4\pi}$ and $\langle iY_1\rangle=\langle i\hat{r}\rangle\sqrt{3/4\pi}$. R and A (KU notation) correspond to $\langle i^J r^J Y_J\rangle$ and $\langle i^L r^L\boldsymbol{\alpha}\cdot\boldsymbol{T}_{L+1}{}^L\rangle$ respectively.

On the other hand, if CVC is correct, we must replace $B^{(J)}$ by $B_{CVC}^{(J)}$ according to Eq. (13):

$$B_{CVC}^{(J)}=\langle f|[H_N-H_{II},\ i^J\sum_j r_j{}^J Y_{JM}(\hat{\boldsymbol{r}}_j)\tau_{\pm j}]|i\rangle/\langle f|i^J\sum_j r_j{}^J Y_{JM}(\hat{\boldsymbol{r}}_j)\tau_{\pm j}|i\rangle$$

$$=-(W_0\mp 2.5m_e)\mp\langle|[H_c,\ i^J\sum_j r_j{}^J Y_{JM}(\hat{\boldsymbol{r}}_j)\tau_{\pm j}]|\rangle/\langle i^J r^J Y_J\rangle \qquad (22a)$$

$$\overset{AF}{=}-\left(W_0\mp 2.5m_e\pm 1.2\frac{\alpha Z}{R}\right)\ \text{for}\ e^{\mp}. \qquad (22b)$$

If we set $J=1$, Eqs. (21) and (22) are reduced to Eqs. (5) and (9).

2.2. *Magnetic interactions*

First let us assume that the nuclear potentials are velocity-independent. As a consequence of the Golden rule,[6] $\boldsymbol{p}\to\boldsymbol{p}-C_V\tau_{\pm}\boldsymbol{L}$, we are led to the interaction Hamiltonian, which is quite analogous to the radiative interaction.

$$H_I{}^{(1)}\{\boldsymbol{L}\}=-(C_V/4AM)\sum_{j,k}\{(\boldsymbol{p}_j-\boldsymbol{p}_k)\cdot(\tau_{\pm j}\boldsymbol{L}_j-\tau_{\pm k}\boldsymbol{L}_k)$$
$$+(\tau_{\pm j}\boldsymbol{L}_j-\tau_{\pm k}\boldsymbol{L}_k)\cdot(\boldsymbol{p}_j-\boldsymbol{p}_k)\}-(C_V/2M)(\mu_p-\mu_n+1)\sum_j(\boldsymbol{\sigma}_j\cdot\mathrm{rot}\,\boldsymbol{L}_j)\tau_{\pm j}. \qquad (23)$$

The last term is referred to as weak magnetism.[8] Again let us study the plane wave case. Inserting the expansion Eq. (15) into Eq. (23), we obtain the expression for ordinary magnetic moments,

$$\mathcal{M}_J{}^{(ord)}=(C_V/2M(J-1)!)\sum_j\tau_{\pm j}[(1/J+1)\{(\boldsymbol{u}\times\hat{\boldsymbol{k}}\cdot\boldsymbol{l}_j)(\hat{\boldsymbol{k}}\cdot\boldsymbol{r}_j)^{J-1}$$
$$+(\hat{\boldsymbol{k}}\cdot\boldsymbol{r}_j)^{J-1}(\boldsymbol{u}\times\hat{\boldsymbol{k}}\cdot\boldsymbol{l}_j)\}+(1+\mu_p-\mu_n)(\boldsymbol{u}\times\hat{\boldsymbol{k}}\cdot\boldsymbol{\sigma}_j)(\hat{\boldsymbol{k}}\cdot\boldsymbol{r}_j)^{J-1}], \qquad (24)$$

where

$$\boldsymbol{l}_j=\boldsymbol{r}_j\times(\boldsymbol{p}_j-\sum_k\boldsymbol{p}_k/M).$$

The ordinary moment $\mathcal{M}_J{}^{(ord)}$ does not contain the complicated effects of velocity-dependent potentials or exchange currents.

It is instructive to compare the expression Eq. (24) with the conventional notations in forbidden beta decays. For $J=1$,

$$\mathcal{M}_1=(C_V/2M)\sum_j\tau_{\pm j}\{(\boldsymbol{u}\times\hat{\boldsymbol{k}}\cdot\boldsymbol{l}_j)+(1+\mu_p-\mu_n)(\boldsymbol{u}\times\hat{\boldsymbol{k}}\cdot\boldsymbol{\sigma}_j)\}$$

which corresponds to $\langle|\boldsymbol{\alpha}\times\boldsymbol{r}|\rangle\overset{NR}{=}\langle\sigma\rangle/M-\langle|\boldsymbol{r}\times\boldsymbol{p}|\rangle/M$. The only difference is a new factor $(1+\mu_p-\mu_n)=4.7$ appearing in the right-hand. Usually $\langle|\boldsymbol{\alpha}\times\boldsymbol{r}|\rangle$ is classified as a second forbidden nuclear matrix element, but its rank is one, so that it gives a small correction to the allowed beta decay. By virtue of the factor $1+\mu_p-\mu_n$, the so-called weak magnetism[8] has been used for the purpose to check the validity of CVC.

Generally it is clear that $\mathcal{M}_J$ corresponds to $\langle ir^J\boldsymbol{\alpha}\cdot\boldsymbol{T}_J{}^J\rangle$.

Besides the ordinary magnetic moment $\mathcal{M}_J{}^{(ord)}$, there should exist several kinds of complicated but possibly small modifications, which cannot be uniquely determined by the requirement of gauge invariance. It is known that electric moments depend only on the charge distribution, while magnetic moments are related also to the current distribution. At any rate situation is completely the same as in the radiative transition,[6] in so far as CVC is assumed. Since every knowledge of the radiative transition is easily translated in the beta transition, only some important information is recapitulated here.

The principal point is that any momentum-dependence of the nuclear potential modifies the form of nuclear magnetic moments. For instance, velocity-dependent term in V must be modified according to the standard prescription, $\boldsymbol{p}\rightarrow\boldsymbol{p}-C_V\tau_{\pm}\boldsymbol{L}$. The reasonably well-established examples of exchange currents are the space exchange current,[11] which arises from space exchange potentials, and the spin exchange current.[12] The latter is closely connected with the spin-exchange potentials, but the explicit form cannot be obtained by the standard prescription, since momentum-independent term involving rot$\boldsymbol{L}$ may appear in $H^{(1)}\{\boldsymbol{L}\}$ without affecting the gauge invariance. Exchange currents are probably two-body effects and of short range comparable to the nuclear force range.

Another example is the quenching effect of the intrinsic anomalous magnetic moments due to the Pauli principle.[13] If such an effect is real, similar effect should occur in beta decay, too.[1]

Current information[6] in nuclear physics shows that the exchange moment is different from zero, but not big enough to change the qualitative aspect of ordinary magnetic moments.

§ 3. Conclusions and discussions

(*a*) *Electric beta decay*

It was shown in § 2 that the conventional matrix element $\langle i^{J-1}r^{J-1}\boldsymbol{\alpha}\cdot\boldsymbol{T}_J{}^{J-1}\rangle$ should be replaced by a slightly different nuclear matrix element:

$$J\langle i^{J-1}r^{J-1}\boldsymbol{\alpha}\cdot\boldsymbol{T}_J{}^{J-1}\rangle \underset{\longrightarrow}{\text{CVC}} \langle|[H_N-H_{II},\, i^J\sum_j r_j{}^J Y_{JM}(\hat{\boldsymbol{r}}_j)\tau_{\pm j}]|\rangle$$

$$=-(W_0\mp 2.5m_e)\langle i^J r^J Y_J\rangle \mp\langle|[H_c,\, i^J\sum_j r_j{}^J Y_{JM}(\hat{\boldsymbol{r}}_j)\tau_{\pm j}]|\rangle$$

$$\underset{=}{AF} -\left(W_0\mp 2.5m_e\pm 1.2\frac{\alpha Z}{R}\right)\langle i^J r^J Y_J\rangle \qquad (25)$$

for $e^{\mp}$.

The ratio $B^{(J)}$ as shown in Eqs. (21) and (22) is useful to check the validity of CVC hypothesis. It is highly desirable to investigate many non-unique forbidden transitions experimentally.

In this paper "the finite nucleon size effect" was omitted, although the electron scattering experiment by Hofstadter revealed the fact that nucleons have finite extensions ($\sim 1/(2m_\pi)$). It seems improbable[1] for such an effect to

Table I. Vector nuclear matrix elements

	allowed	1st-	2nd-	J-th forbidden
Conventional theory (bare-nucleon coupling theory)	longitudinal part—its retarded effect			
	$\langle 1\rangle$	$\langle ir\rangle$	$\langle r^2 Y_2\rangle$	$\langle i^J r^J Y_J\rangle$
		transverse part—its retarded effect		
		$\langle \alpha\rangle$	$\langle ir\boldsymbol{\alpha}\cdot\boldsymbol{T}_2^1\rangle$	$\langle i^{J-1} r^{J-1}\boldsymbol{\alpha}\cdot\boldsymbol{T}_J^{J-1}\rangle$
	↑ magnetic effect			
	[small correction]		$\langle \lvert\boldsymbol{\alpha}\times\boldsymbol{r}\rvert\rangle$	$\langle i^J r^J\boldsymbol{\alpha}\cdot\boldsymbol{T}_J^J\rangle$
			etc.	etc.
	allowed	1st-	2nd-	J-th forbidden
CVC theory	longitudinal part—its retarded effect			
	$\langle 1\rangle$	$\langle ir\rangle$	$\langle r^2 Y_2\rangle$	$\langle i^J r^J Y_J\rangle=\langle \mathcal{D}_J\rangle$
		transverse part—its retarded effect		
		$\langle \lvert[H_N-H_{II}, \mathcal{D}_1]\rvert\rangle$	$\langle \lvert[H_N-H_{II}, \mathcal{D}_2]\rvert\rangle$	$\langle \lvert[H_N-H_{II}, \mathcal{D}_J]\rvert\rangle$
	↑ weak magnetism			
	[small correction] including space- and spin-exchange currents, quenching effect, etc.			
			$\mathcal{M}_1$	$\mathcal{M}_{J-1}$
			etc.	etc.
	All the nuclear matrix elements are understood that they include the finite nucleon size effect.			

be significant in beta decay, but it is easily taken into account if we replace the density operator $\rho(\boldsymbol{r})=\sum_{i=1}^{A}\delta(\boldsymbol{r}-\boldsymbol{r}_i)\tau_{\pm i}$ by $\rho'(\boldsymbol{r})=\sum_{i=1}^{A}f(\boldsymbol{r}-\boldsymbol{r}_i)\tau_{\pm i}$ ($f(r)$ stands for finite extension).

(*b*) *Magnetic beta decay*

Since the situation is completely analogous to the radiative transitions, the ordinary magnetic transition operator, Eq. (24), should be modified by the space-exchange, spin-exchange contributions, quenching effect, and so on.

However, the weak magnetic beta decay is usually not important. The reason lies in the fact that in beta decay the axial vector interaction is equally important, which causes the strong magnetic beta decay.

All the results are tabulated in Table I.

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