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THERMAL AND MECHANICAL DAMPING OF SOLAR p-MODES

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ABSTRACT

Nonadiabatic effects associated with the transfer of energy and with turbulent stresses add small imaginary parts, $\omega_i^{(1)}$ and $\omega_i^{(2)}$, to solar p-mode eigenfrequencies. Numerical calculations have shown that these quite different processes make comparable contributions to ω_i at frequencies well below the acoustic cutoff at ω_{ac}. We derive analytic expressions which reveal the connection between $\omega_i^{(1)}$ and $\omega_i^{(2)}$. Our estimates yield $\omega_i \propto \omega^8$ for $\omega \ll \omega_{ac}$, in good agreement with the numerical calculations. However, the observed line width is proportional to $\omega^{4.2}$ at low frequencies. We suspect that there is an unmodeled component of perturbed convective energy transport or of turbulent viscosity that makes an important contribution to ω_i at $\omega \ll \omega_{ac}$.
Subject headings: radiative transfer — Sun: oscillations — turbulence

1. INTRODUCTION

Ando & Osaki (1975) investigated the effects of radiative transfer on the linear stability of solar p-modes. They found overstable modes excited by the opacity mechanism acting in the hydrogen ionization zone. Goldreich & Keeley (1977) showed that turbulent viscosity could stabilize the modes, although the margin of stability is very small for modes with periods longer than 5 minutes.[2] The similarity between the magnitudes of the thermal and mechanical contributions to the imaginary parts of the mode eigenfrequencies, ω_i, is surprising. It suggests a connection between two different physical mechanisms.

We explain this apparent coincidence in this note. Our emphasis is on simplicity. Thus, we treat radial modes, make the quasi-adiabatic approximation for the perturbations, and describe turbulent convection by the mixing length model. In § 2 we provide analytic estimates for the thermal and mechanical contributions to ω_i. The implications of our results are discussed in § 3.

2. DAMPING RATES

We relate all perturbation variables to the radial displacement eigenfunction, ξ, which may be taken to be real within the quasi-adiabatic approximation. We normalize ξ such that

$$4\pi\omega^2 \int_0^R dr\, r^2 \rho \xi^2 = 1 . \tag{1}$$

It is convenient to define a mode mass by

$$M_\omega = \frac{1}{\omega^2 \xi^2(R)} . \tag{2}$$

With this definition the (observable) mean square surface velocity, $v^2(R)$, is related to the energy contained in the mode, E, by $E = M_\omega v^2$.

Contributions to the imaginary parts of the mode eigenfrequencies due to energy transfer[3] and turbulent stresses are given by

$$\omega_i^{(1)} = \frac{1}{2} \int_0^R dr \frac{\partial \Delta L}{\partial r} \frac{\Delta T}{T} \tag{3}$$

(Cox 1980; Unno et al. 1989), and

$$\omega_i^{(2)} = 4\pi\omega^2 \int_0^R dr\, r^2 \rho \nu_H \left(\frac{\partial \xi}{\partial r}\right)^2 \tag{4}$$

(Goldreich & Keeley 1977), respectively. With our sign convention, positive values of ω_i correspond to damping. Here, Δ denotes a Lagrangian perturbation, the total luminosity $L = L_r + L_c$ is the sum of radiative and convective components, and ν_H is the turbulent viscosity. We set $\nu_H \sim v_H H$, where v_H is the turbulent velocity of eddies of size H.[4] The expressions for $\omega_i^{(1)}$ and $\omega_i^{(2)}$ look very different. However, we shall see that they are closely related.

We need some basic properties of the p-mode eigenfunctions. These are easily obtained from the adiabatic wave equation for radial oscillations,

$$\frac{\partial^2(r^2\xi)}{\partial r^2} + \frac{\partial}{\partial r}\left[\ln\left(\frac{\rho c^2}{r^2}\right)\right]\frac{\partial(r^2\xi)}{\partial r} + \left[\frac{\omega^2}{c^2} - \frac{r^2}{c^2}\frac{\partial}{\partial r}\left(\frac{g}{r^2}\right)\right](r^2\xi) = 0 . \tag{5}$$

Here, c^2 is the adiabatic sound speed, and g is the gravitational acceleration. The wave equation (5) would have a singular point at the solar surface if the temperature vanished there. Because the photosphere has finite temperature, the Sun only traps acoustic waves whose frequencies are below the cutoff frequency

$$\omega_{ac} \approx \frac{c}{2H_t} , \tag{6}$$

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[2] Other authors have also reported a rough balance between the nonadiabatic effects of radiative and convective processes (e.g., Cox et al. 1989 and references therein).

[3] Equation (3) describes radiative damping if $\Delta L_c = 0$.

[4] We take H equal to the local pressure scale height.

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where H_t is the scale height at the photosphere.[5] The photospheric temperature is effectively zero for waves with $\omega \ll \omega_{ac}$, and their eigenfunctions closely correspond to regular solutions of the wave equation. This implies

$$\left(\frac{1}{\xi}\frac{\partial \xi}{\partial r}\right)_R \approx \left(\frac{\omega^2}{g}\right) \approx \frac{1}{2H_t}\left(\frac{\omega}{\omega_{ac}}\right)^2 . \quad (7)$$

2.1. *Thermal Damping*

Most of the contribution to $\omega_i^{(1)}$ comes from the top of the convection zone and the lower portion of the radiative atmosphere. Turbulent convection presents a barrier to detailed modeling. This is especially true in the thin layer at the top of the hydrogen ionization zone where the mode of energy transfer switches from convection to radiation.

To estimate $\omega_i^{(1)}$ from equation (3), we relate ΔL to the strain, $\partial\xi/\partial r$. Both L_r and L_c are functions of ρ, T, and $T_{,r} \equiv \partial T/\partial r$. In addition, L_c depends on the effective gravity and thus includes a contribution from the inertial term $\omega^2\xi$. We set $L = L(\rho, T, T_{,r}, g)$ from which it follows that

$$\Delta L = \left(\rho\frac{\partial L}{\partial \rho}\right)\frac{\Delta\rho}{\rho} + \left(T\frac{\partial L}{\partial T}\right)\frac{\Delta T}{T} + \left(T_{,r}\frac{\partial L}{\partial T_{,r}}\right)\frac{\Delta T_{,r}}{T_{,r}} + \left(g\frac{\partial L}{\partial g}\right)\frac{\omega^2 \xi}{g} . \quad (8)$$

We apply the quasi-adiabatic approximation to express the dimensionless Lagrangian variations in equation (8) in terms of ξ. This approximation is valid where $4\pi R^2 \omega P H/L \gg 1$. Higher in the atmosphere the flux perturbation is essentially frozen, that is $|\partial \Delta L/\partial r| \ll |\Delta L|/H$. However, regions where $\partial\Delta L/\partial r$ is small make small contributions to $\omega_i^{(1)}$. For the solar p-modes of interest to us the adiabatic approximation becomes adequate slightly below the top of the convection zone. Here,

$$\frac{\Delta\rho}{\rho} \approx -\frac{\partial\xi}{\partial r} \sim -\frac{\omega^2\xi}{g} , \quad (9)$$

$$\frac{\Delta T}{T} = -(\Gamma_3 - 1)\frac{\partial\xi}{\partial r} \sim -\frac{\omega^2\xi}{g} , \quad (10)$$

where

$$\Gamma_3 \equiv 1 + \frac{\rho}{T}\left(\frac{\partial T}{\partial \rho}\right)_s . \quad (11)$$

The derivation of $\Delta T_{,r}/T_{,r}$ is slightly more involved. We find

$$\frac{\Delta T_{,r}}{T_{,r}} = \frac{1}{T_{,r}}\frac{\partial \Delta T}{\partial r} - \frac{\partial\xi}{\partial r} \sim -\frac{\omega^2\xi}{g} . \quad (12)$$

In arriving at the final forms for the Lagrangian perturbations, we take $\omega \ll \omega_{ac}$ so that $\omega^2/g \ll 1/H$. Also, we discard factors of order unity and do not distinguish among the pressure, density, and temperature scale heights.

From equations (8)–(12), it follows that

$$\Delta L \sim L\frac{\omega^2\xi}{g} , \quad (13)$$

$$\frac{\partial \Delta L}{\partial r} \sim \frac{\Delta L}{H} . \quad (14)$$

[5] The expression for ω_{ac} would be exact if the atmosphere were isothermal above the photosphere.

The latter scaling holds because H is the scale length of the coefficients in equation (8). We note that equations (13) and (14) are satisfied to order of magnitude without implication as to sign.

We now possess all of the ingredients necessary to evaluate $\omega_i^{(1)}$. Taking the local scale height, H, as the width of the damping region, a simple calculation yields

$$\omega_i^{(1)} \sim \frac{L\omega^2}{M_\omega g^2} \sim \left(\frac{L}{M_\omega c_t^2}\right)\left(\frac{\omega}{\omega_{ac}}\right)^2 , \quad (15)$$

where L is the solar luminosity and c_t is the sound speed at the photosphere. Equation (15) provides an order of magnitude estimate for $\omega_i^{(1)}$ but does not specify its sign. Its derivation assumes that there is no delicate cancellation between excitation in the hydrogen ionization zone and damping higher in the solar atmosphere.

An alternate derivation of equation (15) proceeds by integrating equation (3) by parts to yield

$$\omega_i^{(1)} \sim \frac{\Delta L(R)}{2}\frac{\Delta T(R)}{T(R)} - \frac{1}{2}\int_0^R dr\,\Delta L\frac{\partial}{\partial r}\left(\frac{\Delta T}{T}\right) . \quad (16)$$

Using the scalings given by equations (7), (10), and (13), it follows that the first term on the right-hand side reproduces equation (15), and the second term is smaller than the first by the factor $(\omega/\omega_{ac})^2$. This derivation has the advantage of relating $\omega_i^{(1)}$ to observable quantities.. However, it suffers from lack of rigor in that it applies the quasi-adiabatic approximation outside its range of validity.

Although one can only guess at the form of ΔL_c, it is worth stressing its Lagrangian character. For example, calculations based on $\delta L_c = 0$, where δ denotes an Eulerian perturbation, can lead to unphysically large positive values for $\omega_i^{(1)}$. To see how this can occur, note that $\delta L_c = 0$ implies

$$\Delta L_c = \frac{\partial L^c}{\partial r}\xi . \quad (17)$$

Near the top of the convection L_c drops abruptly from L to 0 so there

$$\Delta L_c \sim -\frac{L}{H}\xi , \quad (18)$$

which is larger, by a factor $(\omega_{ac}/\omega)^2$, than ΔL given by equation (13). Because the freezing of ΔL sets in close to the top of the convection zone, equation (18) leads to

$$\Delta L(R) \sim -\frac{L(R)}{H_t}\xi(R) . \quad (19)$$

Substituting this value for $\Delta L(R)$ in equation (16), and using equation (10) to eliminate $\Delta T(R)/T(R)$, we obtain

$$\omega_i^{(1)} \sim \frac{L}{M_\omega c_t^2} , \quad (20)$$

which is definitely positive, and larger by the factor $(\omega_{ac}/\omega)^2$ than the correct estimate given by equation (15).

2.2. *Mechanical Damping*

The dominant contribution to the integral in equation (4) comes from a critical layer in which the characteristic lifetime of the energy-bearing eddies, $\tau_H \sim H/v_H$, is related to the mode frequency by $\omega\tau_H \sim 1$. Inertial range eddies produce additional

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damping in lower layers. However, according to the mixing length model this additional damping is small so, in keeping with the order of magnitude spirit of this paper, we ignore it. Since the turbulence is subsonic, the mode is evanescent in the critical layer.

The convective velocity is related to the luminosity through the mixing length model as

$$L_c \sim 4\pi R^2 \rho v_H^3 . \quad (21)$$

It follows that the dynamic viscosity in the critical layer may be written as

$$\rho v_H \sim \frac{L}{4\pi R^2 H \omega^2} , \quad (22)$$

where we have taken $L_c \approx L$ in the convection zone.

To evaluate $\omega_i^{(2)}$, we take the local scale height as the thickness of the damping region and evaluate the integrand in equation (4) with the aid of equations (2), (7), and (22). This procedure yields

$$\omega_i^{(2)} \sim \frac{L\omega^2}{M_\omega g^2} \sim \left(\frac{L}{M_\omega c_t^2}\right)\left(\frac{\omega}{\omega_{ac}}\right)^2 , \quad (23)$$

which is identical to the expression given for $\omega_i^{(1)}$ by equation (15).

3. DISCUSSION

We have revealed the connection between the thermal and mechanical contributions to ω_i that previously appeared as a mysterious result of numerical calculations. Next, we examine how well the calculated values of ω_i fit the observationally determined line widths.

At low frequencies, $\omega \lesssim \omega_{ac}/2$, $M_\omega \propto \omega^{-6}$. Therefore, equation (15) and (23) imply $\omega_i \propto \omega^8$. The magnitude and frequency dependence for ω_i given by the analytic formulae are in satisfactory agreement with numerical results obtained by Ando & Osaki (1975), Goldreich & Keeley (1977), Christensen-Dalsgaard & Frandsen (1982), and Kidman & Cox (1984). Near the peak of the acoustic spectrum at 3 mHz, the calculated line widths have similar magnitudes to the measured values. However, observations by Libbrecht (1988) establish that $\omega_i \propto \omega^{4.2}$ for $\omega \ll \omega_{ac}$. Thus, the theoretical line widths decay much too rapidly at low frequencies.

What is the explanation for this discrepancy? Theoretical estimates of the contribution to ω_i from radiative diffusion are fairly reliable. However, the same cannot be said about those due to the convective transport of energy and the turbulent stresses associated with it. We conclude that there is an unmodeled component of damping associated with turbulent convection. It remains for future investigations to pinpoint why current theoretical estimates yield too small values for ω_i at low frequencies.

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