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ABSTRACT

A numerical solution to the problem of thermal elastohydrodynamic lubrication of line contacts was obtained by using a finite difference formulation. The solution procedure consists of simultaneous solution of the thermal Reynolds equation, the elasticity equation, and the energy equation subject to appropriate boundary conditions. Pressure distribution, film shape, and temperature distribution were obtained for fully flooded conjunctions, a paraffinic lubricant, and various dimensionless speed parameters while the dimensionless load and materials parameters were held constant. Reduction in the minimum film thickness due to thermal effects (as a ratio of thermal to isothermal minimum film thickness) is given by a simple formula as a function of the thermal loading parameter Q:

$$\frac{H_{min}}{H_{min,I}} = \frac{10}{10 + Q^{0.4}}$$

Plots of pressure distribution, film shape, temperature distribution, and flow are shown for some representative cases.

INTRODUCTION

Elastohydrodynamic lubrication of nonconformal contacts has been an important aspect of research in the field of tribology for the last two decades primarily because of its paramount importance in the lubrication of heavily loaded contacts in machine elements, viz roller bearings, ball bearings, gears, and traction drives. The variation of viscosity with pressure and the elastic deformation associated with the high pressures generated in the contact region are the major causes of the complexity attributed to the problem. The first major effort in arriving at a numerical solution of the problem was made by Grubin and Vinogradova (1949), who incorporated both elastic deformation and viscosity-pressure characteristics in the inlet zone analysis of the hydrodynamic lubrication of nonconformal line contacts. Elastic deformation in the contact region was approximated by the Hertzian deformation of the solid in that region.

Dowson and Higginson (1961) obtained a numerical solution to the problem of elastohydrodynamic lubrication of nonconformal line contacts by incorporating both elasticity and viscous effects in the coupled solution of the

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Reynolds and elasticity equations. They obtained a formula for the minimum film thickness that was based on their theoretical results and that showed the effects of the load, speed, and materials parameters.

Thermal effects in nonconformal line contacts were investigated by Sternlicht et al. (1961) assuming that the heat generated due to viscous shear was carried away totally by the lubricant and that no heat was conducted into the contacting solids. However, in the lubrication of nonconformal contacts and within the practical range of loads and rolling speeds, conduction of heat to the solids appears to be the major mode of heat transfer rather than convection by the lubricant. Cheng (1965) presented a numerical solution to the thermal elastohydrodynamic lubrication of rolling-sliding line contacts that takes into account both the conduction of heat into the solid from the lubricant and convection by the lubricant. Results showed that temperature had no significant effect on the magnitude of the film thickness. However, the frictional force in the case of rolling-sliding contacts was affected by the temperature rise in the film. In a similar paper, Dowson and Whitaker (1965) also produced a complete thermohydrodynamic solution to the lubrication of rolling-sliding line contacts and concluded that the thermal effects on the film thickness were insignificant. However, the rolling speed and the inlet viscosity of the lubricant used were low, and it is not surprising that thermal effects were not observed. On the contrary, the work done by Cheng (1967), Murch and Wilson (1975), and Goksem and Hargreaves (1978) on the effect of viscous shear heating in the inlet zone in a Grubin type of analysis revealed a much larger influence of temperature on the film thickness, particularly at high rolling speeds. Recently, Kaludjercic et al. (1980), in a rigorous simultaneous solution of the thermal Reynolds equation, the elasticity equation, the energy equation, and the transient heat conduction equation in the solids showed that temperature effects on the film thickness can be significant at higher rolling speeds and presented a formula for the reduction in the central and minimum film thicknesses caused by viscous heating in the film.

Recently, Hamrock and Jacobson (1983) presented a solution for line contacts wherein the computations are carried out from the inlet to the outlet of the conjunction as one complete solution. These results formed the basis for the present investigation, and the numerical procedure adopted retains all of the essential features of the procedure developed by Hamrock and Jacobson (1983). The thermal aspect of the problem uses an approach similar to that adopted by Dowson and Whitaker (1965) or Kaludjercic et al. (1980).

In the present work a numerical solution to the problem of thermal elastohydrodynamic lubrication of line contacts is obtained. It consists of simultaneous solution of the thermal Reynolds equation, which takes into account viscosity variations with pressure and temperature, the elasticity equation for the film shape due to a known pressure distribution, the energy equation for temperature due to a known pressure distribution and film shape, and the heat conduction equation for the solid surface temperatures due to a known temperature distribution in the film. Pressure distribution, film shape, and temperature distribution were obtained in fully flooded conjunctions for a paraffinic lubricant and for various dimensionless speed parameters covering the practical range of low to moderately high rolling speeds, while the dimensionless load and materials parameters were held constant. In arriving at a converged solution for any particular case, special attention was paid to assure that the mass flow rate per unit length was held constant throughout the conjunction. This latter effect is important, especially in the inlet zone of a fully flooded conjunction in which the inlet distance extends several Hertzian widths from the center of the contact.

Computer plots of pressure distribution and film shape are characteristic of elastohydrodynamically lubricated line contacts. Thermal effects on the minimum film thickness are incorporated into a simple formula that shows the influence of the dimensionless thermal loading parameter. Sliding had no significant influence on the film thickness but did affect the frictional characteristics.

SYMBOLS

A	constant defined in eq. (26), m^2/N
a	constant defined in eq. (37)
B	$\bar{b}/b$
b	semiwidth of contact, m
C	constant defined in eq. (26), m^2/N
C_p	specific heat of lubricant, J/kg-K
C_{ps}	specific heat of solid, J/kg-K
D	constant defined in eq. (37)
E	modulus of elasticity of solid, N/m^2
E'	effective modulus of elasticity of solid, $2\Big/\left(\frac{1-\nu_a^2}{E_a}+\frac{1-\nu_b^2}{E_b}\right)$, N/m^2
F_a, F_b	dimensionless shear force per unit length
f_a, f_b	shear force per unit length, N/m
G	dimensionless materials parameter, $\alpha E'$
H	dimensionless film thickness, h/R_x
H_{min}	dimensionless minimum film thickness
H_0	dimensionless constant defined in eq. (13)
h	film thickness, m
h_{min}	minimum film thickness, m
k	thermal conductivity of lubricant, W/m-K
k_s	thermal conductivity of solid, W/m-K
ℓ	some characteristic length of contact, m
n	number of nodes in semiaxis of contact
P	dimensionless pressure, p/E'

Pe	Peclet number, $\rho C_p u_s R_x/k$
p	pressure, N/m^2
Q	dimensionless thermal loading parameter, $\gamma \eta_0 u_s^2/k$
Q_x	dimensionless mass flow rate per unit length, $q_x/\rho_0 u_s R_x$
q_x	mass flow rate per unit length, kg/s-m
R_x	effective radius in direction of motion, m
r	radius of curvature, m
S	slide-to-roll ratio, $2(u_a - u_b)/(u_a + u_b)$
s	geometrical separation, m
T	dimensionless temperature, t/t_0
t	temperature, K
t_0	inlet lubricant temperature, K
U	dimensionless speed parameter, $\eta_0 u_s/E'R_x$
u	fluid particle velocity, m/s
$\bar{u}$	dimensionless fluid particle velocity, u/u_s
u_a, u_b	solid surface velocities, m/s
u_d	sliding velocity, $u_a - u_b$, m/s
u_s	surface velocity in direction of motion, $(u_a + u_b)/2$, m/s
V	fluid element volume, m^3
W	dimensionless load parameter, $w_z/E'R_x$
w	load, N/m
X	dimensionless coordinate, x/b
x	coordinate in direction of motion, m
Z	dimensionless coordinate in direction of film thickness, z/h
z	coordinate in direction of film thickness, m
α	pressure-viscosity coefficient of lubricant, m^2/N
δ	elastic deformation, m
γ	temperature-viscosity coefficient of lubricant, K^{-1}
ε	thermal expansivity of lubricant, $\frac{1}{V}\left(\frac{\partial V}{\partial t}\right)_p$, K^{-1}
η	viscosity of lubricant, $N\text{-}s/m^2$
η_0	viscosity of lubricant at ambient pressure, $N\text{-}s/m^2$
$\bar{\eta}$	dimensionless viscosity of lubricant, η/η_0
μ	coefficient of friction

ν	Poissons ratio for solid
ρ	density of lubricant, kg/m^3
ρ_0	density of lubricant at ambient pressure and inlet oil temperature, kg/m^3
$\bar{\rho}$	dimensionless density of lubricant, ρ/ρ_0
ρ_s	density of solid, kg/m^3

Subscripts:

a	solid a
b	solid b
s	solid
x	coordinate in direction of motion
z	coordinate in direction of film thickness

THEORY

The approach used in solving the isothermal elastohydrodynamic aspect of the problem is similar to that used by Hamrock and Jacobson (1983). The approach used in solving the thermal aspect of the problem is similar to that adopted by Dowson and Whitaker (1965) and Kaludjercic et al. (1980).

Reynolds Equation

The generalized thermal Reynolds equation derived by Dowson (1962) and written in the present form by Fowles (1970) is written for line-contact problems as follows:

$$\frac{\partial}{\partial x}\left(m_2 \frac{\partial p}{\partial x}\right) = u_b \frac{\partial}{\partial x}(m_3) + \frac{\partial}{\partial x}\left[\frac{m_1}{f_0}(u_a - u_b)\right] \tag{1}$$

where

$$m_2 = \frac{f_1}{f_0} m_1 - \int_0^h \rho\left(\int_0^z \frac{z}{\eta}\, dz\right) dz; \quad m_1 = \int_0^h \rho\left(\int_0^z \frac{dz}{\eta}\right) dz$$

$$m_3 = \int_0^h \rho\, dz; \quad f_1 = \int_0^h \frac{z}{\eta}\, dz\ ; \ f_0 = \int_0^h \frac{dz}{\eta}$$

Letting

$$X = \frac{x}{b}\ ; \ \bar{\rho} = \frac{\rho}{\rho_0}\ ; \ \bar{\eta} = \frac{\eta}{\eta_0}\ ; \ H = \frac{h}{R_x}\ ; \ P = \frac{p}{E'} \tag{2}$$

where

$$\frac{1}{R_x} = \frac{1}{r_{ax}} + \frac{1}{r_{bx}} \tag{3}$$

and

$$E' = \frac{2}{\dfrac{1 - \nu_a^2}{E_a} + \dfrac{1 - \nu_b^2}{E_b}} \tag{4}$$

equation (1) can be written in dimensionless form as

$$\frac{\partial}{\partial X}\left(M_2 H^3 \frac{\partial P}{\partial X}\right) = U\left(\frac{b}{R_x}\right)\frac{\partial}{\partial X}\left[\frac{u_d}{u_s} H\left(\frac{M_1}{F_0}\right) + \frac{u_b}{u_s}(M_3 H)\right] \tag{5}$$

where

$$U = \frac{\eta_0 u_s}{E' R_x} \quad ; \quad M_2 = \frac{F_1}{F_0} M_1 - \int_0^1 \bar{\rho}\left(\int_0^Z \frac{Z}{\eta}\, dZ\right) dZ \tag{6}$$

$$M_1 = \int_0^1 \bar{\rho}\left(\int_0^Z \frac{dZ}{\eta}\right) dZ \; ; \; M_3 = \int_0^1 \bar{\rho}\, dZ \; ; \qquad F_1 = \int_0^1 \frac{Z}{\eta}\, dZ \; ; \; F_0 = \int_0^1 \frac{dZ}{\eta}$$

Boundary conditions for equation (5) are

$$P = 0 \; ; \quad X = -\infty \; ; \quad P = \frac{\partial P}{\partial X} = 0 \quad \text{at} \quad X = X_{exit} \tag{7}$$

Figure 1 shows the radii of the rollers used in defining equation (3). It is assumed that convex surfaces, as shown in figure 1, exhibit positive curvature and concave surfaces, negative curvature. Therefore, if the center of curvature lies within the solid, the radius of curvature is positive; if the center of curvature lies outside the solid, the radius of curvature is negative.

Film Shape

The film shape can be written simply as

$$h(x) = h_0 + s(x) + \delta(x)$$

where

h_0 constant

$s(x)$ separation due to geometry of undeformed solids

$\delta(x)$ elastic deformation

The separation due to the geometry of the two undeformed rollers shown in figure 1(a) can be described by an equivalent cylindrical solid near a plane, as shown in figure 1(b). The geometrical requirement is that the separation of the two rollers in the initial and equivalent situations should be the same at equal values of x. Therefore the separation due to the undeformed geometry of the two rollers can be written, using the parabolic approximation, as

$$s(x) = \frac{x^2}{2R_x} \tag{9}$$

Figure 2 shows a rectangular area of uniform pressure. From Timoshenko and Goodier (1951) the elastic deformation at a point $\bar{x}$ on the surface of a semi-infinite solid subjected to pressure at the point x_1 can be written as

$$\delta(\bar{x}) = -\frac{2}{\pi E'} \int_{-\bar{b}}^{\bar{b}} p \ln (\bar{x} - x_1)^2 \, dx_1 \tag{10}$$

If the pressure is assumed to be uniform over the rectangular area, the pressure can be put in front of the integral. Therefore equation (10) results in the following (Hamrock and Jacobson, 1983):

$$\delta(\bar{x}) = \frac{2}{\pi} PD \tag{11}$$

where

$$D = b \left[(\bar{X} - B) \ln (\bar{X} - B)^2 - (\bar{X} + B) \ln (\bar{X} + B)^2 + 4B(1 - \ln b)\right]$$

and

b Hertzian semiwidth of contact

$\bar{b}$ equals b/n

n number of nodes within semiwidth of contact

B equals $\bar{b}/b$

Now the term $\delta(\bar{x})$ in equation (11) represents the elastic deformation, at a point $\bar{x}$, caused by a rectangular area of uniform pressure and width $2\bar{b}$. If the conjunction is divided into a number of equal rectangular areas, the total deformation at a point $\bar{x}$ due to contributions of various rectangular areas of uniform pressure in the conjunction can be evaluated numerically. The total elastic deformation caused by the rectangular areas of uniform pressure within a conjunction can be written as

$$\delta_k(x) = \frac{2}{\pi} \sum_{i=1,2,\ldots} P_i D_j \qquad j = |k - i| + 1 \tag{12}$$

Therefore substituting equations (9) and (12) into equation (8) while writing the film thickness in the dimensionless form gives the following:

$$H_k = \frac{h}{R_x} = H_0 + \frac{1}{R_x}\left[\frac{X^2}{2}\left(\frac{b^2}{R_x}\right) + \frac{2}{\pi}\sum_{i=1,2,\ldots} P_i D_j\right] \tag{13}$$

where

$$j = |k - i| + 1$$

Energy Equation

The temperature distribution within the lubricant film is determined from the solution of the energy equation subject to appropriate boundary conditions. The energy equation for line-contact problems can be expressed, neglecting convection across the film and conduction along the film, as follows:

$$\frac{\partial}{\partial z}\left(k\frac{\partial t}{\partial z}\right) = \rho C_p u \frac{\partial t}{\partial x} - t\varepsilon u \frac{\partial p}{\partial x} - \eta\left(\frac{\partial u}{\partial z}\right)^2 \tag{14}$$

where

$$u = u_b + \frac{\partial p}{\partial x}\int_0^z \frac{z}{\eta}\,dz - \frac{\partial p}{\partial x}\frac{f_1}{f_0}\int_0^z \frac{dz}{\eta} + \frac{u_d}{f_0}\int_0^z \frac{dz}{\eta} \tag{15}$$

$$\frac{\partial u}{\partial z} = \frac{1}{\eta}\left[\frac{u_d}{f_0} + \left(z - \frac{f_1}{f_0}\right)\frac{\partial p}{\partial x}\right] \tag{16}$$

With the following substitutions and assuming that lubricant properties such as thermal conductivity, specific heat, and thermal expansivity do not vary with temperature or pressure, the energy equation can be written in dimensionless form as

$$\frac{\partial^2 T}{\partial Z^2} = N_{cv}\bar{\rho}\bar{u}H^2\frac{\partial T}{\partial X} - N_{ac}T\bar{u}H^2\frac{\partial P}{\partial X} - N_{vd}\bar{\eta}\left(\frac{\partial \bar{u}}{\partial Z}\right)^2 \tag{17}$$

where

$$T = t/t_0; \quad \bar{u} = u/u_s; \quad N_{cv} = \frac{\rho_0 C_p R_x^2 E'U}{k\eta_0}\left(\frac{\pi}{8W}\right)^{1/2}$$

$$N_{ac} = \left[\frac{\varepsilon(R_x E')^2}{k\eta_0}\right] U \left(\frac{\pi}{8W}\right)^{1/2}; \quad N_{vd} = \frac{(R_x E')^2}{\eta_0 t_0 k} U^2$$

The following expressions for fluid velocity and its derivative in the dimensionless form are used in the energy equation:

$$\bar{u} = \bar{u}_b + \frac{H^2}{U}\left(\frac{\pi}{8W}\right)^{1/2} \frac{\partial P}{\partial X}\left(\int_0^Z \frac{Z}{\eta}\, dZ - \frac{F_1}{F_0}\int_0^Z \frac{dZ}{\eta}\right) + \frac{\bar{u}_d}{F_0}\int_0^Z \frac{dZ}{\eta} \tag{18}$$

$$\frac{\partial \bar{u}}{\partial Z} = \frac{1}{\eta}\left[\frac{\bar{u}_d}{F_0} + \frac{H^2}{U}\left(\frac{\pi}{8W}\right)^{1/2}\left(Z - \frac{F_1}{F_0}\right)\frac{\partial P}{\partial X}\right] \tag{19}$$

Boundary conditions for the energy equation are given as

$$T = 1.0\ , \quad X = -\infty\ ; \qquad T(X,0) = T_b\ ; \qquad T(X,H) = T_a \tag{20}$$

where T_a and T_b are the surface temperatures of the two solids and are evaluated by solving transient heat conduction equations for solids for a known temperature distribution in the film.

Surface Temperature Equations

Carlsaw and Jaeger (1959) solved the transient heat conduction equation for the case of linear heat flow and developed the expression for the solid surface temperature rise of a semi-infinite body under the action of a moving heat source. Following this, the surface temperature rise in the thermal elastohydrodynamic lubrication of line contacts can be expressed as

$$t_b = \frac{1}{(\pi k_s \rho_s u_b C_{ps})^{1/2}} \int_{-\infty}^{X} k\left(\frac{\partial t}{\partial z}\right)_{z=0} \frac{d\zeta}{(x-\zeta)^{1/2}} \qquad \text{for solid b} \tag{21a}$$

$$t_a = \frac{1}{(\pi k_s \rho_s u_a C_{ps})^{1/2}} \int_{-\infty}^{X} k\left(\frac{\partial t}{\partial z}\right)_{z=h} \frac{d\zeta}{(x-\zeta)^{1/2}} \qquad \text{for solid a} \tag{21b}$$

These equations are valid for constant thermal conductivity of the solid and for the dimensionless parameter

$$\frac{u\ell}{2}\left(\frac{\rho C_p}{k}\right)_s > 10$$

In a lubricated contact and for the practical range of load and speed parameters, this condition is well satisfied.

Equations (21a) and (21b) can be expressed in dimensionless form as

$$T_b(X,0) = \frac{k}{R_x}\left(\frac{b}{\pi \rho_s C_{ps} u_b k_s}\right)^{1/2} \int_{-\infty}^{X} \frac{1}{H}\left(\frac{\partial T}{\partial Z}\right)_{Z=0} \frac{d\zeta}{(X - \zeta)^{1/2}} \tag{22a}$$

$$T_a(X,H) = \frac{k}{R_x}\left(\frac{b}{\pi \rho_s C_{ps} u_a k_s}\right)^{1/2} \int_{-\infty}^{X} \frac{1}{H}\left(\frac{\partial T}{\partial Z}\right)_{Z=H} \frac{d\zeta}{(X - \zeta)^{1/2}} \tag{22b}$$

Viscosity-Pressure-Temperature Relationship

The isothermal viscosity-pressure relationship employed in the present analysis as proposed by Barus (1893) is given as

$$\eta = \eta_0 \, e^{\alpha p} \tag{23}$$

The complete viscosity relationship can be expressed as

$$\eta = \eta_0 \, e^{[\alpha p + \gamma (t_0 - t)]} \tag{24}$$

In dimensionless form it is written as

$$\bar{\eta} = e^{[GP + \gamma t_0 (1.0 - T)]} \tag{25}$$

The viscosity model used for the sliding friction calculation is

$$\bar{\eta} = \exp\left[\frac{GP}{1.0 + 0.01\,GP} + \gamma t_0 (1.0 - T)\right]$$

Density-Pressure-Temperature Relationship

The isothermal density-pressure relationship used by Dowson, Higginson, and Whitaker (1962) has been employed in this work. For the temperature variation it is assumed that within the range of temperatures considered, a linear model applies. Therefore the complete density relationship used can be expressed as

$$\rho = \rho_0 \left(1.0 + \frac{Ap}{1.0 + Cp}\right) \left[1.0 - \epsilon (t - t_0)\right] \tag{26}$$

In the dimensionless form it is expressed as

$$\bar{\rho} = \left(1.0 + \frac{AE'P}{1.0 + CE'P}\right) \left[1.0 - \epsilon t_0 (T - 1.0)\right] \tag{27}$$

Computational Procedure

To solve the thermal elastohydrodynamic lubrication problem, it is necessary to solve the isothermal cases first. The computational procedure developed by Hamrock and Jacobson (1983) for the solution of isothermal elastohydrodynamic lubrication of line contacts was adopted here to arrive at the isothermal pressure distribution and film shape in the conjunction. The isothermal pressure distribution and the film shape thus obtained were used as initial guesses for thermal cases. Equations (5), (13), and (17) were written in the finite difference form by using a standard central difference scheme.

Figure 3 shows the placement of the uniform nodes within the conjunction. The number of nodes within a semicontact width was 120 throughout all calculations. However, the numbers of nodes in the inlet and outlet regions were different for higher rolling speed cases. The system of simultaneous equations ((5), (13), (17), and (22)) was solved by the standard Gauss-Seidel iterative method. All fluid property integrals were evaluated by using the trapezoidal rule. The cycle for automatic computation procedure is shown in the flow chart in figure 4.

Computational difficulties were encountered in solving the energy equation and in evaluating the surface temperature near the pressure spike. Since the film temperatures are mostly governed by the local viscosity and the pressure gradient, it is not surprising that such difficulties were encountered. Therefore, in some cases, temperatures near the pressure spike could not be evaluated. In general, temperatures near the spike were much higher than temperatures at the center of the contact. Therefore, in the computer plots of temperature, temperatures near the spike have been suppressed in order to clearly depict the temperatures before and after the pressure spike.

In the case of highest rolling speed, the temperature calculation in the beginning of the inlet zone also gave rise to numerical problems. This is due to the very high film thickness and very high recirculation flow caused by the rapidly converging film thickness. Therefore, in that case, temperatures in the first few nodes were not evaluated. However, these numerical difficulties are likely to occur because of the highly nonlinear nature of the energy equation coupled with the nonlinear thermal Reynolds equation. None of these difficulties affected the film thickness calculations in those cases since the magnitude of the film thickness and the film shape are not decided by these regions at all. The coupled solutions were considered to be converged when the combined solution conserved the flow from the inlet to the exit region besides satisfying the convergence criteria associated with various equations. The numerical procedures for solving the energy equation and evaluating surface temperatures fail to converge for high rolling speeds when sliding is introduced, even for slide-to-roll ratios of 0.1 and 0.2. This is due to high temperatures generated within the contact region and perhaps also to the viscosity-temperature-pressure relationship used in the present calculations. Dowson and Whitaker (1965) and Cheng (1965) have used a different pressure-temperature relationship in their solution. However, Dowson and Whitaker (1965) did not carry out computations for higher rolling speeds.

Mass Flow Rate

The mass flow rate per unit length for the elastohydrodynamically lubricated contacts with temperature effects can be written as

$$q_x = (u_a - u_b)\,\frac{m_1}{f_0} + u_b m_3 - m_2\,\frac{dp}{dx} \tag{28}$$

In the dimensionless form the flow rate is written as

$$Q_x = \frac{q_x}{\rho_0 u_s R_x} = \left[\frac{u_d}{u_s}\left(\frac{M_1 H}{F_0}\right) + \frac{u_b}{u_s} M_3\ H\right] - \frac{R_x}{b\bar{U}}\left(M_2 H^3\,\frac{\partial P}{\partial X}\right) \tag{29}$$

Force Components

Normal force components per unit length acting on the solids can be written as (fig. 5):

$$w_z = w_{az} = w_{bz} = \int p\ dx$$

In dimensionless form

$$W = \frac{w_z}{E'R_x} = \frac{b}{R_x}\int P\ dX \tag{30}$$

The tangential force component for solid a is zero. For solid b it can be expressed as

$$w_{bx} = -\int p\ dh$$

In dimensionless form it can be expressed as

$$W_{bx} = \frac{w_{bx}}{E'R_x} = \int H\,\frac{dP}{dX}\,dX \tag{31}$$

Shear Forces

The shear force per unit length of the roller is written as

$$f_{b,a} = \int_{-\infty}^{X_{exit}} \eta\left(\frac{\partial u}{\partial z}\right)_{z=0,h} dx \tag{32}$$

where

$$\frac{\partial u}{\partial z} = \frac{1}{\eta}\left[\frac{u_d}{f_0} + \left(z - \frac{f_1}{f_0}\right)\frac{\partial p}{\partial x}\right]$$

Therefore the shear forces on the roller surfaces can be expressed in dimensionless form as

$$F_a = \int_{-\infty}^{X_{exit}} \left[\left(U \frac{b}{R_x} \right) \frac{u_d}{u_s H F_0} + H \left(1.0 - \frac{F_1}{F_0} \right) \frac{\partial P}{\partial X} \right] dX \qquad \text{for solid a} \tag{33}$$

$$F_b = \int_{-\infty}^{X_{exit}} \left[\left(U \frac{b}{R_x} \right) \frac{u_d}{u_s H F_0} - H \frac{F_1}{F_0} \frac{\partial P}{\partial X} \right] dX \qquad \text{for solid b} \tag{34}$$

For equilibrium to be satisfied, the following must be true:

$$F_a - F_b + W_{bx} = 0$$

$$W_{az} - W_{bz} = 0$$

and the coefficient of friction can be written as

$$\mu = -\frac{F_a}{W} = \frac{-F_b + W_{bx}}{W} \tag{35}$$

RESULTS AND DISCUSSION

Input data for the system of rollers, the physical material properties, and the lubricant physical properties used in the computer calculations are presented in table I. Computed isothermal film thickness, thermal film thickness, and rolling traction coefficient are shown in table II. Table III presents the computed results of midfilm temperature rise, surface temperature rise, and coefficient of friction for combined rolling-sliding cases.

Representative pressure distribution and film shape are shown in figure 6. Figure 6(a) shows the pressure distribution and film shape for the low dimensionless rolling speed of 1×10^{-11}, and figure 6(b) shows the corresponding pressure distribution and film shape for the high dimensionless rolling speed of 1.344×10^{-10} (or 8.0 m/sec). The pressure distribution and film shape exhibit the characteristic pressure spike and nip in the film shape near the pressure spike, where the nip is that portion of the film shape from the tip of the pressure spike to the outlet of the conjunction. The film shape in the contact region is flatter for low rolling speed than for high rolling speed. This is due to the greater film thickness and the pronounced hydrodynamic effect on the pressures in the contact region for the high rolling speed. It is noticeable too in the pressure distribution in the contact region, which departs significantly from the Hertzian pressures when compared with low rolling speeds. This causes a shift in the location of the pressure spike toward the center of the contact. However, the natures of the film shape and pressure profile are not altered by the thermal effects, and no appreciable changes are noticed when compared with respective isothermal solutions.

A computer plot of the mass flow rate from the inlet to the outlet is shown in figure 7. Conservation of mass flow rate is evident and was used as the final criterion for the converged solution. Conservation of flow in the inlet region is an important aspect since the pressure distribution and film shape in the inlet region are responsible for the film shape within the conjunction and to a greater extent for the minimum film thickness.

Temperature distributions in the conjunction for pure rolling cases are presented in figures 8(a) and (b) for two dimensionless rolling speeds, 1×10^{-11} and 1.344×10^{-10}, respectively. In pure rolling cases the maximum temperature rise for the midfilm temperature occurs on the far left in the beginning of the inlet zone. Surface temperatures rise from the beginning of the inlet zone, reach a maximum, and then drop to a lower value before the start of the contact region. In pure rolling cases temperatures are much lower in the contact region than in the inlet zone. For higher rolling speeds much higher temperatures are attained, as is evident from the comparison of figures 8(a) and (b). Viscous shear heating in pure rolling is predominant in the inlet zone and is pronounced at high rolling speeds, thus resulting in significant thermal effects on film thickness for higher rolling speeds.

With sliding, the temperature distribution in the conjuction is altered totally, as is evident from the figure 9. Sliding causes severe heating due to viscous effects in the contact region only, where the viscosities are high, and does not introduce significant heating in the inlet region. Both the midfilm temperature and the surface temperature rise considerably due to shear heating as a result of sliding in the contact region. Sliding results in different surface temperatures in the solids, the slower surface attaining higher temperature than the faster surface. However, for the sliding cases computed, no appreciable change was noticed in the film thickness, even for very high slide-to-roll ratios of 0.3 and 0.5. A decompression cooling effect is visible in the temperature distribution after the pressure spike.

The effect of temperature on rolling and sliding traction is shown in figures 10(a) and (b), respectively. As the rolling speed and slide-to-roll ratio increase, traction coefficients decrease with respect to their isothermal values.

In calculating sliding friction, however, a different pressure-viscosity model was used since the friction coefficients calculated with the model used in computing the pressure and temperature distribution were unreasonably high. This fact was mentioned by Dowson and Whitaker (1965) as a reason for using a different model for thermal analysis than for isothermal analysis.

Minimum Film Thickness Formula

Besides the usual dimensionless parameters associated with the isothermal film thickness calculations, the only other dimensionless parameters that influence the magnitude of film thickness in thermal elastohydrodynamic lubrication of rectangular contacts are

(1) Thermal loading parameter $\quad Q = \dfrac{\gamma\eta_0 u_s^2}{k}$

(2) Peclet number $\quad Pe = \dfrac{\rho C_p u_s R_x}{k}$

with the dimensionless group as $Pe\ W^{3/2}$. However, the influence of the dimensionless group of the Peclet number $Pe\ W^{3/2}$ on the minimum film thickness is negligible. Therefore the functional relationship can be suitably represented (as the ratio of thermal to isothermal minimum film thickness) by

$$\frac{H_{min}}{H_{min,I}} = \frac{1.0}{1.0 + f(Q)} \tag{36}$$

or

$$\frac{H_{min}}{H_{min,I}} = \frac{1.0}{1.0 + DQ^a} \tag{37}$$

where the constants D and a are determined by the linear regression analysis of the computed data for the minimum thickness.

The formula for the thermal elastohydrodynamic minimum film thickness is obtained as

$$\frac{H_{min}}{H_{min,I}} = \frac{10}{10 + Q^{0.4}} \tag{38}$$

with a regression coefficient of 0.97811, where

$$H_{min,I} = 3.07\ U^{0.71} G^{0.57} W^{-0.11} \tag{39}$$

as obtained from Hamrock and Jacobson (1983).

Table IV compares the thermal and isothermal minimum film thickness ratios obtained from equation (38) with those calculated from the Kaludjercic et al. (1980) and Murch and Wilson (1975) formulas. There is good correlation with the values given by Kaludjercic's formula, but Murch and Wilson's formula overestimates the thermal effect at higher rolling speeds. It is understandable since the analysis done by Murch and Wilson (1975) is only approximate. It is a Grubin type of analysis of the inlet zone for an incompressible fluid and constant surface temperature.

CONCLUSIONS

A numerical solution to the problem of thermal elastohydrodynamic lubrication of line contacts has been achieved. It calls for simultaneous solution of the thermal Reynolds equation, the elasticity equation, the energy equation, and the transient heat conduction equation in the solids. The following conclusions were reached:

1. Thermal effects on the minimum film thickness are negligible at low rolling speeds.
2. At high rolling speeds there can be significant reduction in the minimum film thickness due to viscous shear heating in the conjunction. Thermal effects are incorporated in terms of the dimensionless thermal loading parameter Q into isothermal film thickness equations as follows:

$$\frac{H_{min}}{H_{min,I}} = \frac{10}{10 + Q^{0.4}}$$

3. Sliding has negligible effect on the film thickness or film shape, especially at low rolling speeds.

4. Thermal effects considerably reduce both rolling traction and sliding traction, especially at high rolling speeds and for the combination of rolling with sliding.

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TABLE I. - INPUT DATA, MATERIAL PROPERTIES, AND LUBRICANT PROPERTIES

Radius of equivalent roller on plane, cm	1.11125
Inlet temperature of lubricant, K	313
Inlet viscosity of lubricant, Pa-s	0.0411
Inlet density of lubricant, kg/m^3	866
Pressure-viscosity coefficient of lubricant, α, GPa^{-1}	22.76
Temperature-viscosity coefficient of lubricant, K^{-1}	0.04666
Pressure-density coefficients of lubricant, GPa^{-1}	
A	0.582744
C	1.68348
Thermal expansivity of lubricant, K^{-1}	6.5×10^{-4}
Thermal conductivity of lubricant, W/m-K	0.12
Specific heat of lubricant, kJ/kg-K	2.0
Thermal conductivity of steel rollers, W/m-K	52
Specific heat of steel rollers, J/kg-K	460
Density of steel rollers, kg/m^3	7850
Elastic modulus of steel rollers, GPa	200
Poissons ratio of steel rollers	0.3

TABLE II. - ISOTHERMAL FILM THICKNESS, THERMAL FILM THICKNESS, AND ROLLING TRACTION COEFFICIENT

[Dimensionless load, W, 2.0478×10^{-5}; dimensionless materials parameter, G, 5000.]

Case	Dimensionless speed, U	Dimensionless isothermal minimum film thickness, H_{min}	Dimensionless thermal minimum film thickness, H_{min}	Ratio of thermal to isothermal film thickness	Isothermal rolling traction coefficient	Thermal rolling traction coefficient
1	1.000×10^{-11}	20.015×10^{-6}	20.015×10^{-6}	1.0000	0.537×10^{-3}	0.537×10^{-3}
2	2.000	35.384	34.581	.9773	.683	.683
3	3.000	44.978	43.881	.9756	.836	.836
4	5.000	65.144	61.995	.9516	.968	.968
5	7.575	87.943	83.123	.9452	1.074	1.051
6	10.000	113.660	105.240	.9259	1.7097	1.563
7	13.466	138.060	125.740	.9108	1.9097	1.678
8	20.000	177.880	156.710	.8810	2.1610	1.733

TABLE III. - MIDFILM TEMPERATURE RISE, SURFACE TEMPERATURE RISE, AND COEFFICIENT OF FRICTION FOR ROLLING-SLIDING CASES

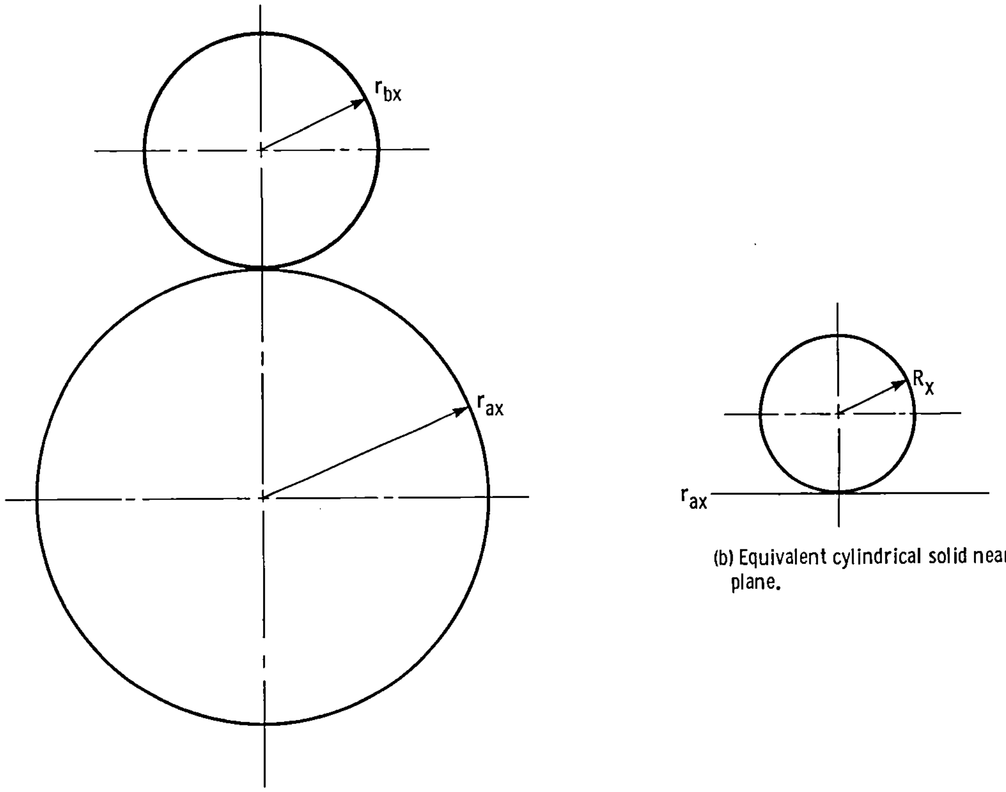
[Dimensionless rolling speed, U, 1×10^{-11}; dimensionless materials parameter, G, 5000; dimensionless load parameter, W, 2.0478×10^{-5}.]

Slide-to-roll ratio	Sliding coefficient of friction	Dimensionless mid-film temperature rise at center of contact	Dimensionless solid surface temperature rise at center of contact	
			Faster surface	Slower surface
0	0	0	0	0
.04	.0145	$.528 \times 10^{-3}$	$.124 \times 10^{-3}$	$.124 \times 10^{-3}$
.08	.0319	3.552	1.615	1.638
.16	.0384	13.799	6.416	6.736
.20	.0460	20.217	9.359	9.975
.30	.0547	38.000	17.460	19.320
.50	.0677	73.142	33.040	39.490

TABLE IV. - THERMAL AND ISOTHERMAL MINIMUM FILM THICKNESS RATIOS OBTAINED FROM EQUATION (38) AND FROM KALUDJERCIC AND MURCH AND WILSON FORMULAS

[Dimensionless load parameter, W, 2.0478×10^{-5}; dimensionless materials parameter, G, 5000.]

Dimensionless speed parameter, U	Dimensionless thermal loading parameter, Q	Dimensionless Peclet group parameter, Pe $W^{3/2}$	Ratio of thermal to isothermal film thickness		
			Present formula	Kaludjercic et al. (1980) formula	Murch and Wilson (1975) formula
1.000×10^{-11}	0.564×10^{-2}	0.883×10^{-2}	0.9875	0.9940	0.9890
2.000	2.256	1.766	.9785	.9876	.9764
3.000	5.076	2.649	.9705	.9812	.9616
5.000	14.100	4.415	.9563	.9682	.9299
7.575	32.360	6.688	.9401	.9517	.8880
10.000	56.400	8.298	.9263	.9361	.8489
13.466	102.270	11.890	.9083	.9150	.7953
20.000	225.600	17.660	.8784	.8767	.7040



(a) Two undeformed rollers.

(b) Equivalent cylindrical solid near plane.

Figure 1. - Rollers and equivalent roller.

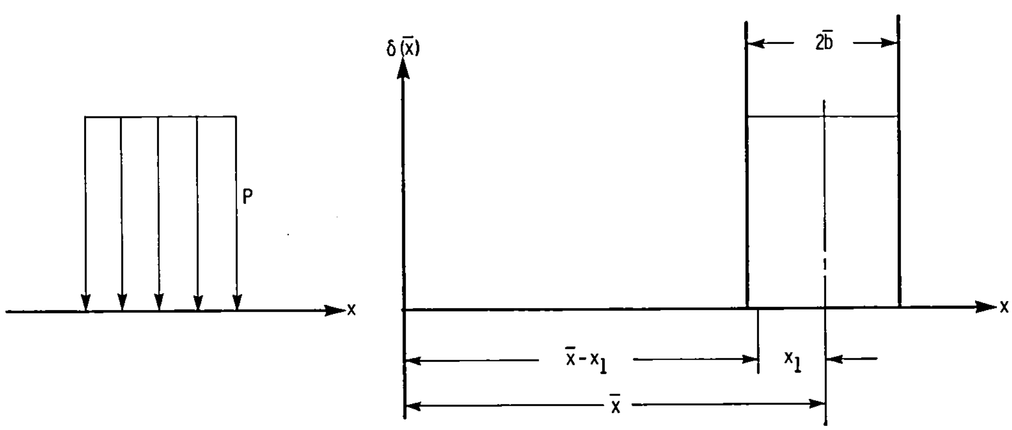


Figure 2. - Surface deformation of semi-infinite body subjected to uniform pressure over a rectangular area.

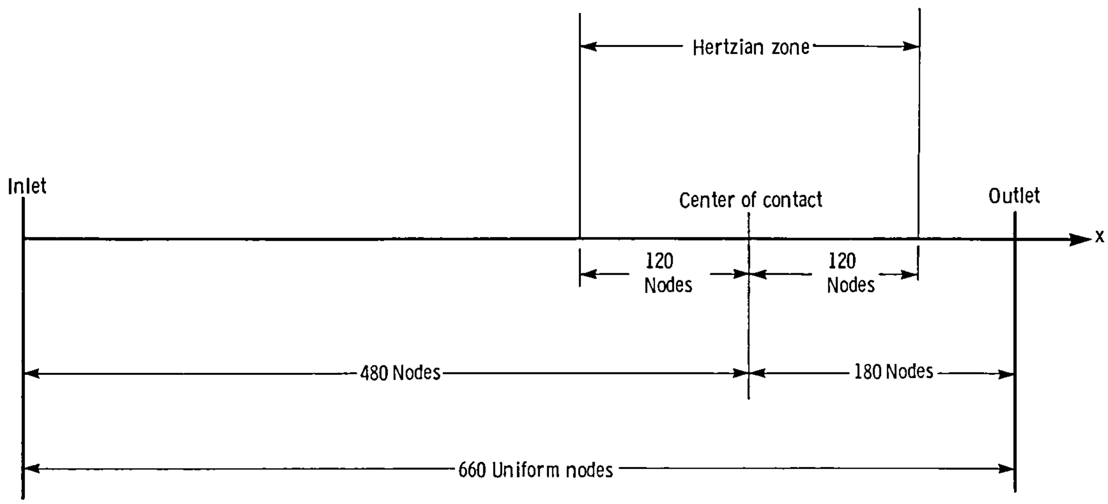


Figure 3. - Sample nodal structure for numerical calculations.

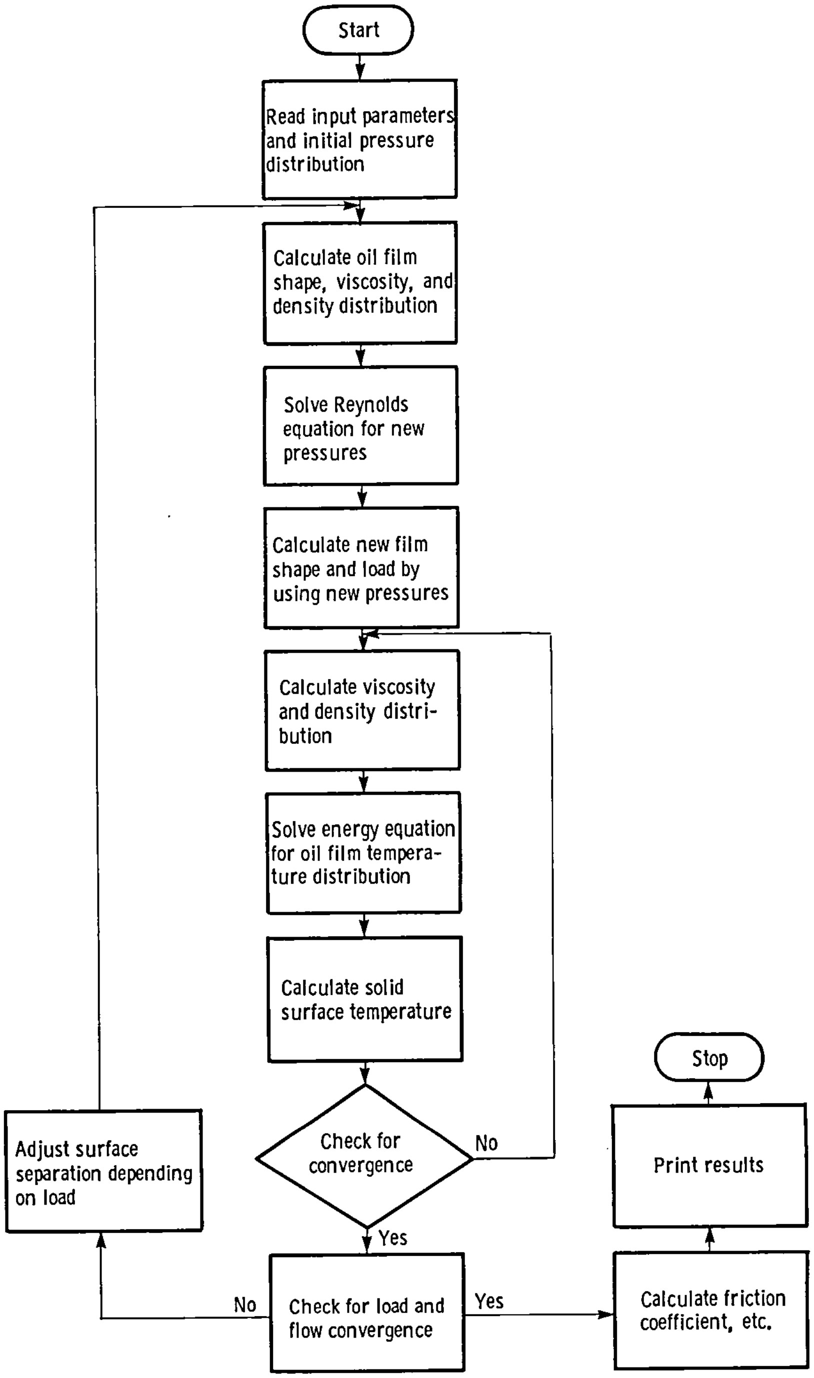


Figure 4. - Flow chart for computational procedure used in thermal elastohydrodynamic studies.

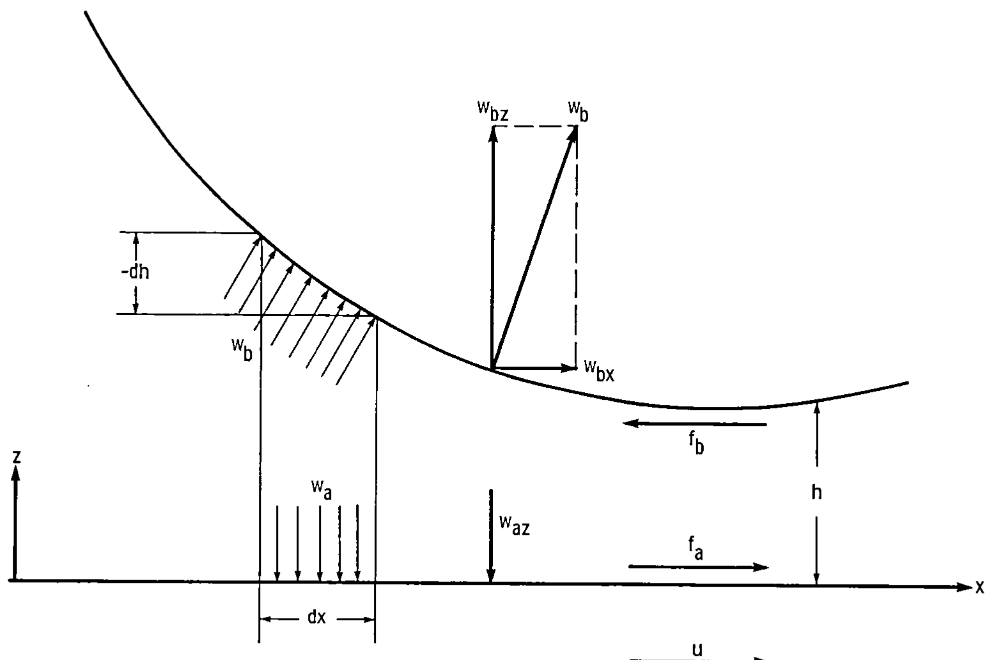


Figure 5. - Force components.

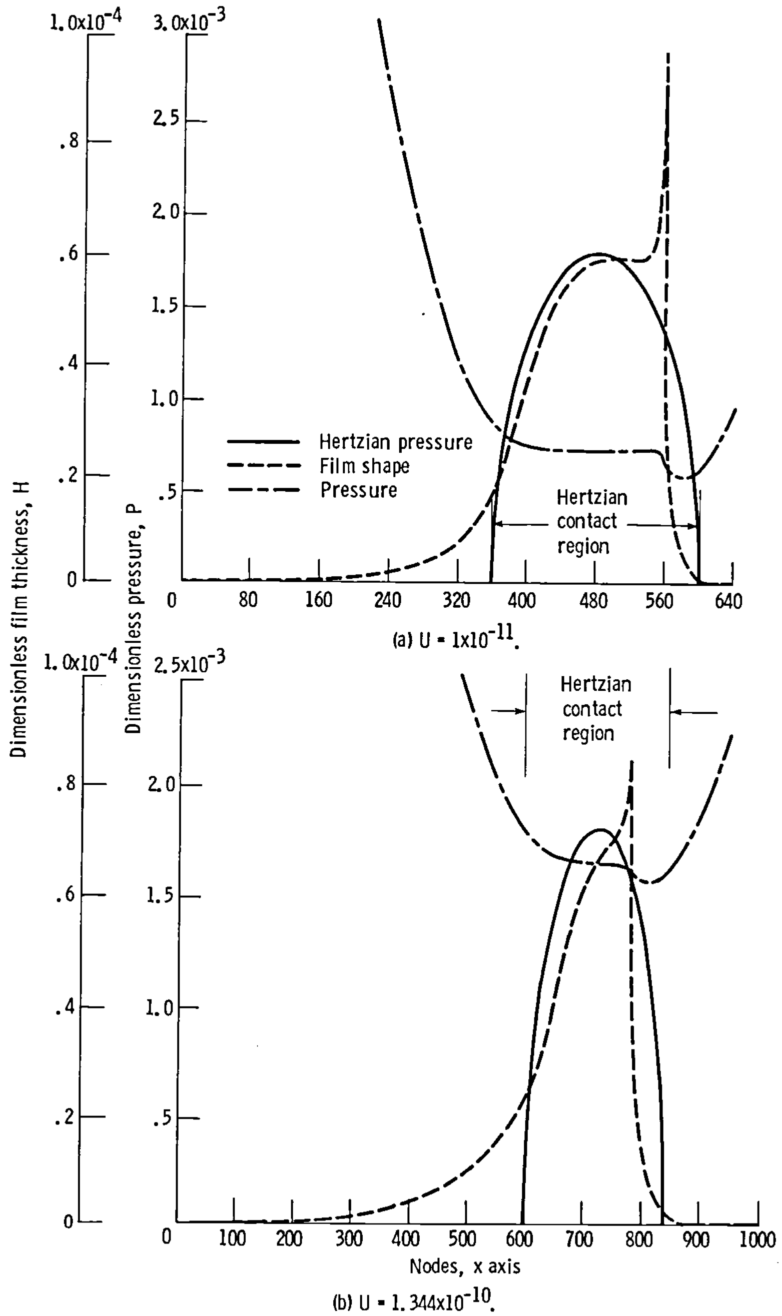


Figure 6. - Variation of dimensionless pressure and film thickness for dimensionless materials parameter G of 5000, dimensionless load parameter W of 2.0478×10^{-5}, and two dimensionless speed parameters U.

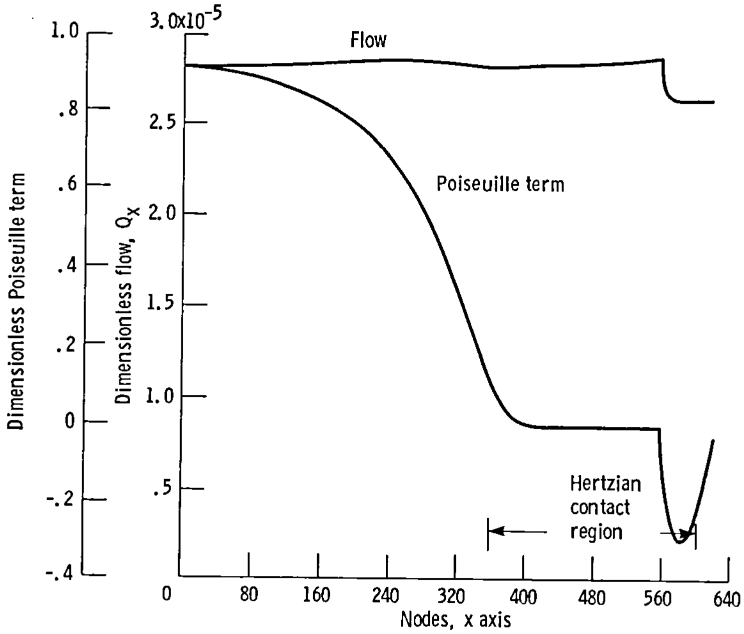


Figure 7. - Representative dimensionless flow and Poiseuille term for dimensionless materials parameter G of 5000, dimensionless load parameter W of 2.0478×10^{-5}, and dimensionless speed parameter U of 1×10^{-11}.

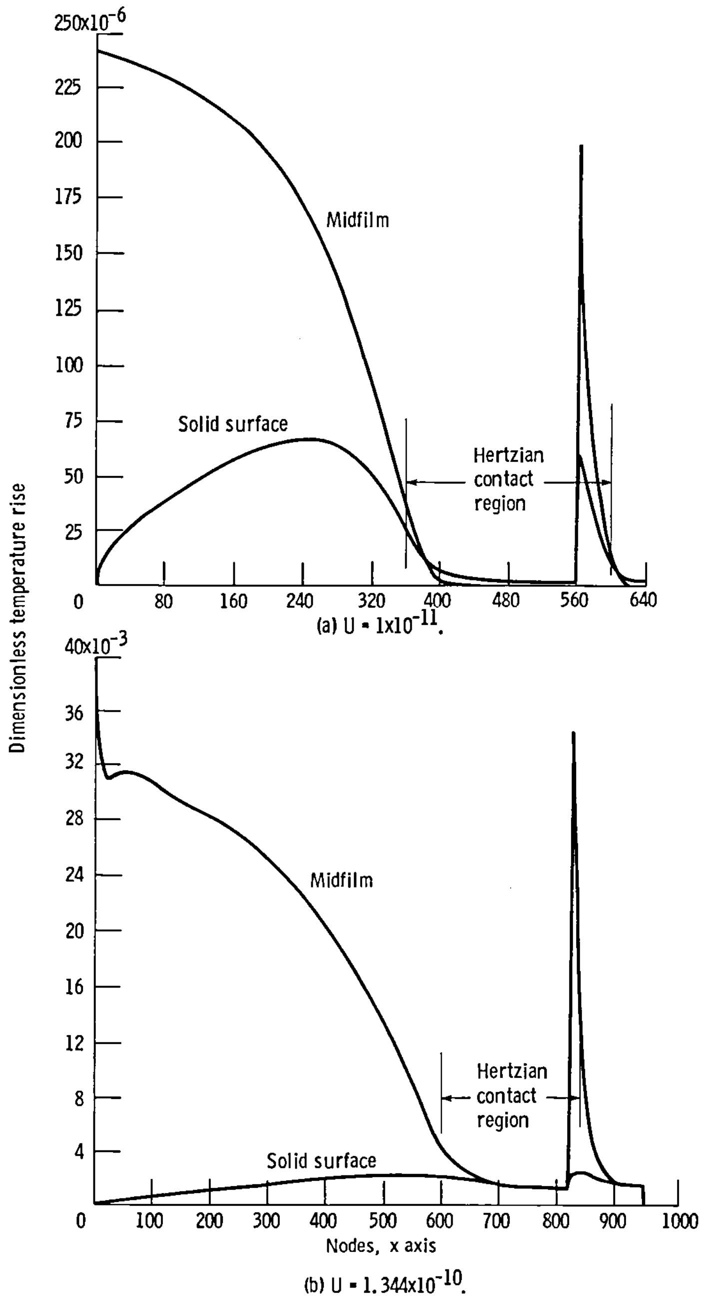


Figure 8. - Variation of dimensionless midfilm and solid surface temperature rise along lubricant film in pure rolling for dimensionless materials parameter G of 5000, dimensionless load parameter W of 2.0478×10^{-5}, and two dimensionless speed parameters U.

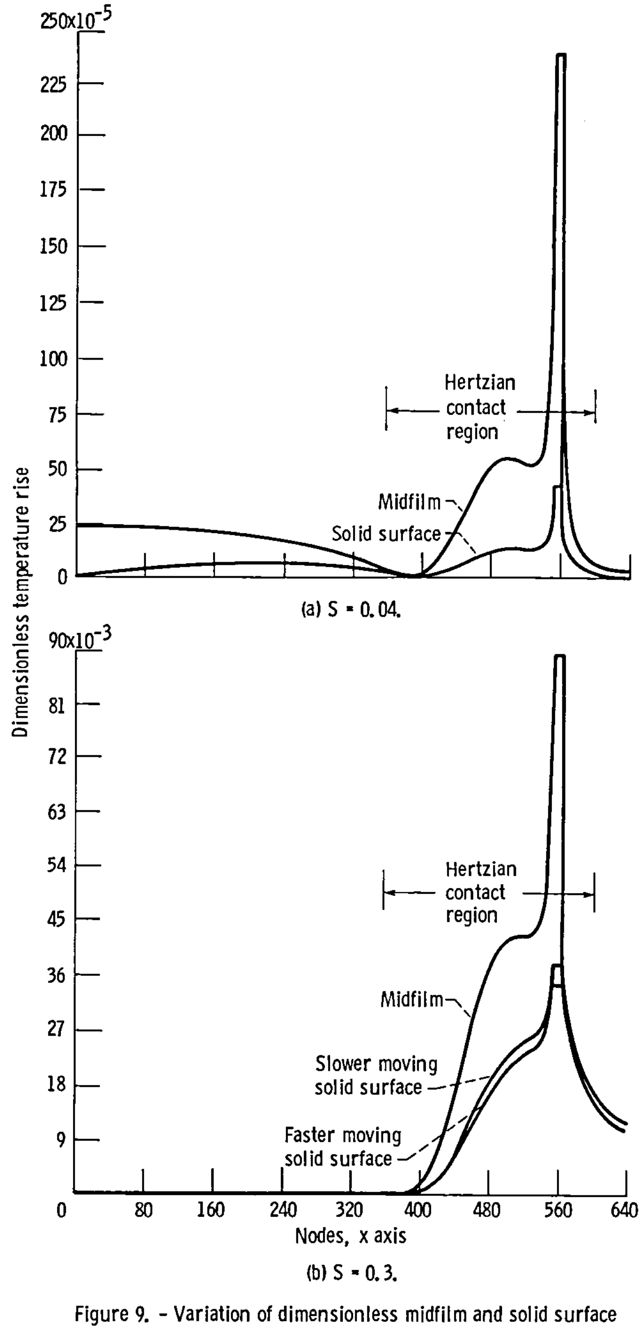
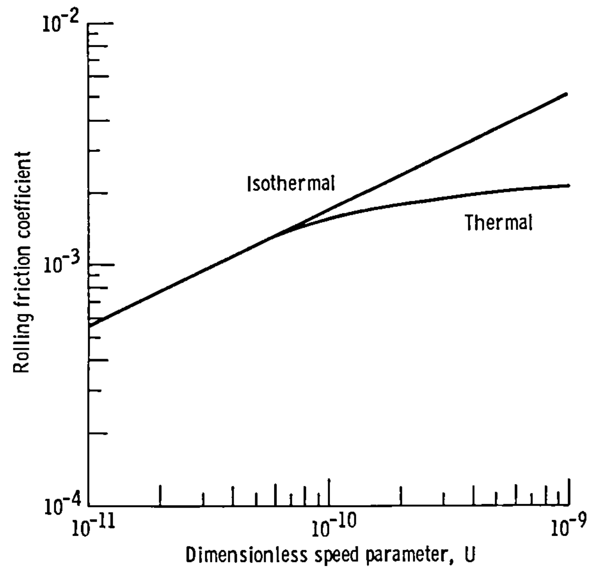
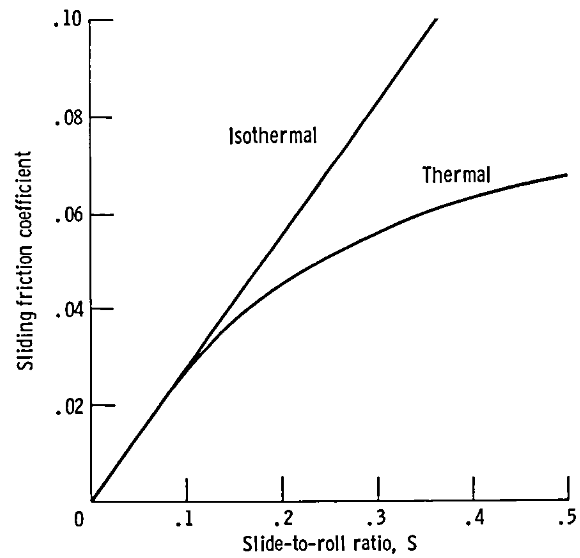


Figure 9. - Variation of dimensionless midfilm and solid surface temperature rise along lubricant film in rolling-sliding case for dimensionless materials parameter G of 5000, dimensionless load parameter W of 2.0478×10^{-5}, dimensionless speed parameter U of 1×10^{-11}, and two slide-to-roll ratios S.



(a) Variation of pure rolling friction coefficient with dimensionless speed parameter for dimensionless materials parameter G of 5000 and dimensionless load parameter W of 2.0478×10^{-5}.



(b) Variation of sliding friction coefficient with slide-to-roll ratio for dimensionless materials parameter G of 5000, dimensionless load parameter W of 2.0478×10^{-5}, and dimensionless speed parameter U of 1×10^{-11}.

Figure 10. - Effect of temperature of rolling and sliding traction.

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15. Supplementary Notes

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16. Abstract

A numerical solution to the problem of thermal elastohydrodynamic lubrication of line contacts was obtained by using a finite difference formulation. The solution procedure consists of simultaneous solution of the thermal Reynolds equation, the elasticity equation, and the energy equation subject to appropriate boundary conditions. Pressure distribution, film shape, and temperature distribution were obtained for fully flooded conjunctions, a paraffinic lubricant, and various dimensionless speed parameters while the dimensionless load and materials parameters were held constant. Reduction in the minimum film thickness due to thermal effects (as a ratio of thermal to isothermal minimum film thickness) is given by a simple formula as a function of the thermal loading parameter Q:

$$\frac{H_{min}}{H_{min,I}} = \frac{10}{10 + Q^{0.4}}$$

Plots of pressure distribution, film shape, temperature distribution, and flow are shown for some representative cases.

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