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THERMAL PERTURBATION OF THE SUN*

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ABSTRACT

An investigation of thermal perturbations of the solar convective zone via changes in the mixing length parameter have been carried out, with a view toward understanding the possible solar radius and luminosity changes that have been cited in the literature. The results show that: (a) a single perturbation of α is probably not the cause of the solar radius change given in ref. 2, and (b) the parameter $W \equiv d\ell nR_{\odot}/d\ell nL_{\odot}$ can not be characterized by a single value, as has been implied in recent work (refs. 2-5)

INTRODUCTION

Recent observations of possible changes of the solar luminosity (ref. 1) and radius (ref. 2) have spurred theoreticians to try to model the physical mechanisms that might produce such changes. One possible mechanism involves thermal perturbations of the solar convection zone. Such perturbations can be modeled (to first order) by perturbing the mixing length parameter α(equal to the ratio of the mixing length to the pressure scale height), used in the standard mixing length theory of convection. In this paper we present the results of such an analysis. Other work in this area can be found in references 2 to 5.

METHOD OF ANALYSIS

The stellar evolution code described by Endal and Sofia (ref. 6) was used for this investigation, utilizing the following assumptions and input data:

(1) Spherically symmetric Sun in hydrostatic equilibrium
(2) Latest Los Alamos opacities and equations of state (ref. 7)
(3) Nuclear reaction rates of Fowler, et al. (ref. 8)
(4) Standard mixing length theory of convection.
(5) The use of 700 interior zones and a small ($6\times10^{-8}M_{\odot}$) static envelope, to insure numerical accuracy.

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A one solar mass ZAMS model was evolved to the present age of the Sun ($T_\odot = 4.7 \times 10^9$ yrs.). A match to the present luminosity and radius of the Sun was found by varying the initial hydrogen abundance, X, and the value of α. The adopted values were X = 0.71242 and α = 2.21772. A sequence was then calculated such that the time step was slowly decreased until ΔT = 1 yr. at $T_\odot$. A perturbation of α in the range 0.05% to 4% was then introduced, and the full non-linear, time-dependent evolution followed either for 200 yrs. (with ΔT = 1 yr.) or for 6×10^7 yrs. (allowing the time step to increase), at which time the normal evolutionary effects dominated the solution. In order to test the effect of varying the size of the small time step, similar sequences were calculated using timesteps of 0.75, 2, and 5 yrs. No difference in the resulting models was seen. The final results are discussed in the next section.

RESULTS

The results of this analysis are summarized in Table I. We can divide the results into three areas, which are discussed below. In the following discussion we adopt the following definitions:

$$\Delta(x) \equiv x_{new} - x_{old} = \text{change in } x.$$

$$\delta(x) \equiv [\Delta(x)*100]/x_{old} = \text{percent change in } x.$$

(1) Solar Luminosity: As can be seen in Table I, the percentage change in $L_\odot$ is of the same sign as $\delta(\alpha)$, and of a much larger value than the corresponding $\delta(R_\odot)$(here $R_\odot \equiv$ solar radius at optical depth 2/3). The relation between $\delta(L_\odot)$ and $\delta(\alpha)$ is linear for $0.75 \leq \delta(\alpha) \leq 4.00$, and can be written down as: $\delta(L_\odot) = 0.76 \cdot \delta(\alpha)$. The small change in slope at $\delta(\alpha) = 0.75$ will be discussed later. The characteristic time scale to recover its initial value is a thermal time scale of $\approx 6 \times 10^6$ yrs.

(2) Solar Radius: In the solar photosphere ($\tau \lesseqgtr 2/3$), $\delta(r)$ shows an initial change of the same sign as $\delta(\alpha)$, followed by a subsequent relaxation toward the new equilibrium radius (larger for $\delta(\alpha)$ negative, smaller for $\delta(\alpha)$ positive) on a much more rapid time scale than $\delta(L_O)$. For $\delta(\alpha) = 2$, the time scale to cross its original value is $\approx$ 200 yrs. For $\tau > 2/3$, (r) simply shows an immediate relaxation toward the new equilibrium value. As Figure 1 shows, $\delta(R_\odot)$, and $\delta(L_\odot)$ to a lesser extent, changes its behavior with $\delta(\alpha)$ at $\delta(\alpha) = 0.75$. The reason for this can be traced to the finite interpolation scheme used for the envelope boundry conditions. Thus the values for $\delta(R_\odot)$ and $\delta(L_\odot)$ are probably not reliable below $\delta(\alpha) = 0.75$.

(3) W: A parameter used by investigators in the field to characterize the changes of radius and luminosity due to any perturbation is $W \equiv d\ell n\ R_\odot / d\ell n\ L_\odot = \delta(R_\odot)/\delta(L_\odot)$. Figure 2 shows a plot of W_0 vs $\delta(\alpha)$ for the data in Table I. Here $W_0 = W$ for the first model after the perturbation is introduced. As noted, the values of W are not reliable for $\delta(\alpha) < 0.75$. We see that (a) since $\delta(R_\odot)$ changes on a much faster time scale than $\delta(L_\odot)$, W will change with time (and will change sign, e.g. after $\approx$ 200 yrs. for $\delta(\alpha) = 2\%$), and (b) as Figure 2 shows, the value of W_0 changes with $\delta(\alpha)$. Hence, there really is no single value for W, as has been implied by earlier investigations.

CONCLUSION

The main conclusions that can be drawn from this investigation are as follows:

(1) $\delta(L_\odot) = 0.76\ \delta(\alpha)$, while $\delta(R_\odot)$ displays a marked non-linear behavior with $\delta(\alpha)$.

(2) There is no one value of W which generally characterizes thermal perturbations of the solar convective zone.

(3) Even for $\delta(\alpha) = 5$, $\delta(R_\odot) \approx 2.10^{-3}$ while $\delta(L_\odot) \approx 3$! Thus for $\delta(R_\odot) = 2.5\mathrm{x}10^{-2}$ (ref. 2), the extremely large $\delta(L_O)$ implied shows that a single perturbation of α is probably not the cause of the radius change, although further work for $\delta(\alpha) < 0.75$, and for a series of random α perturbations is definitely indicated.

(4) Since our results for $\delta(\alpha) < 0.75$ are probably incorrect, a value of W near 0.075 for small $\delta(\alpha)$ is not precluded. Thus our results show that by taking into account the full time-dependent, non-linear behavior of the problem, the entire range of W quoted in the literature (refs. 2 to 5) may be generated.

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Table 1

$\delta\alpha$	$\delta R_{\odot}$	$\delta L_{\odot}$	$W = \frac{\delta R_{\odot}}{\delta L_{\odot}}$
.05	3.818×10^{-5}	.0379	1.007×10^{-3}
.1	7.644×10^{-5}	.0758	1.008
.3	2.303×10^{-4}	.2277	1.015
.75	5.802×10^{-4}	.5701	1.018
.8	5.974×10^{-4}	.6055	.987
.85	6.146×10^{-4}	.6410	.959
.9	6.319×10^{-4}	.6764	.934
.95	6.493×10^{-4}	.7119	.912
1.0	6.667×10^{-4}	.7473	.892
1.25	7.577×10^{-4}	.9256	.819
1.5	8.460×10^{-4}	1.103	.767
2.0	10.262×10^{-4}	1.459	.703
2.5	12.119×10^{-4}	1.815	.668
3.0	14.033×10^{-4}	2.173	.646
3.5	16.038×10^{-4}	2.533	.633
4.0	18.047×10^{-4}	2.893	.624

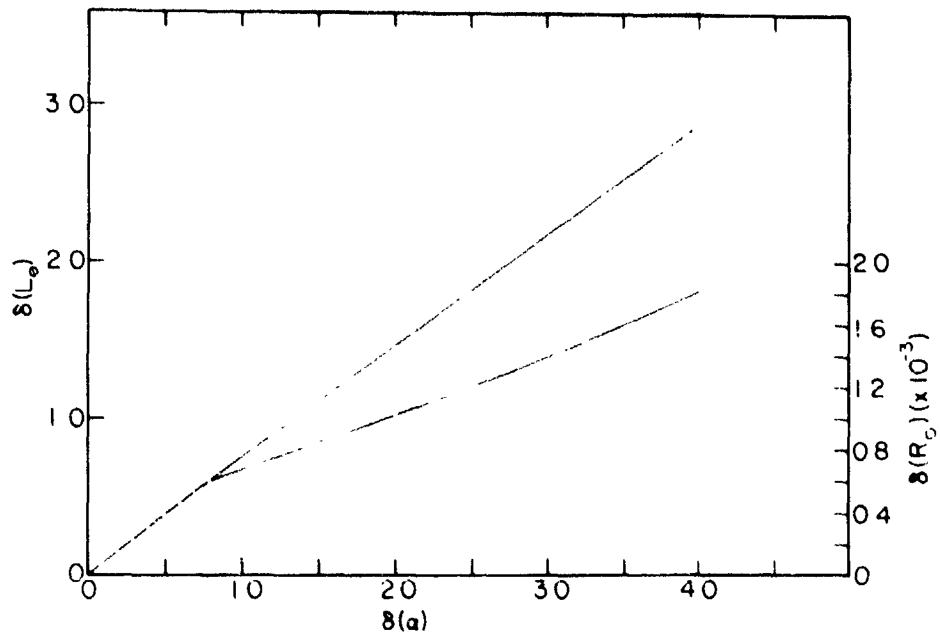

Figure 1. The relation between $\delta(L_\theta)$ and $\delta(\alpha)$, (solid line) and between $\delta(R_\theta)$ and $\delta(\alpha)$, (interrupted line). The data for $\delta(\alpha) < 0.75$ is unreliable (dashed line).

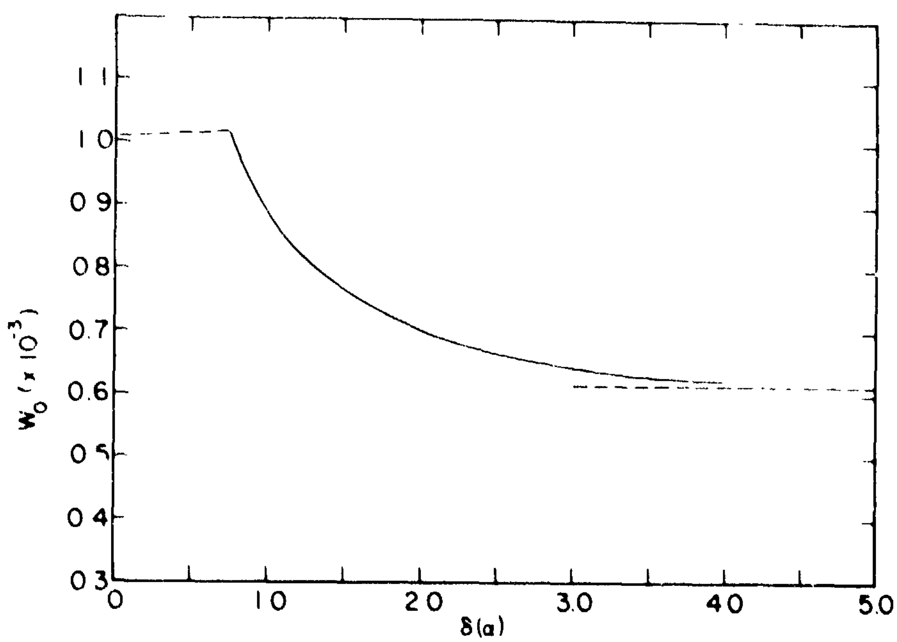

Figure 2. The relation between W_θ and $\delta(\alpha)$. The data for $\delta(\alpha) < 0.75$ is unreliable (dashed line). The dashed line for $\alpha > 3$ is only intended as a guide for the eye.