THERMAL RADIATION IN DISPERSE SYSTEMS: AN ENGINEERING APPROACH

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ISBN: 978-1-56700-268-3

Printed in the United States of America 1 2 3 4 5 6 7 8 9 0

Library of Congress Cataloging-in-Publication Data
Dombrovsky, L. A.
Thermal radiation in disperse systems: an engineering approach / Leonid A. Dombrovsky, Dominique Baillis.
p. cm.
ISBN 978-1-56700-268-3 (hard cover)
1. Heat--Radiation and adsorption. 2. Heat--Transmission. 3. Heat engineering. I. Baillis, Dominique. II. Title.
QC331.D663 2011
621.402'2--dc22
2010028983

Direct inquires to Begell House, Inc., 50 Cross Highway, Redding, Connecticut

To partners in our life
Galina Dombrovskaya
and Christophe Baillis,
and to our five children:
Konstantin, Kirill,
Maxime, Pierre, and Thomas

The monograph provides a systematic consideration of diverse problems of thermal radiation in disperse systems. A set of problems discussed in some details includes the thermal radiation of two-phase combustion products in rocket engines, the spectral radiative properties of advanced thermal insulations, the microwave thermal radiation of disperse systems on the sea surface, and the thermal radiation in a multiphase medium, formed in the case of hypothetic severe accident of a nuclear reactor. The theoretical models developed by the authors are mainly based on the Mie theory for the radiative properties of single particles and approximate methods for the radiation transfer in anisotropically scattering media. The experimental characterization of dispersed materials like foams, porous ceramics, fibrous and nanoporous insulations is based on directional-hemispherical measurements in a wide infrared spectral range and the mathematical identification procedure for the material radiative properties.

A wide use of simplified theoretical models and relatively simple computational and experimental procedures form the so-called engineering approach which appears to be very useful in solving many practical problems. The examples for the solutions of several particular problems are also presented in the book. Therefore, this book can be considered as a manual on applied radiative and combined heat transfer problems. It is destined for students, engineers, and researchers in the field of heat transfer. Numerous references presented in the book enable an interested reader to undertake a further study of specific thermal radiation problems in disperse systems.

Dombrovsky L.A.
Baillis D.

There are only two ways to live your life.
One is as though nothing is a miracle.
The other is as though everything is a miracle.

Albert Einstein

Photo by Sasha Shevelko

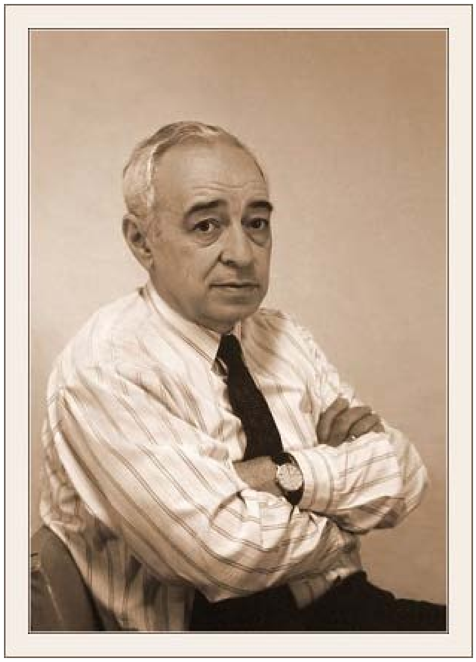


Photo by Leonid Dombrovsky

Leonid A. Dombrovsky is a Chief Researcher of the Joint Institute for High Temperatures of the Russian Academy of Sciences. He received the Candidate (PhD) degree in 1974 from the Moscow Institute of Physics and Technology and the Doctor of Sciences degree in 1990 from the Research Institute of Thermal Processes, Moscow, Russia. His research interests have been focused on theoretical modelling of radiative and combined heat transfer in disperse systems including wide-range spectral properties of particles and fibers, modified differential approximations for radiative transfer, and the problem-oriented computer codes for solving combined heat transfer problems. He has published more than 150 research papers, mainly in refereed journals, and the monograph Radiation Heat Transfer in Disperse Systems (Begell House, New York, 1996). The Fifth Radiation Symposium on Radiative Transfer (Bodrum, Turkey, 2007) was dedicated to Leonid Dombrovsky in recognition of his valuable contributions to the radiation research field.

Dominique Baillis is a Professor of the INSA Lyon Thermal Science Center (CETHIL). She received the PhD degree in 1995 from INSA (National Institute of Applied Sciences) Lyon, France and the Professor degree in 2007 from the same institution. Her research interests have been focused on experimental identification and theoretical modeling of thermal radiative and conductive properties of diverse disperse systems such as advanced thermal insulations, fibrous materials, porous ceramics, foams, and nanoporous materials. She has published more than 50 research papers. Most of Professor Baillis's research has been carried out jointly with students. She has guided the work of more than 13 doctoral and 10 master students coming from many countries of the world, during last 14 years. She has also directed research of 8 postdoctoral researchers.

Preface

A number of technological processes and natural phenomena are accompanied by heat transfer concerned with media thermal radiation. Generally, thermal radiation is thought to be relevant only at high temperatures. This widespread error is easily overcome if we recall, for example, that the weather and climate on our planet are mainly determined by thermal radiation of cloudy atmosphere and the Earth surface. Few people are aware that the quality of an ordinary sleeping-bag is connected with changing of radiation transfer conditions in a fibrous material.

In the examples mentioned, as well as in many other cases, thermal radiation emission, absorption, and scattering take place in a medium containing numerous particles of size comparable with the radiation wavelength. Such media are customary called the disperse systems. One has to solve radiation heat transfer problems for disperse systems in highly different applications such as heat transfer in solid propellant rocket engines and solar chemical reactors, characterization of advanced composite coatings and highly porous thermal insulations, microwave remote sensing of the ocean surface with breaking waves, and spacecraft thermal control by use of a liquid droplet radiator. The geometrical scales of particles, bubbles, and pores in the above mentioned thermal radiation problems and in many other problems may vary in a very wide range — from nanometers in some advanced materials to several millimeters or even greater in the microwave applications. As a result, both the experimental technique and theoretical modeling should be based on a general physical analysis of electromagnetic waves interaction with single particles and adequate description of the radiation propagation in complex disperse systems. It goes without saying that direct simulation of the radiation emission, absorption, and scattering based on the first principles is impractical at the moment and one should find alternative engineering approaches by using the known solutions to some simplified problems. Of course, a correct choice or elaboration of an approximate model which is appropriate to the problem to be solved depends on personal skill and experience of a researcher in this field. In our book, we were trying to do our best to help our young colleagues in improving their knowledge and qualification in thermal radiation problems specific for various disperse systems.

One should remember the usual error of some people who are starting to work in heat transfer modeling. They think that all the problems can be solved by more and more computational skill in combination with great possibilities of the present-day supercomputers. We have several arguments which show that this ambitious point of view is not correct:

- In many problems, radiative heat transfer is not a sole transfer mode and it should be considered simultaneously with the conduction and convective heat transfer. The complex hydrodynamic processes and phase change in the medium components makes a rigorous mathematical statement of such transient combined problems too complicated for the direct numerical simulation.

- The spectral radiative properties of substances are not well-known especially at very low or high temperatures as well as in the regions of extreme values of other physical parameters. The uncertainties in these properties limit the resulting accuracy of the radiation field calculations and make the use of detailed numerical simulation to be not so important.

To our mind, the understanding of physics and the use of relatively simple theoretical models are very important components of the engineering approach to experimental and computational study of thermal radiation in disperse systems.

The known textbooks on thermal radiation are not focused on detailed analysis of radiative properties of various disperse systems and do not give practical examples of solving the radiative and combined heat transfer problems. In this book, we were trying to bridge a gap between the ordinary university education and the research and engineering work. The contents of the book is determined by research and teaching experience by the authors in this field. To make reading easier, we avoid detailed derivations and give the minimal mathematical transformations. All these details can be found in the referenced archive papers.

Of course, the analysis of some problems considered in the book is not so detailed. A reader could find these sections as a kind of starting points which still await his or her further research contributions. But we hope that our efforts were sufficient to pave the way for engineers and researchers in the field of thermal radiation and combined heat transfer in disperse systems. By including a large number of references for further reading, the book may also be used as a reference book by the practicing engineer.

The authors are grateful to the colleagues Wojciech Lipiński, Jaona Randrianalisoa, Remi Coquard, Herve Kamdem Tagne, Sylvain Lallich, Mathilde Loretz, Aurélie Kaemmerlen, Ségoléne Gauthier, Luis Moura, and Mikhail Davydov for their contribution to the studies presented in Sections 1.5.3.1, 1.7.2, 3.3–3.9, and 4.11 of the book. The interest to our joint project and support by professors Jean-Francois Sacadura, Dany Escudie, Truc-Nam Dinh, Leonid Zaichik, Yuri Zeigarnik, and Alexander Leontiev is much acknowledged.

Introduction

The physical basis of the majority of solutions considered in this book is the notion of radiation transfer in an absorbing and scattering medium as some macroscopic process, which can be described by a phenomenological transfer theory and radiative transfer equation for spectral radiation intensity. It is of great importance that the problems, to which the radiation transfer theory can be applied, are quite numerous and contain thermal radiation of various disperse systems. In the book, we use the following main assumptions concerning disperse system properties and radiative transfer:

- Radiation propagation is more rapid than any change of physical parameters, therefore the radiation intensity field is quasi-steady;
- Radiative properties of the medium do not depend directly on the radiation intensity, but they vary only with temperature;
- Wave polarization can be ignored in radiative transfer calculations;
- Radiation scattering is not accompanied by any frequency variation.

In many (but not all) cases, we assume also that characteristics of absorption and scattering of radiation by a small (elementary) volume of the disperse system can be determined from the properties of single particles regardless of any collective effects. The last assumption simplifies the problem, and gives also a chance for direct investigation of the medium composition influence on thermal radiation transfer. The restrictions occurring from this assumption are usually not as significant, as might seem, since the assumption about small collective (dependent scattering) effects remains valid up to a high enough particle concentration.

The radiation transfer theory has been developed by a number of famous scientists working in the physical optics, astrophysics, nuclear reactor theory, and heat transfer theory. The mathematical theory was created containing up-to-date analytical and numerical methods. Numerous particular publications dealt with computational methods applied to radiative transfer problems. Together with the development of the radiation transfer theory, significant achievements took place in theoretical investigations of particle radiative properties for various disperse systems. Properties of particles comparable with the wavelength turned out to be diverse and complex. Many applied investigations and well-known monographs were published on this subject. In order to solve the practical problems of thermal radiation in disperse systems, one needs to combine achievements of both the scattering theory and radiation transfer theory. A reasonable choice of the method for solving the radiative transfer equation depends upon the medium properties. On the other hand, the requirements for completeness and accuracy of single particle properties are determined by essential precision of radiation flux calculations. This was also reflected in the book.

While solving many practical heat transfer problems, one should take into account not only the thermal radiation but also heat transfer by conduction and convection in the medium. Most general problems of combined radiative-convective heat transfer are very complicated and their solution is possible only by employing approximate computational models for radiative transfer. Therefore, more attention was given to errors resulting from these approximate models.

As the measurements of radiative characteristics of diverse dispersed materials are very important to develop and validate appropriate theoretical models, a special emphasis is put on recent overviews of experimental characterization of spectral radiative properties of disperse systems and comparison between experimental and theoretical results. The examples of the simultaneous use of both theoretical analysis and experimental identification to understand unusual radiative properties of quite different porous materials of complex morphology are expected to be interesting for potential readers to know the complete set of tools employed in this field.

It is natural that the choice of material for this book corresponds to the field of practical work by the authors. The book is divided into four chapters. Chapter 1 deals with computational models for radiative transfer in disperse systems. The main attention is given to simple approximate models, both traditional and modified, which have a clear physical sense and enable one to derive some useful analytical solutions to classic problems. Computer codes based on these approximate models for radiative transfer are widely used in engineering practice especially in combined heat transfer calculations. The error of various approximations is analyzed in some details by comparison with exact analytical and numerical solutions. Approximate models presented in Chapter 1 form a basis of solutions obtained for applied problems considered in the book. A detailed numerical simulation of radiative transfer using the discrete ordinate method and the Monte Carlo procedure as applied to disperse systems is also discussed in this chapter.

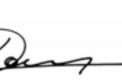
Spectral radiative properties of single particles and fibers are considered in some detail in Chapter 2. The theoretical part of this chapter includes the Mie solution for homogeneous spherical particles and more general solutions for hollow and core-mantled spheres. We give also the known solution for arbitrary illumination of long cylinders. This solution is widely employed in modeling the radiative properties of single fibers and highly porous fibrous materials. A complete set of equations is presented for homogeneous, hollow, and two-layered cylinders. The main limiting cases of the general theory (the Rayleigh and Rayleigh–Gans scattering, the geometrical optics, and the anomalous diffraction approximation) are considered in Section 2.2. Absorption and scattering of the visible, infrared, and microwave radiation by single particles and fibers of various substances are analyzed in Section 2.3. Thermal radiation from nonisothermal particles and the radiation from a particle to ambient medium through narrow concentric gap are considered in Sections 2.4 and 2.5. The radiative properties of polydisperse systems and applicability of monodisperse approximation are discussed in the last section of Chapter 2.

Chapter 3 presents an engineering approach for both theoretical prediction and experimental determination of spectral radiative properties of quite different dispersed materials containing the morphology elements of arbitrary shape. A general theoretical basis of radiative properties determination and present-day principles of experimental characterization with identification procedure are recalled. Physical limitations of independent scattering theory are also discussed in this chapter. Experimental and computational results, approximate theoretical models and engineering estimates important for potential applications are presented for porous materials such as cellular foams, fibrous materials, ceramics, polymer coatings containing microspheres, and nanoporous aerogel superinsulations. The materials under investigation can be applied in advanced energy and combustion systems, such as low-NO_x combustion burners, solar thermal energy systems or specific applications requiring lightness and high insulating efficiencies. Thus the characterization of radiative properties of such dispersed materials plays an important role in many engineering systems.

Some radiative and combined heat transfer problems in various disperse systems are considered in Chapter 4. These problems include the main results for radiation heat transfer in solid-propellant rocket engines (Section 4.1), the problems of radiative cooling of particle flow in vacuum (Section 4.2), the combined heat transfer in boundary-layer flows (Section 4.3), the thermal microwave radiation of foam and water sprays produced by breaking ocean waves (Section 4.4), the radiative-conductive heat transfer in composite coatings, fibrous materials, and foam insulations (Sections 4.5, 4.6), the radiative effects in a semi-transparent liquid containing gas bubbles (Section 4.7), the effects of nonuniform absorption and heating of semi-transparent particles by an external radiation (Section 4.8, 4.9), and the thermal radiation modeling in multiphase flows with high-temperature nonisothermal particles (Sections 4.10 and 4.11).

Chapters 1, 2, and 4 are only partially based on the revised material of the previous book by Leonid Dombrovsky "Radiation Heat Transfer in Disperse Systems" (Begell House, New York, 1996). These chapters include some new results obtained in the period from 1994 to 2010. Sections 1.7 and 4.6.2, which are written by Dominique Baillis, are also presented in Chapters 1 and 4. As to Chapter 3, it involves mainly a systematic presentation of the research by a group of Dominique Baillis. Sections 3.6–3.9 of this chapter are written by Leonid Dombrovsky on the basis of the research work, which has been done in cooperation with Dominique Baillis and her students.

For a topic which is broad as the one considered in this book, it is very difficult to be comprehensive. However, we hope that enough key references are cited in the book to enable an interested reader to undertake a more detailed study of specific thermal radiation problems in disperse systems.



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Dominique Baillis Doermann

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1. Computational Models for Radiative Transfer in Disperse Systems

In this Chapter, we will not consider the nature and basic laws of thermal radiation because this general knowledge is given in the well-known textbooks by Sparrow and Cess [1], Siegel and Howell [2], and Modest [3]. Nevertheless, before proceeding to mathematical formulation of radiation transfer problems for scattering media, it is reasonable to recall some definitions of the main physical quantities.

The radiation energy in the wavelength interval (λ, $\lambda + d\lambda$), passing per time dt in the solid angle $d\vec{\Omega}$ near the direction $\vec{\Omega}$ through the area $d\sigma$ located at the point $\vec{r}$ and oriented perpendicular to $\vec{\Omega}$, is equal to $I_\lambda(\vec{r}, \vec{\Omega})d\lambda\, dt\, d\sigma\, d\vec{\Omega}$, where the function $I_\lambda(\vec{r}, \vec{\Omega})$ is the spectral intensity of radiation. This function is the most general characteristic of the radiation field in the case of unpolarized (randomly polarized) radiation. The polarization of electromagnetic waves is usually not important in the problems concerning thermal radiation and it is sufficient to use the scalar function $I_\lambda(\vec{r}, \vec{\Omega})$ instead of the Stokes parameters [4]. The details concerning description of polarized radiation can be found in the classic book by Chandrasekhar [5] and in the monographs by van de Hulst [6] and Bohren and Huffman [7] which are very close to the problems under consideration and should be included by a reader in a shortlist of the most important handbooks. The spectral intensity of equilibrium (the so-called "black-body") thermal radiation of an isothermal medium is given by the Planck function: $I_\lambda = B_\lambda(T)$. The black-body radiation is homogeneous and isotropic, i.e., independent of both the coordinate $\vec{r}$ and direction $\vec{\Omega}$. For radiating medium, a deviation of the function $I_\lambda(\vec{r}, \vec{\Omega})$ from the intensity of equilibrium radiation at a local temperature $T(\vec{r})$ is described by radiative transfer equation.

Absorption and scattering of the radiation in a medium are described by the spectral coefficients α_λ and σ_λ, respectively, by the extinction coefficient $\beta_\lambda = \alpha_\lambda + \sigma_\lambda$, and by the scattering function $\Phi_\lambda(\vec{\Omega}'\vec{\Omega})$ called also the scattering phase function or indicatrix of scattering. The latter function presents the angular intensity distribution for the radiation scattered by a small (elementary) volume of the medium by one act of scattering. The scattering function satisfies the normalizing condition:

$$\frac{1}{4\pi}\int_{(4\pi)}\Phi_\lambda\left(\vec{\Omega}'\vec{\Omega}\right)d\vec{\Omega}' = 1 \tag{1.1}$$

Note that the coefficients α_λ, σ_λ, and β_λ are also referred to the medium elementary volume. It is assumed that absorption and scattering characteristics of a small element of the medium can be determined on the basis of the so-called far-field single-scattering approximation. This assumption, which is also known as the independent scattering approximation, is correct in many applied problems concerning rarefied disperse systems when positions of single particles are random and uncorrelated and the average distances between the neighboring particles are greater than the particle size and radiation wavelength. Note that single particles we are talking about may look as aggregates or clusters of some primary particles. The physical sense of the above formulated assumption has been considered in some details in the recent paper by Mishchenko et al. [8].

The above definitions of the absorption and scattering characteristics of a medium correspond to the continuous model of the radiation propagation in the medium. It is a widely used approach, which appears to be fairly good in many practical problems concerning thermal radiation in disperse systems. But there are some specific cases such as particulate debris beds or large-scale cellular structures when the classical continuum theory may not be appropriate and the discrete transfer models are physically more adequate to the real processes [9, 10].

1.1 The Radiative Transfer Equation

In the continuum theory, the radiative transfer equation (RTE) for an absorbing, scattering and radiating medium can be written as follows [2, 3]:

$$\vec{\Omega}\nabla I_\lambda\left(\vec{r},\vec{\Omega}\right)+\beta_\lambda I_\lambda\left(\vec{r},\vec{\Omega}\right)=\frac{\sigma_\lambda}{4\pi}\int_{(4\pi)} I_\lambda\left(\vec{r},\vec{\Omega}'\right)\Phi_\lambda\left(\vec{\Omega}'\vec{\Omega}\right)d\vec{\Omega}'+\alpha_\lambda B_\lambda(T) \qquad (1.2)$$

The physical meaning of Eq. (1.2) is evident: variation of the spectral radiation intensity in direction $\vec{\Omega}$ takes place due to extinction by absorption and by scattering in other directions, as well as due to scattering from other directions (integral term) and thermal radiation of the medium. The coefficients α_λ, σ_λ, β_λ, function $\Phi_\lambda(\vec{\Omega}'\vec{\Omega})$, and temperature T depend on the coordinate $\vec{r}$. For simplicity, Eq. (1.2) is not written for the general case of an arbitrary medium. The following assumptions are used:

- The medium is isotropic, i.e., coefficients of RTE do not depend on direction;
- Every small (elementary) volume of the medium is characterized by only one temperature for all the medium components;
- There is no refraction in the host medium (the index of refraction is equal to unity);
- The medium properties do not depend directly on radiation intensity.

Only the last of these assumptions will not be revised in this book because we are not going to consider the nonlinear radiation effects. Of course, variation of the RTE coefficients with temperature or local composition of the medium will be considered and it may be the result of the radiation heat transfer. But these effects are quite different from the direct effect of radiation field on the medium radiative properties. The other assumptions from the above list will be revised in subsequent sections of the book by studying the specific problems of radiative transfer in disperse systems.

For understanding a mathematical formulation, it is usually convenient to imagine an adequate physical picture of the problem. To our mind, the RTE seems to be clear if we imagine numerous randomly located particles suspended in a gas. For example, one can assume that there is no any external radiation but only the own thermal radiation of the particle cloud. The hot particles not only emit thermal radiation, but absorb and scatter the radiation emitted by other particles. The radiation can also be emitted and absorbed by a host gaseous medium. There is a local thermal equilibrium between the particles and ambient gas. It means that the temperatures of gas and all the particles in a small volume are the same. A small single-scattering volume of the particle cloud contains many particles but an average distance between them is much greater than the particle size and radiation wavelength, so that there are no any wave (interference) effects and the absorption and scattering characteristics of single particles are additive. Obviously, one can easily define the local absorption and scattering coefficients as well as the scattering function of this medium. Gases have no contribution to scattering which is determined by the presence of particles. The above physical picture is realistic but it is only the simplest variant of diverse disperse systems. In more complex problems, it is not obvious if the ordinary RTE is applicable.

The most general analysis should be based on the electromagnetic theory. It is especially important for relatively dense disperse systems. The relation between the rigorous electromagnetic theory and the phenomenological radiation transfer theory is a subject of theoretical studies during several decades. We do not consider this problem in the book assuming that RTE is applicable to the majority of the engineering problems to be considered. Nevertheless, we can recommend important references [11–16] concerning this matter.

It may be of interest to remember the history of developing the radiation transfer theory including the analytical and computational methods. The astrophysics, particularly the study of star photospheres, was the first branch of science which initiates the theoretical foundations and analytical methods of the radiation transfer theory in the beginning of the 20^{th} century [5, 17]. A new very important period was related to the nuclear physics because transport of neutrons is described by a similar equation. Some well-known analytical and numerical methods of the general neutron/photon transport theory have been developed at that time [18–23]. Computational modeling of neutron transport in nuclear reactors was one of the first engineering applications of the theory. In parallel with the nuclear studies, the radiation transfer theory was used and further developed by researchers working in the field of atmospheric optics and thermal radiation transfer in the atmosphere [24–28]. Starting from the 60^{th} of the last century, the high-temperature processes in thermal engineering provide a great field for modifications and practical applications of the radiation transfer theory. One can remember such problems as radiation heat transfer in furnaces and combustion chambers of rocket engines, the radiative heating of space vehicles in the atmosphere, the radiative-conductive heat transfer in material processing and in highly porous thermal insulations [29–43]. The specific of these problems is combined heat transfer by radiation, conduction, and convection. The interaction of different heat transfer modes makes the mathematical formulation too complicated for analytical study or direct numerical simulation. It was a motivation of the interest to the use and modification of simplified approaches developed by physicists and mathematicians in astrophysics and nuclear engineering. It is interesting that the present-day period in the history of radiation transfer theory and its applications are characterized by increasing efforts in solving the specific problems of medical imaging and diagnosis. This problem appears to be very complicated. The new developments concerning the optical tomography and the propagation of infrared radiation in a biological tissue can be found in the literature [44–55].

Note that radiative properties of dense disperse systems characterized by very small scattering can be studied on the basis of quite different approach. It is an electrodynamic model of effective absorbing and refracting medium. The classical model of this type for optical properties of colloidal solutions and particularly semi-transparent metal films has been developed by Maxwell-Garnett [56]. The main idea of this model was that small metal particles in a dielectric host medium can be considered as single dipoles and the composite disperse medium is treated as a homogeneous medium with some optical constants. These optical constants are determined by use of the Lorentz–Lorenz relation for dielectrics with known dipole moments of molecules [4]. We will also employ this approach for some specific problems of those considered in the book. One can find the details of the effective medium theories in the literature [57–60].

Let us return to radiative transfer equation (1.2). Obviously, one needs a boundary condition to complete the problem formulation. Basically, the boundary condition on a body surface should take into account angular characteristics of surface reflection and emission [2, 3]. The Fresnel reflection [4] can be considered to formulate the boundary condition at ideally smooth interface between the substances having the known optical constants. We will use the Fresnel relations in analysis of some model problems. As far as the above discussed problem of the thermal radiation of a particle cloud is concerned, one can use the simplest boundary condition of zero external radiation at the cloud surface:

$$I_\lambda\left(\vec{r},\vec{\Omega}\right)=0 \qquad \vec{\Omega}\cdot\vec{n}<0 \tag{1.3}$$

where $\vec{n}$ is the external normal to the boundary surface. In further analysis, we will need not only the radiation intensity but also some other characteristics of the spectral radiation field. First of all, we introduce the spectral energy density of radiation:

$$E_\lambda\left(\vec{r}\right)=\frac{1}{c}\int_{(4\pi)} I_\lambda\left(\vec{r},\vec{\Omega}\right)d\vec{\Omega} \tag{1.4}$$

where c is the velocity of light. For brevity, the name "spectral radiation energy density" is also used for the quantity

$$I_\lambda^0(\vec{r}) = cE_\lambda(\vec{r}) = \int_{(4\pi)} I_\lambda(\vec{r},\vec{\Omega})d\vec{\Omega} \tag{1.5}$$

which is also called spectral irradiance. Note that $I_\lambda^0 = 4\pi B_\lambda(T)$ for the black-body radiation at temperature T. We will also use the spectral radiation flux given by

$$\vec{q}_\lambda(\vec{r}) = \int_{(4\pi)} I_\lambda(\vec{r},\vec{\Omega})\vec{\Omega}\, d\vec{\Omega} \tag{1.6}$$

The spectral radiation flux from the black-body surface radiating in vacuum is equal to $\vec{q}_\lambda = \pi B_\lambda(T)\vec{n}$, where $\vec{n}$ is the external normal to this surface. Aside from spectral characteristics of the radiation field, one can also employ the corresponding integral quantities:

$$I(\vec{r},\vec{\Omega}) = \int I_\lambda(\vec{r},\vec{\Omega})d\lambda \quad I_0(\vec{r}) = \int I_\lambda^0(\vec{r})d\lambda \quad \vec{q}(\vec{r}) = \int \vec{q}_\lambda(\vec{r})d\lambda \tag{1.7}$$

where the integration is performed over the whole spectrum (formally, from 0 to ∞). In the case of the equilibrium radiation in a medium, $I = \sigma T^4/\pi$ and $I_0 = 4\sigma T^4$, where $\sigma = 5.67\cdot 10^{-8}$ W/(m$^2\cdot$K^4) is the Stefan–Boltzmann constant. The integral thermal radiation flux from the black-body surface is equal to $\vec{q} = \sigma T^4\vec{n}$.

Returning to the radiative transfer equation (1.2), we will integrate it over all values of the solid angle. Taking into account Eqs. (1.1), (1.5), and (1.6), we find the following important expression for divergence of the spectral radiation flux

$$\nabla\vec{q}_\lambda = \alpha_\lambda\left[4\pi B_\lambda(T) - I_\lambda^0(\vec{r})\right] \tag{1.8}$$

which can be treated as a local balance of the spectral radiation energy. A similar equation for integral characteristics of the radiation field

$$\nabla\vec{q} = \int \alpha_\lambda\left[4\pi B_\lambda(T) - I_\lambda^0(\vec{r})\right]d\lambda \tag{1.9}$$

is also called the radiative energy conservation equation [1–3]. We should recall the so-called gray model when radiative properties of a medium and boundary surfaces are assumed to be independent of the wavelength. The gray model is currently employed in engineering practice for some preliminary estimates. In the gray approximation, Eq. (1.9) is written as follows:

$$\nabla\vec{q} = \alpha\left[4\sigma T^4 - I_0(\vec{r})\right] \tag{1.10}$$

One can see from the above definitions that radiation intensity is not only a function of the position $\vec{r}$ but depends also on the direction $\vec{\Omega}$. The exact numerical solution to multi-dimensional RTE is an extremely complicated problem even in the case of simple boundary conditions. The problem would be most easily solved if scattering is not taken into consideration and hence the integral term is equal to zero. Unfortunately, this assumption is unacceptable for us since scattering by particles is one of the main features of radiative transfer in disperse systems. For scattering media, the RTE is integrodifferential, but employing various approximations for the scattering function simplifies the integral term [1–3, 32, 33, 37, 38].

1.2 Transport Approximation

One of the following methods is commonly used in approximation of the scattering function: expansion in a series of the Legendre functions or description of the main maxima of the scattering function by several delta-functions with some weight coefficients. Some combinations of these two approaches are also employed [3, 38]. The simplest approximations of each method are well known. If one is restricted to two terms in expansion in terms of the Legendre functions, the result is the linear-anisotropic approximation:

$$\Phi_\lambda(\mu_0) = 1 + 3\bar{\mu}_\lambda \mu_0 \qquad \mu_0 = \vec{\Omega}' \cdot \vec{\Omega} \tag{1.11}$$

where $\bar{\mu}_\lambda$ is the asymmetry factor of scattering:

$$\bar{\mu}_\lambda = \frac{1}{4\pi} \int_{(4\pi)} \left(\vec{\Omega}' \cdot \vec{\Omega}\right) \Phi_\lambda \left(\vec{\Omega}' \cdot \vec{\Omega}\right) d\vec{\Omega}' \tag{1.12}$$

The linear-anisotropic approximation loses any physical sense with $\bar{\mu}_\lambda > 1/3$, i.e., at large scattering anisotropy, since $\Phi_\lambda(\mu_0)$ becomes negative when $\mu_0 < -1/(3\bar{\mu}_\lambda)$. The other drawback of the linear approximation (1.11) should also be noted: the linear dependence on the cosine of the scattering angle is too far from the typical scattering functions of disperse systems [38]. Note that the linear-anisotropic approximation has been widely used in early papers to estimate the role of anisotropic scattering in model radiative transfer problems [3, 32, 38].

If one takes into account only the forward and backward scattering, presenting the scattering function as a linear combination of the Dirac delta-functions $\delta(1 + \mu_0)$ and $\delta(1 - \mu_0)$, the simplest "back-scattering" model is derived. This approximation, in which the integral term in the RTE disappears, gives good results in some one-dimensional problems [61] but cannot be applied to description of scattering in multi-dimensional problems for inhomogeneous and nonisothermal disperse systems.

The well-known transport approximation appears to be highly successful method [18, 38, 62]. According to this approximation, the scattering function is replaced by a sum of the isotropic component and the term describing the peak of forward scattering:

$$\Phi_\lambda(\mu_0) = \left(1 - \bar{\mu}_\lambda\right) + 2\bar{\mu}_\lambda \delta\left(1 - \mu_0\right) \tag{1.13}$$

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