



THERMAL RADIATION INTERACTION WITH UNSTEADY MHD FLOW PAST A VERTICAL POROUS PLATE IMMERSSED IN A POROUS MEDIUM

Md Abdus Samad and Mohammad Mansur Rahman

Department of Mathematics, University of Dhaka, Dhaka-1000, Bangladesh, Email: mansurdu@yahoo.com, Fax: 0088 02 8615583

Abstract

A study of unsteady MHD free convection flow through a porous vertical flat plate immersed in a porous medium in presence of magnetic field with radiation has been analyzed. Introducing a time dependent suction to the plate, a similarity procedure has been adopted by taking a time dependent similarity parameter. In this analysis we consider a Darcy-Forchhemier model and the corresponding momentum and energy equations have been solved numerically, for cooling and heating of the plate by employing Nachtsheim-Swigert iteration technique along with the sixth order Runge-Kutta integration scheme. Non-dimensional velocity and temperature profiles are then presented graphically for different values of the parameter entering into the problem. During the process of numerical computations the skin-friction coefficient (viscous drag) and the rate of heat transfer (Nusselt number), which are of physical interest, are sorted out and presented in the form of tables.

Keywords: Thermal radiation, MHD, Unsteady, Suction, Porous medium

Nomenclature

c_f	skin friction coefficient	t	time
c_p	specific heat at constant pressure	u	velocity along x -axis
Da	local Darcy number	v	velocity along y -axis
Ec	Eckert number	v_0	suction parameter
Fs	local Forchhemier number	$v_0(t)$	time dependent suction velocity
Fs_1	modified Forchhemier number	x	coordinate along the plate
Gr	local Grashof number	y	coordinate normal to the plate
g	acceleration due to gravity		
M	local magnetic field parameter	Greek	
N	radiation parameter	α	thermal diffusivity
Nu	Nusselt number	β	coefficient of volume expansion
n	nonnegative integer	δ	characteristic length scale
Pr	Prandtl number	ρ	density of the fluid
q_r	radiative heat flux	μ	coefficient of dynamic viscosity
Re	local Reynolds number	ν	coefficient of kinematic viscosity
T	temperature within the boundary layer	σ_1	Stefan-Boltzmann constant
$T(t)$	temperature at the plate	κ	thermal conductivity
T_∞	temperature of the ambient fluid	η	similarity parameter
		θ	dimensionless temperature
		$\Delta\eta$	step size

1. Introduction

The effect of free convection on the accelerated flow of a viscous incompressible fluid past an infinite vertical porous plate with suction has many important technological applications in the astrophysical, geophysical and engineering problems. The heating of rooms and buildings by the use of radiators is a familiar example of heat transfer by free convection. Heat losses from hot pipes, ovens etc surrounded by cooler air, are at least in part, due to free convection. The problem of heat transfer in a vertical channel has been studied in recent years as a model for the re-entry problem. This is due to the significant role of thermal radiation in surface heat transfer when convection heat transfer is similar, particularly in free convection problems involving absorbing emitting fluids. Soundalgekar and Takhar (1981) studied radiation effects on free convection flow of a gas past a semi-

infinite flat plate. Hossain and Takhar (1996) studied the effect of radiation using the Rosseland diffusion approximation on mixed convection along a vertical plate with uniform free stream velocity and surface temperature. Ali et al. (1984) studied radiation effect on natural convection flow over a vertical surface in a gray gas. Following Ali et al. Mansour (1990) studied the interaction of mixed convection with thermal radiation in laminar boundary layer flow over a horizontal, continuous moving sheet with suction/injection. Whereas Albraba et al. (1992) studied the same problem considering magnetic effect taking into account the binary chemical reaction and Soret - Dufour effects.

Seigel (1958) first studied transient free convection flow past a semi-infinite vertical plate by an integral method. Since then many researchers have been published papers on free convection flow past a semi-infinite vertical plate. Soundalgekar et al. (1981) studied free convection flow past a vertical porous plate. Yamamoto et al. (1976) investigated the acceleration of convection in a porous permeable medium along an arbitrary but smooth surface. Raptis (1983) studied free convection in a porous medium bounded by an infinite plate. Raptis and Perdakis (1985) studied numerically free convection flow through a porous medium bounded by a semi-infinite vertical porous plate. Sattar (1992) studied the same problem and obtained analytical solution by the perturbation technique adopted by Singh and Dikshit (1988). Sattar et al. (2000) studied unsteady free convection flow along a vertical porous plate embedded in a porous medium. Very recently, Alam and Rahman (2005) studied MHD free convection flow and mass transfer along a vertical porous plate in a porous medium considering Soret-Dofour effects. Sattar and Kalim (1996) studied the effects of unsteady free convection interaction with thermal radiation in a boundary layer flow. El-Arabawy (2003) studied the effect of suction/injection on a micropolar fluid past a continuously moving plate in the presence of radiation. Recently, Ferdows et al. (2004) investigate numerically the thermal radiation interaction with convection in a boundary layer flow at a vertical plate with variable suction. In the present paper we investigate the thermal radiation interaction on an absorbing emitting fluid permitted by a transversely applied magnetic field past a moving vertical porous plate embedded in a porous medium with time dependent suction and temperature. The similarity solutions are then obtained numerically for various parameters entering into the problem and discussed them from the physical point of view.

2. Mathematical Formulation

Let us consider the problem of an unsteady MHD free convection flow of a viscous, incompressible and electrically conducting fluid along a vertical porous flat plate under the influence of a uniform magnetic field. The flow is assumed to be in the x -direction, which is taken along the plate in the upward direction and y -axis normal to the plate. Initially it is assumed that the plate and the fluid are at a constant temperature T_∞ at all points. At time $t > 0$ the plate is assumed to be moving in the upward direction with the velocity $U(t)$ and there is a suction velocity $v_0(t)$ taken to be a function of time, the temperature of the plate raised to $T(t)$ where $T(t) > T_\infty$. The plate is considered to be of infinite length, all derivatives with respect to x vanish and so the physical variables are functions of y and t only. The flow configuration and coordinate system are shown in Fig. 1.

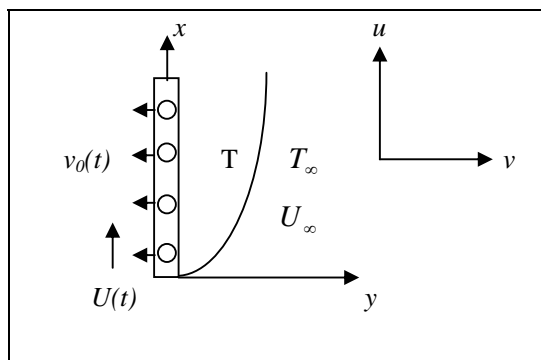


Fig. 1: Flow configuration and coordinate system.

The fluid is considered to be gray; absorbing-emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radioactive heat flux in the x -direction is considered negligible in comparison to the y -direction.

Assuming that the Boussinesq and boundary-layer approximations hold and using the Darcy-Forchheimer model, the governing equations for the problem are as follows:

Continuity equation

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g_0 \beta (T - T_\infty) - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{k} u - \frac{b}{k} u^2 \tag{2}$$

Energy equation

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \tag{3}$$

where (u, v) are the components of velocity along the x - and y -directions respectively, t is the time, ν is the kinematic viscosity, ρ is the density of the fluid, g_0 is the acceleration due to gravity, β is the coefficient of volume expansion, B_0 is the magnetic induction, T and T_∞ are the temperature of the fluid within the boundary layer and in the free stream respectively, σ is the electric conductivity, α is the thermal diffusivity and c_p is the specific heat at constant pressure, k is the permeability of the porous medium.

The corresponding boundary conditions for the above problem are given by

$$\left. \begin{aligned} u = U(t), v = v_0(t), T = T(t) \text{ at } y = 0, \\ u = 0, T = T_\infty \text{ as } y \rightarrow \infty. \end{aligned} \right\} \text{ for } t > 0 \tag{4}$$

By using Rosseland approximation q_r takes the form

$$q_r = - \frac{4\sigma_1}{3k_1} \frac{\partial T^4}{\partial y} \tag{5}$$

where σ_1 , the Stefan-Boltzmann constant and k_1 , the mean absorption coefficient. It is assumed that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{6}$$

Using (5) and (6) in equation (3) we have

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{4\sigma_1 T_\infty^3}{\rho c_p k_1} \frac{\partial T}{\partial y} \tag{7}$$

In order to obtain a similarity solution in time of the problem, we introduce a similarity parameter δ as

$$\delta = \delta(t), \tag{8}$$

such that δ is a length scale.

With this similarity parameter, a similarity variable is then introduced as

$$\eta = \frac{y}{\delta}. \tag{9}$$

In terms of this length scale, a convenient solution of the equation (1) can be taken as

$$v = v(t) = - \frac{\nu}{\delta} v_0,$$

where v_0 is the mass transfer parameter, which is +ve for suction and -ve for injection.

Following Sattar and Hossain (1992) $U(t)$ and $T(t)$ are now consider to have the following form:

$$\left. \begin{aligned} U(t) = U_0 \delta_*^{2n+2} \\ T(t) = T_\infty + (T_0 - T_\infty) \delta_*^{2n}, \end{aligned} \right\} \tag{10}$$

where n is a non-negative integer and U_0, T_0 are respectively the free stream velocity and mean temperature.

Here $\delta_* = \frac{\delta}{\delta_0}$, where δ_0 is the value of δ at $t = t_0$.

Now to make the equations (2) and (7) dimensionless, we introduce the following transformations:

$$\left. \begin{aligned} u &= U(t)f(t) = U_0 \delta_*^{2n+2} f(\eta), \\ T &= T_\infty + (T_0 - T_\infty) \delta_*^{2n} \theta(\eta). \end{aligned} \right\} \quad (11)$$

Using equations (8), (9), and (11) the equations (2) and (7) are become [using the analysis of Hashimoto (1957), Sattar et al. (2000) and Sattar and Maleque (2000)]

$$f'' + (2\eta + \nu_0) f' - (4n + 4 + M + \frac{1}{Da}) f + Gr \theta - \frac{Fs_1}{Da} f^2 = 0 \quad (12)$$

$$\theta'' + (2\eta + \nu_0) \left(\frac{3N Pr}{3N + 4} \right) \theta' - \left(\frac{12nN Pr}{3N + 4} \right) \theta + \left(\frac{3N Pr}{3N + 4} \right) Ec f'^2 = 0 \quad (13)$$

where $Gr = \frac{g_0 \beta (T_0 - T_\infty) \delta_0^2}{\nu U_0}$ is the local Grashof number, $M = \frac{\sigma B_0^2 \delta^2}{\rho \nu}$ is the local magnetic parameter,

$Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $Da = \frac{k}{\delta^2}$ is the local Darcy number, $Re = \frac{\nu_0 \delta}{\nu}$ is the local Reynolds

number, $Fs = \frac{b}{\delta}$ is the Forchhemier number and $Fs_1 = \frac{b}{\delta} \left(\frac{\delta}{\delta_0} \right)^{2n+2} Re$ is the modified Forchhemier number,

$N = \frac{\kappa \kappa_1}{4\sigma_1 T_\infty^3}$ is the radiation number.

The corresponding boundary conditions for $t > 0$ are given by

$$\left. \begin{aligned} f &= 1, \theta = 1 \text{ at } \eta = 0, \\ f &= 0, \theta = 0 \text{ as } \eta \rightarrow \infty. \end{aligned} \right\} \quad (14)$$

4. Numerical Computation

The numerical solutions of the nonlinear differential Equations (12)-(13) under the boundary conditions (14) have been performed by applying Nachtsheim-Swigert (1965) iteration technique along with the sixth order Runge-Kutta integration scheme. We have chosen a step size of $\Delta\eta = 0.01$ to satisfy the convergence criterion of 10^{-6} in all cases. The value of η_∞ was found to each iteration loop by $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞ to each group of parameters ν_0, M, n, Pr, Gr, Da , and Fs_1 determined when the value of the unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than 10^{-6} .

In order to verify the effects of the step size ($\Delta\eta$) we ran the code for our model with three different step sizes as $\Delta\eta = 0.01, \Delta\eta = 0.005, \Delta\eta = 0.001$ and in each case we found excellent agreement among them. Fig. 2 shows the velocity profiles for different step sizes.

5. Results and Discussion

For the purpose of discussing the results, the numerical calculations are presented in the form of non-dimensional velocity and temperature profiles. Numerical computations have been carried out for different values of the parameters entering into the problem. The values of Grashof number (Gr) are taken to be large from the physical point of view. The large Grashof number values correspond to free convection problem. The effects of suction parameter ν_0 on the velocity and temperature profiles are shown in Fig. 3 and Fig. 4 respectively. From Fig. 3 we found that the velocity decreases with the increase of suction for cooling of the plate and increases for the heating of the plate. It is also clear that suction stabilizes the boundary layer growth. Fig. 4 reveals that temperature decreases with the increase of the suction parameter.

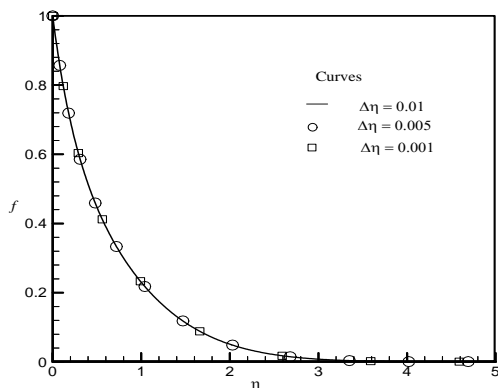


Fig. 2: Velocity profiles for different step sizes

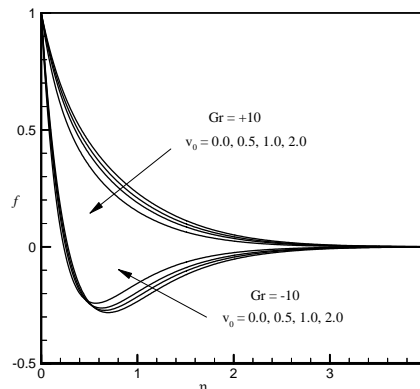


Fig. 3: Velocity profiles for different values of suction parameter (v_0)

Fig. 5 and Fig. 6 show the effects of Prandtl number (Pr) on the velocity as well as temperature profiles. From Fig. 5 we see that for cooling plate velocity profiles decrease with the increase of Pr whereas these profiles increase with the increase of Pr for a heating plate. For cooling plate Pr has decreasing effect on the temperature profiles.

The effects of radiation parameter (N) on the velocity profile is shown in the figure 6 for both cooling and heating plates. This figure shows that velocity decreases with the increase of the radiation parameter. This parameter has reverse effects on the heating plate. Fig. 8 shows the effect of N on the temperature profiles. For large N , it is clear that temperature decreases more rapidly with the increase of N . Therefore using radiation we can control the flow characteristic and temperature distribution. The effect of magnetic field parameter on the velocity profiles are shown in Fig. 9. It is observed from this figure that the magnetic field has decreasing effect on the velocity field for cooling plate and increasing effect for heating plate. Magnetic field lines act as a string to retard the motion of the fluid as –consequence the rather heat transfer increases.

Fig. 10 and Fig. 11 show the effect of non-negative integer n on the velocity and temperature profiles. From Fig. 10 we see that velocity profiles decrease for the cooling plate while it increases for the heating plate with the increase of n . Here $n = 0$ case represents the velocity as well as temperature is time independent. The nonzero values of n represents the case of time dependent velocity and temperature. Analyzing Figs. 10 and 11 we can say flow characteristics strongly depend on the values of n .

Fig. 12 shows the effects of Darcy number (Da) on the velocity profiles for cooling as well as heating of the plate. For a cooling plate fluid velocity increases, whereas for a heating plate it decreases with increase of Da . Darcy number is the measurement of the porosity of the medium. As the porosity of the medium increases, the value of Da increases. For large porosity of the medium fluid gets more space to flow as a consequence its velocity increases.

The effect of the modified Forchhemier number on the velocity field is shown in Fig. 13. It is observed from this figure that modified Forchhemier number has decreasing effect on the velocity field for the cooling plate while increasing effect for the heating plate.

Finally, the effects of the above mentioned parameters on the skin-friction coefficient and the Nusselt number are shown in Tables I-II. These effects as observed from the Tables I-II are found to agree with the effects on the velocity and temperature profiles hence any further discussions about them seem to be redundant.

6. Conclusions

In this paper we have studied the thermal radiation interaction with unsteady MHD boundary layer flow past a continuously moving vertical porous plate immersed in a porous medium. From the present study we can make the following conclusions:

- (i) The suction stabilizes the boundary layer growth.
- (ii) The velocity profiles increase whereas temperature profiles decrease with an increase of the free convection currents.
- (iii) Using magnetic field we can control the flow characteristics and heat transfer.
- (iv) Radiation has significant effects on the velocity as well as temperature distributions.

- (v) Flow characteristics strongly depend on the nonnegative integer n .
- (vi) Large Darcy number leads to the increase of the velocity profiles.

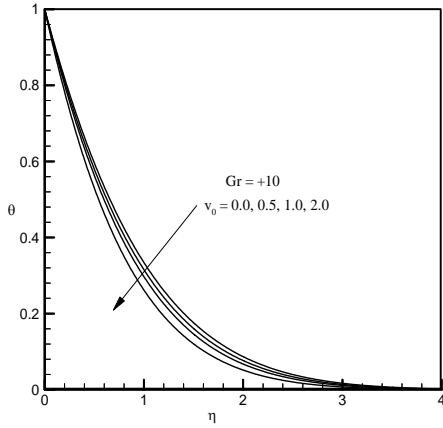


Fig. 4: Temperature profiles for different values of suction parameter (v_0)

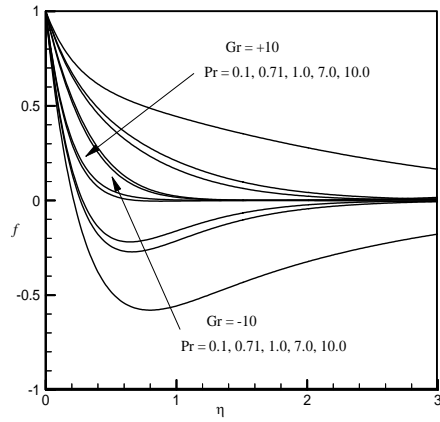


Fig. 5: Velocity profiles for different values of Prandtl number (Pr)

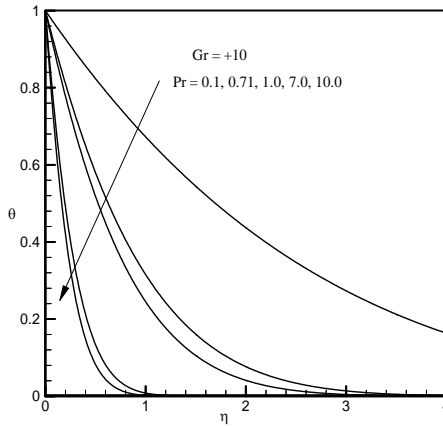


Fig. 6: Temperature profiles for different values of Prandtl number (Pr)

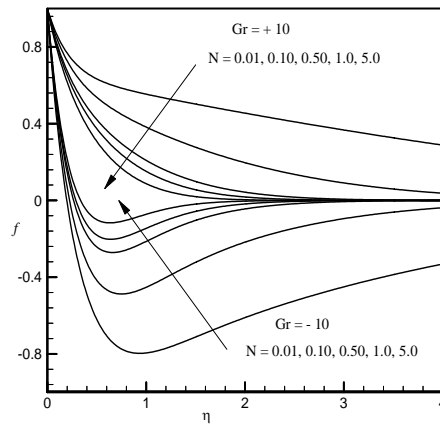


Fig. 7: Velocity profiles for different values of radiation (N)

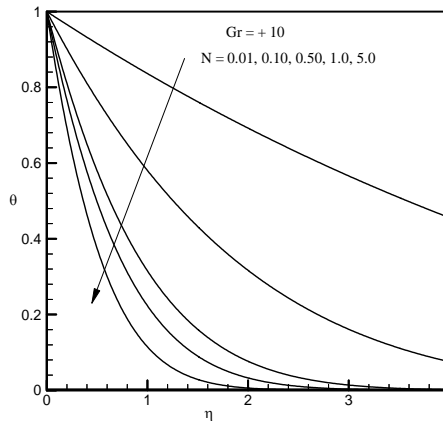


Fig. 8: Temperature profiles for different values of radiation parameter (N)

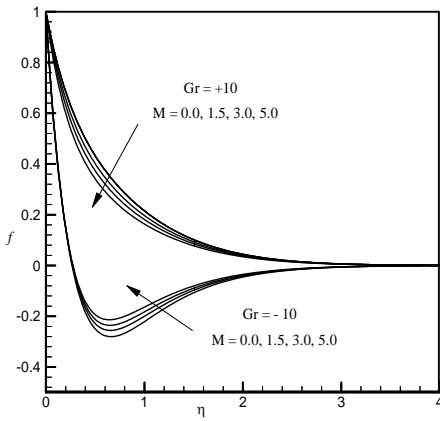


Fig. 9: Velocity Profiles for values of Magnetic parameter (M)

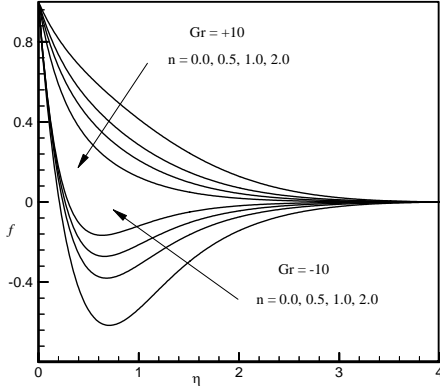


Fig. 10: Velocity profiles for different values non-negative integer (n)

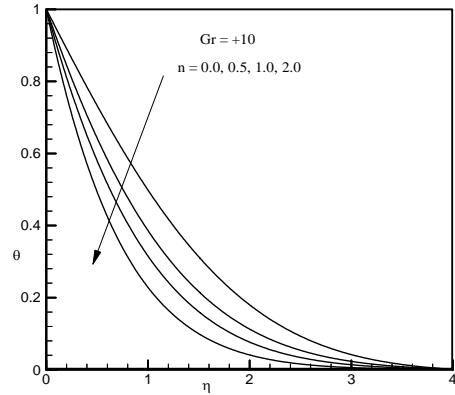


Fig. 11: Temperature profiles for different Values of non-negative integer (n)

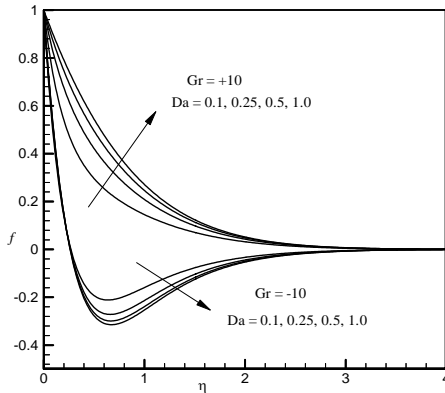


Fig. 12: Velocity profiles for different values of Darcy number (Da)

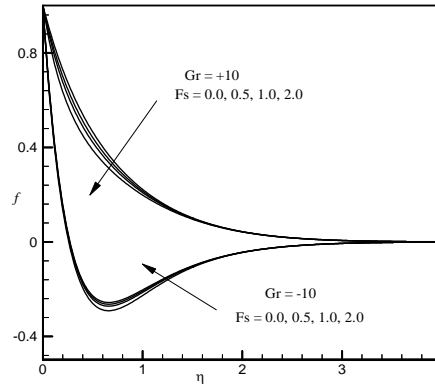


Fig. 13: Velocity profiles for different Values of Forchhemier number (Fs_1).

Table I: Skin-friction coefficients and rate of heat transfer for different values of $Ec=0.2$, $N=0.5$, $M=0.5$, $Fs_1=1.0$, $n=1.0$ and $Da=0.25$.

v_0	Gr	Pr	c_f	N_u
0.0	10	0.71	-2.0239	0.96755
0.5	10	0.71	-2.20139	1.01875
1.0	10	0.71	-2.39638	1.07183
2.0	10	0.71	-2.84188	1.18337
0.0	-10	0.71	-6.16298	0.84126
0.5	-10	0.71	-6.51201	0.88650
1.0	-10	0.71	-6.87269	0.93390
2.0	-10	0.71	-7.62243	1.03513
0.5	10	0.1	-1.84548	0.37878
0.5	10	0.71	-2.20139	1.01875
0.5	10	1.0	-2.29305	1.21177
0.5	10	7.0	-2.94205	3.25812
0.5	10	10.0	-3.06843	3.91936
0.5	-10	0.1	-7.04351	0.35009
0.5	-10	0.71	-6.51201	0.88650
0.5	-10	1.0	-6.39626	1.04436
0.5	-10	7.0	-5.66090	2.75038
0.5	-10	10.0	-5.52584	3.32275

Table II: Skin-friction coefficients and rate of heat transfer for different values of $v_0=0.5$, $Gr=10$, $Pr=0.71$, $Ec=0.2$, $Fs_1=1.0$.

M	N	n	Da	c_f	N_u
0.0	0.5	1	0.25	-2.11351	1.01995
1.5	0.5	1	0.25	-2.37175	1.01636
3.0	0.5	1	0.25	-2.61501	1.01279
5.0	0.5	1	0.25	-2.91936	1.011811
0.5	0.01	1	0.25	-1.71445	0.17387
0.5	0.10	1	0.25	-1.92589	0.51118
0.5	0.50	1	0.25	-2.20139	1.01875
0.5	1.0	1	0.25	-2.32422	1.28088
0.5	5.0	1	0.25	-2.51338	1.74547
0.5	0.5	0.0	0.25	-1.16364	0.54941
0.5	0.5	0.5	0.25	-1.73516	0.81536
0.5	0.5	1.0	0.25	-2.20139	1.01875
0.5	0.5	2.0	0.25	-2.95768	1.33587
0.5	0.5	1	0.1	-3.68945	0.99792
0.5	0.5	1	0.25	-2.20139	1.01875
0.5	0.5	1	0.5	-1.53694	1.02612
0.5	0.5	1	1.0	-1.14466	1.02969

References

- Alabraba, M. A., Bestman, A. R. and Ogulu, A. (1992): Laminar Convection in Binary Mixer of Hydromagnetic Flow with Radiative Heat Transfer. *Astrophys. Space Sci.*, Vol.195, pp. 431-445.
- Alam, M. S. and Rahman, M. M. (2005): Dufour and Soret Effects on MHD Free Convective Heat and Mass Transfer Flow Past a Vertical Porous Flat Plate Embedded in a Porous Medium. *J. Nav. Arc. Mar. Eng.*, Vol.2(1), pp.55-65.
- Ali, M. M., Chen, T. S. and Armaly, B. F. (1984): Natural Convection Radiation Interaction in Boundary Layer Flow Over Horizontal Surfaces. *AIAA Journal.*, Vol.22(12), pp. 797-1803.
- El-Arabawy, H. A. M. (2003): Effect of Suction/Injection on a Micropolar Fluid Past a Continuously Moving Plate in the Presence of Radiation. *Intl. J. Heat Mass Trans.*, vol.46, pp.1471-1477.
- Ferdows, M., Sattar, M. A. and Siddiki, M. N. A. (2004): Numerical Approach on Parameters of the Thermal Radiation Interaction with Convection in a Boundary Layer Flow at a Vertical Plate with Variable Suction. *Thammasat Intl. J. Sci. Tech.*, Vol.9(3), pp.19-28.
- Hashimoto, H. (1957): Boundary Layer Growth on a Flat Plate with Suction or Injection, *J. Phys. Soc. Japan*, Vol.22, pp.7-21.
- Hossain, M. A. and Takhar, H. S. (1996): Radiation Effect on Mixed Convection Along a Vertical Plate with Uniform Surface Temperature. *Heat and Mass Trans.*, Vol.31(4), pp.243-248.
- Mansour, M. A. (1990): Radiative and Free Convection Effects on the Oscillatory Flow Past a Vertical Plate. *Astrophys. Space Sci.*, Vol.166, pp.269-275.
- Nachtsheim, P. R. and Swigert, P. (1965): Satisfaction of the Asymptotic Boundary Conditions in Numerical Solution of the System of Non-linear Equations of Boundary Layer Type, NASSA TND-3004.
- Raptis, A. (1983): Unsteady Free Convection Flow Through a Porous Medium. *Intl. J. Eng. Sci.*, vol.21(4), pp.345-349.
- Raptis, A. and Perdikis, P. C. (1985): Unsteady Flow Through a Porous Medium in the Presence of Free Convection. *Intl. Comm. Heat Mass Trans.*, Vol.12, pp.697-704.
- Sattar, M. A. (1992): Free and Forced Convection Flow Through a Porous Medium Near the Leading Edge. *Astrophys. Space Sci.*, Vol.191, pp.323-328.
- Sattar, M. A., and Hossain, M. M. (1992): Unsteady Hydromagnetic Free Convection Flow with Hall Current and Mass Transfer along an Accelerated Porous Plate with Time Dependent Temperature and Concentration, *Can. J. Phys.*, Vol.70, pp.369-375.
- Sattar, M. A. and Kalim, H. (1996): Unsteady Free Convection Interaction with Thermal Radiation in a Boundary Layer Flow Past a Vertical Porous Plate. *J. Math. Phys. Sci.*, vol.30(1), pp.25-37.
- Sattar, M. A. and Maleque, M. A. (2000): Unsteady MHD natural convection flow along an accelerated porous plate with hall current and mass transfer in a rotating porous medium, *J. Energy, Heat and Mass Transfer.*, Vol.22, pp.67-72.
- Sattar, M. A., Rahman, M. M. and Alam, M. M. (2000): Free Convection Flow and Heat Transfer Through a Porous Vertical Flat Plate Immersed in a Porous Medium. *J. Energy Res.*, Vol.22(1), pp.17-21.
- Siegel, R. (1958): *Trans. Amer. Soc. Mech. Eng.*, Vol.80, pp.347.
- Singh, A. K. and Dikshit, C. K. (1988): Hydromagnetic Flow Past a Continuously Moving Semi-Infinite Plate for Large Suction. *Astrophys. Space Sci.*, Vol.148, pp.249-256.
- Soundalgekar, V. M. and Takhar, H. S. (1981): MHD Flow Past a Vertical Oscillating Plate. *Nucl. Eng. Des.*, Vol.64(1), pp.43-48.
- Soundalgekar, V. M., Vighnesam, N. V. and Pop, I. (1981): Combined Free and Forced Convection Flow Past a Vertical Porous Plate. *Intl. J. Energy Res.*, Vol.5, pp.215-226.
- Yamamoto, K. and Iwamura, N. (1976): Flow with Convection Acceleration Through a Porous Medium. *J. Eng. Math.*, Vol.10(1), pp.41-54.