Thermal ρ and σ mesons from chiral symmetry and unitarity

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We study the temperature evolution of the ρ and σ mass and width, using a unitary chiral approach. The one-loop $\pi\pi$ scattering amplitude in chiral perturbation theory at $T\neq 0$ is unitarized via the inverse amplitude method. Our results predict a clear increase with T of both the ρ and σ widths. The masses decrease slightly for high T, while the $\rho\pi\pi$ coupling increases. The ρ behavior seems to be favored by experimental results. In the σ case, it signals chiral symmetry restoration.

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One of the outstanding phenomena related to heavy ion collisions is the flatness of the dilepton spectrum near the mass of the ρ meson, which is so clearly visible in many processes involving hadrons and electromagnetic probes. This flatness has been observed by the HELIOS and CERES collaborations [1,2] and has been the subject of widespread discussion. Dileptons and photons provide neat signals of the early stages of the quark-gluon plasma and its subsequent evolution into a hadron gas [3]. In fact, the most credible explanation of the absence of a prominent hill in the dilepton spectrum is a change in the mass and width of the ρ due to its interactions with the hot hadron gas [4–7]. Since the baryons, with a large forward momentum, have almost escaped the central collision region, this gas is composed mainly of pions. Our aim is to study the thermal evolution of the ρ mass M_ρ and width Γ_ρ, from the first principles of chiral symmetry and unitarity in $\pi\pi$ scattering.

What happens to the ρ in extreme conditions is a hadronic physics problem, involving non perturbative physics, and hence difficult to be treated. Prior to this work, a copious number of models and estimations have appeared. In most of them Γ_ρ increases with temperature, simply as a consequence of stimulated emission in the pion thermal bath or, equivalently, because the effective phase space increases [8,9]. This behavior is often interpreted as a deconfining effect, or hadron "melting." As for the mass, vector meson dominance (VMD) implies that M_ρ changes very little at low temperatures [8,10,11]. As T approaches the critical temperature, earlier works claimed that M_ρ increases [8,11,12] but the analysis of experimental dilepton data seems to favor a decreasing behavior [5,7]. Let us remark that in all these works, the ρ is introduced as an explicit degree of freedom and often a dilute pion gas is assumed, so that the thermal effects appear, to leading order, only through the pion distribution function and not through the interaction details. Other approaches include the NJL model [13], where M_ρ and Γ_ρ slightly decrease (but there is a spurious quark threshold near M_ρ) as well as $q\bar{q}$ wave-function analysis in the π channel yielding a decreasing width [14].

In this work we will use a thermal treatment of the effective degrees of freedom, the pions in the aftermath of the collision at moderate temperatures. The guiding fundamental principles will be just chiral symmetry and unitarity. We will build on a previous work [15] where the $T\neq 0$ $\pi\pi$ scattering amplitude has been calculated to one loop in chiral perturbation theory (ChPT). Demanding unitarity, we will construct a nonperturbative amplitude reproducing the expected behavior for thermal resonances. Our amplitude has the correct analytic structure, without spurious cuts, and resonances are not introduced by hand.

The most general framework comprising the QCD chiral symmetry breaking pattern is ChPT [16,17], see [18] for reviews, where observables are calculated as expansions in $p/(4\pi f_\pi)$, p denoting any pion energy scale (including the temperature) and $f_\pi \simeq 92.4$ MeV. Despite its success, ChPT is limited to low energies (usually, less than 500 MeV) and low temperatures and it is not able to generate resonances. Thus, over the past few years, there has been a growing interest to extend the ChPT applicability range to higher energies and to reproduce resonances within a unitary chiral approach, which we briefly review. At $T=0$, unitarity for the S matrix ($S^\dagger S=1$) implies the following relation for partial waves:

$$\mathrm{Im}\, a_{IJ}(s)=\sigma(s)|a_{IJ}(s)|^2, \tag{1}$$

for $s>4m_\pi^2$ and below other inelastic thresholds, where $\sigma(s)=\sqrt{1-4m_\pi^2/s}$ is the two-pion phase space and a_{IJ} denotes the projection of the $\pi\pi$ elastic amplitude with isospin I and total angular momentum J in the center-of-mass (c.m.) frame. Equation (1) is only satisfied *perturbatively* within ChPT, i.e., if we write the perturbative series for any partial wave as $a=a_2+a_4+\cdots$ where a_k is $\mathcal{O}(p^k)$, then one has $\mathrm{Im}a_2=0$, $\mathrm{Im}a_4=\sigma a_2^2$ and so on. Hence, deviations from Eq. (1) are more severe at high energies, and in particular near the resonance region, where the bounds imposed by unitarity are saturated. The ChPT series, which essentially behaves as a polynomial, is unbounded and cannot reproduce resonances, which show up as poles of the amplitude in the complex plane.

In fact, from Eq. (1), any partial wave should satisfy $a=1/(\mathrm{Re}\ a^{-1}-i\sigma)$ on the real axis below inelastic thresholds. A unitarization method is just one way of approximating $\mathrm{Re}\ a^{-1}$, thus introducing some model dependency, but since we want to ensure chiral symmetry, and the correct low-energy behavior at $T=0$, we will use the one-loop ChPT result. This is called the inverse amplitude method (IAM) at $T=0$, which can be recast as $a^{IAM}=a_2^2/(a_2-a_4)$ [19]. The single channel IAM amplitude satisfies Eq. (1) exactly and at low energies it follows the ChPT result up to one loop. In addition, the IAM reproduces the scattering data for real energies above the two-pion threshold up to 1 GeV, where the elastic approximation breaks down, and it can be continued into the complex s plane, yielding correct σ and ρ poles in the second Riemann sheet. We point out that the IAM is nothing but the [1,1] Padé approximant of the ChPT series in squared energy, mass, or temperature over f_π^2.

As long as they contain the $\mathcal{O}(p^4)$ tree level terms, other chiral unitary approximations, both for SU(2) or SU(3) ChPT, either based on the IAM with coupled channels [20], or the IAM with higher orders [21], or inspired in Lippmann-Schwinger or Bethe-Salpeter equations [22], or mixed formalisms [23] yield equivalent results for the ρ and σ channels. In particular, they reproduce the experimental phase shifts with compatible sets of chiral parameters, and they generate poles associated to the σ and ρ resonances whose position in the second Riemann sheet agrees for all the above mentioned methods, which therefore describe resonances with the same masses and widths. These unitarized approaches also allow to study finite baryon density effects on the change of the sigma properties in the nuclear medium [24] that suggested a decrease on both the sigma mass and width as the nuclear density increases. As a consequence of these effects, it is expected [25] a shift of strength of the two-pion invariant mass distribution in $\gamma N\rightarrow N\pi^0\pi^0$, which has been recently confirmed experimentally [26]. In particular, using a chiral unitary approach, this shift is interpreted as an in medium modification of the σ pole towards lower masses and widths. Nevertheless, since in this work we are interested in the ρ and σ mesons it is enough to work with the single channel IAM to $O(p^4)$ that we have just described.

Back to $T\neq 0$, the thermal amplitude can be defined by considering $T=0$ initial and final asymptotic states and calculating the $T\neq 0$ four-pion Green's function [15]. To one loop in ChPT, and in the $\pi\pi$ c.m. frame (at rest with the thermal bath) it satisfies the *perturbative* unitarity relation [15],

$$\mathrm{Im}\, a_4(s;T)=\sigma_T(s)[a_2(s)]^2, \tag{2}$$

where

$$\sigma_T(s)=\sigma(s)[1+2n_B(\sqrt{s}/2)] \tag{3}$$

is the thermal phase space and $n_B(x)=[\exp(x/T)-1]^{-1}$ is the Bose-Einstein distribution function. Recall that the lowest order a_2 is T independent.

Therefore, the natural unitarized version of the thermal amplitude in ChPT should be

$$a^{IAM}(s;T)=\frac{a_2^2(s)}{a_2(s)-a_4(s;T)}, \tag{4}$$

which satisfies the exact elastic unitarity condition

$$\mathrm{Im}\, a^{IAM}(s;T)=\sigma_T(s)|a^{IAM}(s;T)|^2, \tag{5}$$

and reproduces the low energy results of (thermal) ChPT in Ref. [15]. Besides, as we will see below, it has the proper analytical behavior and, for the appropriate values of the chiral parameters, it is able to reproduce resonances like the ρ as poles in the second Riemann sheet.

Some remarks are in order here: We are assuming that the exact thermal version of Eq. (1) holds, feature reproduced by the IAM in Eq. (5). This assumption will prove to be reasonable in view of the results shown below. Nevertheless, it is important to remark that such assumption implies, in particular, that only two-pion states are available in the thermal bath. This is equivalent to a dilute gas approximation. In other words, the n_B term in Eq. (3) must remain small compared to one so that we can neglect higher orders in density like $\mathcal{O}(n_B^2)$ which would spoil the simple algebraic unitarity relation given by Eq. (2) [15]. This implies, for instance that $\rho-\pi$ scattering, which in our approach is regarded as a three-pion effect, would be suppressed by the low density. Note that, alternatively, we can view Eq. (4) also as the [1,1] Padé approximant of ChPT when counting the powers of momenta, masses, or temperature over f_π^2, since T is $\mathcal{O}(p)$ in the chiral expansion. Again, this counting would be spoiled for large $n_B(\sqrt{s})$, which typically weights the thermal corrections. Let us finally remark that the IAM has been extended to deal with other intermediate states, describing successfully the $T=0$ data in all meson-meson channels up to 1.2 GeV, within a coupled channel formalism [20]. In such a case, the amplitudes satisfy a matrix version of the unitarity relation in Eq. (1). This would be the natural extension of our approach in order to deal with other intermediate coupled states, like K or η which could be relevant at high temperatures [27]. However, it would require a thermal generalization of the matrix unitarity relation and the one-loop calculation of the additional coupled amplitudes, which lie beyond the scope of this work.

Before proceeding to the detailed calculation of the IAM thermal amplitude in Eq. (4), we will provide a simple argument as to why our method can actually give rise to the expected thermal behavior for the ρ width. As it is well known, in most cases (like the ρ) a resonant behavior can be reproduced on the real axis by means of a Breit-Wigner parametrization of the partial waves:

$$a^{BW}(s;T)=\frac{R_T(s)}{s-M_T^2+i\Gamma_T M_T}, \tag{6}$$

where M_T and Γ_T are the thermal mass and width of the resonance [9] and $R_T(s)$ is a smooth real function near $s=M_T^2$, which can be related to the $\rho\pi\pi$ vertex (see below). The parametrization in Eq. (6) applies only for $s\simeq M_T^2$ and for narrow resonances ($\Gamma_T\ll M_T$). Comparing Eq. (4) with Eq. (6) at $s=M_T^2$ one readily gets Re $a_4(M_T^2)=a_2(M_T^2)$ (the resonance mass condition) and, using Eq. (5), $\Gamma_T M_T=-R_T(M_T^2)\sigma_T(M_T^2)$. Therefore, assuming that the thermal corrections to R_T and to M_T are much smaller than those to Γ_T, i.e., $R_T \simeq R_0$ and $M_T \simeq M_0$ we would get

$$\Gamma_T \simeq \Gamma_0[1+2n_B(M_0/2)]. \tag{7}$$

Hence, in this limit the thermal IAM yields an increasing resonance width driven only by the available thermal phase space Eq. (3) for a ρ at rest [8,9]. The above result takes into account the stimulated emission $\rho \to \pi\pi$ and absorption $\pi\pi \to \rho$ from the thermal bath [15] and gives the dominant effect at very low temperatures, as our full analysis below confirms. This approximation indicates that the unitarity requirements on the amplitude capture the qualitative thermal resonance behavior. Note that, from the resonance mass condition, taking $M_T \simeq M_0$ is equivalent to ignoring the T dependence in Re $a_4(s;T)$.

Therefore, by using the full thermal amplitude $a_4(s;T)$ in Ref. [15], we will calculate below both the M_T corrections and the deviations from Eq. (7). Moreover we will find the analytic continuation of the amplitude to the complex plane, so that we can describe the resonances as poles of the thermal amplitude. This is particularly important for the σ, whose description in terms of Eq. (6) is not so appropriate due to its large width.

Let us note that the breaking of Lorentz invariance of the thermal formalism allows for a definition of a "transversal" and a "longitudinal" mass, however, in our case, since we are working in the c.m. frame, where the ρ is at rest with the thermal bath, both mass definitions coincide [11].

In the c.m. frame, the thermal one-loop amplitude can be written in terms of the loop functions [15,32]:

$$\Delta J_0^s(s;T) = -\frac{1}{\pi^2}\int_{m_\pi}^{\infty} dE \frac{\sqrt{E^2-m_\pi^2}\,n_B(E)}{s-4E^2},$$

$$\Delta J_0^{tu}(t;T) = \frac{1}{4\pi^2\sqrt{-t}}\int_0^{\infty} dq \frac{q n_B(E_q)}{E_q} \ln\left|\frac{2q+\sqrt{-t}}{\sqrt{-t}-2q}\right|,$$

$$\Delta J_2^{tu}(t;T) = \frac{1}{4\pi^2\sqrt{-t}}\int_0^{\infty} dq q E_q n_B(E_q) \ln\left|\frac{2q+\sqrt{-t}}{\sqrt{-t}-2q}\right|, \tag{8}$$

for real $s>4m_\pi^2$ and real $t<0$, where $\Delta F(T) \equiv F(T) - F(0)$, $E_q^2 = q^2 + m_\pi^2$, $J_0^s(s;0)$ is given in Ref. [17] (after the standard $\overline{MS}-1$ renormalization) and $J_{0,2}^{tu}(t;0)$ can be written in terms of $J_0^s(t;0)$. Note that on the real axis the only imaginary part comes from Im $J_0^s(s+i\epsilon;T) = \sigma_T(s)/16\pi^2$ for $s>4m_\pi^2$, thus ensuring Eq. (2) [33].

We have calculated the IAM amplitude, Eq. (4), from the one-loop thermal $a_4(s;T)$. The phase shifts for different temperatures are shown for the ρ channel $I=J=1$ in Fig. 1. Note the excellent agreement with scattering data at $T=0$, where we have fitted the SU(2) low-energy constants in the a_{00}, a_{11}, a_{20} channels, yielding the following values for the standard dimensionless and scale independent SU(2) chiral parameters defined in Refs. [17,18] $\bar{l}_1 = -0.3$, $\bar{l}_2 = 5.6$, $\bar{l}_3 = 3.4$, and $\bar{l}_4 = 4.3$, compatible with the recent determination

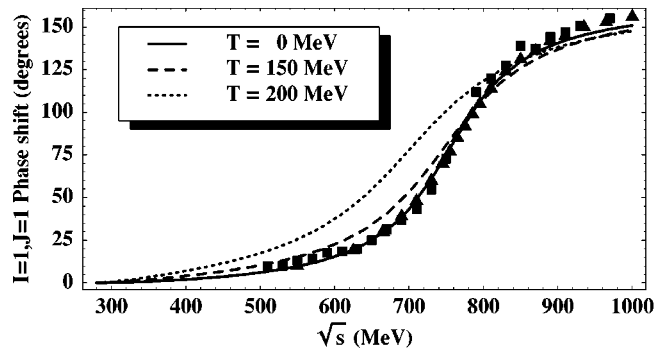

FIG. 1. $I=J=1$ phase shift for different temperatures. For the data see Ref. [19], and references therein.

in Ref. [28]. The ρ mass and width can now be estimated using $\delta_{11}(M_\rho) = 90^o$ and $\Gamma_\rho(s) \simeq M_\rho(1-s/M_\rho^2)\tan\delta_{11}$ near $s = M_\rho^2$ [19]. Thus we obtain $M_0 = 770$ MeV and $\Gamma_0 = 159$ MeV. All finite T results are now predictions. As T increases, Γ_T grows, as shown in Fig. 2. The curves are shown only below the validity limit of our approach which naively is set by $2n_B(M_\rho/2) < 1$ yielding $T < 300$ MeV. We remark that the validity of one-loop SU(2) ChPT has been estimated to reach about $T \simeq 150$ MeV [27], but with the unitarization methods we are able to reach higher temperatures, as long as the density factors remain small (see our previous discussion). To lie on the conservative side, we are only showing results up to 200 MeV, where $2n_B(M_\rho/2) \simeq 0.3$. Let us still note that the deviations from the naive phase space correction in Eq. (7) are clearly sizable already at $T \simeq 100$ MeV, precisely when thermal effects start being significant, the full calculation giving a higher value for the width than Eq. (7). The mass changes little up to $T \simeq 200$ MeV, consistently with previous analysis [7,8,10,12–14]. It grows slightly up to $T \simeq 100$ MeV ($M_{100} \simeq 775.5$ MeV) and then decreases for higher T. In addition, in the narrow resonance approximation (which becomes less reliable as T increases) we have $R_T = g_T^2(4m_\pi^2 - M_T^2)/48\pi$, g_T being the effective coupling in the VMD $\rho\pi\pi$ vertex [6,8] with a thermal $\rho(g_0 \simeq 6.2)$. Therefore, from the IAM Γ_T, M_T we find the behavior of g_T plotted in Fig. 2. At low

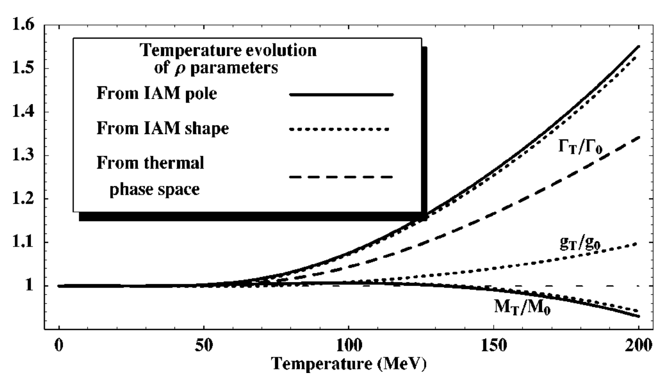

FIG. 2. Temperature evolution of the ρ mass, width, and $\rho\pi\pi$ coupling. The dashed line corresponds to Eq. (7), the dotted line to the real axis IAM, Eq. (4), and the solid line is the IAM pole position. The M_0, Γ_0, g_0 values are given in the text.

T, $g_T \lesssim g_0 (g_{50}/g_0 \simeq 0.9991)$ in agreement with the chiral low-T analysis in Ref. [6] and it grows for higher T. The corrections are more important at finite density [29].

Although the direct experimental measurement is the dilepton spectrum, $\pi\pi$ scattering is still a very interesting process since it is strongly constrained by unitarity. In particular, this provides relevant information about the ρ pole position, which has to be a common feature for all other processes where the ρ resonance appears. That is why we now turn to study the analytic continuation of the amplitude to the complex plane. The analytic continuation of the $T=0$ J_0^s is straightforward [17,19]. However, due to the loss of Lorentz covariance in the thermal bath, we need the analytic continuations of Eqs. (8) which are somewhat more subtle. Since $\Delta J_0^s(s;T)$ is already written as an analytic function for Im $s \neq 0$, the same expression is straightforwardly continued to the complex plane. However, for the others we find

$$\Delta^{\pm} J_0^{tu}(t;T) = \frac{1}{4\pi^2\sqrt{-t}} \left\{ \int_0^\infty dq \frac{q n_B(E_q)}{E_q} \ln\left[\frac{2q+\sqrt{-t}}{\sqrt{-t}-2q}\right] \pm i\pi T \ln(1-e^{-R(t)/T}) \right\}, \tag{9}$$

$$\Delta^{\pm} J_2^{tu}(t;T) = \frac{1}{4\pi^2\sqrt{-t}} \left\{ \int_0^\infty dq q E_q n_B(E_q) \ln\left[\frac{2q+\sqrt{-t}}{\sqrt{-t}-2q}\right] \pm i\pi T[R^2(t)\ln(1-e^{-R(t)/T}) - 2TR(t)\mathrm{Li}_2(e^{-R(t)/T}) - 2T^2 \mathrm{Li}_3(e^{-R(t)/T})] \right\}, \tag{10}$$

where $R(t)=\sqrt{m_\pi^2 - t/4}$, $\Delta^{+(-)}$ denote the analytic continuation for Im $t>0(<0)$ and $\mathrm{Li}_n(z)$ is the polylogarithmic function, analytic except for a branch cut for real $z>1$ [30]. It is not difficult to check that $\Delta J_{0,2}^{tu}(t;T)$ coincides with Eq. (8) at $t \pm i\epsilon$ with real $t<0$, and they have a branch cut only for real $t>4m_\pi^2$. Thus, as it happened for $T=0$, both a_4 and a^{IAM} have a right (unitarity) cut for real $s>4m_\pi^2$ and a left cut for $s<0$ coming, respectively, from ΔJ_0^s and $\Delta J_{0,2}^{tu}$ [34]. Finally, using Eq. (5), the analytic continuation of the amplitude a^{II} into the second Riemann sheet across the right cut is given by $a^{II}(s;T) = a^{IAM}(s;T)/[1-2i\sigma_T(s) a^{IAM}(s;T)]$.

For $I=J=1$, we find the pole corresponding to the ρ resonance. Its position on the complex plane as a function of T is shown in Fig. 3. Let us recall that the definition of the pole position in terms of the resonance mass and width is $s_{pole} = (M - i\,\Gamma/2)^2$, which coincides with the pole of Eq. (6) for a narrow resonance. In particular, at $T=0$, we have $M_0 = 755$ MeV and $\Gamma_0 = 152$ MeV. The results are also plotted

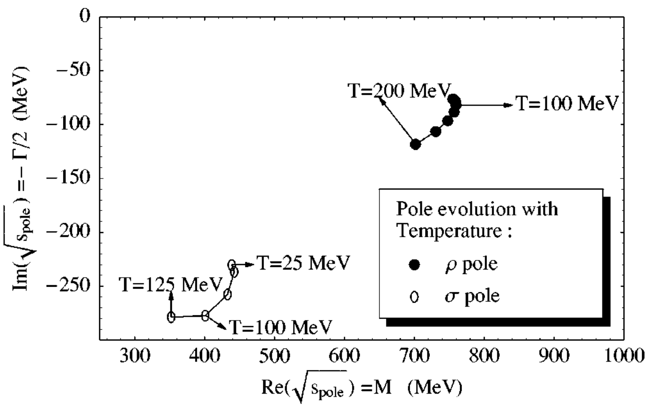

FIG. 3. Position of the ρ and σ poles in the complex plane, with increasing temperature.

in Fig. 2, where we see that the evolution of the pole mass and width agrees with our previous real axis calculation. For $I=J=0$, the observed pole corresponds to the σ and is plotted in Fig. 3, too. The width also increases, essentially by the increase of phase space and $M_\sigma(T)$ decreases with T, as expected from chiral symmetry restoration [31]. Once again, the applicability of our approach is limited by $2n_B(M_\sigma/2) \simeq 1$, i.e., $T<180$ MeV.

The main conclusions of this work are the following. We have shown, using only chiral symmetry and unitarity, that the thermal width of the ρ and σ mesons at rest with the thermal bath grow with temperature, while their thermal masses decrease slightly. They can be read off from the real and imaginary parts of the pole position of the thermal $\pi\pi$ elastic scattering amplitude in the corresponding channels. For that purpose, we have unitarized and calculated the analytic continuation to the complex plane of the amplitude on the real axis above threshold analyzed in Ref. [15]. For the case of the ρ we have also estimated its thermal mass, width, and effective $\pi\pi$ coupling from the unitarized amplitude in the real axis. At low temperatures, the thermal widths increase slightly according to the thermal phase space, while the masses and the effective $\rho\pi\pi$ vertex remain almost constant. For higher T, our analysis gives sizable decreasing mass corrections, an increasing effective vertex, as well as significant deviations from the phase space contribution, yielding higher thermal widths. The σ mass shows a decreasing behavior compatible with chiral symmetry restoration. Our results agree with recent theoretical and experimental analysis, up to temperatures of 250 MeV and they shed light on the dilepton spectrum problem in relativistic heavy ion collisions.

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[32] In the notation of Ref. [15], $\Delta J_0^s(s)$ and $\Delta J_{0,2}^{tu}(t)$ correspond to $\Delta J_0(\sqrt{s},\vec{0})$ and $\Delta J_{0,2}(0,\sqrt{-t})$, respectively.
[33] Remember that $t(s,x)=(x-1)(s-4m_\pi^2)/2$, $u(s,x)=t(s,-x)$ with x being the cosine of the scattering angle.
[34] For $s<0$ there is always a finite region in $x\in[-1,1]$ such that $t,u(s,x)>4m_\pi^2$.