

1 **Thermal stratification effects on flow over a generic urban**
2 **canopy**

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6 **Abstract** The influence of local surface heating and cooling on flow over urban-
7 like roughness is investigated using large-eddy simulations (LES). By adjusting the
8 incoming or outgoing heat flux from the ground surface, various degrees of local ther-
9 mal stratification, represented by a Richardson number (Ri_τ), were attained. Drag and
10 heat transfer coefficients, turbulence structure, integral length scales, and the strength
11 of quadrant events that contribute to momentum and heat fluxes were obtained and
12 are compared with locally stable, neutral and unstable flows. With increasing Ri_τ , or
13 equivalently as the flow characteristics change from local thermal instability to sta-
14 bility, a gradual decline in the drag and heat transfer coefficients is observed. These
15 values are found to be fairly independent of the type of thermal boundary condition
16 (constant heat flux or constant temperature) and domain size. The maps of anisotropy
17 invariants showed that for the values of Ri_τ considered, turbulence structures are al-
18 most the same in shape for neutral and unstable cases but differ slightly from those in
19 the stable case. The degree of anisotropy is found to decrease as Ri_τ increases from
20 -2 to 2.5 . Compared to the neutral case, the integral length scales are shortened in
21 the streamwise and vertical direction by ground cooling, but enhanced in the vertical
22 direction with ground heating. Quadrant analysis showed that increase in floor heat-
23 ing increases the strength of ejections above the canopy. However, the contributions
24 of updrafts or downdrafts to heat flux are found not to be significantly influenced by
25 the type of local thermal stratification for the values of Ri_τ considered. The transport
26 mechanisms of momentum and heat above the canopy are found to be very similar in
27 both locally unstable and stable flows.

28 **Keywords** Correlations · Drag coefficient · Heat transfer coefficient · Quadrant
29 events · Turbulent structures

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1 Introduction

Do the effects of thermal stratification have a dominant role on the structure of turbulence and mechanisms of pollutant transport in and above roughness canopies of various morphologies? To investigate this, numerous field, wind-tunnel and computational studies have been conducted, especially in the last two to three decades. The field studies included several vegetation (e.g. Gao et al., 1989) and urban (e.g. Christen et al., 2007) areas to understand the similarities and differences in the transport of momentum and heat over the two kinds of canopies. One of the similarities that was observed is that sweep events contribute most to the momentum flux below and immediately above the canopy height and ejection events dominate further above the canopy; these events are considered to be the signatures of the large coherent structures. Li and Bou-Zeid (2011) discussed in detail the dissimilarity of momentum, temperature and water vapour transport with increasing instability from measurements over a vineyard and a lake. However, it is difficult to obtain comprehensive, spatially detailed measurements from the field owing to instrument limitations and the impossibility of obtaining repeated and controlled conditions; wind-tunnel and computational studies can therefore be particularly useful.

The simplest geometry, yet challenging if thermal stratification is included, is two-dimensional (2-D) street canyons. Allegrini et al. (2013), Huizhi et al. (2003), Kovar-Panskus et al. (2002), for example, have studied such cases in wind tunnels and shown that surface heating greatly influences the number and intensity of vortices within the canyon. Similar observations have also been made from various computational studies (e.g. Cai, 2012; Kim and Baik, 1999; Park et al., 2012). In the case of 3-D roughness morphologies, by adjusting the temperatures of the approach flow and the floor of a wind-tunnel, Uehara et al. (2000) created a thermally stratified atmospheric boundary layer over square arrays of roughness obstacles. They showed that a stable atmosphere results in weak cavity eddies whilst unstable conditions enhances the strength of cavity eddies. Using LES, Inagaki et al. (2012) simulated a complete day time atmospheric boundary layer over a square array of cubes with ground and roof heating and showed that the turbulent organized structures above the canopy are correlated to the strong upward motion that occurs within the cavity of the arrays. All these ‘generic’ urban canopy investigations clearly imply that the dispersion of pollutants might be affected by surface heating. Computational studies on field sites like DAPPLE (Dispersion of Air Pollution and its Penetration into the Local Environment) have certainly suggested that weak unstable conditions in the approach flow have notable effects on scalar dispersion (Xie et al., 2013).

It is necessary to quantify the effects of such thermal stratification on street and/or neighborhood scale flows in order to provide required parameters for city or regional scale modelling. For this purpose, we first performed computations to simulate passive scalar dispersion from a surface area source in an array of uniform and random height blocks (Boppana et al., 2010), followed by simulation of heat transfer from the strongly heated leeward surface of a large building (Boppana et al., 2013). These computations showed good agreement with the wind-tunnel experiments of Pascheke et al. (2008) and Richards et al. (2006) respectively. The former LES study had no buoyancy and the latter included its effects on the surrounding flow. These previous

75 investigations led naturally to the current LES study where, instead of heating a single
 76 surface of an isolated obstacle, the entire ground surface (i.e. all streets, in direct
 77 contact with the atmosphere) is uniformly heated (see Fig. 1) or cooled and the re-
 78 sulting buoyancy effects are included to model the flow over an array of staggered
 79 cubes. It is to be noted that, in this study, thermal stratification in a fully-developed
 80 boundary layer is a result of surface heating or cooling within the bottom canopy,
 81 which is rather different to the case of a thermally stratified approach flow over an
 82 unheated region (e.g. Xie et al., 2013).

83 The overall goal of the present paper is to obtain insights on the effects of uniform
 84 ground heating or cooling on the flow over an array of uniform height staggered
 85 buildings. To address this, the following objectives were formulated: (1) to quantify
 86 the effects of thermal stratification on the surrounding flow, including the turbulence
 87 structure, and (2) to determine the similarities and/or differences in momentum and
 88 heat transport for stable, neutral and unstable stratified flows *via* assessment of the
 89 affects of stratification on surface drag and heat transfer coefficients. We present the
 90 numerical description in Sect. 2, followed by the results and conclusions in Secs. 3
 91 and 4 respectively.

92 2 Numerical Details and Settings

93 The filtered continuity and Navier–Stokes equations governing unsteady incompress-
 94 ible flow are

$$\frac{\partial u_i}{\partial x_i} = 0, \quad (1a)$$

95 and,

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho} \left(\frac{\partial p}{\partial x_i} + \delta_{i1} \frac{\partial \langle P \rangle}{\partial x_1} \right) + \frac{\partial}{\partial x_j} \left(\frac{\tau_{ij}}{\rho} + \nu \frac{\partial u_i}{\partial x_j} \right) + f \delta_{i3}. \quad (1b)$$

96 The resolved-scale velocity and pressure are respectively given by u_i and p with u ,
 97 v and w the streamwise, lateral and vertical velocity components respectively. The
 98 flow was driven by a constant mean streamwise pressure gradient $\partial \langle P \rangle / \partial x$ and δ_{i1} is
 99 the Kronecker-delta. $f \delta_{i3}$ is the body force due to thermal buoyancy and is estimated
 100 using the Boussinesq approximation. ρ and ν are the density and kinematic viscosity
 101 of the fluid. τ_{ij} is the subgrid-scale (SGS) Reynolds stress and was handled using
 102 the Smagorinsky model in conjunction with a Lilly damping function near the walls.
 103 We set Smagorinsky’s constant $C_s = 0.1$ since this was found to provide satisfactory
 104 results in our earlier computations (Boppana et al., 2010).

105 In the streamwise (x) and lateral (y) directions, periodic boundary conditions were
 106 employed. Stress free conditions were imposed on the top of the domain, i.e.,

$$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0; \quad w = 0. \quad (2)$$

107 No slip conditions were set on the bottom surface ($z = 0$) and on all faces of the
 108 roughness elements.

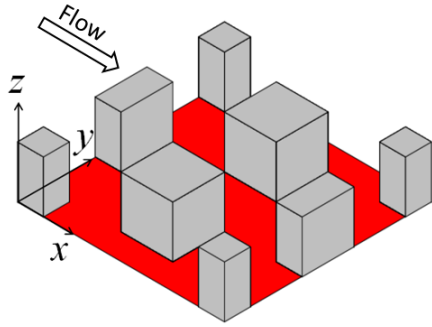


Fig. 1 Sketch of 3-D view of computational domain. All the bottom surface between cubes is heated or cooled.

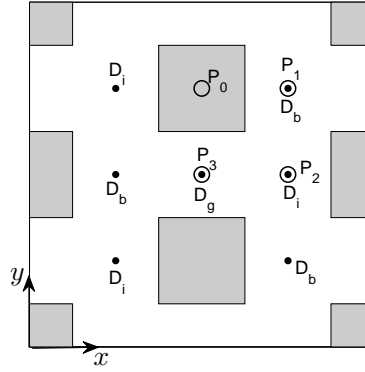


Fig. 2 Plan view of computational domain. The four typical locations, P_{0-3} are identified by ‘circles’ and data at ‘dots’ D_b, D_i, D_g are used for quadrant analysis in Sect. 3.5.

109 The filtered governing equation for temperature is

$$\frac{\partial T}{\partial t} + \frac{\partial u_j T}{\partial x_j} = \frac{\partial}{\partial x_j} \left((k_s + k_m) \frac{\partial T}{\partial x_j} \right), \quad (3)$$

110 where T is the resolved-scale temperature. k_s is the subgrid turbulent diffusivity and
 111 is given by v_s/Pr_s , where v_s is the subgrid viscosity and Pr_s is the subgrid Prandtl
 112 number whose value was set to 0.9. k_m is the molecular diffusivity and is defined
 113 as v/Pr_m , where Pr_m is the molecular Prandtl number whose value was set to 0.71
 114 in our computations. Periodic boundary conditions were specified in the streamwise
 115 and spanwise directions. The stable stratification in the computational domain was
 116 obtained by specifying a negative heat flux at the bottom surface and the same was
 117 set to enter through the top surface. Similarly, the unstable stratification was obtained
 118 by specifying a positive heat flux at the bottom surface of the computational domain
 119 and the same was set to leave through the top surface. These computations were
 120 done on a domain size of $L_x \times L_y \times L_z = 4h \times 4h \times 6h$ (D4), where $h = 0.2$ m is the
 121 cube height. Whilst this domain is probably too small to capture adequately the long
 122 streamwise rolls known to exist in the outer flow, earlier work has demonstrated that it
 123 is sufficient for domain-independent mean flow fields, particularly within the canopy
 124 region. For example, based on two-point measurements on an array of the same con-
 125 figuration, Castro et al. (2006) showed that the integral length scales are constant in
 126 the region $2 \leq z/h \leq 4$ and are $3h, 0.8h$ and h in x, y and z directions respectively.
 127 Also, the DNS study by Coceal et al. (2006) showed that the mean flow field is in-
 128 dependent of the domain sizes $4h \times 4h \times 4h, 8h \times 8h \times 4h$ and $4h \times 4h \times 6h$. 3-D and
 129 plan views of the computational domain are shown in Figs. 1 and 2 respectively. A
 130 finite volume approach was followed to discretize the flow and temperature equa-
 131 tions. The monotone advection and reconstruction scheme (STAR-CD, 2007) with a
 132 blending factor of 0.9 was used for the spatial convective terms and the central differ-
 133 ence scheme was used for the spatial diffusive terms of (1) and (3). A second-order

backward implicit scheme was used for discretizing the time-dependent term. The computational domain D4 consisted of hexahedral cells and the grid resolution was $h/16$. The driving force was the constant streamwise pressure gradient in Eq. (1) on every cell and is given by

$$\frac{\partial \langle P \rangle}{\partial x} = \frac{\rho u_\tau^2}{L_z} \quad (4)$$

where u_τ is the total wall friction velocity. The Reynolds number (Re_τ) based on the total wall friction velocity and h was approximately 1200. The Reynolds number (Re) based on h and the streamwise velocity at h varied from 3000 to 5000. The initial duration of most of the simulations was approximately $200e_t$ where $e_t = h/u_\tau$ is the eddy turn-over time. The averaging duration varied from $200e_t$ to $400e_t$ depending on how rapidly the shear and dispersive stresses converged. All the computations were carried out using STAR-CD version 4.14 (STAR-CD, 2007).

Sensitivity tests were done by conducting a further four independent sets of computations. They are

1. D4T - constant temperature instead of constant heat flux was specified on the top and bottom surfaces of the computational domain D4.
2. D4S - As an alternative means of achieving steady state for energy in the computational domain, constant heat sink (source) for unstable (stable) stratification was specified in all computational cells in D4 instead of a constant heat flux boundary condition on the top surface.
3. D16 - the domain size was $8h \times 8h \times 10h$ with constant heat flux on the top and bottom surfaces of the domain. The vertical resolution varied geometrically from $h/64$ at $z = 0$ to $h/16$ at the building height i.e $z = h$, and in the remaining parts of the domain $h/16$ was used.
4. D64 - the domain size was $16h \times 16h \times 10h$ with constant heat flux on the top and bottom surfaces of the domain. A uniform resolution of $h/16$ was set throughout the domain.

A summary of all computations is given in Table 1.

3 Results

The first objective stated at the end of the Sect. 1 is addressed by determining the drag and heat transfer coefficients, displacement height d and roughness length z_0 for various Ri_τ in Sect. 3.1 and 3.2 respectively. By analysing the Reynolds stress anisotropy map, spatial correlations, quadrant and octant events for stable, neutral and unstable cases, the second objective is addressed and the details are presented in the latter subsections.

3.1 Drag and heat transfer coefficients

The degree of thermal heating or cooling can be characterized by the Richardson number Ri_τ defined as

$$Ri_\tau = \frac{gh(T_b - T_{z=0})}{T_b u_\tau^2} \quad (5)$$

Type of instability	Domain size	Type of thermal boundary condition	$q_{z=0}$ (Wm^{-2}) or $T_{z=0}$ (K)	Ri_τ	C_d	C_h
Stable	$4h \times 4h \times 6h$ (D4)	constant heat flux	-3	0.8775	0.0739	0.0066
			-8	2.5099	0.0645	0.0057
			-10	3.1986	0.0662	0.0057
			-12.5	4.1042	0.0628	0.0054
			-15	4.9978	0.0618	0.0053
			-18	6.1868	0.0569	0.0049
			-25	8.9943	0.0552	0.0046
Unstable	$4h \times 4h \times 6h$ (D4)	constant heat flux	1	-0.2737	0.0758	0.0071
			3	-0.7909	0.0779	0.0075
			8	-2.0472	0.0812	0.0079
			12.5	-3.0969	0.0856	0.0084
			25	-6.0259	0.0959	0.0091
			50	-11.6382	0.1158	0.0104
			100	-22.3893	0.1552	0.0125
Unstable	$4h \times 4h \times 6h$ (D4T)	constant temperature	293.35	-0.2703	0.0765	0.0072
			294	-0.7788	0.0796	0.0074
			297	-3.1465	0.0868	0.0084
			307	-11.0886	0.1155	0.011
Unstable ^a	$4h \times 4h \times 6h$ (D4S)	constant heat flux	3	-0.7343	0.0761	0.008
			8	-1.9416	0.0811	0.0083
Stable ^b	$4h \times 4h \times 6h$ (D4S)	constant heat flux	-8	2.0722	0.0688	0.0072
			-12.5	3.2955	0.064	0.0068
Unstable	$8h \times 8h \times 10h$ (D16)	constant heat flux	8	-1.504	0.0791	0.0106
			25	-4.1804	0.0862	0.0125
Unstable	$16h \times 16h \times 10h$ (D64)	constant heat flux	3	-0.75	0.0814	0.0081
			8	-1.9387	0.0817	0.0084
Neutral	D4	-	-	0	0.0759	-
	D16	-	-	0	0.0762	-
	D64	-	-	0	0.0816	-

Table 1 Summary of computational cases.

a – To establish a steady state for energy, constant heat sink is specified throughout the domain.

b – To establish a steady state for energy, constant heat source is specified throughout the domain.

171 where g is the acceleration due to gravity and T_b is the bulk temperature, which is the
172 average temperature over the whole domain. It is to be noted that Ri_τ is not known *a*
173 *priori*, but is an outcome of the computation that depends on the specified boundary
174 conditions. The values of Ri_τ along with the resulting coefficients are listed in Table 1.
175 Instead of using the bulk or gradient Richardson numbers to represent the degree of
176 thermal stratification, a frictional Richardson number is used here because the former
177 two depend on domain size and particularly good accuracy in determination of the
178 flux gradients, respectively. In the conventional definition of Ri_τ , which is often used
179 in (open) channel flows (e.g. Armenio and Sarkar, 2002; Dong and Lu, 2005; García-
180 Villalba and del Álamo, 2011), the density or temperature difference between the two
181 surfaces and channel half height are used. This definition is modified here for two
182 reasons: (i) because a roughness height is a more appropriate characteristic length
183 and (ii) similar to the bulk velocity, temperature distribution inside the domain also
184 depends on domain height. Therefore, the temperature difference between the ground
185 surface and bulk temperature instead of that at the top surface is used.

186 The thermal impact on the surrounding flow can be quantified using drag (C_d)
 187 and heat transfer (C_h) coefficients defined here as

$$C_d = \frac{u_\tau^2}{u_{z=h}^2} \quad (6)$$

188

$$C_h = \frac{q_{z=0}}{c_p \rho u_{z=h} (T_b - T_{z=0})} \quad (7)$$

189 where c_p is the specific heat capacity at constant pressure and $q_{z=0}$ is the heat flux
 190 at the ground surface. Note that when constant heat flux was specified on the bottom
 191 surface, $T_{z=0}$ is the spatially and temporally averaged non-uniform surface tempera-
 192 ture. Similarly when constant temperature was specified on the ground surface, $q_{z=0}$
 193 is the spatially and temporally averaged non-uniform surface heat flux. The procedure
 194 for obtaining $T_{z=0}$ or $q_{z=0}$ (STAR-CD, 2007) was as follows:

$$T^+ = \begin{cases} Pr_m z^+ & \text{if } z^+ \leq z_T^+ \\ (Pr_s + Pr_m)(u^+ + P) & \text{if } z^+ > z_T^+ \end{cases} \quad (8)$$

195 where

$$T^+ = \frac{c_p \rho (T_{z=0} - T_{z_1}) u_*}{q_{z=0}} \quad (9)$$

196 and

$$u^+ = \begin{cases} z^+ & \text{if } z^+ \leq z_u^+ \\ \frac{1}{\kappa} \ln(E z^+) & \text{if } z^+ > z_u^+ \end{cases} \quad (10)$$

Here $z^+ = z_1 u_* / \nu$, T_{z_1} is the temperature at the near-wall grid point, z_1 is the distance
 from the wall to the centre of the near-wall grid point, u_* is the near-wall friction
 velocity determined by Spalding's law (Shih et al., 1999) and P is the sub-layer resis-
 tance factor (Jayatilika, 1969). z_u^+ and z_T^+ satisfy the following equations:

$$z_u^+ - \frac{1}{\kappa} (E z_u^+) = 0 \quad (11)$$

$$Pr_m z_T^+ - (Pr_s + Pr_m) \left[\frac{1}{\kappa} \ln(E z_T^+) + P \right] = 0 \quad (12)$$

197 where E is an empirical coefficient whose value was set to 9. It was observed in our
 198 computations that most of the near-wall grid points lie within the viscous sublayer.

199 For the basic case, D4, Figs. 3a and b show an increase in C_d and C_h as the
 200 thermal stratification changed from stable to unstable. For $Ri_\tau < 0$, a similar increas-
 201 ing trend was also found by Cheng and Liu (2011) and Kanda et al. (2007) in 2-D
 202 street canyons and the COSMO (Comprehensive Outdoor Scale Model) experiments
 203 respectively. Such an increase is due to a gradual increase in the strength of the tur-
 204 bulence motions, as illustrated by the data in Fig. 8a (discussed later). In comparison
 205 with the flow over smooth terrain, stability effects on the flow over a rough surface
 206 are likely to be lower because of the dominant influence of the mechanical turbulence
 207 generated by the roughness elements. However, the assumption that urban flows may
 208 be considered as neutral or nearly neutral in urban dispersion models (Britter and
 209 Hanna, 2003) is probably invalid, as the results presented above suggest that stratifi-
 210 cation effects are not negligible.

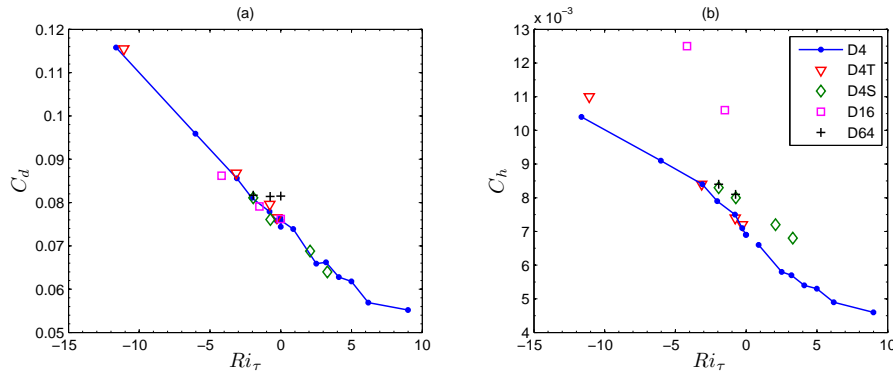


Fig. 3 Variation of (a) C_d and (b) C_h with Ri_τ . For the legend details, see Table 1.

211 3.1.1 Sensitivity checks

212 As mentioned in Sect. 2, the sensitivity tests were done by performing computations
 213 on different domain sizes and grid resolution. In Figs. 3a and b, the values of C_d and
 214 C_h from D16 and D64 are also shown. It can be observed that C_d from D4 and D16
 215 are in good agreement. Both D4 and D16 show gradual increase in C_d with Ri_τ , while
 216 the drag coefficient from D64 remains constant as Ri_τ decreases from 0 to -1.94 but
 217 is anyway quite close to the results from the smaller domains. Figure 3(b) shows that
 218 the values of C_h from D64 are approximately 7% larger and those from D16 are ap-
 219 proximately 41% larger than D4. The significant increase seen in D16 can perhaps be
 220 partly attributed to domain size effects but, much more importantly, is a direct result
 221 of the much finer resolution near the ground surface. Although we have shown that
 222 it is necessary to employ fine resolution near the surface to predict scalar transfer co-
 223 efficients very accurately (Boppa et al., 2010), to save on expensive computational
 224 time (which would be particularly demanding for D64) an identical uniform resolu-
 225 tion of $h/16$ was enforced in all D4 and D64 cases. These computations show that
 226 C_d is fairly insensitive to both domain size and resolution but the estimation of C_h is
 227 indeed significantly affected by the mesh resolution. Therefore, the variation of C_h
 228 with Ri_τ shown here should be considered as a qualitative indicator only.

229 Figures 3a and b also show that the two types of thermal boundary conditions, i.e
 230 constant heat flux (D4) and constant temperature (D4T) on bottom and top surfaces
 231 of the computational domain, yield very similar values of C_d and C_h . Even though
 232 a constant heat flux (temperature) boundary condition at the bottom of a rough wall
 233 yields a non-uniform distribution of temperature (heat flux) around the obstacles,
 234 this study confirms that the integral quantities are not significantly affected by the
 235 different physics at the ground surface.

236 To establish a steady state for energy, all D4 unstable (stable) computations had
 237 constant heat flux entering (leaving) through the ground surface and leaving (en-
 238 tering) through the top surface of the computational domain. But this can also be
 239 achieved by specifying constant sink (source) in all cells of the computational domain
 240 for unstable (stable) cases and these simulations are classified as D4S. The differences

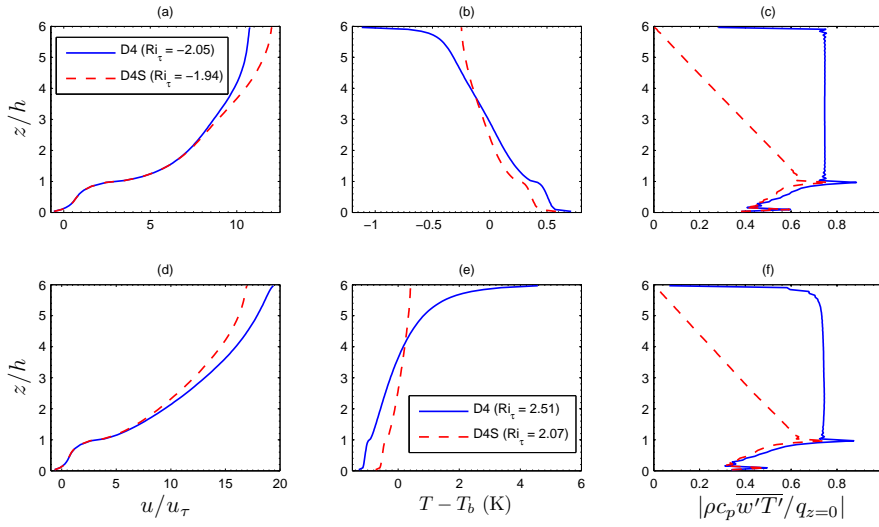


Fig. 4 Comparison of temporally and spatially averaged profiles of (a & d) normalized streamwise velocity, (b & e) temperature difference and (c & f) normalized vertical turbulent heat flux for D4 and D4S. Top row: unstable, bottom row: stable. For the legend details, see Table 1.

241 in the vertical distribution of turbulent heat flux for D4 and D4S are shown in Fig. 4c
 242 for both stable and unstable cases. Figure 3a shows that the drag coefficient is not
 243 affected by the way in which steady state for energy is achieved, but the values of C_h
 244 from D4S in Fig. 3b are found to be 25% larger than in D4 for the stable case, while
 245 only 5% larger in the unstable case. The reason for such differences can be explained
 246 from the temporal and spatial mean of the temperature difference, shown in Fig. 4b. It
 247 can be observed that the temperature variation with height is very much dependent on
 248 the way in which steady state for energy is achieved. This in turn affects the flow field
 249 and can be seen in the spatial and temporal mean profiles of streamwise velocity in
 250 Fig. 4a. This brief numerical test suggests that heat transfer coefficients are sensitive
 251 to the way in which steady state for energy in the computational domain is realised.
 252 It would be quite challenging if not impossible to set up heat sinks or sources away
 253 from boundaries in a wind-tunnel experiment, and in any case such sources or sinks
 254 are not possible physically without the action of additional flow variables, like mois-
 255 ture content. Further analysis in this current study is therefore restricted to cases with
 256 constant heat flux boundary conditions on the top and bottom surfaces.

257 3.1.2 A note on domain size and its influence on dispersive stresses

258 Dispersive stresses, denoted by $\langle \widetilde{uw} \rangle$ in the case of shear stress, arise due to spatial
 259 inhomogeneities in the flow. Therefore, their presence is expected below the canopy
 260 but not far above. In the case of D4, the dispersive stresses above the canopy were
 261 very small. But in the case of D64, it was observed that the dispersive stresses above
 262 the canopy persisted even after a time average duration of $1000e_t$. This is because
 263 D64 is conducive to the development of streamwise rolls that are larger in scale than

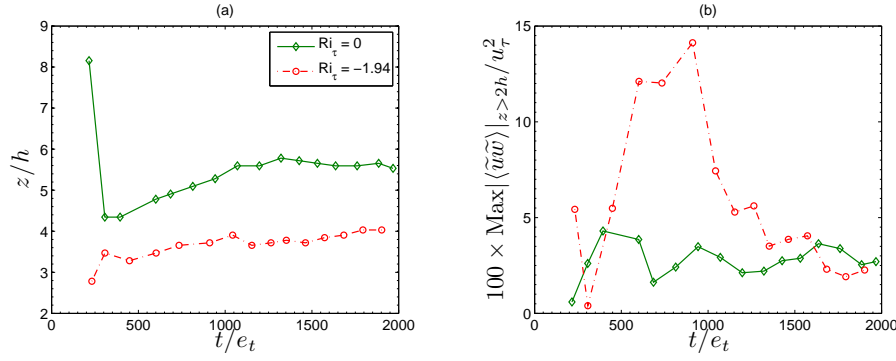


Fig. 5 (a) Location of maximum dispersive stress and (b) percentage variation of maximum dispersive stress with time mean duration. The initial duration for $Ri_\tau = 0$ and -1.94 are $200e_t$ and $400e_t$ respectively.

264 are allowed by domain D4. Such slow evolving mean longitudinal rolls are clearly
 265 shown in the DNS study of Coceal et al. (2006) for the neutral case. But it was
 266 also shown that for a sufficiently long averaging time i.e. $400e_t$, these dispersive
 267 stresses above the canopy disappear. It was observed in the current study that the
 268 dispersive stresses above $z/h = 2$ exhibit non-monotonic behaviour with increasing
 269 averaging time. This can be seen in Fig. 5b, where the percentage variation with
 270 averaging time of maximum dispersive stress for $z > 2h$ is shown. It can be observed
 271 that the maximum dispersive stress above the canopy appears to be converging to
 272 approximately 2.5% of the wall stress (or approximately 5% of the shear stress at
 273 that height) and the location at which it occurs is around $z/h = 5.5$ and 4 for neutral
 274 and unstable cases respectively. In a systematic set of investigations conducted by
 275 Fishpool et al. (2009) in a turbulent channel flow at $Re_\tau = 410$, it was observed that
 276 (i) the spanwise inhomogeneities persisted even when the domain length was increased
 277 from $2\pi\delta$ to 62δ , where 2δ is the channel depth and (ii) these features remained, with
 278 a large magnitude, for time averaging in excess of $10\delta/u_\tau$ (Fishpool et al., 2009,
 279 called δ/u_τ the ‘friction time scale’). Detailed investigations are being carried out on
 280 D64 to determine the averaging time required for the dispersive stresses to completely
 281 disappear (if they do) and the reason for their existence over long durations.

282 3.2 Determination of pressure distribution, d and z_0

283 It was observed in Sect. 3.1 that the increase in the drag coefficient with decreasing
 284 Ri_τ is correlated with an increase in the turbulent kinetic energy. More directly,
 285 however, it is the pressure difference between the windward and leeward sides of the
 286 cubes which determine the (form) drag. The vertical profiles of time- and laterally-
 287 averaged pressure coefficients (C_p) were obtained for various Ri_τ and are shown in
 288 Fig. 6a. The pressure coefficient is defined as

$$C_p = \frac{(p_w - p_l)}{\frac{1}{2}\rho u_{z=h}^2}, \quad (13)$$

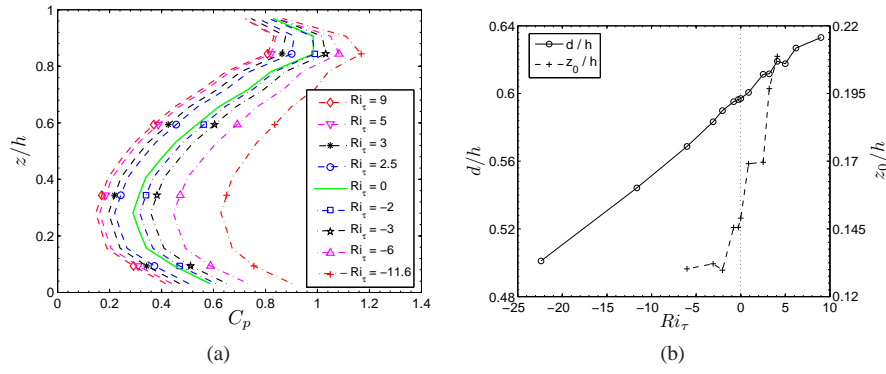


Fig. 6 Variation of (a) temporally and laterally averaged normalised pressure coefficient, and (b) mean displacement height and roughness length with Ri_τ .

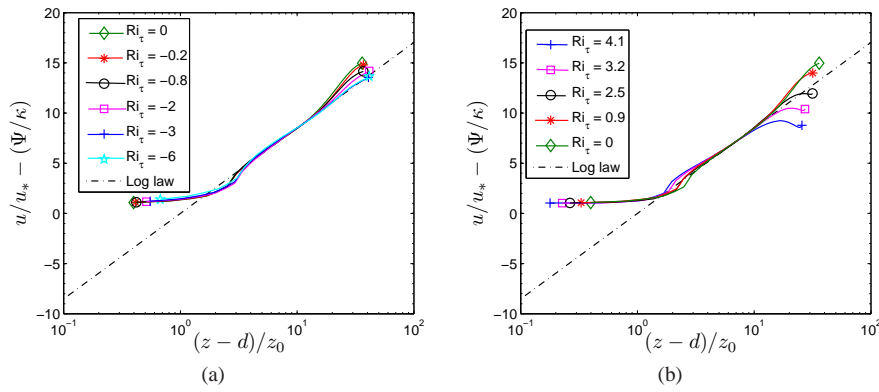


Fig. 7 Spatial- and temporal-averaged mean streamwise velocity profiles in log-linear form for various Ri_τ (a) neutral and unstable flows (b) neutral and stable flows.

289 where p_w and p_l are the pressures on the windward and leeward faces of the cube
 290 respectively. It can be observed that there is a notable increase in the values of C_p with
 291 ground heating and a slow decrease with ground cooling. The form drag, C_{pd} , can be
 292 obtained by integrating Eq. 13 with respect to z , and in all cases is approximately 85%
 293 of C_d . (The remaining drag component arises from frictional forces on the ground and
 294 the top and sides of the cubes, see Leonardi and Castro (2010) for a discussion on this
 295 point.)

296 The most sensible definition of the zero plane displacement height, d , is that it
 297 is the height at which the surface drag acts (Jackson, 1981). Assuming that frictional
 298 forces are negligible, this can be written (Coceal et al., 2006) as,

$$d = \frac{\int_z z(p_w - p_l) dz}{\int_z (p_w - p_l) dz}. \quad (14)$$

299 With the data shown in Fig. 6a this suggests that d decreases with increase in heating,
 300 as confirmed in Fig. 6b. Although the change only amounts to some 25% over the

301 range of Ri_τ covered, one would expect corresponding, but larger, changes in the
 302 roughness length z_0 , which was indeed found to be the case and can be seen in Fig. 6b.
 303 The procedure of obtaining z_0 at various Ri_τ is briefly described below.

304 The wind speed profile for non-neutral condition is given by (Stull, 2009):

$$\frac{u}{u_*} = \frac{1}{\kappa} \left[\ln \left(\frac{z-d}{z_0} \right) + \Psi \left(\frac{z-d}{L} \right) \right], \quad (15)$$

305 where κ is von Kármán's constant and L is the Obukhov length defined as

$$L = - \frac{\left[\overline{u'w'^2}_{z=0} + \overline{v'w'^2}_{z=0} \right]^{3/4}}{\kappa (g/\overline{T}_v) (\overline{w'T'_v})_{z=0}} \equiv - \frac{u_*^3}{\kappa (g/\overline{T}_{z=0}) (q_{z=0}/\rho c_p)}. \quad (16)$$

306 Here primed quantities denote deviation from their respective mean values, \overline{T}_v is
 307 mean virtual potential temperature and $\overline{w'T'_v}$ is the mean kinematic virtual potential
 308 temperature flux in the vertical direction. The stability function $\Psi((z-d)/L)$ is typ-
 309 ically given as (Stull, 2009)

$$\Psi \left(\frac{z-d}{L} \right) = \begin{cases} 4.7(z-d)/L & \text{for } Ri_\tau > 0 \\ -2 \ln \left[\frac{1+\gamma}{2} \right] - \ln \left[\frac{1+\gamma^2}{2} \right] + 2 \tan^{-1}(\gamma) - \frac{\pi}{2} & \text{for } Ri_\tau < 0, \end{cases} \quad (17)$$

310 where

$$\gamma = \left[1 - \zeta \frac{z-d}{L} \right]^{1/4} \quad \text{where } \zeta = 15. \quad (18)$$

311 In Eq. 15, $\Psi = 0$ yields the standard logarithmic law for neutral (rough-wall) flow,
 312 with u_* the surface friction velocity. (Note that the addition of the non-neutral term
 313 (Ψ) in Eq. 15 breaks the usual monotonic correspondence between z_0 and u_* , so that
 314 for $Ri_\tau \neq 0$ z_0 may rise when u_* falls or *vice versa*.) For a pressure-driven channel flow
 315 Coceal et al. (2006) derived $u_* = u_\tau \sqrt{(1-d/L_z)}$ to account for the linear variation
 316 in shear stress from $z/h = d$ to L_z which otherwise is constant in the surface layer of
 317 the atmospheric boundary layer. ζ in Eq. 18 is changed to 16 such that the resulting
 318 Ψ agreed with that given in Table 1.1 of Kaimal and Finnigan (1994).

319 Using d from Eq. 14, the values of κ and z_0 are obtained as fitting parameters of
 320 Eq. 15 for neutral flow. The necessary value of κ was found to be 0.27, which is 34%
 321 lower than the classical value of 0.41. A similar discrepancy from the classical value
 322 was also reported by Cheng and Castro (2002), Coceal et al. (2007), Leonardi and
 323 Castro (2010) to name a few.

324 By fixing κ as 0.27 and using the computed value of d for each Ri_τ , z_0 was
 325 deduced by fitting the measured u profile to Eq. 15 over a height range of $z/h = 1.5$
 326 to 2.5 - approximately chosen such that the variations of individual estimates of z_0
 327 from the velocity at a specific height in this range was less than 10%. However, for
 328 $Ri_\tau < -6$ and > 4.1 , the variation of z_0 in the above mentioned range of z/h exceeded
 329 10% and hence these data are not included in Fig. 6b.

330 Figure 7 shows the vertical variation of spatial- and temporal-averaged velocity
 331 profiles for neutral, stable and unstable cases. It can be observed that the LES data is
 332 not incompatible with the log-linear form and that for increasing $|Ri_\tau|$ the data appear
 333 to shift gradually to the right of the neutral case; this movement is found to be slightly
 334 stronger in stable flows.

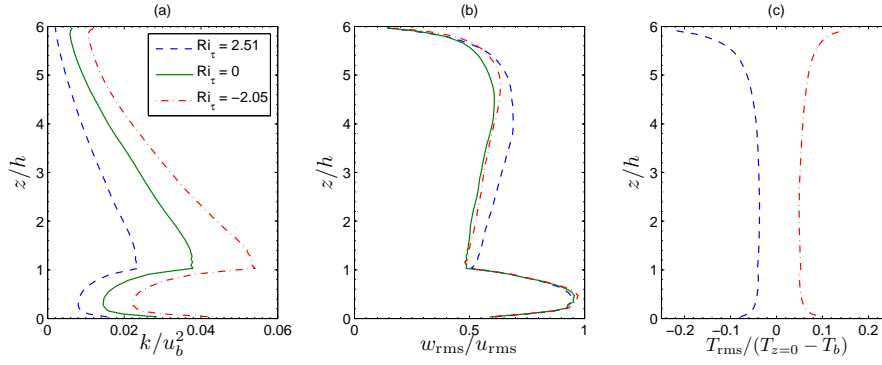


Fig. 8 Temporal and spatial mean of (a) turbulent kinetic energy normalized with bulk velocity, (b) ratio of vertical to streamwise Reynolds stresses and (c) normalized temperature fluctuations for $Ri_\tau \approx -2, 0$ and 2.5.

3.3 Turbulence level and Reynolds stresses anisotropy

Some effects of thermal stratification on the turbulence field are shown in Fig. 8 for D4. Increase in the normalized turbulent kinetic energy with decrease in Ri_τ is evident in Fig. 8a. The ratio of vertical to streamwise fluctuations in Fig. 8b is found to be nearly the same for neutral and unstable cases thus suggesting that this structural parameter is not affected by ground heating, at least within the range $0 > Ri_\tau \geq -2$. However, for the stable case at $Ri_\tau = 2.5$ the ratio is found to be slightly larger than in the neutral and unstable cases. This indicates that the turbulence structural characteristics of the stable case are different to those of neutral and unstable cases. Therefore, further exploration of turbulence structure have been carried out and are discussed in the following paragraphs. Figure 8(c) shows that the normalized temperature fluctuations are almost constant throughout the domain height, except near the bottom and top surfaces where the temperature gradients are inevitably strongest because of proximity to the imposed boundary conditions.

The anisotropy of the time mean Reynolds stresses is often used as an indicator of turbulence structure and this is shown using Lumley's anisotropy invariant map, AIM (Pope, 2011). Figure 9 shows AIM for various Ri_τ and for four typical locations, as identified by Castro et al. (2006) and indicated as P_{0-3} in Fig. 2. The AIM is obtained from the second and third principle invariants of the stress tensor b_{ij} , $6\eta^2 = -2II_b = b_{ij}b_{ji}$ and $6\xi^3 = 3III_b = b_{ij}b_{jk}b_{ki}$, where

$$b_{ij} = \frac{\langle \overline{u_i u_j} \rangle}{2k} - \frac{\delta_{ij}}{3}. \quad (19)$$

The vertical axis η of the AIM gives the magnitude of the anisotropy and the horizontal axis ξ represents the shape of anisotropy (i.e. distinguishing qualitatively between 'rod-like' and 'disc' shaped turbulent eddies). The linear sides of the triangle originating from $(\xi, \eta) = (0, 0)$ represent axisymmetric turbulence and the origin indicates isotropy. $\xi > 0$ implies 'rod-like' shaped turbulence where two eigenvalues of the Reynolds stress tensor are smaller than the third one and $\xi < 0$ refers to 'disc'

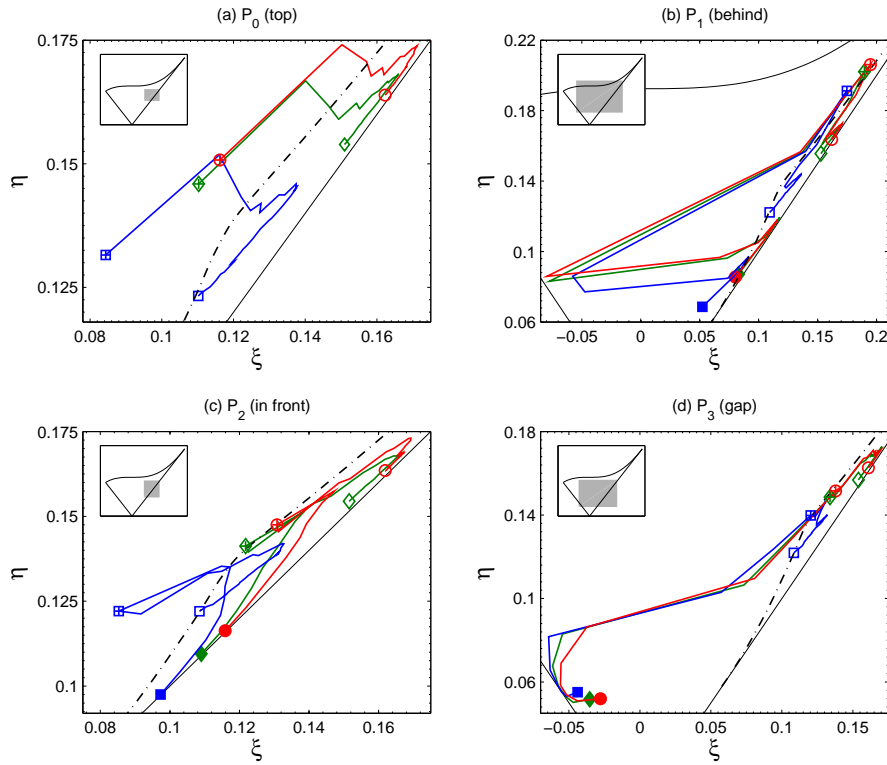


Fig. 9 AIM at typical locations for $Ri_\tau \approx -2$ (circles), 0 (diamonds) and 2.5 (squares). For clarity, only the immediate regions occupied by the data are shown, with solid black lines indicating boundaries of the Lumley triangle where appropriate; the inset figures show these regions in a grey shade, in relation to the entire Lumley triangle. Filled symbols are at $z/h = 0.5$, open symbols with an internal '+' are at $z/h = 1$ and clear open symbols are at $z/h = 3$. The dash-dot line near the right outline of the Lumley triangle is the logarithmic and core region data ($30 \leq z^+ \leq 180$) from smooth wall turbulent channel flow with $Re_\tau = 180$ (Busse and Sandham, 2012); here the data approach $(\xi, \eta) = (0, 0)$ with increasing distance from the wall.

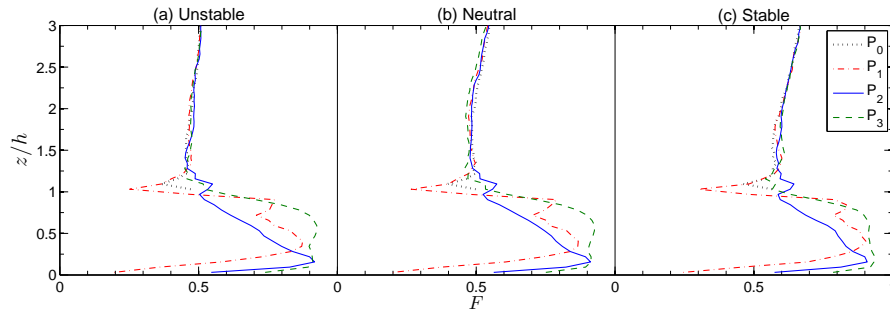


Fig. 10 The anisotropy function for (a) $Ri_\tau = -2$ (unstable), (b) 0 (neutral) and (c) 2.5 (stable) cases at typical locations.

361 shaped turbulence where two eigenvalues are greater than the third eigenvalue of the
 362 Reynolds stress tensor. The upper curve of the triangle represents two-component
 363 turbulence where one of the eigenvalues is zero.

364 For clarity, data within the AIM are shown only for $0.5 \leq z/h \leq 3$ in Fig. 9. The
 365 data shown at each typical location P_i , where $i = 0-3$, are temporal and spatial means
 366 at the four identical locations in the computational domain. Comparison of AIM data
 367 for P_1 and P_2 for $Ri_\tau = 0$ with the experimental values of Castro et al. (2006) show
 368 qualitative agreement (not shown here).

369 It is observed that the shapes of profiles for neutral and unstable cases are very
 370 similar and they differ mostly in the magnitude of anisotropy. The structure of the
 371 anisotropy for the stable case is found to be slightly different to that of neutral and
 372 unstable cases. At all four typical locations, the magnitude of anisotropy is found
 373 to be generally lower in a stable and higher in an unstable case, and this is very
 374 evident in the profiles at P_0 and P_2 . For $z/h > 1.2$ (i.e. in the log-linear region of
 375 the mean velocity profile), the profiles at all four locations are on or close to the right
 376 outline of the Lumley map just as they are in the log and core region of a smooth-wall
 377 turbulent channel flow at $Re_\tau = 180$ (Busse and Sandham, 2012). This suggests an
 378 axisymmetric nature of turbulence with predominantly ‘rod-like’ shaped eddies. With
 379 increasing z/h above the canopy the data tend to move towards the origin, just as they
 380 do in the smooth-wall channel flow. However, note that, unlike the data in the neutral
 381 and unstable cases that are very close to right outline of the Lumley map, stable case
 382 data are a little further away from the right boundary. Overall, we conclude that even
 383 with surface heating or cooling the turbulence structure in the log region (i.e. above
 384 the urban canopy) is not very different to that in the log region of flow over smooth
 385 surfaces. This indicates that for $z/h > 1.2$, the turbulent structure is similar to that of
 386 smooth-wall boundary layer. The fact that in neutral flows urban-type roughness does
 387 not have a large effect on turbulence structure at least qualitatively within the log law
 388 has previously been noted by Coceal et al. (2006). Based on the field measurements,
 389 same observation was made by Roth et al. (2013) and this is conceptually shown in
 390 the Fig. 6 of their article. It is interesting that the same seems to be true for cases of
 391 moderate ground heating or cooling. The data suggest that changes become apparent
 392 soonest for stable cases but, in any case, one would not expect the same conclusion
 393 to hold if Ri_τ were to increase to very large magnitudes.

394 As expected, the shapes of profiles at the lower heights (between $z/h = 0.5$ and
 395 1.2) differ significantly at the various locations. At P_1 and with z/h increasing from
 396 0.5, the turbulence structure becomes more ‘disc’ shaped, which could be due to the
 397 recirculation region, and again changes back to ‘rod-like’ shape as the profile reaches
 398 the canopy height. With increasing z/h at P_2 , the turbulence structure appears to drift
 399 gradually away from the ‘rod-like’ shape and revert back to this shape for $z/h > 1$.
 400 At P_3 , where the mean flow field experiences ‘channeling’ effects, the presence of
 401 side-walls appears to encourage the turbulence structure to be more ‘disc’ shaped,
 402 which is counter-intuitive.

403 A direct measure of the degree of isotropy in the turbulence is provided by the
 404 parameter $F = 1 + II_b + 27III_b$; $F = 0$ and 1 represents two-component and isotropic
 405 turbulence respectively. The values of this parameter at the four typical locations and
 406 for various Ri_τ are shown in Fig. 10. As expected, the values of F vary considerably

below the canopy, but not above where the flow is essentially homogeneous in x and y . Owing to the strong three-dimensional effects, the turbulence below the canopy becomes increasingly isotropic as z approaches zero, especially at P_1 and P_3 , until very close to the wall when of course eddies are strongly constrained vertically. Such high values of F were also observed in the wind-tunnel experiments of Castro et al. (2006) for the neutral case. Perhaps surprisingly, the values of F below the canopy are found to be almost same for stable, neutral and unstable cases. This must be due to the very high turbulence intensities caused by shear and the wake of the cubes, which are not strongly reduced by surface heating or cooling. But above the canopy, the stable case shows slightly larger values of F compared to neutral and unstable cases.

The above analysis was also carried out for case D64 with $Ri_\tau = 0$ and -2 ; the corresponding figures (not shown here) show qualitatively similar behaviour to that for D4. Differences were most evident above the canopy, no doubt because of the non-zero dispersive stresses there.

3.4 Spatial correlations

In order to determine the influence of thermal stratification on the integral length scales of the turbulent structures, two-point velocity correlations were computed. The spatial correlation for streamwise velocity in the streamwise direction is given by (e.g. Castro et al., 2006)

$$R_{uu}(\Delta x) = \frac{\overline{u'(x)u'(x+\Delta x)}}{\overline{\sigma'_u(x)\sigma'_u(x+\Delta x)}}. \quad (20)$$

The two-point correlation of vertical velocity in the vertical direction is obtained by replacing u and x in Eq. (20) with w and z respectively. Figure 11 shows these computed correlations for D4; the streamwise spatial correlations are shown at $z/h = 1.28$ and the vertical spatial correlations are obtained by specifying $z/h = 1.53$ as a fixed reference. It is observed in this figure that $R_{uu}(\Delta x)$ does not tend to zero at $\Delta x = 2$, which is half of the streamwise domain length. This suggests that the domain length is not sufficient to capture the longest eddy structures. Nonetheless, we can make some deductions from the data.

Figure 11 shows that $R_{uu}(\Delta x)$ for the stable case is lower than that of the neutral and heated cases. The streamwise integral length scale has clearly been significantly reduced by ground cooling, but appears not to be influenced by ground heating. The reason for such a strong influence on streamwise length scales by stable stratification is not yet completely understood, although it is well known that stability generally weakens turbulence fields. The profiles of $R_{ww}(\Delta z)$ indicate that the vertical integral length scales are marginally increased and decreased by ground heating and cooling respectively. This is expected because the size of the vertical structures is enhanced by thermal plumes due to buoyancy in an unstable case and reduced in the case of stable stratification. These spatial correlations suggest that the turbulent structures are smaller in stable stratification when compared to neutral and unstable cases. As smaller structures tend to be more isotropic, this observation is consistent with the

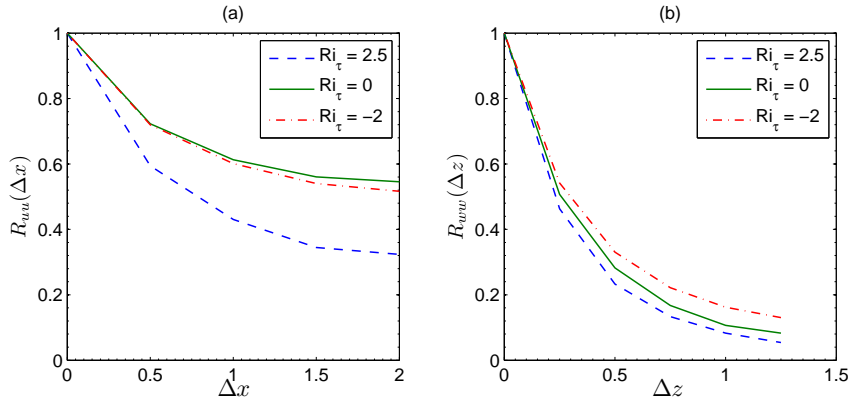


Fig. 11 Spatial correlations of (a) u in x direction and (b) w in z direction for different Ri_τ .

447 implications of the AIM discussed in Sect. 3.3. The spatial correlations from D64 for
 448 neutral and unstable cases are, incidentally, found to be similar to those of D4.

449 3.5 Quadrant and Octant Analysis

450 The occurrence and contribution of various intermittent events to the transfer of mo-
 451 mentum and heat is often deduced using quadrant analysis. According to this, the
 452 events are classified as follows

$$\begin{aligned}
 \text{Q1: } & u' > 0, w' > 0; & \theta' > 0, w' > 0 \\
 \text{Q2: } & u' < 0, w' > 0; & \theta' < 0, w' > 0 \\
 \text{Q3: } & u' < 0, w' < 0; & \theta' < 0, w' < 0 \\
 \text{Q4: } & u' > 0, w' < 0; & \theta' > 0, w' < 0
 \end{aligned}
 \tag{21}$$

453 where primed quantities refer to fluctuating values (about their respective time-means).
 454 In the case of momentum flux, ‘Q2’ refers to movement of low-speed fluid in the up-
 455 ward direction (referred as ‘ejections’) and ‘Q4’ refers to movement of high-speed
 456 fluid in the downward direction (referred as ‘sweeps’). In the case of stable stratifi-
 457 cation, ‘Q2’ refers to those events where cold fluid moves in the upward direction
 458 (termed as ‘updrafts’) and ‘Q4’ refers to those events where hot fluid moves in the
 459 downward direction (termed as ‘downdrafts’). In the case of unstable stratification,
 460 ‘Q1’ refers to ‘updrafts’ where hot fluid is ejected and ‘Q3’ refers to ‘downdrafts’
 461 where cold fluid moves in the downward direction. The difference in the frequency
 462 of occurrence of sweeps and ejections, and downdrafts and updrafts, and their pro-
 463 portional contribution to total momentum and heat fluxes (often referred to as ‘flux
 464 fraction’, but here we use the term ‘strength’) are shown in Fig. 12. The method used
 465 to obtain the frequency and strength of momentum and heat flux for various events
 466 is explained in detail in Boppana et al. (2013). The values shown in Figs. 12a, d are
 467 obtained using a time average of $330e_t$ and a spatial average of data at all the seven
 468 locations shown as dots in Fig. 2 and identified as D_b , D_i and D_g . The values shown

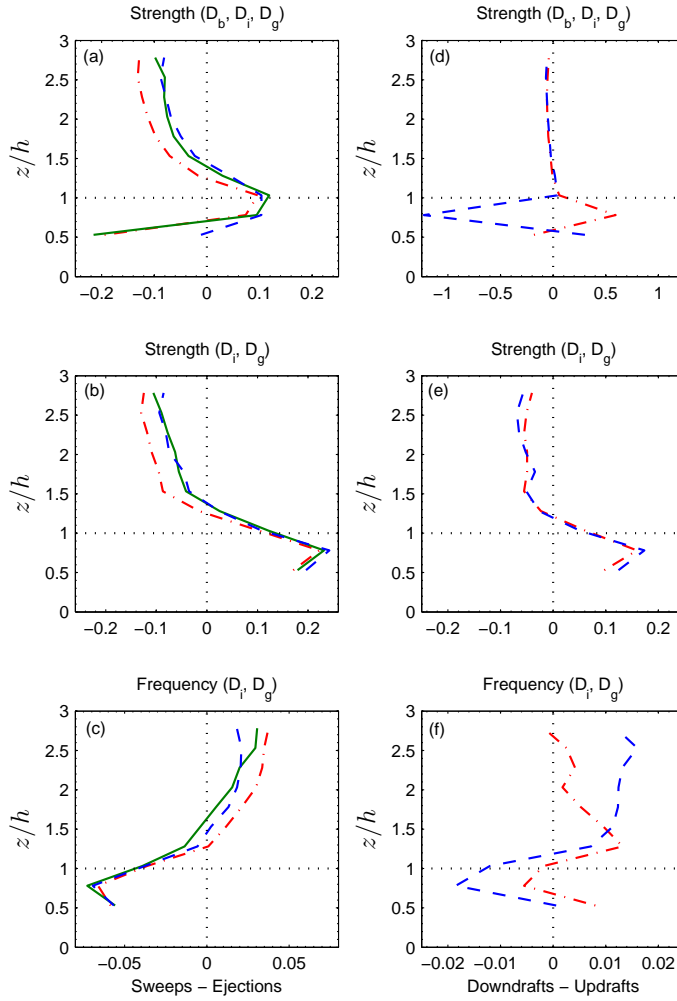


Fig. 12 Left column: differences in the contributions to momentum flux by sweeps (Q4) and ejections (Q2) (a), (b) and the difference in their frequency of occurrence (c); right column: differences in the contributions to heat flux by downdrafts (Q4 - stable, Q3 - unstable) and updrafts (Q2 - stable, Q1 - unstable) (d), (e), and the difference in their frequency of occurrence (f). Dash-dot lines: unstable ($Ri_\tau = -2$); solid lines: neutral ($Ri_\tau = 0$); dash lines: stable ($Ri_\tau = 2.5$). The time and spatial average of data from D_b , D_i and D_g locations (shown in Fig. 2) are used in (a) and (d), and the average of data from D_i and D_g are used in (b), (c), (e) and (f).

469 in Figs. 12b, c, e and f are from a time and spatial average of the four locations D_i
 470 and D_g which do not lie in the recirculating regions immediately behind the cubes.

471 The time and spatial average of data from all seven locations shows that ejections
 472 are stronger above the canopy (Fig. 12a), but below the canopy ejections dominate at
 473 $z/h \approx 0.5$ whilst, for $0.5 \leq z/h \leq 1$, sweeps contribute more to the momentum flux.
 474 Such a non-monotonic behaviour below the canopy is a result of the strong influence

475 of the recirculating region in the wake of the cubes. This influence is also observed
 476 in the strength of events contributing to heat flux (Fig. 12d). As suggested in the
 477 DNS study by Coceal et al. (2007), it is instructive to obtain the temporal and spatial
 478 mean from all locations in the computational domain. But as the available data here
 479 is limited to seven locations, the data from the three locations behind the cubes (D_b)
 480 have been excluded in some of the results shown so as to prevent the strong influence
 481 from the recirculation region biasing the results of the quadrant analysis. Figure 12b
 482 then shows that momentum flux is dominated by sweeps below the canopy, which is
 483 consistent with the observations made in the DNS study. (Including the three ‘behind
 484 cube’ profile locations destroys that consistency.) Further analysis will therefore be
 485 based on the time and spatial average data from the D_i and D_g locations only, shown
 486 in Figs. 12b, c, e and f.

487 Below the canopy, the strength and frequency of momentum flux events in Figs.
 488 12b,c are found to be the same for unstable, neutral and stable cases. This implies
 489 that the mechanical turbulence generated by the roughness elements has a much
 490 stronger influence than the local thermal stratification. Further above the canopy,
 491 thermal stratification, especially for the unstable case, appears to have a notable ef-
 492 fect as the strength of ejections and the frequency of sweeps is enhanced. In the field
 493 study of Christen et al. (2007), point measurements from a tower in an urban street
 494 canyon showed qualitatively similar behavior except that the strength of ejections be-
 495 gins to dominate sweeps at $z/z_h = 1.9$ for an unstable case and at $z/z_h = 2.5$ for a
 496 near-neutral case, whereas sweeps dominated throughout the measurement height i.e.
 497 $0.5 \leq z/z_h \leq 2.5$ in the stable case (z_h is an average building height). The reason for
 498 these minor differences between field experiments and LES could be partly attributed
 499 to the urban morphometry, different Ri_τ , prevailing meteorological conditions (e.g.
 500 large-scale turbulent motions (Michioka et al., 2011) and wind direction) in the field.

501 Similar to the momentum flux contributions in stable and unstable cases, down-
 502 drafts contribute more to the heat flux below the canopy and updrafts are stronger
 503 above the canopy. Figure 12f suggests the reverse behaviour in the frequency of
 504 events. The field study of Christen et al. (2007) showed similar behaviour in the
 505 strength of events, but the stratification effects were found to be strong above the
 506 canopy unlike this study, probably for reasons similar to those mentioned above.

507 The same analysis was carried out on time series data corresponding to a duration
 508 of $2000e_t$ and from eight locations situated in front of the cubes in D64. The strength
 509 and frequency of events were found to be qualitatively very similar to those described
 510 above for the D4 domain.

511 From the above analysis, it is understood that for both stable and unstable cases,
 512 above the canopy ejections and updrafts contribute more to the momentum flux and
 513 heat flux respectively, whereas within the canopy sweeps and downdrafts dominate.
 514 Sweeps and downdrafts occur more often above the canopy, whilst ejections and up-
 515 drafts are more frequent within the canopy. But it is not immediately clear if the
 516 updraft (downdraft) and ejection (sweep) events are correlated. Inagaki et al. (2012)
 517 showed that the horizontal distribution of ejection and sweep events at the building
 518 height is similar to the distribution of updraft and downdraft events suggesting that
 519 these events might be correlated. To determine this quantitatively, octant analysis (as
 520 used by Dupont and Patton, 2012, on a vegetation canopy) has been conducted. Based

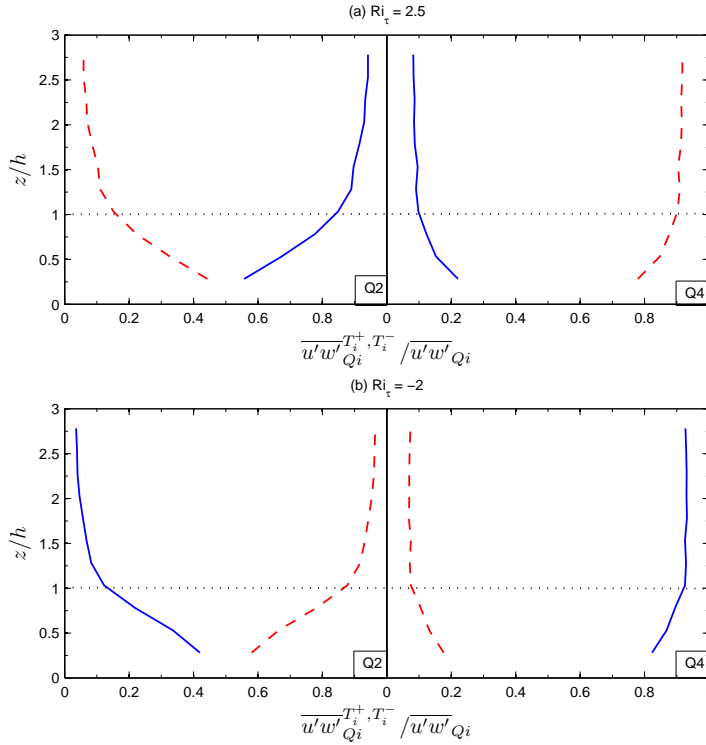


Fig. 13 Momentum flux associated with positive (dashed line) and negative (positive line) temperature fluctuations in quadrants 2 and 4 for (a) stable and (b) unstable cases. The dotted line indicates the canopy top.

521 on the sign of temperature fluctuations, the momentum flux from a quadrant ‘ Q_i ’ is
 522 split further such that

$$\overline{(u'w')}_{Q_i} = \overline{(u'w')}_{Q_i}^{T_i^+} + \overline{(u'w')}_{Q_i}^{T_i^-}, \quad (22)$$

523 where T_i^+ and T_i^- correspond to positive and negative temperature fluctuations re-
 524 spectively in the quadrant Q_i . The two right hand terms in the above equation are
 525 normalized with their respective quadrant momentum fluxes and are shown in Fig. 13.
 526 For both stable and unstable cases the ‘updrafts’ contribution to the momentum flux
 527 is found to be larger in ‘Q2’ and the ‘downdrafts’ contribution is found to be larger
 528 for ‘Q4’. This suggests that updrafts (downdrafts) and ejections (sweeps) are well
 529 correlated, which implies at least some degree of similarity in momentum and heat
 530 transport for such flows.

531 4 Conclusions

532 The effects of local thermal stratification on the atmospheric flow in and above urban
533 canopies have been investigated by conducting large-eddy simulations on flow past
534 an array of staggered cubes, with the ground surface subjected to uniform cooling or
535 heating. The global thermal influences have been quantified by computing drag and
536 heat transfer coefficients. With increase in ground surface heating, characterised by
537 $-23 < Ri_\tau < -0.2$, a gradual increase in C_d and C_h was observed. Specification of
538 either constant heat flux or constant temperature boundary condition on the ground
539 surface yielded similar values of C_d and C_h , despite the different physics of flow
540 and heat very close to the ground surface. With increase in ground surface cooling,
541 i.e. $0 < Ri_\tau < 9$, a gradual decline in C_d and C_h was observed. The steady increase
542 in C_d and C_h with decrease in Ri_τ is linked with an increase in turbulent kinetic
543 energy due to buoyancy. The sensitivity tests included computations with different
544 domain sizes, grid resolution and means of achieving the steady state for energy in
545 the computational domain. These showed that C_d was relatively insensitive to all
546 these, but the estimates of C_h were found to be very sensitive to resolution in the
547 near-wall region, not surprisingly.

548 The structure of the turbulence for $Ri_\tau = -2, 0$ and 2.5 was then quantitatively
549 analysed by exploring the Reynolds stresses, spatial correlations and the results of
550 quadrant and octant analyses. The turbulence intensity was found to be significantly
551 affected by ground heating and cooling. However, the anisotropy invariant maps implied
552 that the shape of the turbulent structures remained very similar for neutral and
553 unstable cases, but differed slightly in the stable case. From the two-point spatial
554 correlations it was observed that the turbulent integral length scales of the structures
555 are reduced in both streamwise and vertical directions by stable stratification when
556 compared to the neutral case; only the vertical integral length scale was found to
557 be increased by ground heating. The quadrant analysis showed that ground heating
558 (cooling) enhances (reduces) the contribution of ejections to momentum flux above
559 the canopy whereas the contribution of updrafts and downdrafts to heat flux are found
560 to be very similar. Octant analysis showed that the strength of ejections (sweeps) and
561 updrafts (downdrafts) are well correlated, thereby suggesting that the transport mechanisms
562 of momentum and heat flux are similar above the canopy, probably because of
563 the prevailing large-scale structures although no attempt has yet been made to study
564 the correlated spectral content between ejections and updrafts in order to delineate
565 scale effects.

566 This study has shed some light on the effects of local thermal stratification on
567 the aerodynamic coefficients and turbulent structure of flow over an idealised urban
568 canopy. It would be useful to know whether the general conclusions outlined above
569 apply also to different kinds of roughness morphology, and to what extent they are
570 affected by differential surface heating arising for example from radiation. Coupled
571 with the present results, this might then be a further step towards understanding and
572 modelling the pollutant dispersion in significantly non-neutral urban boundary layers.

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