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Thermally induced mode coupling in rare-earth doped fiber amplifiers

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We present a simple semianalytical model of thermally induced mode coupling in multimode rare-earth doped fiber amplifiers. The model predicts that power can be transferred from the fundamental mode to a higher-order mode when the operating power exceeds a certain threshold, and thus provides an explanation of recently reported mode instability in such fiber amplifiers under high average-power operation. We apply our model to a simple step-index fiber design, and investigate how the power threshold depends on various design parameters of the fiber.   © 2012 Optical Society of America

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Large-mode-area rare-earth doped fiber amplifiers are well suited for high-power operation due to their large surface area to active volume ratio and low peak intensity in the core. Recently, however, it has been reported that a major limitation to the power scalability of these fiber amplifiers is the onset of mode instability as the average signal power reaches a certain threshold, which typically is on the order of a few hundred W up to about 1 kW [1].

Previous numerical investigations, based on a beam propagation approach [2,3], have discussed the importance of a thermally induced refractive index grating due to beating between the fundamental mode (FM) and a higher-order mode (HOM). While such a self-induced grating has the correct period to couple the FM and HOM, and thus lead to mode instability, a phase-lag between the grating and the intensity oscillation is required for efficient power transfer between the modes, as pointed out in [4]. Beam propagation codes are, however, numerically very intensive, especially when the transient thermal response is taken into account, and formulating a simplified model, which captures the essential physics of the thermally induced mode coupling, is desirable. In this Letter, we present such a model.

We assume that the electric field can be written as a superposition of two normalized modes $\psi_1(x, y)$ and $\psi_2(x, y)$ with propagation constants β_1, β_2,

$$E = a_1(z)\psi_1 e^{i(\beta_1 z - \omega_1 t)} + a_2(z)\psi_2 e^{i(\beta_2 z - \omega_2 t)} + c.c., \quad (1)$$

where ψ_1 is the FM and ψ_2 is the HOM. This field expansion is inserted into the scalar wave equation along with the relative permittivity, which is given by

$$\varepsilon(\mathbf{r}, t) = \varepsilon_f(\mathbf{r}_\perp) - i\frac{g(\mathbf{r})\sqrt{\varepsilon_f}}{k} + \Delta\varepsilon(\mathbf{r}, t), \quad (2)$$

where ε_f is the real relative permittivity of the fiber, g is the bulk gain coefficient due to the rare-earth doping, k is the vacuum wavenumber, taken to be the same for both field components, and the subscript $\perp$ denotes the transverse coordinates x, y. $\Delta\varepsilon = \eta\Delta T$ is the thermally induced perturbation, where ΔT is the increase in

temperature relative to a given reference temperature, which is typically the temperature of the cooling fluid, and η is the thermo-optic coefficient of the fiber material.

Our goal is to obtain coupled mode equations for the mode amplitudes $a_i(z)$. To obtain the coupling coefficients, we consider the heat equation under the assumption of a slow longitudinal variation of the temperature

$$\rho C \frac{\partial \Delta T}{\partial t} - \kappa \nabla_\perp^2 \Delta T(\mathbf{r}, t) = Q(\mathbf{r}, t), \quad (3)$$

where ρ is the density, C is the specific heat capacity, and κ is the thermal conductivity of the fiber material. The heat source term Q is related to the intensity I by

$$Q(\mathbf{r}, t) = \left(\frac{\lambda_s}{\lambda_p} - 1\right) g(\mathbf{r}) I(\mathbf{r}, t), \quad (4)$$

where λ_s and λ_p are the signal and pump wavelengths, respectively. Fourier transforming Eq. (3) with respect to time yields

$$\nabla_\perp^2 \Delta\tilde{T}(\mathbf{r}, \omega) - q(\omega)\Delta\tilde{T}(\mathbf{r}, \omega) = -\frac{\tilde{Q}(\mathbf{r}, \omega)}{\kappa}, \quad (5)$$

where $q = i\rho C\omega/\kappa$. This equation can be solved by the appropriate Green's function G [5]:

$$\Delta\tilde{T}(\mathbf{r}, \omega) = \frac{1}{\kappa} \iint G[\mathbf{r}_\perp, \mathbf{r}'_\perp, q(\omega)]\tilde{Q}(\mathbf{r}', \omega)\mathrm{d}^2\mathbf{r}'_\perp. \quad (6)$$

By inserting the field expansion in Eq. (1) into Eq. (4) and Fourier transforming with respect to time, we can obtain $\Delta\tilde{T}$ from Eq. (6). Inverting the Fourier transform then yields $\Delta\varepsilon(\mathbf{r}, t)$, which we need to derive the coupled mode equations for the mode amplitudes. This derivation is straightforward, so we simply state the result for the coupled mode equations in terms of the power P_i in mode i:

$$\frac{\partial P_1}{\partial z} = -\chi_1(\Delta\omega)g(z)P_2 P_1 + \Gamma_1 g(z)P_1, \quad (7)$$

$$\frac{\partial P_2}{\partial z} = \chi_2(\Delta\omega)g(z)P_2P_1 + \Gamma_2 g(z)P_2, \qquad (8)$$

where $\Delta\omega = \omega_1 - \omega_2$ and the coupling constants χ_i are given by

$$\chi_{1,2}(\Delta\omega) = \frac{\eta k^2}{\kappa\beta_{1,2}}\mathrm{Im}[A(\Delta\omega)]\left(1 - \frac{\lambda_s}{\lambda_p}\right). \qquad (9)$$

In the derivation of Eqs. (7) and (8), we have neglected the spatio-temporal oscillations of g, which occur if the gain is highly saturated. The quantity A is determined by the Green's function and the mode functions through

$$A = \iint \psi_1(\mathbf{r}_\perp)\psi_2(\mathbf{r}_\perp)$$
$$\times \iint_{\Omega_d} G(\mathbf{r}_\perp, \mathbf{r}'_\perp, \Delta\omega)\psi_1(\mathbf{r}'_\perp)\psi_2(\mathbf{r}'_\perp)\mathrm{d}^2\mathbf{r}'_\perp \mathrm{d}^2\mathbf{r}_\perp, \qquad (10)$$

where Ω_d denotes the doped cross section of the fiber and the outer integral is over the whole fiber cross section. This quantity can be calculated numerically for any given fiber design for which the Green's function and mode functions can be determined. From the property of the Green's function $G(\mathbf{r}_\perp, \mathbf{r}'_\perp, -\omega) = G(\mathbf{r}_\perp, \mathbf{r}'_\perp, \omega)^*$, we find the important property of the coupling constant $\chi_i(-\Delta\omega) = -\chi_i(\Delta\omega)$. Finally, the quantities Γ_i appearing in Eqs. (7) and (8) are given by the overlap integral

$$\Gamma_i = \frac{k}{\beta_i}\iint_{\Omega_d}\sqrt{\varepsilon_f(\mathbf{r}_\perp)}\psi_i(\mathbf{r}_\perp)^2\mathrm{d}^2\mathbf{r}_\perp, \qquad (11)$$

and determine the overlap between mode i and the doped region of the fiber.

In this Letter, we consider a simple water-cooled step-index fiber design for which analytical expressions for the Green's function and the fiber modes are known. The boundary condition for the Green's function is taken to be a Dirichlet boundary condition, which implies that the temperature of the fiber surface is fixed at the temperature of the coolant. The integrals in Eq. (10) are evaluated by standard numerical methods. We consider first a fiber with a V parameter of 3 and calculate the coupling constant between the LP_{01} and LP_{11} modes for varying core sizes R_c and a fixed outer radius R of 500 μm. The core refractive index is taken to be 1.45 and is assumed to have a uniform ytterbium doping. The signal and pump wavelengths are assumed to be 1030 and 975 nm, respectively. Figure 1 shows the coupling constant χ_1, which is indistinguishable from χ_2, as a function of $\Delta f = \Delta\omega/2\pi$ for core radii of 10, 20, and 40 μm. As expected, the coupling constant vanishes for $\Delta f = 0$. However, for a small frequency difference on the order of 1 kHz, depending on core size, the coupling constant is positive and a transfer of power from LP_{01} to LP_{11} can occur. Interestingly, the maximum value of the coupling constant is largely insensitive to the core size, and only the position and width of the peak vary significantly.

We now investigate the dependence of the coupling constant on the V parameter of the fiber by calculating χ for a fixed $R_c = 20$ μm and varying V with all other
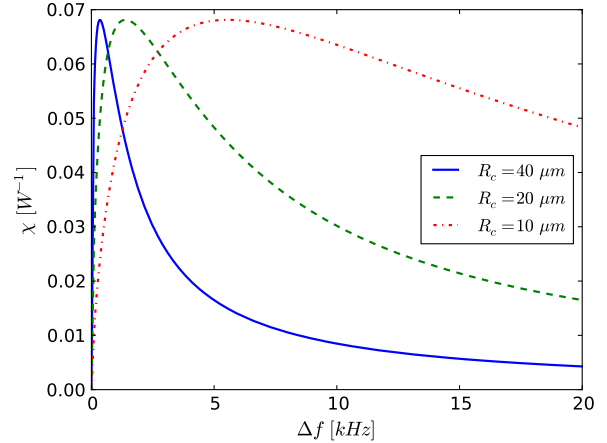

Fig. 1. (Color online) Coupling constant χ for $LP_{01} - LP_{11}$ coupling as a function of Δf for varying R_c and $V = 3$.

parameters the same as before. The results are shown in Fig. 2. Here we see a significant increase in χ as V increases. This can be understood by considering Eq. (10), from which it is clear that the coupling constant depends on the overlap of the two modes. For $V = 2.5$, which is just above cut-off for LP_{11}, the overlap between the two modes is small, and as V increases, the overlap, and hence the coupling constant, increases as well.

Finally, we investigate the dependence of the coupling constant on the size of the rare-earth doped region. We do this by considering a fiber with fixed $V = 3$ and $R_c = 20$ μm but varying radius of the doped region of the core R_{Yb}. The results are shown in Fig. 3. It is seen that reducing the size of the doped region causes a significant reduction in the coupling constant, which again is expected from Eq. (10), since the inner integral is taken over the decreasing doped region, while the mode functions remain unchanged.

To study the onset of mode instability in the fiber amplifiers, we solve Eqs. (7) and (8) under the assumption $P_2 \ll P_1$. We thus obtain

$$\frac{P_2(L)}{P(L)} \approx \frac{P_2(0)}{P_1(0)}\exp\left(\frac{\chi(\Delta\omega)}{\Gamma_1}\Delta P - \Delta\Gamma g_{av}L\right), \qquad (12)$$
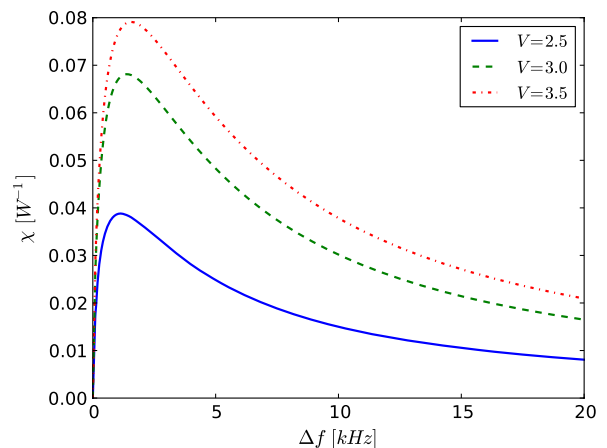

Fig. 2. (Color online) Coupling constant χ for $LP_{01} - LP_{11}$ coupling as a function of Δf for varying V and $R_c = 20$ μm.
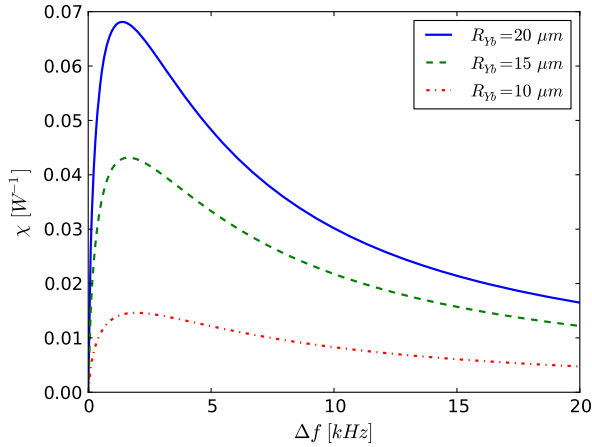
Fig. 3.   (Color online) Coupling constant χ for $LP_{01} - LP_{11}$ coupling as a function of Δf for varying R_{Yb} and $R_c = 20\ \mu m$, $V = 3$.

Table 1.   Power Threshold

R_c [μm]	R_{Yb} [μm]	V	P_{th} [W]
10	10	3	440
20	20	3	458
40	40	3	479
20	20	3.5	401
20	20	2.5	813
20	15	3	586
20	10	3	1035

where $P_i(L)$ is the output power in mode i, $P_i(0)$ is the input power in mode i, $P = P_1 + P_2$ is the total power, $\Delta P = P(L) - P(0)$, $\Delta\Gamma = \Gamma_1 - \Gamma_2$, and g_{av} is the gain coefficient averaged over the length of the fiber L. Note that the solution depends only on the total gain $g_{\mathrm{av}}L$ and not on the precise z-dependence of g, which is influenced by gain saturation and thus is not known *a priori*. As discussed above, the coupling constant χ vanishes for $\Delta\omega = 0$. We therefore assume that the seed responsible for the mode instability is quantum noise. To obtain the power in the HOM, we assume that the equivalent input power spectral density of the quantum noise is $\hbar\omega$ [6] and integrate Eq. (12) over all frequencies to obtain the HOM content at the output $x_o = P_2(L)/P(L)$:

$$ x_o \approx \frac{e^{-\Delta\Gamma g_{\mathrm{av}}L}}{P_1(0)} \int_{-\infty}^{\infty} \hbar\omega \, \exp\left(\frac{\Delta P}{\Gamma_1}\chi(\omega_1 - \omega)\right) d\omega. \quad (13) $$

The integral in Eq. (13) can be approximated by Laplace's method, which yields the following approximate expression for the HOM content:

$$ x_o \approx \hbar\omega_1 \sqrt{\frac{2\pi\Gamma_1}{|\chi''(\omega_0)|}} \frac{P(L)^{\left(\frac{\Gamma_2}{\Gamma_1} - \frac{3}{2}\right)}}{P_1(0)^{\frac{\Gamma_2}{\Gamma_1}}} \, \exp\left(\frac{\chi_0}{\Gamma_1}P(L)\right), \quad (14) $$

where χ_0 and ω_0 are the maximum value of χ and the corresponding frequency, χ'' is the second derivative of χ, and we have assumed $\Delta P \approx P$. Given a threshold value x_{th} of the HOM content, Eq. (14) can be solved numerically for the corresponding threshold power P_{th}.

We have calculated the threshold power for a threshold HOM content $x_{\mathrm{th}} = 0.05$ for the various values of the design parameters of the step-index fiber models considered in this Letter. The input power $P_1(0)$ was taken to be 1 W. The results are summarized in Table 1 and show a significant variation of the threshold power with fiber design. Lowering the V parameter to reduce the overlap between LP_{01} and LP_{11} is clearly effective in increasing the power threshold. Similarly, reducing the area of the doped region is effective in reducing the mode instability. The variation of the threshold power with core size is quite small. It is important to note, however, that our model does not take the thermally induced change of the mode functions ψ into account. This effect can be significant for very-large-mode fibers with very small index contrasts, and will lead to stronger confinement and overlap of the mode functions [3], thus increasing the nonlinear coupling constant and decreasing the power threshold.

In summary, we have formulated a simple semianalytical model of thermally induced mode coupling in multimode rare-earth doped fiber amplifiers. We have applied the model to a step-index fiber, but it can be applied to any fiber design for which the mode functions and the thermal Green's functions can be determined, either analytically or numerically. Because of the approximations made in the derivation of the model, the predicted threshold power should not be expected to be in perfect agreement with experiments, but we expect that the model provides a good basis for comparing the performance of different fiber designs.

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