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Thermodynamic control of tropical rainfall

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SUMMARY

In 1987, Neelin and Held showed how, in the tropics, surface heat fluxes and infrared radiation control atmospheric convergence, and hence rainfall, by means of their joint effect on the supply of moist static energy to the troposphere. They also showed that for a given rate of supply of moist static energy, the strength of convergence is inversely proportional to a 'gross moist stability', which is related to the humidity of the troposphere and to the difference between the height of the environmental minimum in moist static energy and the elevation of maximum vertical mass-flux. The present paper extends Neelin and Held's analysis to the non-equilibrium case by invoking the somewhat speculative hypothesis that rainfall is primarily controlled by the mean saturation-deficit of the troposphere. The relaxation time of the atmosphere to the Neelin–Held equilibrium is found to be a strong function of the existing saturation-deficit under this hypothesis.

KEYWORDS: Convection Numerical modelling Thermodynamics Tropical rainfall

1. INTRODUCTION

The means by which the large-scale circulation of the atmosphere controls convection and precipitation is still a matter of controversy. The present paper attempts to illuminate this matter by extending the ideas of Neelin and Held (1987) (hereafter NH87).

NH87 related the change in the moist static energy of air passing through convective systems to the net input of energy into the troposphere by surface fluxes and radiation. This allowed them to infer how horizontal divergence is related to forcing by radiation and surface heat flux. Their analysis, however, is useful only in the steady state. The present paper extends the analysis to the non-equilibrium case.

Moist static energy and moist entropy have quite similar properties in the atmosphere—the main difference being that the former is approximately conserved between hypothetical static states with zero kinetic energy before and after a dynamic event, whereas the latter is conserved during dynamic events to the extent that they operate in reversible adiabatic fashion. Neither quantity is perfectly conserved between real atmospheric states, however, so that the choice of which to use is somewhat a matter of taste. The present paper recasts the arguments of NH87 in terms of entropy and the closely related quantity equivalent potential temperature.

Figure 1 shows the entropy currents pertinent to the analysis of NH87, who note that, in a steady situation,

$$I_{\mathrm{es}} - I_{\mathrm{ed}} = I_{\mathrm{e\,out}} - I_{\mathrm{e\,in}}. \quad (1)$$

In other words, the import of entropy (or moist static energy) into the control volume of Fig. 1 by surface convective and radiative fluxes, I_{es}, minus the net radiative loss of entropy to outer space, I_{ed}, equals the net export of entropy by lateral mass currents, $I_{\mathrm{e\,out}} - I_{\mathrm{e\,in}}$.[†] However, we can write $I_{\mathrm{e\,out}} - I_{\mathrm{e\,in}} = M\delta s_{\mathrm{m}}$, where M is the rate at which mass passes through the control volume and δs_{m} is the difference between the mass-weighted means of moist entropy entering and leaving the box. Solving for M, we find

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[†] We ignore here the production of entropy in the box, which must be considered in a full accounting. Entropy production is formally included in the next section.

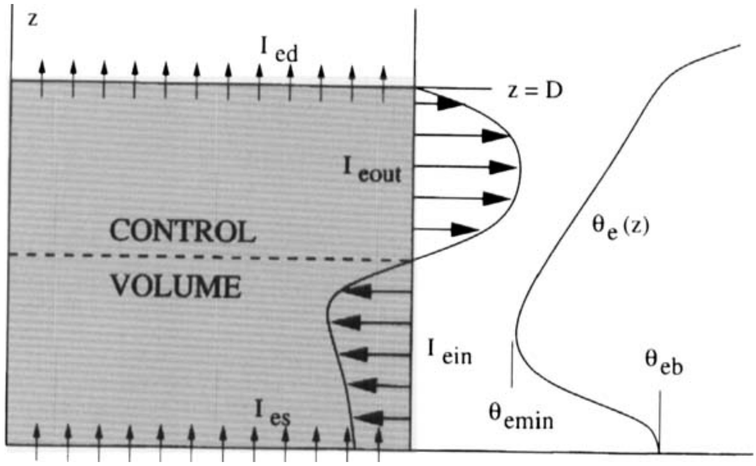

Figure 1. Entropy currents pertinent to a 1987 analysis of Neelin and Held. The control volume is a region of the troposphere, defined by the shaded box with top at the tropopause $z = D$. Note that, in this case, the profile of θ_e is that of the environment outside the control volume. The dashed line indicates the level of convective non-divergence.

that

$$M = (I_{es} - I_{ed})/\delta s_m. \tag{2}$$

Thus, for a given $I_{es} - I_{ed}$, the mass current M through the box and hence the net upward mass current through the level of non-divergence is inversely proportional to δs_m. This quantity in turn is likely to scale with the difference between surface and minimum values of equivalent potential temperature in the environmental sounding, $\theta_{eb} - \theta_{e\,min}$. Therefore, the smaller $\theta_{eb} - \theta_{e\,min}$ the larger is M. Moist environments with near-neutral stability, and hence smaller $\theta_{eb} - \theta_{e\,min}$, respond more readily to net entropy forcing $I_{es} - I_{ed}$. NH87 characterize such environments as having small gross moist stability (GMS).

When GMS is positive, the minimum in the environmental θ_e profile generally occurs below the level of non-divergence for convection, as illustrated in Fig. 1. In this way, the inflow into convection has smaller average values of θ_e than the outflow.*

It is not obvious that the steady-state assumption inherent in the analysis of NH87 can be justified on day-to-day timescales. However, the introduction of additional information bolsters the analysis. Observational studies (Raymond and Wilkening 1985; Brown and Zhang 1997; DeMott and Rutledge 1998) and numerical studies (Tompkins and Craig 1998; Raymond and Torres 1998) suggest that the precipitation produced by convection is a strong function of the relative humidity of the atmosphere, with higher humidities resulting in heavier precipitation. The main reason for this is the sensitivity of convection to entrainment by environmental air—the moister this air, the less the tendency for it to evaporate condensed water in the updraught, thereby reducing both liquid-water content and updraught buoyancy.

NH87 are silent on the possibility that GMS might be negative. However, negative GMS would appear to be very unlikely, since it would be subject to spontaneous positive feedbacks in which precipitation results in the import of moist static energy from the

* One can imagine cases where this relationship does not hold, but they tend not to occur in soundings of the real atmosphere.

environment into the precipitating region, which generates heavier precipitation, and so on. Let us ignore this possibility.

Now, let us introduce a rather extreme working hypothesis, namely that the precipitation rate over tropical oceans depends only on the mean saturation deficit of the troposphere. This hypothesis and some other relatively minor assumptions together allow the moist entropy budget of the troposphere to be closed in the non-steady case, resulting in a model for the evolution of precipitation rate and tropospheric humidity over tropical oceans. The hypothesis may be seen as much too simple, and, indeed, only a small amount of quantitative evidence supports it at present. However, in an analysis of numerical simulations of the tropics, Raymond and Torres (1998) found that rainfall rate was more closely correlated with saturation deficit than with any other plausible variable, far more closely, for example, than with surface convergence or sea–air heat-flux. Given its simplicity and plausibility, the consequences of the hypothesis seem worth exploring.

2. Non-equilibrium model

The governing equations for equivalent potential temperature θ_e, and total-cloud-water mixing-ratio r_t (vapour plus cloud droplets plus small ice-crystals) are

$$\frac{\partial \rho \theta_e}{\partial t} + \nabla \cdot (\rho \mathbf{u} \theta_e) + \frac{\partial \rho w \theta_e}{\partial z} = -\frac{\partial F_e}{\partial z} + G \tag{3}$$

and

$$\frac{\partial \rho r_t}{\partial t} + \nabla \cdot (\rho \mathbf{u} r_t) + \frac{\partial \rho w r_t}{\partial z} = -\frac{\partial F_r}{\partial z} - P, \tag{4}$$

where $\rho(z)$ is the air density, $\mathbf{u}$ and w are the horizontal and vertical wind components (with convective fluctuations smoothed out), F_e is the vertical equivalent potential temperature flux from turbulence, convection and radiation, G is related to the generation of entropy by irreversible processes, F_r is the vertical flux of total cloud-water, and P is the net conversion rate of total cloud water to and from precipitation.

In the tropics, the temperature profile of the atmosphere usually changes only slowly with time, because gravity waves tend to redistribute buoyancy anomalies laterally. Consequently, the saturated equivalent potential temperature θ_{es} and the saturation mixing ratio r_s, are essentially functions of height alone. This allows us to recast Eqs. (3) and (4) into a more useful form.

Let us define the equivalent potential temperature deficit $\Delta\theta_e = \theta_{es} - \theta_e$ and the saturation deficit $\Delta r = r_s - r_t$.* For a fixed temperature profile,

$$\Delta\theta_e \approx \theta_{es} L \Delta r / (C_p T_R), \tag{5}$$

where C_p is the specific heat at constant pressure, L is the latent heat of condensation, and T_R is a constant reference temperature. Consequently, $\Delta\theta_e$ and Δr are proportional to each other, with the proportionality factor being a function only of height.

Let us now average Eqs. (3) and (4) over a control volume of area A and depth D, indicating this average by an overbar. Assuming that $w = 0$ both at the surface and at $z = D$, the top of the control volume, we find

$$-\frac{d\overline{\rho \Delta\theta_e}}{dt} + \frac{M \delta\theta_e}{AD} = \frac{F_{es} - F_{ed}}{D} + \overline{G} \tag{6}$$

* For Δr truly to be the saturation deficit, r_t should be replaced by the vapour mixing ratio. However, to the extent that the conversion of cloud water to precipitation is efficient, the error should be small.

and

$$-\frac{\mathrm{d}\overline{\rho\Delta r}}{\mathrm{d}t}-\frac{M\delta r_{\mathrm{t}}}{AD}=\frac{F_{\mathrm{rs}}-R}{D}. \tag{7}$$

The areally averaged total (convective plus radiative) equivalent potential temperature fluxes at the surface and at $z = D$ are respectively F_{es} and F_{ed}, F_{rs} is the areally averaged surface evaporation-rate, and $R = D\overline{P}$ is the areally averaged rainfall rate.

The control volume average of the horizontal equivalent potential temperature flux-divergence is obtained as follows.

$$\begin{aligned}\frac{1}{AD}\int_0^d\int_A \nabla\cdot(\rho\mathbf{u}\theta_{\mathrm{e}})\,\mathrm{d}A\,\mathrm{d}z &= \frac{1}{AD}\int_0^d \mathrm{d}z\oint \rho\theta_{\mathrm{e}}\mathbf{u}\cdot\mathbf{n}\,\mathrm{d}s\\ &= \frac{-M\theta_{\mathrm{e\,in}}+M\theta_{\mathrm{e\,out}}}{AD}\equiv\frac{M\delta\theta_{\mathrm{e}}}{AD},\end{aligned} \tag{8}$$

where the line integral in the second term is around the periphery of the control volume, $\mathbf{n}$ is a horizontal outward unit normal to the control volume, M is the mass current through the control volume, $\theta_{\mathrm{e\,in}}$ and $\theta_{\mathrm{e\,out}}$ are the weighted averages of equivalent potential temperature flowing into and out of the control volume sides, and where the weighting factor in each case is $\rho\mathrm{u}\cdot\mathbf{n}$, and $\delta\theta_{\mathrm{e}}$ is the difference between these averages. A similar analysis of the total cloud water yields

$$\frac{1}{AD}\int_0^D\int_A \nabla\cdot(\rho\mathbf{u}r_{\mathrm{t}})\,\mathrm{d}A\,\mathrm{d}z=\frac{-Mr_{\mathrm{t\,in}}+Mr_{\mathrm{t\,out}}}{AD}\equiv-\frac{M\delta r_{\mathrm{t}}}{AD}, \tag{9}$$

where the minus sign in the definition of δr_{t} is imposed because generally $r_{\mathrm{t\,out}} < r_{\mathrm{t\,in}}$. So, $\delta\theta_{\mathrm{e}} = \theta_{\mathrm{e\,out}} - \theta_{\mathrm{e\,in}}$ and $\delta r_{\mathrm{t}} = r_{\mathrm{t\,in}} - r_{\mathrm{t\,out}}$.

Using Eq. (5), it is possible to eliminate the time derivatives between Eqs. (6) and (7) and obtain an equation for the mass current M:

$$M = A\left\{\frac{F_{\mathrm{es}}-F_{\mathrm{ed}}+D\overline{G}+\theta_{\mathrm{es}}L(R-F_{\mathrm{rs}})/(C_pT_{\mathrm{R}})}{\delta\theta_{\mathrm{e}}+\theta_{\mathrm{es}}L\delta r_{\mathrm{t}}/(C_pT_{\mathrm{R}})}\right\}. \tag{10}$$

Thus, the mass current in this model is a function of the entropy fluxes at the surface and at the tropopause, of the irreversible generation of entropy, and of rainfall minus evaporation. Since this result is not limited to steady situations it extends the results of NH87 to the non-steady case.

Furthermore, eliminating M between Eqs. (6) and (7) yields

$$-\frac{\mathrm{d}\chi}{\mathrm{d}t}=-\frac{(R-F_{\mathrm{rs}})\delta\theta_{\mathrm{e}}}{D\delta r_{\mathrm{t}}}+\frac{F_{\mathrm{es}}-F_{\mathrm{ed}}}{D}+\overline{G}, \tag{11}$$

where

$$\chi=\overline{\rho\Delta\theta_{\mathrm{e}}}+\frac{\overline{\rho\Delta r\delta\theta_{\mathrm{e}}}}{\delta r_{\mathrm{t}}}\approx\left(\frac{\overline{\theta_{\mathrm{es}}}L}{(C_pT_{\mathrm{R}})}+\frac{\delta\theta_{\mathrm{e}}}{\delta r_{\mathrm{t}}}\right)\overline{\rho\Delta r}. \tag{12}$$

The last step assumes that $\overline{\theta_{\mathrm{es}}\rho\Delta r}\approx\overline{\theta_{\mathrm{es}}}\;\overline{\rho\Delta r}$. Thus, χ is essentially proportional to Δr, and, so, is large for a dry atmosphere and small for a moist one.

Now, let us explore the consequences of linking the rainfall rate to the saturation deficit. One can express the hypothesis that rainfall is a function of the mean saturation deficit in the convective region by assuming that $R = R(\chi)$. Let us give this a definite form with the assumption that

$$R = R_0 \chi_0 / \chi \tag{13}$$

where R_0 is the rainfall rate under a reference condition, which we take to be that associated with radiative—convective equilibrium, and the constant χ_0 is the value of χ occurring under these conditions. The inverse dependence on χ means that the rainfall rate becomes infinite if the atmosphere becomes saturated. The radiative–convective equilibrium precipitation-rate in the tropics is about 4 mm $\mathrm{d}^{-1} \approx 5 \times 10^{-5}$ kg $\mathrm{m}^{-2}\mathrm{s}^{-1}$. A plausible value for χ_0 is 5 K kg m^{-3}.

The time derivative of χ can be related to the time derivative of R using Eq. (13):

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{\mathrm{d}R}{\mathrm{d}\chi}\frac{\mathrm{d}\chi}{\mathrm{d}t} = -\frac{R_0\chi_0}{\chi^2}\frac{\mathrm{d}\chi}{\mathrm{d}t} = -\frac{R^2}{R_0\chi_0}\frac{\mathrm{d}\chi}{\mathrm{d}t}. \tag{14}$$

Combining Eqs. (11) and (14) gives

$$\frac{R_0\chi_0}{R^2}\frac{\mathrm{d}R}{\mathrm{d}t} = -\frac{R\delta\theta_\mathrm{e}}{D\delta r_\mathrm{t}} + \frac{F_\mathrm{rs}\delta\theta_\mathrm{e}}{D\delta r_\mathrm{t}} + \frac{F_\mathrm{es} - F_\mathrm{ed}}{D} + \overline{G}. \tag{15}$$

Equation (15) is a prognostic equation for rainfall rate which, when the time derivative is zero, reduces to something analogous to the steady-state condition of NH87. It may be put into non-dimensional form by defining a dimensionless rainfall-rate $\alpha = R/R_0$ and a dimensionless time $\tau = t/t_0$:

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\tau} = -\alpha^3 + \alpha^2\Delta\phi \tag{16}$$

when we set

$$t_0 = D\chi_0\delta r_\mathrm{t}(R_0\delta\theta_\mathrm{e})^{-1} \tag{17}$$

and define

$$\Delta\phi = \frac{F_\mathrm{rs}}{R_0} + \frac{F_\mathrm{es} - F_\mathrm{ed} + D\overline{G}}{F_\mathrm{en}}, \tag{18}$$

where $F_\mathrm{en} = R_0\delta\theta_\mathrm{e}/\delta r_\mathrm{t}$. If we take $D = 15$ km, $\chi_0 = 5$ K kg m^{-3}, $\delta r_\mathrm{t} = 10$ g kg^{-1}, $\delta\theta_\mathrm{e} = 5$ K, and $R_0 = 5 \times 10^{-5}$ kg $\mathrm{m}^{-2}\mathrm{s}^{-1}$, then $t_0 = 3 \times 10^6$ s ≈ 35 d and $F_\mathrm{en} = 0.025$ kg $\mathrm{m}^{-2}\mathrm{s}^{-1}$K. Converted to heat-flux terms, this value is equivalent to about 25 W m^{-2}. The quantity $\Delta\phi$ is a dimensionless measure of the forcing of rainfall.

3. Model solutions

Let us first examine the steady-state solutions of Eq. (16) and analyse their stability. Then, we solve Eq. (16) exactly for a range of values of the forcing parameter $\Delta\phi$.

(*a*) *Steady solutions*

If we set the time derivative to zero, we readily find that $\alpha = 0$ and $\alpha = \Delta\phi$ satisfy Eq. (16). Physically, these correspond to the case of no rain and to one of steady rain at a rate equal to $\Delta\phi$ times the radiative–convective equilibrium rainfall rate. The latter solution is physically meaningful only when $\Delta\phi \geqslant 0$. Let us call it the equilibrium solution $\alpha_\mathrm{eq} \equiv \Delta\phi$.

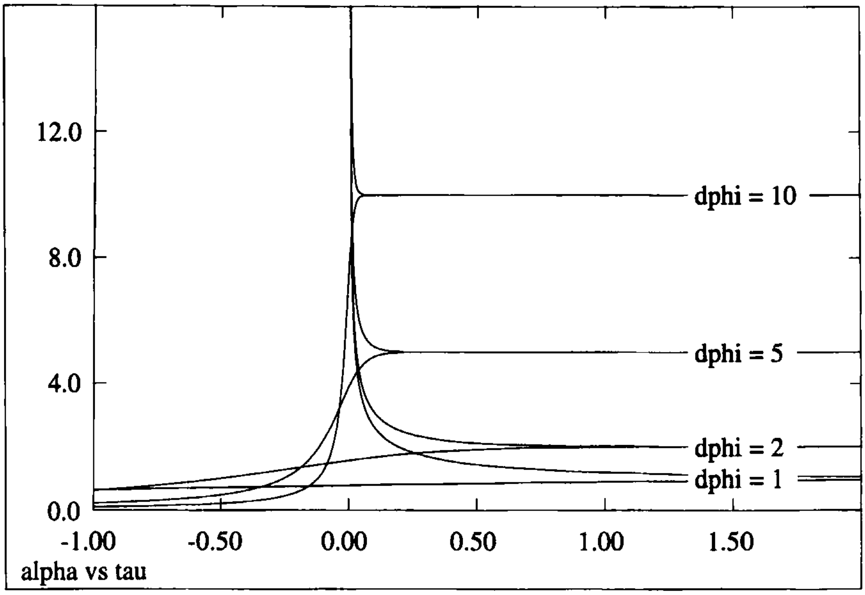

Figure 2. Full solution to the equation for dimensionless precipitation rate α as a function of dimensionless time τ. Solutions beginning both above and below the equilibrium precipitation rate are shown for $\Delta\phi = 1, 2, 5, 10$.

(*b*) *Behaviour of near steady solutions*

If we set $\alpha = \Delta\phi + \alpha'$, substitute into Eq. (16), and linearize in terms of α', we find that

$$\frac{\mathrm{d}\alpha'}{\mathrm{d}\tau} = -\Delta\phi^2\alpha'. \tag{19}$$

This has the solution

$$\alpha' \propto \exp(-\Delta\phi^2\tau), \tag{20}$$

which means that the equilibrium solution is stable, with perturbations decaying on a dimensionless timescale of $1/\Delta\phi^2$. So, the stronger the forcing, the shorter the decay transient.

No linearization about the solution at $\alpha = 0$ is possible—the solution is inherently nonlinear. However, where $|\alpha| \ll |\Delta\phi|$ we have the approximate governing equation

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\tau} = \alpha^2\Delta\phi, \tag{21}$$

which has the solution

$$\alpha = \{\Delta\phi(\tau_0 - \tau)\}^{-1}, \tag{22}$$

where τ_0 is a constant of integration. For positive values of $\Delta\phi$, α approaches infinity asymptotically as τ tends to τ_0. This limit violates the initial approximation, but shows that the $\alpha = 0$ solution is unstable for positive forcing. On the other hand, if $\Delta\phi < 0$, α decays to zero as τ increases, and the steady solution is stable.

To sum up, when $\Delta\phi$ is positive, α tends towards $\alpha_{\mathrm{eq}} = \Delta\phi$, and the tendency is more rapid when $\Delta\phi$ is large. When $\Delta\phi < 0$, α tends towards zero.

(c) *Full solution*

Equation (16) has an exact solution, which can, however, only be expressed explicitly as $\tau = \tau(\alpha)$ rather than vice versa:

$$\tau = \tau_0 + \frac{1}{\Delta\phi^2} \ln\left(\frac{\alpha}{|\Delta\phi - \alpha|}\right) - \frac{1}{\Delta\phi\alpha}. \tag{23}$$

The quantity τ_0 is again a constant of integration. Figure 2 shows this solution, for $\tau_0 = 0$ and for four different values of $\Delta\phi$.

These results confirm the earlier assertion that $\alpha = 0$ is an unstable solution while $\alpha = \alpha_{eq} = \Delta\phi$ is stable when $\Delta\phi > 0$. They also illustrate the dramatic decrease in transient time as $\Delta\phi$ increases, further confirming the results of the linearized analysis.

4. Discussion

The results presented here are based on two hypotheses: that precipitation rate is a unique decreasing function of the mean saturation deficit of the convective environment, and that the temperature profile of the convective environment does not change with time. From these it follows that

(i) rainfall rate tends to relax toward an equilibrium value equal to

$$R_{eq} = \alpha_{eq} R_0 = \Delta\phi R_0 = F_{rs} + \frac{\delta r_t (F_{es} - F_{ed} + D\overline{G})}{\delta\theta_e}. \tag{24}$$

So, in equilibrium, the excess or deficit of precipitation relative to the surface evaporation rate depends on the sign of $(F_{es} - F_{ed} + D\overline{G})$. However, its strength also depends on δr_t and $\delta\theta_e$. The parameter δr_t does not vary much, since mixing ratio typically drops off rapidly with height, so that δr_t is approximately equal to the mean value of the total-water mixing-ratio below the convective level of non-divergence. However, $\delta\theta_e$ depends sensitively on the relative heights of the level of non-divergence and the minimum in environmental θ_e—when the two are of comparable height, $\delta\theta_e$ will be small, whereas it will be larger if the level of non-divergence is much higher than the level of minimum θ_e. Negative values of $\delta\theta_e$ are unlikely to occur in the real atmosphere; on present hypotheses they represent a highly unstable situation.

(ii) Small deviations from this equilibrium state relax on the timescale $t_0/(\Delta\phi^2)$. Since $\Delta\phi$ can vary widely, the relaxation timescale is also widely variable. If $\Delta\phi = 1$, for instance, corresponding to a radiative–convective equilibrium rainfall rate of about 4 mm d^{-1}, the relaxation time is about 35 d according to the estimate made at the end of section 2. In a rainy region with $\alpha_{eq} = \Delta\phi = 6$, however, corresponding to a rainfall rate of about 24 mm d^{-1}, the relaxation time is about one day.

(iii) Even when $\alpha_{eq} \gg 1$, the relaxation time can be large if the initial state is dry and therefore has a low precipitation rate. This is evident in Fig. 2, from the curve for $\Delta\phi = 10$, for which precipitation rate increases only slowly over an extended period and then suddenly jumps to the equilibrium value.

(a) *Vertical velocity and precipitation*

In radiative–convective equilibrium, the surface evaporation rate is precisely equal to the precipitation rate. Furthermore, the mean vertical velocity is precisely zero at all levels. It corresponds to the case in which $\Delta\phi = 1$. When $\Delta\phi > 1$ the equilibrium mean motion in the troposphere is upwards and the equilibrium rainfall rate then exceeds the local evaporation rate; this is because moist air is drawn in laterally at low levels and expelled at upper levels after losing most of its moisture.

When $0 < \Delta\phi < 1$ mean tropospheric motion is downwards and equilibrium rainfall is less than surface evaporation, even though some deep convection may exist. Air moistened by low-precipitation-efficiency convection is exported to other regions. When $\Delta\phi < 0$ no precipitation occurs at all in equilibrium in this picture.

These three regimes correspond well to Gray's (1973) categorization of the tropics into clear regions ($\Delta\phi < 0$), variably cloudy regions with little precipitation ($0 < \Delta\phi < 1$), and heavily precipitating cloud clusters ($\Delta\phi > 1$).

(*b*) *Feedbacks*

Let us remind ourselves that unlike $\Delta\theta_e$, which is defined in the interior of the control volume and is therefore subject to local modification by the clouds there, $\delta\theta_e$ is defined on the boundary of the control volume and is externally determined, at least to the extent that its value is a function of the equivalent potential temperature of inflowing air.* As a consequence, we do not normally consider the possibility that the control-volume convection will itself modify $\delta\theta_e$, by modifying θ_e on lateral control volume boundaries. However, moistening in the interior of the control volume can change the character of the convection there, resulting in a change in the height of the level of convective non-divergence. If this change is downwards, then the value of $\delta\theta_e$ is decreased and the rainfall rate is increased simply because the outflowing air has a smaller value of θ_e relative to the inflowing air (see Fig. 1).

As Raymond, López and López (1998) showed, intensifying tropical cyclones exhibit precisely this sort of transformation. Thus, a nonlinear feedback of this type may be responsible for the intensification of a tropical depression into a tropical storm. This conclusion is consistent with the idea postulated by Bister and Emanuel (1997) that downdraughts decrease during intensification.

Another feedback which may be of considerable importance is the production of extensive middle to upper tropospheric layer clouds by strong convection. They reduce outgoing long-wave infrared radiation (Albrecht and Cox 1975), and so produce larger values of $F_{es} - F_{ed}$, and therefore $\Delta\phi$. The production by convection of extensive layer clouds at middle to upper levels can therefore result in stronger subsequent convection.

(*c*) *Role of convective dynamics*

In order to close the present theory, additional information is needed on how environmental conditions affect the updraught and downdraught mass-flux profiles in convection, and in particular, what determines the level of convective non-divergence. Though it seems likely that the humidity of the environment plays a strong role here, the relationship needs to be better quantified.

Similarly, the hypothesis that the mean saturation deficit is the primary control on precipitation rate needs to be better tested. This hypothesis is almost certainly oversimplified, and a better knowledge of the controls on precipitation will lead to better modelling of rainfall rates.

(*d*) *Quasi-equilibrium theories*

The present work is conceptually related to some of the simpler convective quasi-equilibrium theories, such as that of Betts (1986) and Betts and Miller (1986, 1993) who,

* At upper levels, the outflow θ_e is determined primarily by the convection itself. However, the possible range of variation in θ_e at upper levels is small so long as the temperature profile is assumed to be fixed.

in particular, assumed that there exists a reference relative humidity profile to which the atmosphere relaxes in a very short time, say, less than a day. This relaxation comes about through excess water vapour falling out as precipitation. In this way, the precipitation rate is calculated. However, in very dry conditions, net moistening is needed to bring the relative humidity to its target value, and this would require negative precipitation. The Betts–Miller scheme solves this problem by turning off the parametrization of deep convection and turning on one for shallow, non-precipitating convection. In this way, as in the present model, precipitation rate in the Betts–Miller scheme is a decreasing function of the mean saturation deficit of the atmosphere.

The original quasi-equilibrium theory of Arakawa and Schubert (1974) makes no assumptions about the timescale for humidity adjustment, asserting only that the adjustment times for the effective available potential energies of an ensemble of entraining plumes are short. Thus, the Arakawa–Schubert theory does not conflict with the ideas presented in the present paper. Any relationship between precipitation rate and environmental humidity results from the assumed precipitation model for the convective plumes.

A different type of quasi-equilibrium theory, called boundary-layer quasi-equilibrium (BLQ), was developed by Raymond (1995) and Emanuel (1995), who postulated that the sub-cloud-layer entropy rapidly adjusts to an equilibrium value, resulting in a tightly controlled relationship between surface entropy fluxes and the net upward transport of entropy by convection. The latter is caused primarily by the *downward* transport of low equivalent potential temperature air into the sub-cloud layer. The additional postulate of a deterministic relationship between updraught and downdraught mass-fluxes in convection plus an estimate of the equivalent potential temperature depression in downdraughts relative to boundary-layer air leads to an estimate of updraught mass-fluxes near cloud base in this theory.

Downdraughts are postulated to have greater θ_e depressions in BLQ, and to be more abundant relative to updraughts when the middle troposphere is drier. In this way, a drier middle troposphere must result in less updraught mass-flux for a given surface entropy flux. This result is similar to the conclusion reached from the present model.

(*e*) *Mechanical versus thermodynamic forcing*

The present work postulates that precipitation in the tropics is completely controlled by thermodynamic factors. On sufficiently short time- and space-scales, this is clearly not true. For instance, gust fronts from earlier convection may produce convection and precipitation which would not otherwise exist at that place and time, irrespective of the thermodynamic state. However, ultimately, the aggregate of precipitation over a sufficiently large area and period must be consistent with thermodynamic conservation laws. The border between dynamic and thermodynamic domination of forcing is not yet well established, and the relative importance of these two modes of convective control have yet to be determined. A significant amount of work remains to be done.

5. Further work

The present work extends the ideas of Neelin and Held (1987) to the time-dependent case, with the help of a somewhat speculative hypothesis about the environmental control of precipitation rate. The next step is to test this hypothesis and any others which might be proposed. Also needed is better information about how convective updraughts and downdraughts depend on the convective environment. Both these projects will require the development and execution of sophisticated observational and computational strategies. However, their successful resolution should result in significant progress in solving the long-standing question of what controls moist convection and precipitation in the tropical atmosphere.

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