

Thermodynamic Efficiency and Entropy Production in the Climate System

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Thermodynamics and Climate

- ▶ *Climate is non-equilibrium system, which generates entropy by irreversible processes and keeps a steady state by balancing the energy and entropy fluxes with the environment.*
- ▶ *Climate is FAR from equilibrium: FDT?*
- ▶ *MEPP? No, but...*
- ▶ *We have recently obtained some new results drawing a line connecting thermodynamic efficiency and entropy production*



Today

- ▶ *We present the theory: Lorenz Cycle + Carnot + Entropy Production (L., PRE, 2009)*
- ▶ *We focus on diagnostics describing the global thermodynamic properties of the climate system using PLASIM (U. Hamburg)*
 - ▶ *Onset and decay of snowball conditions due to variations in the solar constant*
 - ▶ *L., Fraedrich, Lunkeit, QJRMS (2010)*
 - ▶ *Impact of CO₂ changes & generalized sensitivities*
 - ▶ *L., Fraedrich, Lunkeit, ACPD (2010)*
 - ▶ *Thermodynamic Bounds from TOA budgets*
 - ▶ *L., Fraedrich, submitted GRL (2010)*



Energy Budget

- Let the total energy of the climatic system be:

$$E(\Omega) = \int_{\Omega} dV \rho e = \int_{\Omega} dV \rho (u + \phi + k),$$

- where ρ is the local density, e is the total energy per unit mass, with u , ϕ and k indicating the **internal**, **potential** and **kinetic energy** components
- Energy budget $\dot{E}(\Omega) = \dot{P}(\Omega) + \dot{K}(\Omega)$



Detailed Balances

WORK

- ▶ *Kinetic energy budget*

$$\dot{W} = C(P, K)$$

$$\dot{K}(\Omega) = -\int_{\Omega} dV \varepsilon^2 + C(P, K) = -\dot{D} + C(P, K)$$

- ▶ *Potential Energy budget*

$$\dot{P}(\Omega) = \int_{\Omega} dV \rho \dot{Q} - \dot{W} \quad \dot{Q} = 1/\rho (\varepsilon^2 - \vec{\nabla} \cdot \vec{H})$$

- ▶ *Total Energy Budget*

$$\dot{E}(\Omega) = \int_{\Omega} dV (-\vec{\nabla} \cdot \vec{H}) = -\int_{\partial\Omega} dS \hat{n} \cdot \vec{H}$$

FLUXES

DISSIPATION (L&F, PRE 2009)

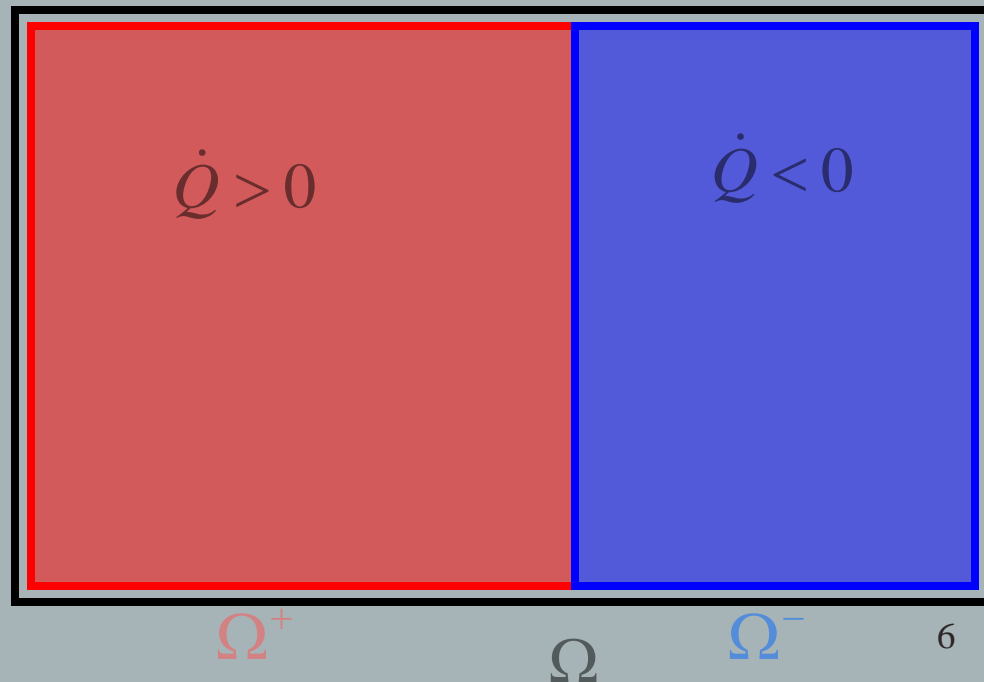


Johnson's idea (2000)

Partitioning the Domain

$$\dot{P}(\Omega) + \dot{W} = \int_{\Omega^+} dV \rho \dot{Q}^+ + \int_{\Omega^-} dV \rho \dot{Q}^- = \dot{\Phi}^+ + \dot{\Phi}^-$$

Better than it seems!



Long-Term averages

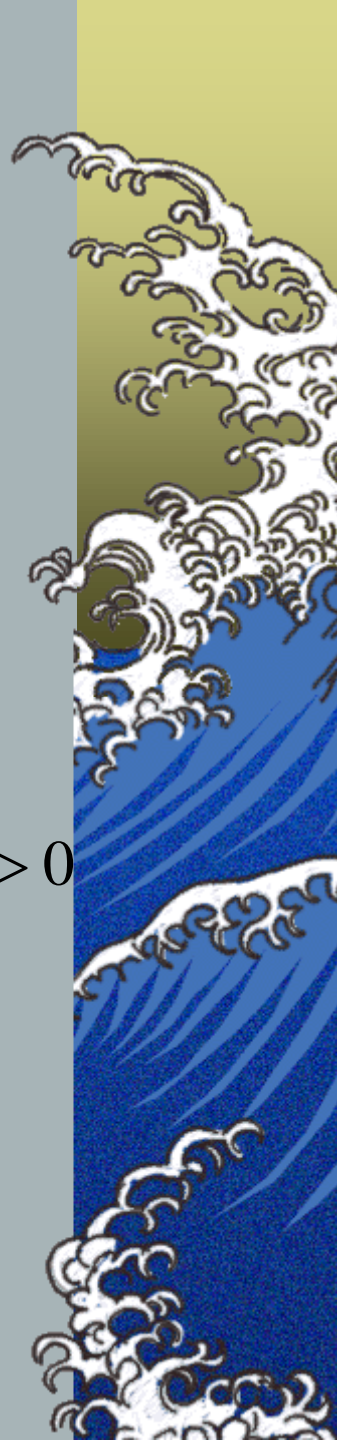
★ *Stationarity:* $\overline{\dot{E}(\Omega)} = \overline{\dot{P}(\Omega)} = \overline{\dot{K}(\Omega)} = 0$

★ *Work = Dissipation* $-\overline{\dot{K}(\Omega)} + \overline{\dot{W}} = \overline{\dot{W}} = \overline{\dot{D}} > 0$

★ *Work = Input-Output* $\overline{\dot{P}(\Omega)} + \overline{\dot{W}} = \overline{\dot{W}} = \overline{\dot{\Phi}^+} + \overline{\dot{\Phi}^-} > 0$

★ *Inequalities come from 2nd law*

★ *A different view on Lorenz Energy cycle* 7



Entropy

- ▶ *Mixing neglected (small on global scale), LTE: $\dot{Q} = \dot{s}T$*
- ▶ *Entropy Balance of the system:*

$$\dot{S}(\Omega) = \int_{\Omega^+} dV \frac{\rho \dot{Q}^+}{T} + \int_{\Omega^-} dV \frac{\rho \dot{Q}^-}{T} = \int_{\Omega^+} dV \rho \dot{s}^+ + \int_{\Omega^-} dV \rho \dot{s}^- = \dot{\Sigma}^+ + \dot{\Sigma}^-$$

- ▶ *Long Term average:*

$$\overline{\dot{S}(\Omega)} = \overline{\dot{\Sigma}^+} + \overline{\dot{\Sigma}^-} = 0 \Rightarrow \overline{\dot{\Sigma}^+} = \left| \overline{\dot{\Sigma}^-} \right| = -\overline{\dot{\Sigma}^-} > 0$$

- ▶ *Note: if the system is stationary, its entropy does not grow \rightarrow balance between generation and boundary fluxes*



Carnot Efficiency

★ *Mean Value Theorem:*

$$\overline{\dot{\Phi}^+} = \overline{\dot{\Sigma}^+} \Theta^+$$

$$\overline{\dot{\Phi}^-} = \overline{\dot{\Sigma}^-} \Theta^-$$

★ *We have*

$$\Theta^+ > \Theta^- > 0$$



Hot Cold reservoirs

★ *Work:*

$$\overline{\dot{W}} = \frac{\overline{\dot{\Phi}^+} + \overline{\dot{\Phi}^-}}{\overline{\dot{\Phi}^+}} \overline{\dot{\Phi}^+} = \frac{\Theta^+ - \Theta^-}{\Theta^+} \overline{\dot{\Phi}^+}$$

★ *Carnot Efficiency:*

$$\eta = \frac{\overline{\dot{\Phi}^+} + \overline{\dot{\Phi}^-}}{\overline{\dot{\Phi}^+}} = \frac{\Theta^+ - \Theta^-}{\Theta^+}$$



Bounds on Entropy Production

Minimal Entropy Production:

$$\overline{\dot{S}_{in}(\Omega)} \geq \overline{\dot{S}_{min}(\Omega)} = \overline{\left(\frac{\int_{\Omega} dV \rho \dot{Q}}{\int_{\Omega} dV \rho T} \right)} \approx \frac{\overline{\dot{W}}}{(\Theta^+ + \Theta^-)/2} \approx \eta \overline{\dot{\Sigma}^+}$$

$\Delta\Theta / (\Theta^+ + \Theta^-) \ll 1$

Efficiency relates minimal entropy production and entropy fluctuations

Min entropy production is due to dissipation:

$$\overline{\dot{S}_{min}(\Omega)} \approx \int_{\Omega} dV \overline{\left(\frac{\varepsilon^2}{T} \right)}$$

and the rest?



Entropy Production

- ▶ *Total entropy production: contributions of viscous dissipation plus heat transport:*

$$\overline{\dot{S}_{in}}(\Omega) = \int_{\Omega} dV \overline{\vec{H} \cdot \vec{\nabla} \left(\frac{1}{T} \right)} + \int_{\Omega} dV \overline{\frac{\varepsilon^2}{T}} \approx \int_{\Omega} dV \overline{\vec{H} \cdot \vec{\nabla} \left(\frac{1}{T} \right)} + \overline{\dot{S}_{min}}(\Omega)$$

- ▶ *We can quantify the “excess” of entropy production, degree of irreversibility with α :*

$$\alpha = \int_{\Omega} dV \overline{\vec{H} \cdot \vec{\nabla} \left(\frac{1}{T} \right)} / \overline{\dot{S}_{min}}(\Omega)$$

- ▶ *Heat Transport downgradient T field increases irreversibility*



MEPP re-examined

- ▶ *Let's look again at the Entropy production:*

$$\overline{\dot{S}_{in}(\Omega)} \approx \overline{\dot{S}_{min}(\Omega)}(1 + \alpha) \approx \eta \overline{\Sigma^+}(1 + \alpha)$$

- ▶ *If heat transport down-gradient the temperature field is strong, η is small*
- ▶ *If the transport is weak, α is small.*



- ▶ *MEPP requires a joint optimization of heat transport and of the production of mechanical work*



Can this be useful?

- ▶ *What we have shown provides a series of diagnostic tools for:*
 - ▶ *Defining thermodynamics of the climate system*
 - ▶ *Validating, intercomparing climate models*
 - ▶ *Analyzing impact of natural and anthropogenic forcing on climate*
 - ▶ *Dynamic Paleoclimatology à la Saltzman*
 - ▶ *Climate Feedbacks*
 - ▶ *radiation \Leftrightarrow dynamics*
 - ▶ *We have tested it together with Fraedrich & Lunkeit on classic climate experiments: Snowball Earth & CO₂ climate sensitivity*



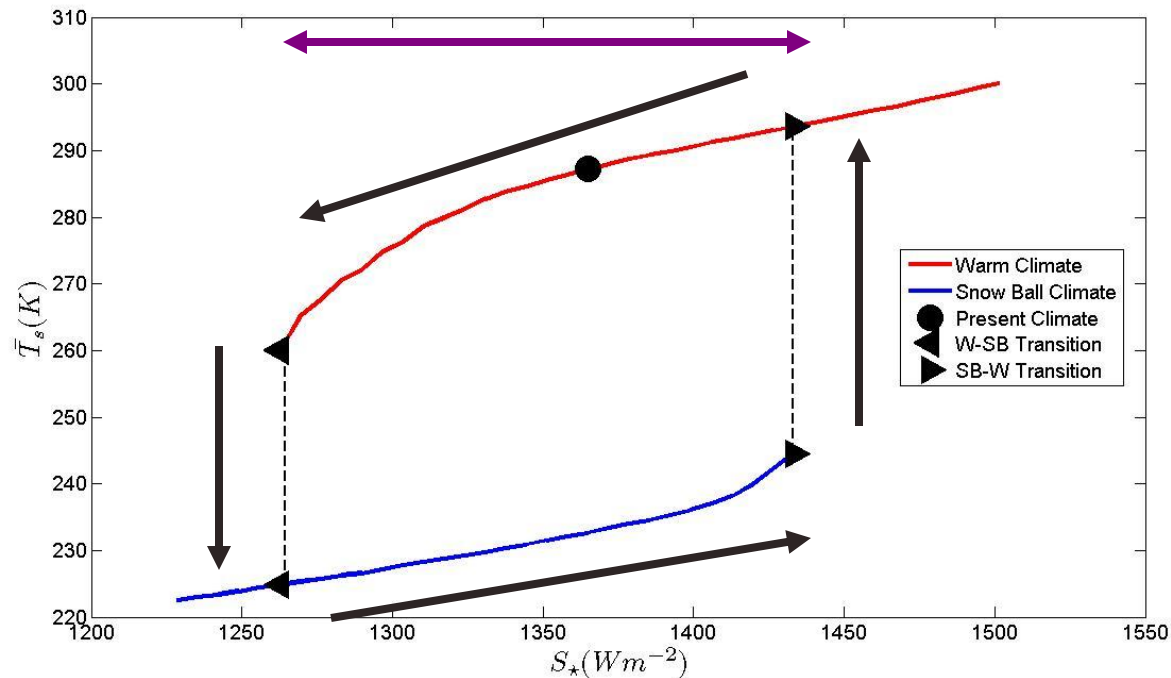
PLASIM

- ▶ *Climate model developed at U. Hamburg (Fraedrich, Lunkeit, Blender, Kirk) from PUMA*
- ▶ *State-of-the art AGCM but T21*
- ▶ *50m mixed-layer swamp ocean with sea ice*
- ▶ *Reasonable present climate*
- ▶ *Good for long simulations, sensitivity tests; can be adapted to studying other planets...*
- ▶ *We test the theory just proposed, try to analyze macro-climatic variability using 1st and 2nd law diagnostics*



Hysteresis experiment

- ▶ In 8000 years we make a swing of the solar constant S_* by $\pm 10\%$ starting from present climate
 - ▶ \rightarrow hysteresis experiment!
- ▶ **Global average surface temperature T_S**
 - ▶ Wide (about 10%) range of S_* with bistable regime
 - ▶ ΔT_S is about 40-50 K
 - ▶ $d T_S / d S_* > 0$ everywhere, almost linear



W

SB



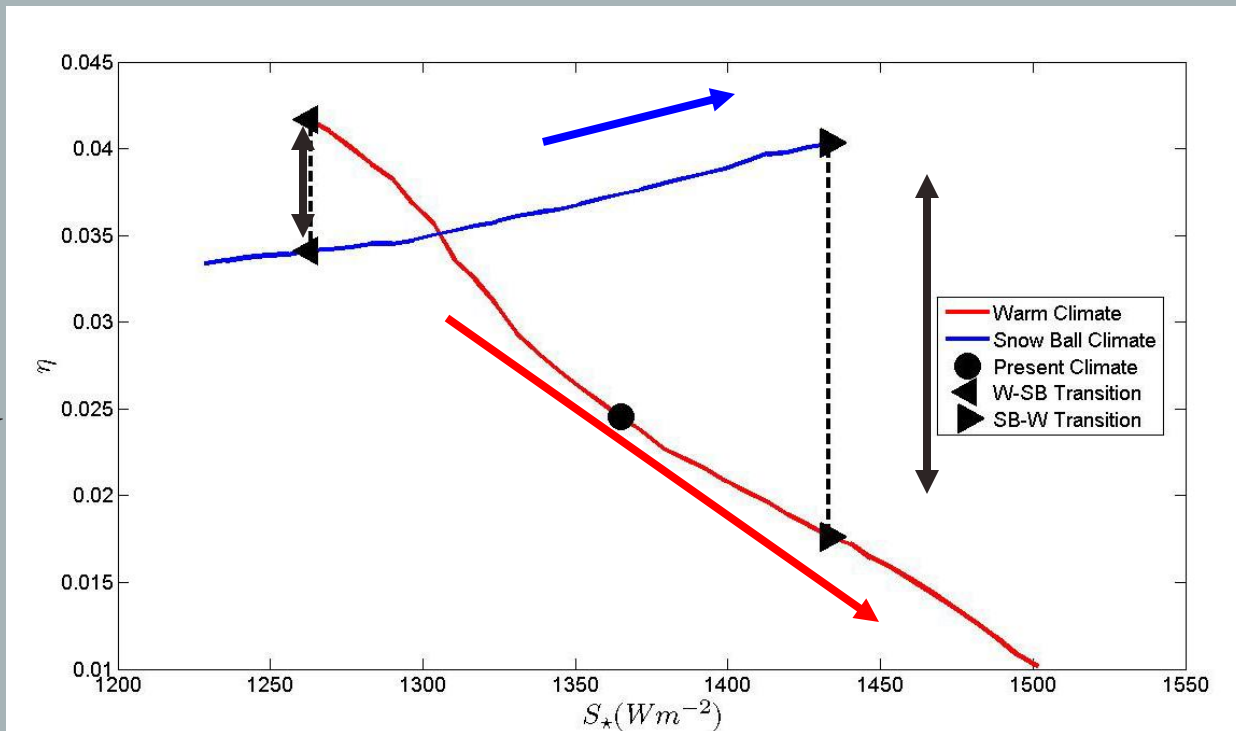
Thermodynamic Efficiency

- ▶ $d\eta/dS_* > 0$ in SB regime
 - ▶ Large T gradient due to large albedo gradient
- ▶ $d\eta/dS_* < 0$ in W regime
 - ▶ System thermalized by efficient LH fluxes
- ▶ η decreases at transitions \rightarrow System more stable

$$\eta = 0.04$$

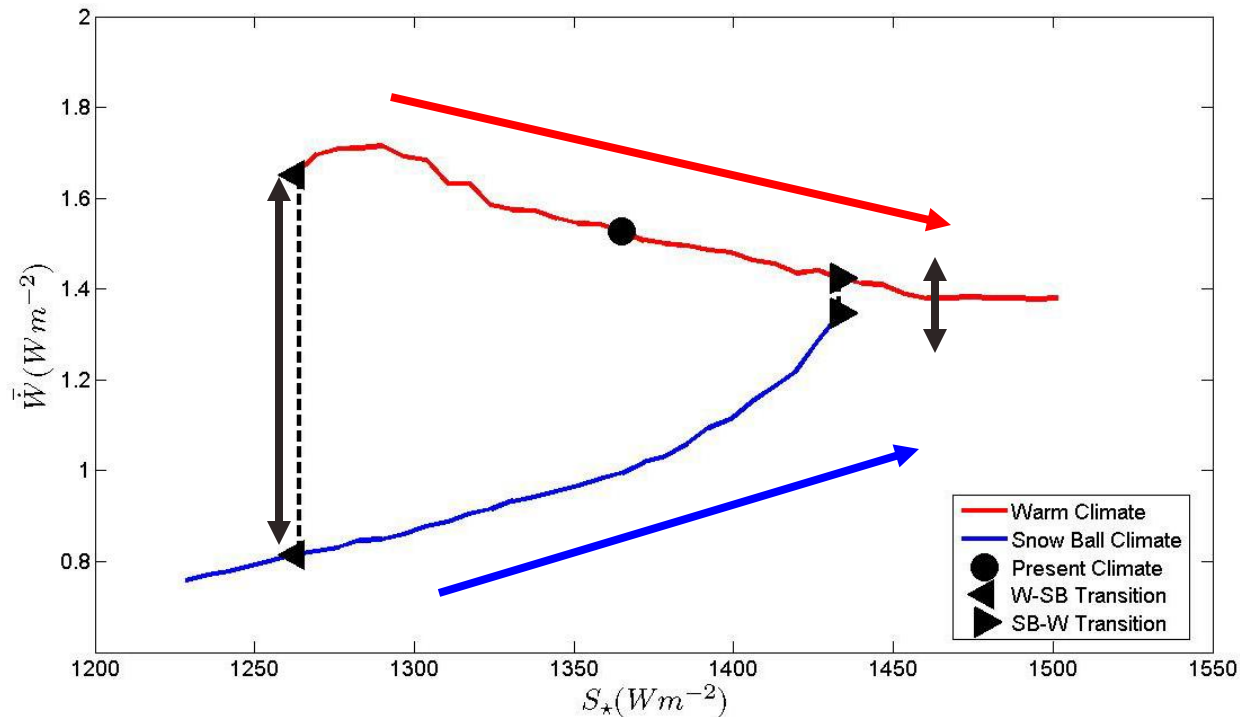
$$\leftrightarrow$$

$$\Delta\theta = 10K$$



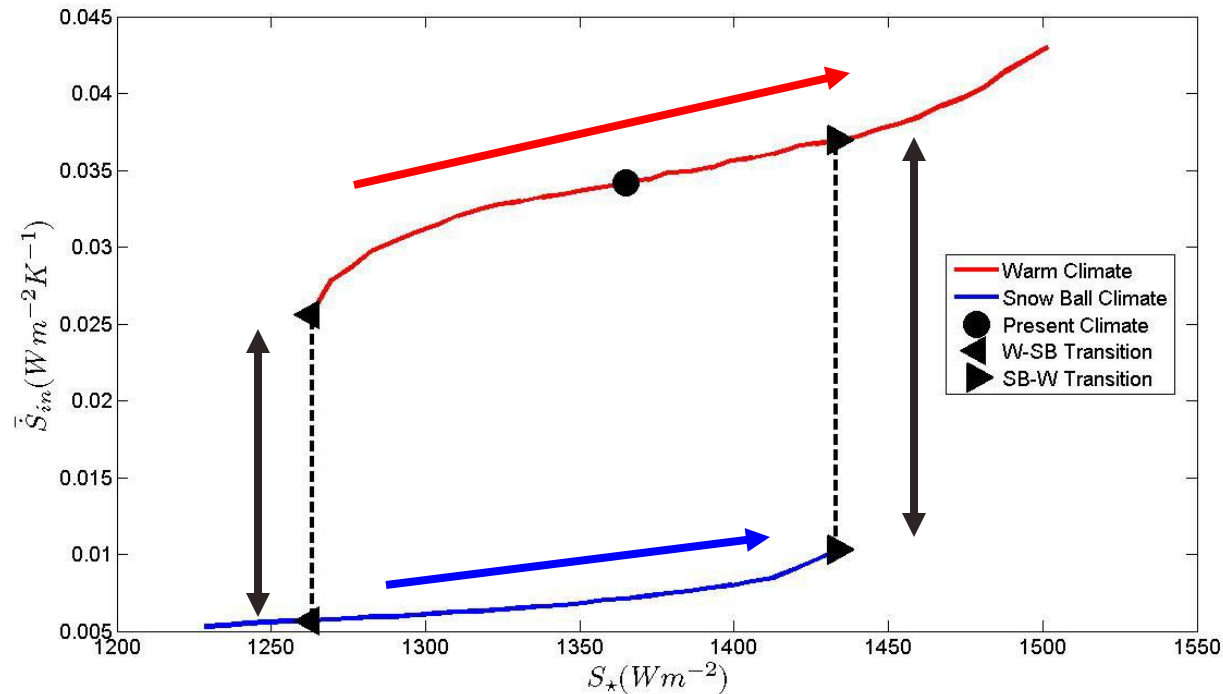
$D=C(P,K)$ - Lorenz energy cycle

- ▶ Energy input increases with S_* in both regimes
- ▶ $dW/dS_* > 0$ in SB regime
 - ▶ Stronger circulation: more input & higher efficiency
- ▶ $dW/dS_* \leq 0$ in W regime
 - ▶ Weaker circulation: more input & much lower efficiency



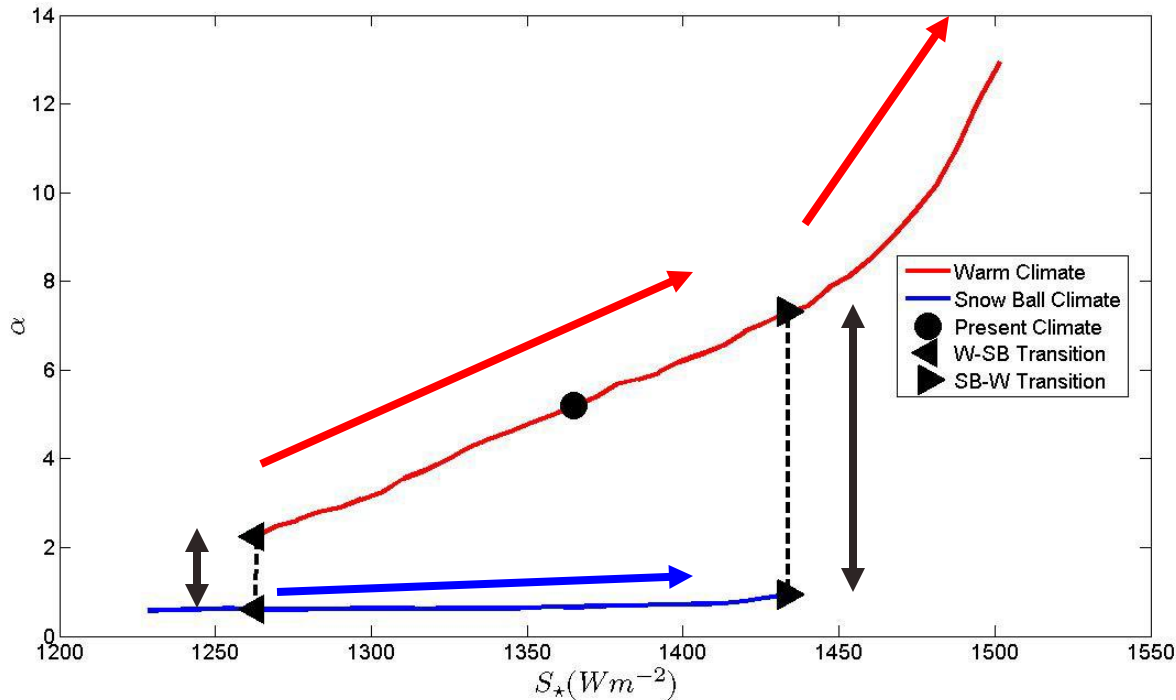
Entropy Production

- ▶ $d S_{in}/d S_* > 0$ in SB & W regime
- ▶ Entropy production is “like” T_S ... but better than T_S !
- ▶ ΔS_{in} is about 400% → benchmark for SB vs W regime
- ▶ S_{in} is an excellent state variable



Irreversibility

- ▶ $d\alpha/dS_* > 0$ in *W* regime
 - ▶ System is *VERY* irreversible; α explodes for high S_*
- ▶ $d\alpha/dS_* \sim 0$ in *SB* regime
 - ▶ α is about 1
- ▶ Again: a qualitative difference between *W* and *SB*

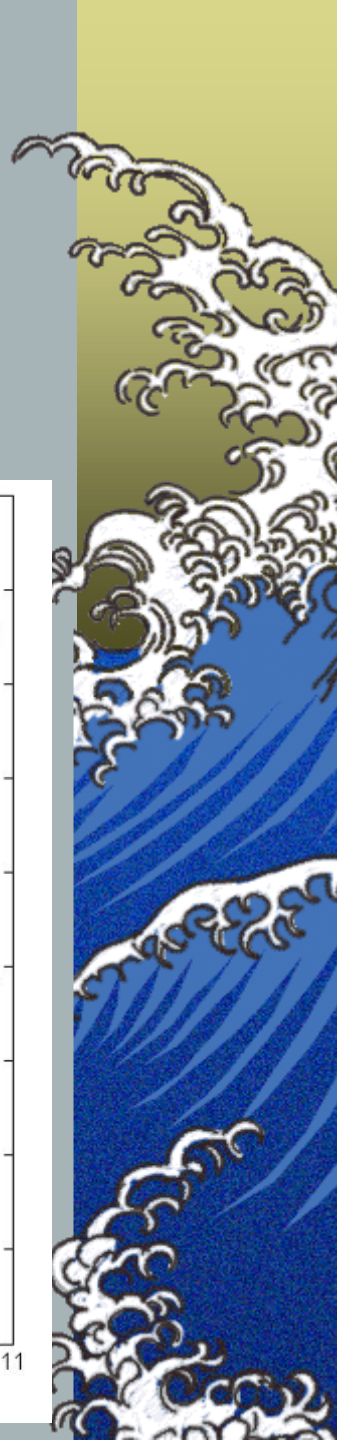
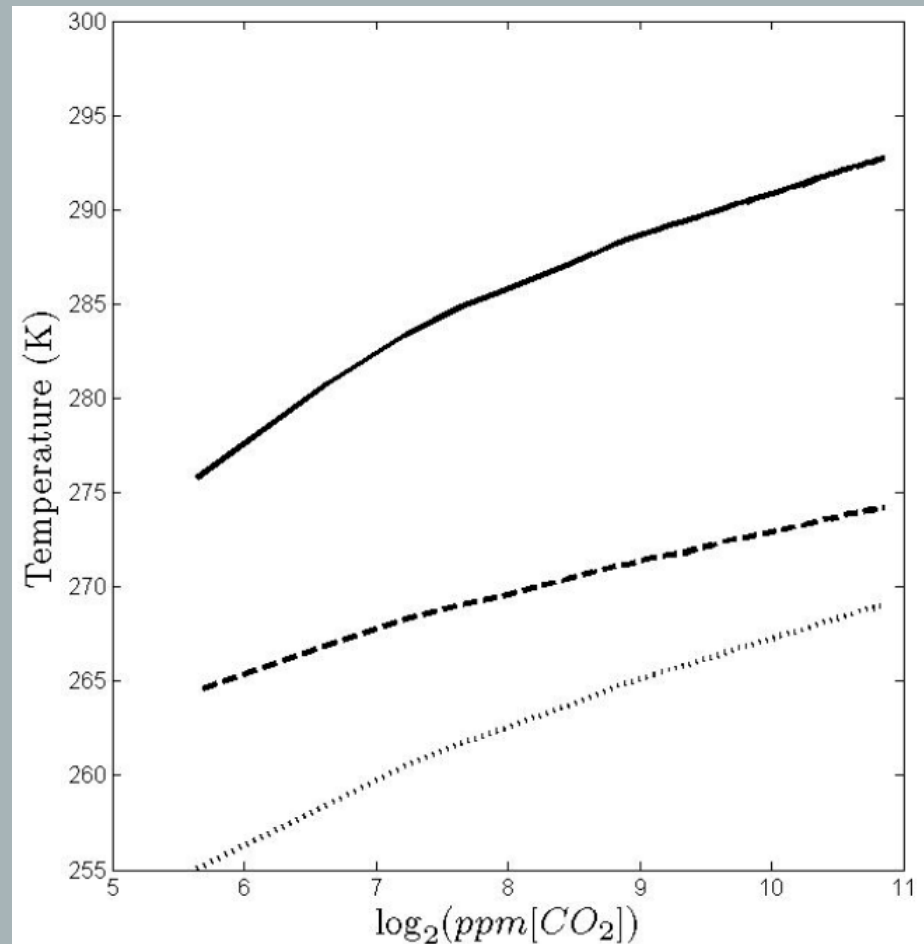


Climate sensitivity experiment

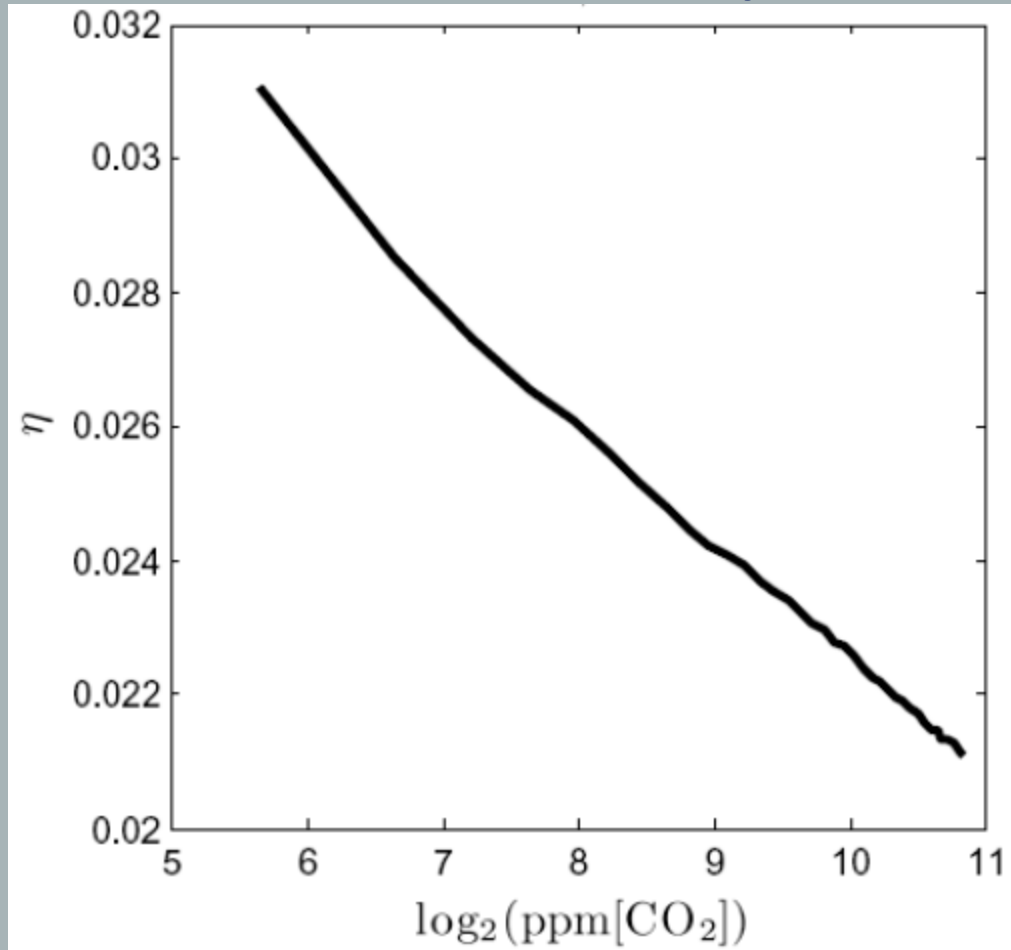
- ▶ We explore the statistical properties of climate resulting from CO_2 concentrations ranging from 50 to 1750 ppm
 - ▶ → no bistability!
 - ▶ → We define a set of generalized thermodynamical sensitivities

▶ Temperature variables

- ▶ Surface temperature has the largest sensitivity
- ▶ Cold bath becomes relatively warmer → linearity wrt $\log([CO_2])$
- ▶ System tends to become more isothermal with higher $[CO_2]$



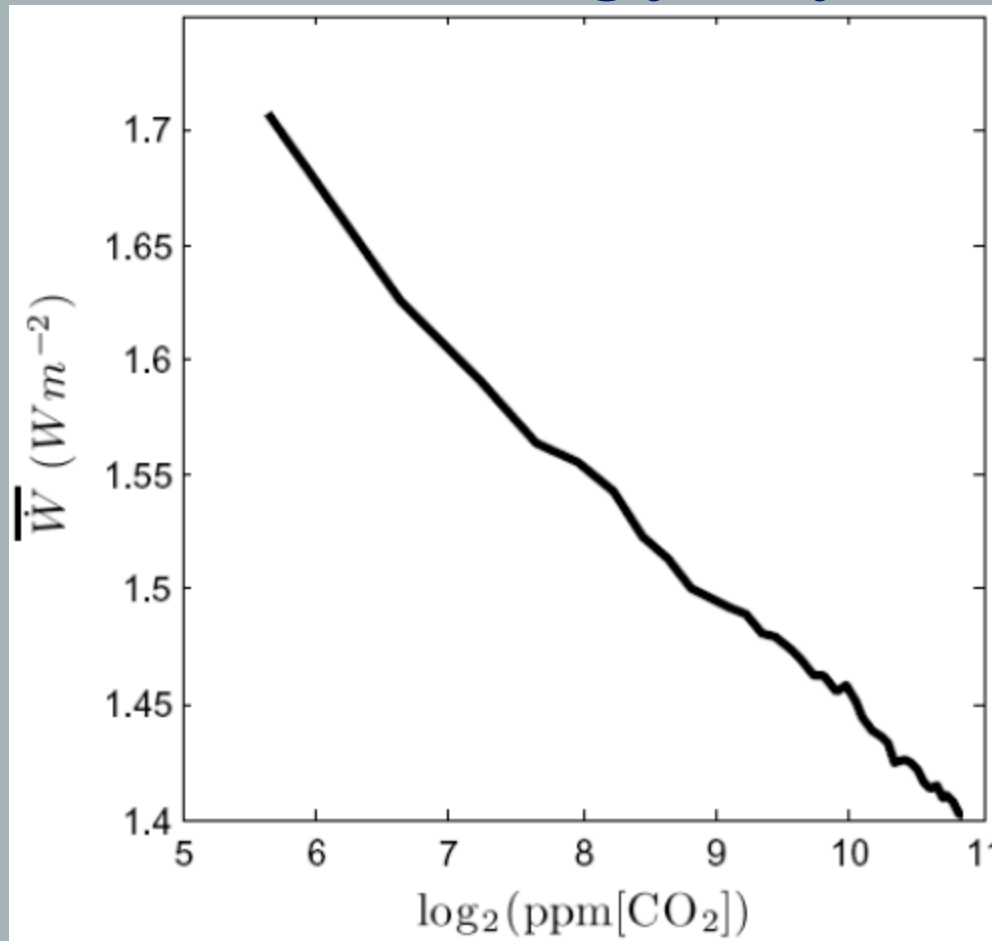
Efficiency



- ▶ *The system becomes more isothermal → The efficiency decreases with increasing CO₂ concentration*
- ▶ *Relative decrease is 6% per CO₂ doubling*
- ▶ *Latent heat fluxes play a crucial role*



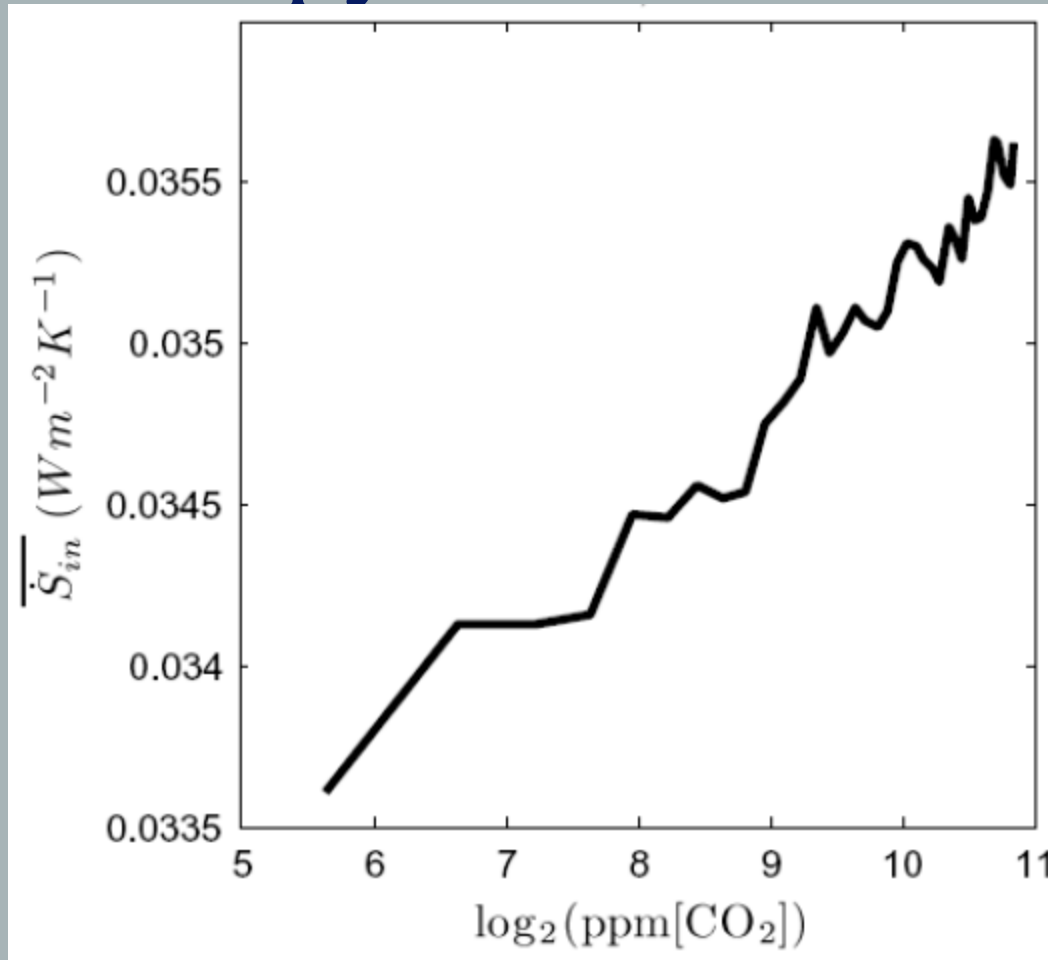
Lorenz Energy Cycle



- Trade-off between higher heat absorption and lower efficiency: Lorenz energy cycle is weaker when $[\text{CO}_2] \uparrow$
- Relative decrease is 3% per $[\text{CO}_2]$ doubling
- Same applies for dissipation: lower surface winds



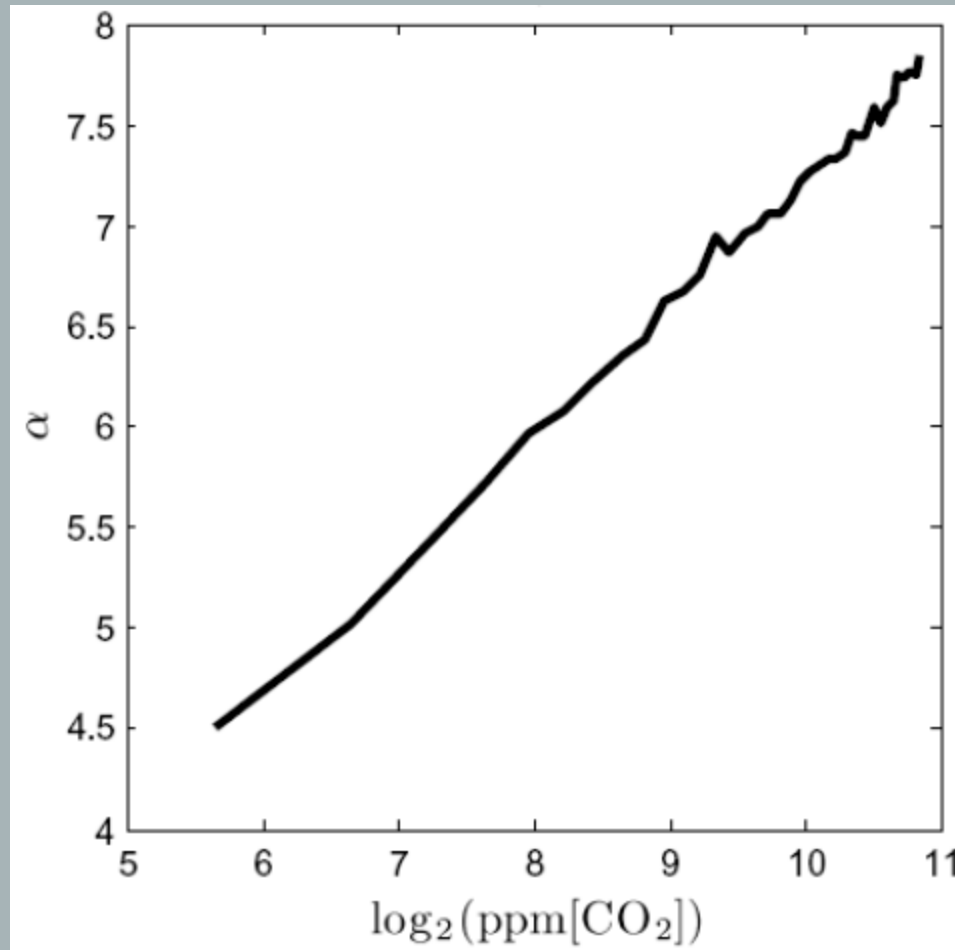
Entropy Production



- ▶ *Lower dissipation & higher temperature → lower entropy production by viscous processes*
- ▶ *Nevertheless, total material entropy production increases*
 - ▶ *Greater role by heat transport contributions → LH fluxes*



Irreversibility



- ▲ *The irreversibility of the system increases with $[\text{CO}_2]$*
- ▲ *The system becomes closer to a “conductive” system producing negligible mechanical work*



Generalized Sensitivities

| Definition | Value |
|-------------------------------------|---|
| $\Lambda_{\overline{T_s}}$ | 2.55 K |
| Λ_{Θ^+} | 1.65 K |
| Λ_{Θ^-} | 2.35 K |
| Λ_{η} | -0.002 |
| $\Lambda_{\overline{\dot{W}}}$ | -0.06 Wm ⁻² |
| $\Lambda_{\overline{\dot{S}_{in}}}$ | 0.0004 Wm ⁻² K ⁻¹ |
| Λ_{α} | 0.7 |



Parameterisations

| Definition | Value |
|-------------------------------------|--|
| $d\Theta^+ / d\bar{T}_s$ | 0.65 |
| $d\Theta^- / d\bar{T}_s$ | 0.92 |
| $d\eta / d\bar{T}_s$ | -0.0008 K^{-1} |
| $d\bar{W} / d\bar{T}_s$ | $-0.024 \text{ Wm}^{-2} \text{ K}^{-1}$ |
| $d\bar{S}_{\text{in}} / d\bar{T}_s$ | $0.00016 \text{ Wm}^{-2} \text{ K}^{-2}$ |
| $d\alpha / d\bar{T}_s$ | 0.275 K^{-1} |



Thermodynamic Bounds

- ▶ *All of this looks good, but we need 3D data!*
- ▶ *Thermodynamic bounds for entropy production and Lorenz energy cycle based only on average TOA 2D radiative fields*
- ▶ *Good for data from planetary objects*

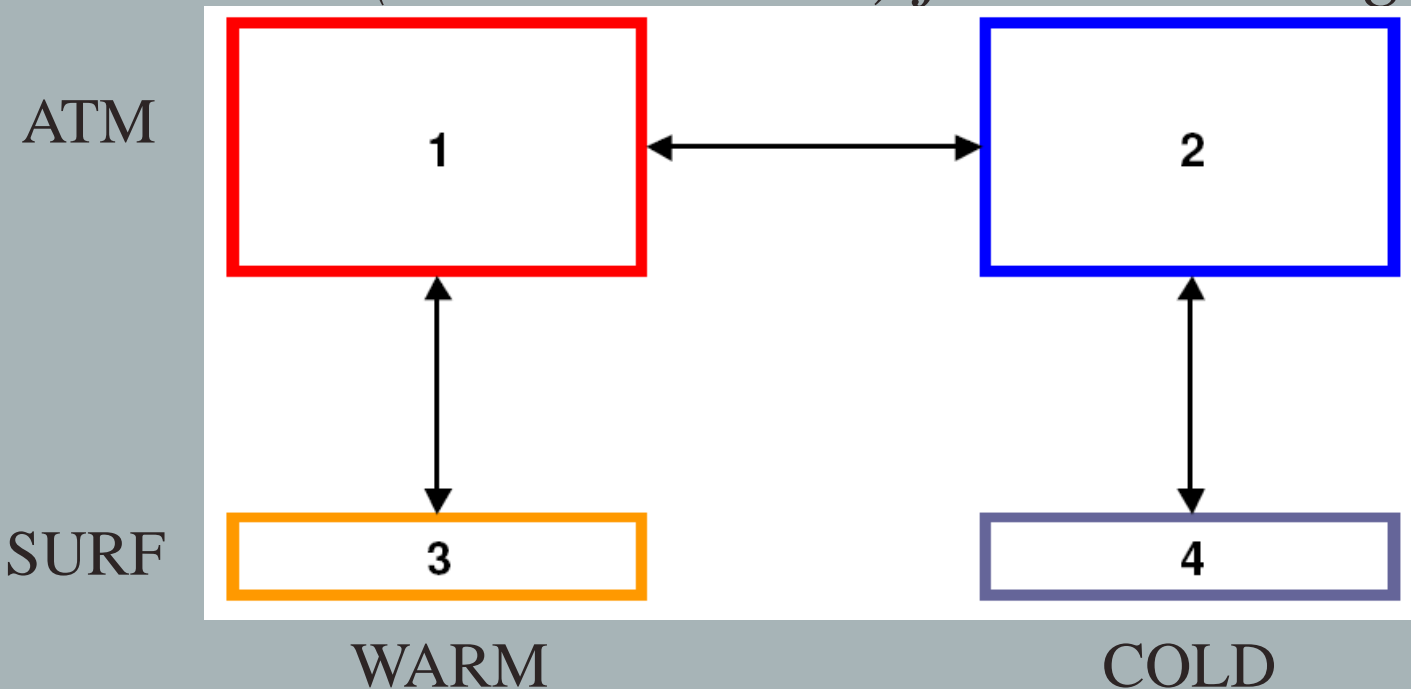
$$\overline{\dot{S}}_{mat} > \overline{\dot{S}}_{mat}^{diff} = \int_A \frac{\overline{\varepsilon^2}}{T_{diss}} d\sigma > \overline{\dot{S}}_{mat}^{hor} = \left(\frac{\langle \overline{R_{net}} \rangle_{>} |A_{>}|}{\langle T_E \rangle_{>}} + \frac{\langle \overline{R_{net}} \rangle_{<} |A_{<}|}{\langle T_E \rangle_{<}} \right)$$

$$\overline{\dot{W}} \geq \overline{\dot{W}}_{min} = -\langle T_E \rangle \left(\frac{\langle \overline{R_{net}} \rangle_{>} |A_{>}|}{\langle T_E \rangle_{>}} + \frac{\langle \overline{R_{net}} \rangle_{<} |A_{<}|}{\langle T_E \rangle_{<}} \right)$$



Vertical Structure

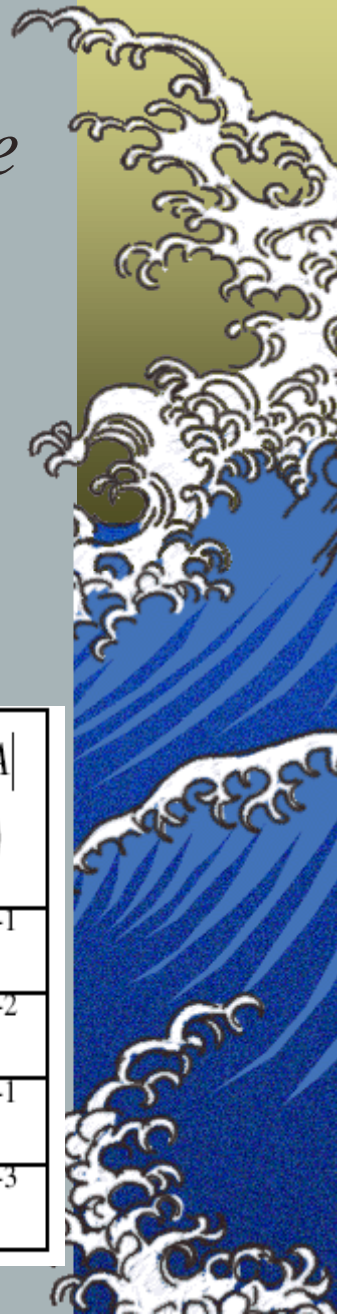
- ▶ *If no vertical temperature structure, the inequalities become identities!*
- ▶ *A minimal model for EP requires 4 boxes (atmo/surf, warm/cold)*
- ▶ *2 Boxes (R. Lorenz etc.) just not enough*



Earth, Mars, Venus, Titan

- ▶ *Bounds can be easily computed from coarse resolution TOA data*
- ▶ *With LW data we obtain effective temperatures*
- ▶ *With SW data we obtain the budgets*

| Celestial Body | $ A (m^2)$ | $\langle T_E \rangle_<(K)$ | $\langle T_E \rangle_>(K)$ | η_h | $\overline{F_{\max}}/ A $ (Wm^{-2}) | $\overline{S_{\text{mar}}^{\text{hor}}}/ A $ ($WK^{-1}m^{-2}$) | $\overline{W_{\min}}/ A $ (Wm^{-2}) |
|----------------|----------------------|----------------------------|----------------------------|----------|--|---|--|
| Earth | 5.1×10^{14} | 248.3 | 259.8 | 0.044 | 10.01 | 3.8×10^{-3} | 9.7×10^{-1} |
| Mars | 1.4×10^{14} | 201.9 | 222.5 | 0.096 | 0.32 | 2.8×10^{-4} | 5.9×10^{-2} |
| Venus | 4.1×10^{14} | 227.9 | 231.0 | 0.014 | 7.60 | 8.8×10^{-4} | 2.0×10^{-1} |
| Titan | 8.3×10^{13} | 82.8 | 85.0 | 0.026 | 0.12 | 3.7×10^{-5} | 3.1×10^{-3} |



Energy Scaling

- ▶ *We can scale the thermo terms with respect to suitable powers of average $SW=S(1-\alpha)$*
- ▶ *Differences will depend on circulation*
- ▶ *All results are within one order of magnitude!*

| Celestial Body | $ A (m^2)$ | $S(1-\alpha)$ (Wm^{-2}) | r | $\langle T_E \rangle_<$ (K) | $\langle T_E \rangle_>$ (K) | η_h | $\overline{H_{max}}/ A $ (Wm^{-2}) | $\overline{\dot{S}_{mat}^{hor}}/ A $ ($WK^{-1}m^{-2}$) | $\overline{W_{min}}/ A $ (Wm^{-2}) |
|----------------|---------------------|--------------------------------|-------|--------------------------------|--------------------------------|----------|---|---|---|
| Earth | $5.1 \cdot 10^{14}$ | 237.0 | 1.000 | 248.3 | 259.8 | 0.044 | 10.01 | 3.8×10^{-3} | 9.7×10^{-1} |
| Mars | $1.4 \cdot 10^{14}$ | 116.7 | 0.492 | 241.0 | 265.6 | 0.096 | 0.65 | 4.8×10^{-4} | 1.2×10^{-1} |
| Venus | $4.1 \cdot 10^{14}$ | 157.3 | 0.664 | 252.5 | 256.0 | 0.014 | 11.41 | 1.2×10^{-3} | 3.0×10^{-1} |
| Titan | $8.3 \cdot 10^{13}$ | 2.81 | 0.012 | 250.9 | 257.6 | 0.026 | 10.08 | 1.0×10^{-3} | 2.6×10^{-1} |

Conclusions

- ▶ *Theoretical framework linking previous finding on the efficiency of the climate system to its entropy production.*
- ▶ *Unifying picture connecting the Energy cycle to the MEPP;*
- ▶ *Test of these results on Snow Ball hysteresis experiment, and some ideas on mechanisms involved in climate transitions;*
- ▶ *Analysis of the impact of $[CO_2]$ increase, with a generalized set of climate sensitivities and a proposal for parameterizations and diagnostic studies*
- ▶ *Thermodynamic bounds seem promising! (new results)*
- ▶ *Issues:*
 - ▶ *Are climate models balanced in terms of energy budgets? No! (Lucarini & Ragone 2010) Can this be a problem? Yes! (Lucarini & Fraedrich 2009)-We have introduced eqs. also for dealing with biases*
 - ▶ *What is the role of the ocean?*
 - ▶ *Climate tipping points?*
 - ▶ *Other celestial bodies?*



References

- ▲ *Lucarini V., Thermodynamic Efficiency and Entropy Production in the Climate System, Phys. Rev. E 80, 021118 (2009)*
- ▲ *Lucarini V., Fraedrich K., Symmetry breaking, mixing, instability, and low-frequency variability in a minimal Lorenz-like system, Phys. Rev. E 80, 026313 (2009)*
- ▲ *Lucarini V., Fraedrich K., Lunkeit F., Thermodynamic Analysis of Snowball Earth Hysteresis Experiment: Efficiency, Entropy Production, and Irreversibility, QJRMS 135, 2-11 (2010)*
- ▲ *Lucarini V., Fraedrich K., Lunkeit F., Thermodynamics of Climate Change: Generalized sensitivities, ACPD 10, 3699-3715 (2010)*
- ▲ *Lucarini V., Ragone F., Energetics of IPCC4AR Climate Models: Energy Balance and Meridional Enthalpy Transports, submitted to Rev. Geophys. also at arXiv:0911.5689v1 [physics.ao-ph] (2010)*
- ▲ *Lucarini V., Fraedrich K., Bounds on the thermodynamical properties of a planet based upon its radiative budget at the top of the atmosphere: theory and results for Earth, Mars, Titan, and Venus, submitted to GRL (2010); also at arXiv:1002.0157v2 [physics.ao-ph] (2010)*

