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Article

Thermoeconomic Optimum Operation Conditions of a Solar-driven Heat Engine Model

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Abstract: In the present paper, the thermoeconomic optimization of an endoreversible solar-driven heat engine has been carried out by using finite-time/finite-size thermodynamic theory. In the considered heat engine model, the heat transfer from the hot reservoir to the working fluid is assumed to be the radiation type and the heat transfer to the cold reservoir is assumed the conduction type. In this work, the optimum performance and two design parameters have been investigated under three objective functions: the power output per unit total cost, the efficient power per unit total cost and the ecological function per unit total cost. The effects of the technical and economical parameters on the thermoeconomic performance have been also discussed under the aforementioned three criteria of performance.

Keywords: thermoeconomic performance; endoreversible; solar-driven heat engine; optimization

Classification: PACS 05.70. Ln, 44.40.+a89., 65.Gh

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1. Introduction

Within the context of finite-time/finite-size thermodynamic theory several authors have studied the optimum performance of endoreversible solar-driven heat engines [1–4]. These authors have employed different regimes of performance, like the optimum thermal efficiency [4], the ecological criterion [5] or the maximum power output [1]. Other authors [6, 7] have recognized that the ecological function is an exergy-based ecological optimization, in the sense that this criterion considers the exergy losses (or dissipation) of the system.

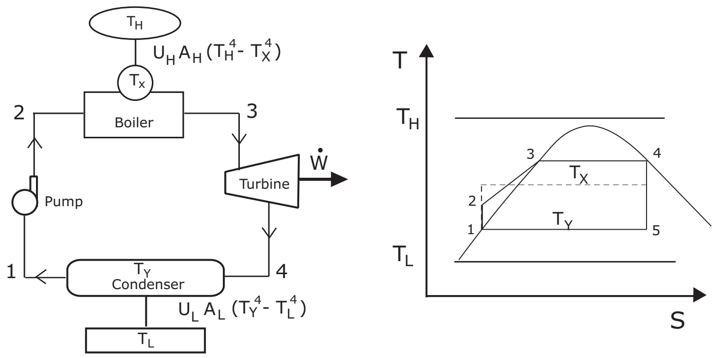
Recently, Sahin [8] studied the optimum performance of an endoreversible solar-driven heat engine. In this study, the author considered that the heat transfer from the hot reservoir to the working fluid is given by radiation, while the mode of heat transfer from the working fluid to the cold reservoir is given by a Newtonian heat transfer law. In another work, Sahin [9] also considered that the heat transfer from the hot reservoir to the working fluid is simultaneously produced by radiation and conduction. In references [8, 9], Sahin calculated the optimum temperatures of the working fluid and the optimum efficiency of the engine operating at maximum power conditions. On the other hand, Sahin and Kodal [10] made a thermoeconomic analysis of an endoreversible heat engine in terms of the maximization of an objective function defined as the quotient of the power output and the total cost involved in the operation of the power plant. Sahin and Kodal [10] considered the total cost as the sum of both investment and fuel costs, and the investment cost of the plant is assumed proportional to the size of the plant and can be taken as proportional to the total heat transfer area. The procedure used by Sahin and Kodal [10] was later applied by Sahin *et al.* [11] to study the thermoeconomics of an endoreversible solar-driven heat engine in terms of the maximization of a profit function defined as the quotient of the power output and the annual investment cost.

More recently, Barranco-Jiménez et al.[12] studied the optimum operation conditions of an endoreversible solar-driven heat engine with different heat transfer laws in the thermal couplings but operating at maximum ecological function conditions. Barranco-Jiménez and Angulo-Brown [13] also studied the thermoeconomics of an endoreversible heat engine model under maximum ecological conditions and by considering different heat transfer laws in a Novikov model. In the present work we follow the procedure by Sahin *et al.* [8, 9, 11] to study the thermoeconomics of an endoreversible solar-driven heat engine, but by using two other objective functions, which are the ecological function [5] and the so-called efficient power [14] function, and we take the total cost as that considered by Sahin *et al.* [11]. Our results can be useful as a general guide for the design of solar energy converters concerning their mode of thermoeconomic performance.

2. Theoretical Model

The endoreversible solar-driven engine model of Sahin *et al.* is shown in Figure 1 [11]. The solar-driven engine is considered to operate according to a Rankine cycle, also given in Figure 1. The endoreversible Rankine cycle works between a heat reservoir of temperature T_H and a heat sink of temperature T_L. To simplify the analysis, the endoreversible Rankine cycle (1-2-3-4-5-1) can be modified by using an entropic average temperature defined by Khaliq [15] for an endoreversible Carnot cycle (1-a-b-5-1). Following Khaliq[15] and Sahin[11] the area under the process 2-3-4 in the T-S diagram of Figure 1 rep-

Figure 1. Schematic diagram of a solar powered heat engine and its $T - S$ diagram [11].



resents the amount of heat added to the Rankine cycle, we can make this area equal to the area under the horizontal line with an entropic average temperature of heat addition. The entropic average temperature can be written as $T_X = \frac{\Delta Q}{\Delta S} = \frac{H_4 - H_2}{S_4 - S_2}$ [15]. The heat transfer from the hot reservoir is assumed to be radiation dominated and the net heat flow rate $\dot{Q}_H$ from the hot reservoir to the engine can be written as [11],

$$\dot{Q}_H = A_H \sigma \left(\epsilon_H T_H^4 - \alpha_H T_X^4 \right) = U_H A_H \left(T_H^4 - T_X^4 \right), \tag{1}$$

where σ is the Stefan-Boltzmann constant, ϵ_H and α_H are the emittance and absorption coefficients of the heat source and A_H is the heat transfer area of the hot side heat exchanger. $U_H = \epsilon_H \sigma$ is the hot side heat transfer coefficient. The emittance will be balanced uniformly by absorption $\epsilon_H = \alpha_H$ for a thin slice of gas [11], that is, for fully concentrated solar radiation. In Equation 1 convective effects are not considered, thus, this model only applies to space applications. One manner of considering convection effects can be made by using the Dulong-Petit heat transfer law. On the other hand, the conduction heat transfer is assumed the main mode of heat transfer to the low temperature reservoir, therefore the heat flux rate $\dot{Q}_L$ from the heat engine to the cold reservoir can be written as

$$\dot{Q}_L = U_L A_L \left(T_Y - T_L \right), \tag{2}$$

where U_L is the cold side heat transfer coefficient and A_L is the heat transfer area of the cold side heat exchanger. Applying the first law of thermodynamics, the power output of the solar driven heat engine is given as

$$W = \dot{Q}_H - \dot{Q}_L = \left[\beta A_R \frac{(T_H^4 - T_X^4)}{T_H^3} - (T_Y - T_L) \right] U_L A_L, \tag{3}$$

where A_R and β are the ratio of the heat transfer areas and the heat conductance parameter respectively, and are defined as

$$A_R = \frac{A_H}{A_L}, \tag{4}$$

and

$$\beta = \frac{U_H}{U_L} T_H^3. \tag{5}$$

These two parameters can be taken as design parameters. The thermal efficiency of the endoreversible heat engine is

$$\eta = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 1 - \frac{T_Y}{T_X}, \tag{6}$$

Applying in Figure 1 the second law of thermodynamics to the reversible part of the model, we obtain

$$\frac{\dot{Q}_H}{T_X} = \frac{\dot{Q}_L}{T_Y}, \tag{7}$$

Substituting Equations 1 and 2 into Equation 7, a relationship between T_Y and T_X is obtained as

$$T_Y = \frac{T_L}{1 - \beta A_R \left(\frac{T_H^4 - T_X^4}{T_X T_H^3} \right)}. \tag{8}$$

In thermoeconomic analysis of power plant models, an objective function is defined in terms of a characteristic function (power output [9, 11, 16], ecological function [5, 12, 13], etc.) and the cost involved in the performance of the power plant. In his early paper, De Vos [16] studied the thermoeconomics of a Novikov power plant model in terms of the maximization of an objective function defined as the quotient of the power output and the performing costs of the plant. In that paper, De Vos considered a function of costs with two contributions: the cost of the investment which is assumed proportional to the size of the plant and the cost of the fuel consumption which is assumed proportional to the quantity of heat input in the Novikov model. Analogously, Sahin and Kodal made a thermoeconomic analysis of a Curzon and Ahlborn [17] model in terms of an objective function which they defined as power output per unit total cost taking into account both the investment and fuel costs [9], while assuming the size of the plant proportional to the total heat transfer area, rather than the maximum heat input previously considered by De Vos [16]. Following the Sahin *et al.* procedure [11], the objective function has been defined as the power output per unit investment cost, because a solar driven heat engine does not consume fossil fuels. In order to optimize power output per unit total cost, the objective function is given by [11]

$$F = \frac{W}{C_i}, \tag{9}$$

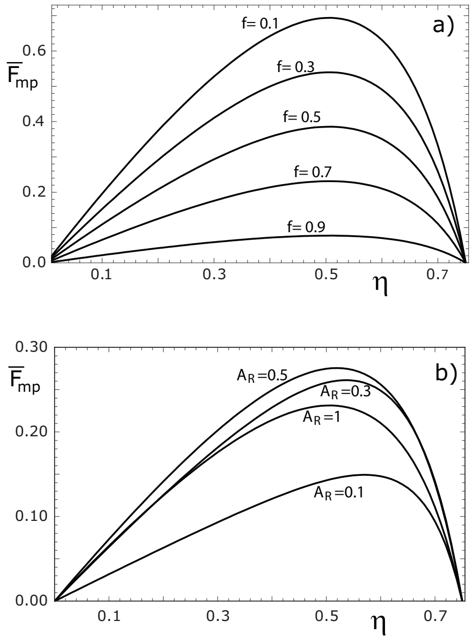
where C_i refers to annual investment cost. The investment cost of the plant is assumed proportional to the size of the plant. The size of the plant can be proportional to the total heat transfer area. Thus, the annual investment cost of the system can be given as [11]

$$C_i = aA_H + bA_L, \tag{10}$$

where the investment cost proportionality coefficients for the hot and cold sides a and b respectively are equal to the capital recovery factor times the investment cost per unit heat transfer area, and their dimensions are $ncu/(year \cdot m^2)$. Substituting Equations 2 and 9 into Equation 10, we obtain a normalized expression for objective function under maximum power regime given by [11]

$$\overline{F}_{mp} = \frac{bF}{U_L T_L} = \frac{\beta A_R \tau \left(1 - \theta^4\right) \left[1 - \frac{1}{\tau[\theta - \beta A_R (1-\theta^4)]}\right]}{\frac{f}{1+f} A_R + 1}, \tag{11}$$

Figure 2. a) Objective function $\overline{F}_{mp}$ with respect to thermal efficiency for several values of f with $\tau = 4$, $\beta = 1$ and $A_R = 1$ and b) $\overline{F}_{mp}$ against η for several values of A_R with $f = 0.7$, $\tau = 4$ and $\beta = 1$.



where $\theta = \frac{T_X}{T_H}$, $\tau = \frac{T_H}{T_L}$, and the parameter f is the relative investment cost of the hot size heat exchanger and defined as [11]

$$f = \frac{a}{a+b}. \tag{12}$$

For our thermoeconomic optimization approach, we define two objective functions, the so-called efficient power [14] and the ecological function [5, 13], both divided by the annual investment cost. These two functions are given by $F_\eta = \frac{\eta W}{C_i}$ and $F_E = \frac{W - T_L \Sigma}{C_i}$ respectively, where Σ is the total entropy production of the engine model. Analogously to Equation 11, the normalized objective functions under both the maximum efficient power regime and the maximum ecological function are given by

$$\overline{F}_\eta = \frac{bF_\eta}{U_L T_L} = \frac{\beta A_R \tau \left(1-\theta^4\right)\left[1 - \frac{1}{\tau[\theta - \beta A_R(1-\theta^4)]}\right]\eta_{th}}{\frac{f}{1+f}A_R + 1}, \tag{13}$$

and

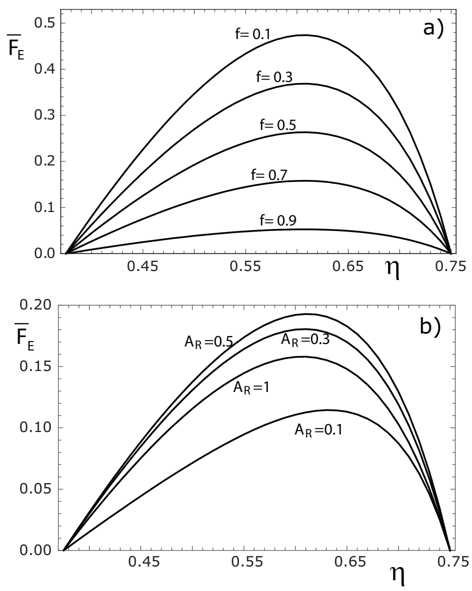
$$\overline{F}_E = \frac{bF_E}{U_L T_L} = \frac{\beta A_R \left(1-\theta^4\right)\left[(1+\tau)\left(\theta - \beta A_R\left(1-\theta^4\right)\right) - 2\right]}{\left[\frac{f}{1+f}A_R + 1\right]\left[\theta - \beta A_R\left(1-\theta^4\right)\right]}, \tag{14}$$

where η_{th} is the thermal efficiency of the endoreversible heat engine given by

$$\eta_{th} = 1 - \frac{1}{\tau\left[\beta A_R\left(1-\theta^4\right)\right]}. \tag{15}$$

In Equation 14 we have applied the second law of thermodynamics to calculate the total entropy production given by $\Sigma = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H}$ (see Figure 1). The normalized thermoeconomic objective functions (Equations 11, 13 and 14) can be plotted with respect to the thermal efficiency (Equations 15) for given values of f and A_R as shown in Figures 2a, 3a and 4a respectively, and in Figures 2b, 3b and 4b for the cases of the maximum power output, maximum ecological function and maximum efficient power conditions respectively. In all cases we use $\tau = 4$, as in [11], where $T_L \approx 300$K and therefore $T_H \approx 1200$K. This value of τ is for comparison with [11], however a more realistic value of T_H could be of the order of 431K [18], which is the effective sky temperature stemming from the dilution of solar energy. As could be seen from Figures 2a–4b, there exists an optimal thermal efficiency that maximizes the objective functions for given f, A_R and τ values. In Figure 5, we show the comparison of the aforementioned three objective functions.

Figure 3. a) Objective function $\overline{F}_E$ with respect to thermal efficiency for several values of f with $\tau = 4$, $\beta = 1$ and $A_R = 1$ and b) $\overline{F}_E$ against η for several values of A_R with $f = 0.7$, $\tau = 4$ and $\beta = 1$.



Since the three objective functions and thermal efficiency depend on the working fluid temperatures (T_X, T_Y), the objective functions given by Equations 11, 13 and 14 can be maximized with respect to T_X and T_Y. This optimization procedure has been numerically carried out in the next section [11, 12].

3. Numerical Results and Discussion

In Figures 2a–4b, we observe how the maximum of the objective functions diminishes while the economical parameter f augments for both the maximum ecological function (see Figure 3a) and the efficient power (see Figure 4a) regimes, in the same way shown by Sahin *et al.* [11] for the case of the maximum power output regime (see Figure 2a). On the other hand, when comparing with each other the profits obtained under the maximum power output, maximum efficient power and the ecological function conditions for the same value of f, it can be observed that the ecological profits are less than the profits in both maximum power output and maximum efficient power regimes, as shown in Figure 5. In Figures

2b, 3b and 4b we can also observe an optimum value of the ratio of heat transfer areas (A_R) at which each objective function has its highest value.

Figure 4. a) Objective function $\overline{F}_\eta$ with respect to thermal efficiency for several values of f with $\tau = 4$, $\beta = 1$ and $A_R = 1$ and b) $\overline{F}_\eta$ against η for several values of A_R with $f = 0.7$, $\tau = 4$ and $\beta = 1$.

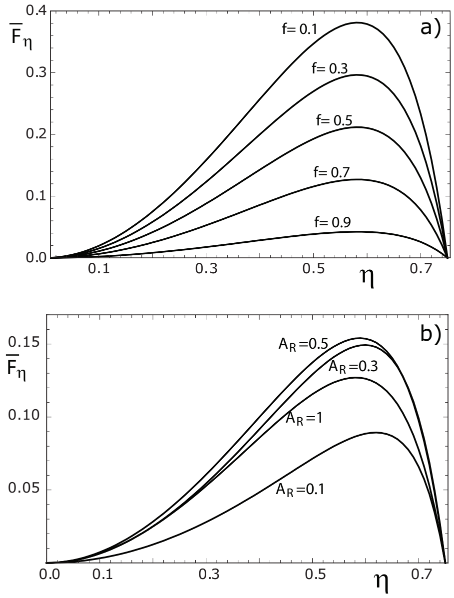
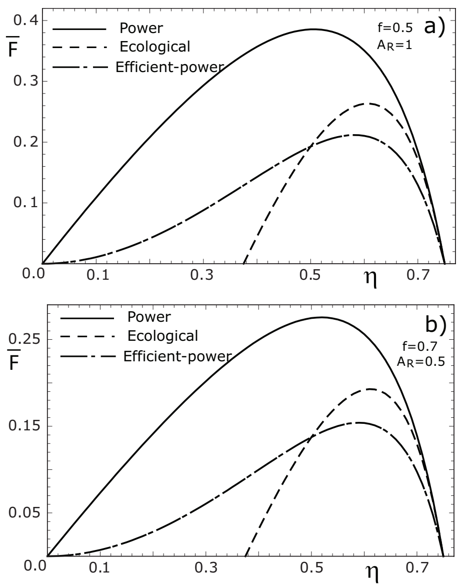
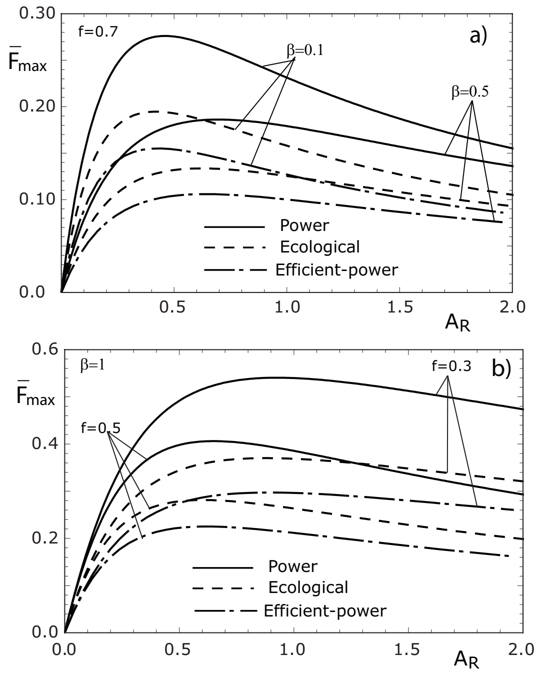


Figure 5. A comparison bewteen the three thermoeconomic objective functions with respect to thermal efficiency. a)with $f = 0.5$, $\tau = 4$, $\beta = 1.$ and $A_R = 1$. b) The same with $A_R = 0.5$, $f = 0.7$, $\tau = 4$ and $\beta = 1$.



These cases are more evident in Figure 6a,b for two different values of β and f. From these figures we can observe that the optimal A_R value depends on the β and f parameters. As we can see in these figures, under the three considered regimes the optimal A_R value slightly decreases for increasing β and f. The variation of the optimal thermal efficiency (η_{opt}) and the optimal power output (W_{opt}) under the maximum ecological function with respect to β for different A_R values are given in Figure 7.

Figure 6. Variation of the three maximum thermoeconomic objective functions with respect to A_R for different β. a) with $f = 0.7$, $\tau = 4$ for $\beta = 0.5$ and $\beta = 0.1$, b) with $\tau = 4$ $\beta = 1$ for $f = 0.5$ and $f = 0.3$.



In Figure 8, we show the comparison of optimal thermal efficiencies and the optimal power output for the three considered regimes. As we can see in Figures 7a and 8a, for the whole range of β values, the optimal thermal efficiencies satisfy

$$\eta_{CA} = 1 - \sqrt{T_L/T_H} \leq \eta_{opt}^{mp} < \eta_{opt}^{ep} < \eta_{opt}^{e} < \eta_C = 1 - T_L/T_H, \tag{16}$$

that is, the Curzon-Ahlborn (η_{CA}) and the Carnot (η_C) efficiencies are the ground and the ceiling of the optimum thermal efficiencies (η_{opt}). Analogously, from Figures 7b and 8b, the optimal power output satisfy

$$W_{opt}^{e} < W_{opt}^{ep} < W_{opt}^{mp}. \tag{17}$$

for the whole range of β values. On the other hand, we can see in Figures 7a and 8a that for given β and A_R values, the optimal thermal efficiencies (η_{opt}) decrease while the optimal power output (W_{opt}) increases. From Figures 7a and 8a it is also seen that the effect on η_{opt} and W_{opt} is more important within the interval $0 < \beta < 1.5$. As A_R increases, the effective β range becomes narrower.

Figure 7. a) Thermal efficiency vs. β for several values of A_R. b) Power output at maximum thermoeconomic ecological function vs. β for different values of A_R ($f = 0.7$ and $\tau = 4$).

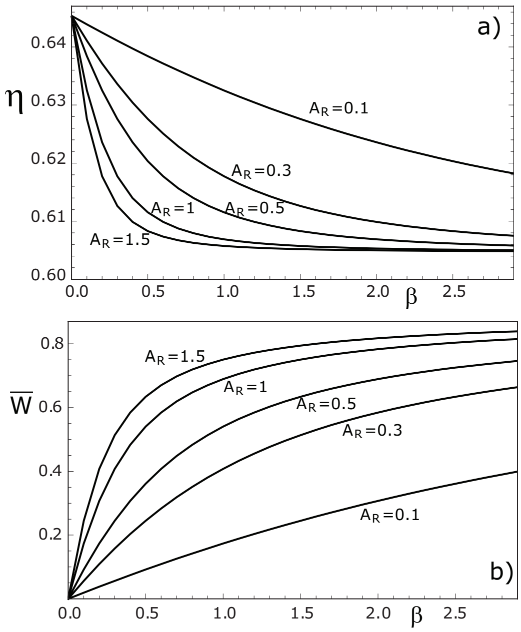
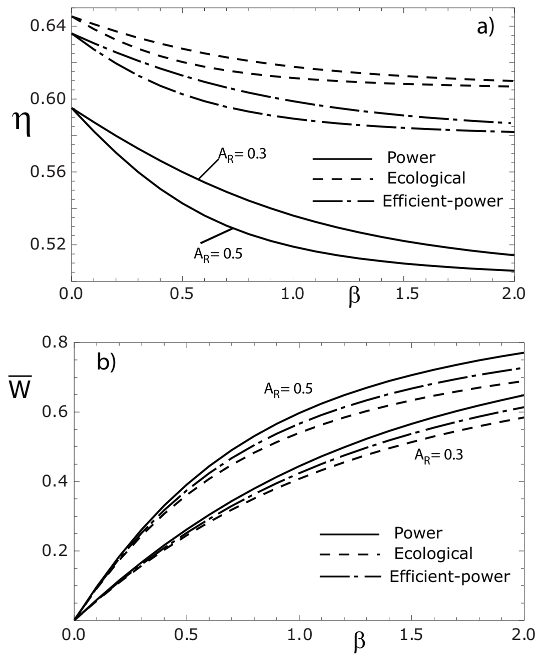


Figure 8. a) Optimal thermal efficiencies vs. β for the three regimes, b) power output vs. β for the three regimes (with $f = 0.7$ and $\tau = 4$). In both cases $A_R = 0.5$, $A_R = 0.3$.



4. Concluding Remarks

In this work, following the Sahin *et al.* procedure, a thermoeconomic performance analysis using finite time/finite size thermodynamics has been carried out for an endoreversible solar-driven heat engine in terms of the maximization of three different characteristic functions. The objective functions have been defined as the characteristic functions (power output, ecological function and efficient power) per unit total investment cost. By the maximization of these three objective functions, the optimum thermoeconomic performance and the corresponding best design parameters of the solar-driven heat engine were determined. In this context, the effects of the economic parameter, f, and the ratio of heat transfer areas, A_R, on the optimal thermoeconomic performance have been investigated. We show how the optimal thermal efficiency under maximum ecological conditions is bigger than the optimal thermal efficiencies under both the maximum power and the maximum efficient power conditions. Moreover, the maximum power output under maximum ecological conditions is less than the corresponding maximum power output under both the maximum power and the maximum efficient power conditions. This result has been systematically observed in all kinds of thermal engine models operating under maximum ecological function conditions.

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References

1. Bejan, A. *Advanced Engineering Thermodynamics*. Addison Wiley: New York, NY, USA, 1988.
2. De Vos, A.; Pauwels, H. On the thermodynamic limit of photovoltaic energy conversion. *Appl. Phys. A* **1981**, *25*, 119–125.
3. Jeter, S.M. Maximum Conversion efficiency for the utilization of direct solar radiation. *Solar Energy* **1981**, *26*, 231–236.
4. Gordon, J.M. On optimized solar-driven heat engines. *Solar Energy* **1988**, *40*, 457–461.
5. Angulo-Brown, F. An ecological optimization criterion for finite-time heat engines. *J. Appl. Phys.* **1991**, *69*, 7465–7469.
6. Chen, L.; Zhu, X.; Sun, F.; Wu, C. Exergy-based ecological optimization of linear phenomenological heat-transfer law irreversible Carnot-engines. *Applied Energy* **2006**, *83*, 573–582.
7. Zhu, X.; Chen, L.; Sun, F.; Wu, C. Exergy-based ecological optimization for a generalized irreversible Carnot refrigerator. *J. Energy Inst.* **2006**, *79*, 42–46.
8. Sahin, A.Z. Optimum operating conditions of solar driven heat engines. *Energ. Conv. Manage.* **2000**, *41*, 1335–1343.
9. Sahin, A.Z. Finite-time thermodynamic analysis of a solar driven heat engine. *Exergy, An International Journal* **2001**, *01*, 122–126.
10. Sahin, B.; Kodal, A. Performance analysis of an endoreversible heat engine based on a new thermoeconomic optimization criterion. *Energ. Conv. Manage.* **2001**, *42*, 1085–1093.
11. Sahin, B.; Ust, Y.; Tamer, Y.; Akcay, I.H. Thermoeconomic analysis of a solar driven heat engine. *Renewable Energy* **2006**, *31*, 1033–1042.

12. Barranco-Jiménez, M.A.; Sánchez-Salas, N.; Angulo-Brown, F. On the optimum operation conditions of an endoreversible heat engine with different heat transfer laws in the thermal couplings. *Revista Mexicana de Física* **2008**, *54*, 284–292.
13. Barranco-Jiménez, M.A.; Angulo-Brown, F. Thermoeconomic optimisation of Novikov power plant model under maximum ecological conditions. *J. Energy Inst.* **2007**, *80*, 96–104.
14. Yilmaz, T. A new performance criterion for heat engines: efficient power. *J. Energy Inst.* **2006**, *79*, 38–41.
15. Khaliq, A. Finite-time heat-transfer analysis and generalized power-optimization of an endoreversible Rankine heat-engine. *Applied Energy* **2004**, *79*, 27–40.
16. De Vos, A. Endoreversible thermoeconomics. *Energ. Conv. Manage.* **1995**, *36*, 1–5.
17. Curzon, F.; Ahlborn, B. Efficiency of a Carnot engine at maximum power output. *Amer. J. Phys.* **1975**, *43*, 22–24.
18. De Vos, A. *Endoreversible Thermodynamics of Solar Energy Conversion*. Oxford University Press: New York, NY, USA, 1992.