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THIN FILM EVAPORATION-THEORY

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ENGINEERING NOTE

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SUBJECT

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By

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NOMENCLATURE

a,b,A,B = constants

$C_1 = G\Phi_1/(16\Phi_1/15 - 3)$

$C_2 = 1.4G\Phi_2/(\Phi_2 + 3)$

c_p = specific heat

f = friction factor

g = gravitational constant

h = heat transfer coefficient

i = enthalpy

k = thermal conductivity

p = pressure

q = heat transfer rate

t = temperature

u,v = horizontal and vertical velocities

x,y = horizontal and vertical positions in film

<u>Subscripts</u>

d = dry-up

i = inlet

m = maximum

o = start of vaporization

t = thermal

T = total

v = vapor or velocity

w = wall

Greek Symbols

δ = film a boundary layer thickness

τ = shear stress

μ, ν = dynamic and kinematic viscosities

ρ = density

Dimensionless Parameters

$G = g\delta_i^3/\nu^2$

$I = i_{lv}/c_p\,(t_w - t_v)$

$Nu = h\delta_i/k$, Nusselt number

$Pr = \nu/\alpha$, Prandtl number

$Q = q_w\delta_i/k(t_w - t_v)$

$T = (t - t_v)/(t_w - t_v)$

$U = u\delta_i/\nu$, Reynolds number

$X = (x - x_o)/\delta_i$

$Y = \delta/\delta_i$

INTRODUCTION

Evaporation of liquids from a heated surface covered by a thin film poses an unusual case, in that heat transfer rates can be much higher than in the pool evaporation condition. Evaporation rates from a thin film are attributed mostly to conduction while in a pool, free convection is the principle mode.

Consider for now, a horizontal, flat plate, held at a steady temperature, with a constant liquid film height on one end supplied by a reservoir at some temperature less than or equal to the plate temperature (Figure 1). As the liquid flows across the plate from the reservoir a velocity boundary-layer developes in a manner that is probably similar to that of a flat plate exposed to a uniform parallel flow. The velocity boundary-layer thickness increases until it is equal to the film thickness (Figure 2). From this location the film thickness δ and velocity boundary-layer thickness δ_v are equal. Flow occurs by virtue of a decreasing film thickness so as to create a negative pressure gradient on the plate surface. Thus, the magnitude of the maximum velocity must vary along the plate whether evaporation occurs or not. However, evaporation must occur to achieve a steady-state condition of the flow. Consequently, at some location all the liquid will be evaporated and the film thickness will be zero.

The temperature profile should develop in a manner similar to the velocity profile (Figure 3). A thermal boundary-layer δ_t increases along the plate until it equals the film thickness δ . If the inlet temperature t_i is less than the liquid vapor temperature t_v, then an additional length is needed to allow the surface temperature of the film to change from t_i to t_v before evaporation begins.

In the region where evaporation occurs, $o < x < x_o$, there is a coupling between heat and mass flow equations. Evaporation caused by heat transfer results in mass-loss from the film thus effecting the mass and momentum flow. A simultaneous solution of the appropriate mass, momentum, and energy equations are required in order to estimate heat transfer rates and location of film dry-up, both of which are important quantities in this type of problem.

One might speculate that the posed phenomena could be solved in three or four separate regions; namely, the velocity development up to evaporation, the temperature development up to the location of a fully-developed profile x_t, temperature development in the region $x_t < x < x_o$, and temperature and velocity development in the evaporation region $x > x_o$. A solution for the latter region will be given herein for laminar and turbulent flow. The entrance region is a much more difficult problem and a solution has not yet been achieved. However, it will be shown that a very small portion of the total heat transferred is in the entrance region.

PROBLEM DESCRIPTION

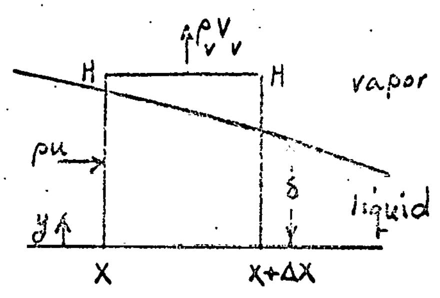
At some location x_o on a horizontal plate of steady temperature t_w, a liquid film of thickness δ_o exists. Both the velocity and temperature profiles are "fully-developed" at this location and evaporation occurs. The problem is to estimate the location of "dry-up" and local heat transfer rates. Underlying assumptions are:

- steady-state heat and mass flow
- constant fluid properties
- negligible heat conduction in the direction parallel to the plate
- no viscous heating
- no nucleate boiling

INTERGAL EQUATIONS

Continuity

The conservation of mass principle is applied to the Δx by H element shown extending from the



wall into the vapor region.

$$\text{(Mass flow out)} - \text{(Mass flow in)} = 0 \qquad (1)$$

$$\left[\int_0^H \rho u\, dy\right]_{x+\Delta x} - \left[\int_0^H \rho u\, dy\right]_x + \rho_v v_v \Delta x = 0 \qquad (2)$$

$$\frac{d}{dx}\left[\int_0^H \rho u\, dy\right] + \rho_v v_v = 0 \qquad (3)$$

Since $\rho u = 0$ for $\delta \le y \le H$ and $\rho_v v_v$ is constant for $y > \delta$, equation (3) becomes

$$\boxed{\rho \frac{d}{dx}\left[\int_0^\delta u\, dy\right] + \rho_v v_v = 0} \qquad (4)$$

Momentum

The conservation of momentum principle applied to the same Δx by H element can be stated as

$$\text{(Momentum out)} - \text{(Momentum in)} = \sum \text{Vector forces} \qquad (5)$$

The direction of momentum and external forces must, of course, be complimentary and are taken in the x-direction. The external forces are made up of pressure and viscous boundary-shear forces.

$$\sum \text{Momentum} = \left[\int_0^H \frac{\rho u^2}{g} dy\right]_{X+\Delta X} - \left[\int_0^H \frac{\rho u^2}{g} dy\right]_X \qquad (6)$$

$$\sum \text{Forces} = -\tau_w \Delta X - \left[\int_0^H p\,dy\right]_{X+\Delta X} + \left[\int_0^H p\,dy\right]_X \qquad (7)$$

Using equations (6) and (7) in (5) yields

$$\frac{\rho}{g}\frac{d}{dX}\left[\int_0^H u^2 dy\right] = -\tau_w - \frac{d}{dX}\left[\int_0^H p\,dy\right] \qquad (8)$$

As in the derivation of equation (4) δ can be substituted for H. Also, $p = p_v + \rho(\delta - y)$ and $dp_v/dx = 0$. Thus

$$\frac{\rho}{g}\frac{d}{dX}\left[\int_0^\delta u^2 dy\right] + \tau_w + \frac{d}{dX}\left[\int_0^\delta \left(p_v + \rho[\delta - y]\right) dy + \int_\delta^H p_v\, dy\right] = 0 \qquad (9)$$

$$\boxed{\frac{\rho}{g}\frac{d}{dX}\left[\int_0^\delta u^2 dy\right] + \tau_w + \frac{\rho}{2}\frac{d\delta^2}{dX} = 0} \qquad (10)$$

Energy

Conservation of energy is required for the ΔX by H element:

(Energy out) - (Energy in) = 0 (11)

The energies in this case are made up of conducted and convected heat in the liquid and vapor convected away from the film. Since conduction in the x-direction is neglected

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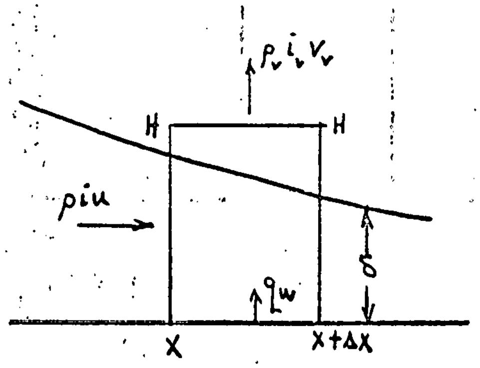

$$\left[\int_0^H \rho i u \, dy\right]_{X+\Delta X} - \left[\int_0^H \rho i u \, dy\right]_X + \rho_v i_v V_v \Delta X - q_w \Delta X = 0 \qquad (12)$$

where i = enthalpy.

Using the continuity equation (4) and the same argument for changing H to δ yields

$$\rho \frac{d}{dx}\left[\int_0^\delta (i - i_v) u \, dy\right] - q_w = 0 \qquad (13)$$

Note that

$$i - i_v = i - i_{sat.\,liq.} + i_{sat.\,liq.} - i_v \qquad (14)$$
$$= c_p(t - t_v) - i_{lv}$$

where i_{lv} is the energy required for phase change from liquid to vapor.

$$\boxed{\rho \frac{d}{dx}\left[\int_0^\delta \left[c_p(t - t_v) - i_{lv}\right] u \, dy\right] - q_w = 0} \qquad (15)$$

Dimensionless Equations

In order to generalize the applicable integral equations (4), (10), and (15) and to reduce the number of variables, the following dimensionless parameters are selected.

$T = (t-t_v)/(t_w-t_v)$

$U = u\delta_i/\nu$

$X = (x-x_o)/\delta_i$

$\Delta = \delta/\delta_i$

$I = i_{lv}/c_p(t_w-t_v)$

$G = g\delta_i^3/\nu^2$

The continuity equation (4) now becomes

$$\frac{d}{dX}\left[\int_0^\Delta U\,dY\right] + \rho_v V_v \delta_i/\mu = 0 \tag{16}$$

The momentum equation (10) becomes

$$\frac{d}{dX}\left[\int_0^\Delta U^2 dY\right] + \left[\frac{\partial U}{\partial Y}\right]_{Y=0} + \frac{G}{2}\frac{d\Delta^2}{dX} = 0 \tag{17}$$

The energy equation (15) becomes

$$\frac{d}{dX}\left[\int_0^\Delta (T-I)U\,dY\right] + P_r\left[\frac{\partial T}{\partial Y}\right]_{Y=0} = 0 \tag{18}$$

Note that $\tau_w = [\mu\,\partial u/\partial y]_{y=0}$ and $q_w = -[k\,\partial t/\partial y]_{y=0}$ are used in equations (17) and (18).

SOLUTIONS - LAMINAR FLOW

Profiles

The integral equations (16), (17), and (18) do not specify laminar or turbulent flow, but it is necessary to specify the velocity and temperature profiles in order to perform the integration. Therefore, a quadratic approximation for both is assumed in the laminar flow case.

Velocity:

In dimensionless form, the velocity profile is given as

$$U(Y) = a_0 + a_1 Y + a_2 Y^2 \tag{19}$$

Boundary conditions are given as

$$U(0) = 0, \quad U(\Delta) = U_m, \quad \partial U(\Delta)/\partial Y = 0 \tag{20}$$

The first condition is the "no-slip at the wall" requirement, the second is a maximum value at the film surface which is unknown at this point, and the third is a zero shear for the free surface. Using these three gives

$$U(Y)/U_m = 2Y/\Delta - Y^2/\Delta^2 \tag{21}$$

$$\partial U(0)/\partial Y = 2U_m/\Delta \tag{22}$$

$$(U/U_m)^2 = 4Y^2/\Delta^2 - 4Y^3/\Delta^3 + Y^4/\Delta^4 \tag{23}$$

$$\int_0^\Delta U^2 dY = \frac{8}{15} U_m^2 \Delta \tag{24}$$

Temperature:

It is assumed that

$$T(Y) = b_0 + b_1 Y + b_2 Y^2 \tag{25}$$

with the boundary conditions given as

$$T(0) = 1, \quad T(\Delta) = 0, \quad (\partial^2 T/\partial Y^2)_{Y=0} = 0 \tag{26}$$

The first two conditions specify the fixed values of temperature in the film and the third is derived from the differential energy equation stated as

$$u\,\partial t/\partial x + v\,\partial t/\partial y = \partial^2 t/\partial x^2 + \partial^2 t/\partial y^2 \qquad (27)$$

At the wall $u = v = \partial t/\partial y = 0$, thus $\partial^2 t/\partial y^2$ must also be zero.

Using the conditions (25) in equation (24) yields

$$T(Y) = 1 - Y/\Delta \qquad (28)$$

$$\partial T(0)/\partial Y = -1/\Delta \qquad (29)$$

$$(T-I)U = U_m\left[2(1-I)Y/\Delta + (I-3)Y^2/\Delta^2 + Y^3/\Delta^3\right] \qquad (30)$$

$$\int_0^\Delta (T-I)U\,dY = U_m\Delta\left(\frac{1}{4} - \frac{2}{3}I\right) \qquad (31)$$

Solutions

If equations (22) and (24) are used in the momentum equation (17), the result is

$$\frac{d}{dX}\left(\frac{8}{15}U_m^2\Delta\right) + 2U_m/\Delta + \frac{G}{2}\frac{d\Delta^2}{dX} = 0 \qquad (32)$$

or

$$\frac{dU_m}{dX} + \left(\frac{U_m}{2\Delta} + \frac{15}{16}\frac{G}{U_m}\right)\frac{d\Delta}{dX} + \frac{15}{8\Delta^2} = 0 \qquad (33)$$

Likewise, by using equations (29) and (31) in the energy equation (18) we obtain

$$\frac{d}{dX}\left(U_m\Delta\Phi_1\right) + Pr/\Delta = 0 \qquad (34)$$

or $$\frac{d\mathcal{U}_m}{dX} + \frac{\mathcal{U}_m}{\Delta}\frac{d\Delta}{dX} + \frac{\Phi_1}{\Delta^2} = 0 \qquad (35)$$

where $\Phi_1 = Pr/(\frac{2}{3}I - \frac{1}{4})$

By combining equations (35) and (33) to eliminate the $1/\Delta^2$ term one obtains

$$\frac{d\mathcal{U}_m}{dX} + \left(\frac{A\,\mathcal{U}_m}{\Delta} + \frac{B}{\mathcal{U}_m}\right)\frac{d\Delta}{dX} = 0 \qquad (36)$$

where $$A = \frac{\frac{4}{15}\Phi_1 - 1}{\frac{8}{15}\Phi_1 - 1}, \quad B = \frac{G\Phi_1}{\frac{8}{15}\Phi_1 - 1}$$

A solution to equation (36) is

$$\mathcal{U}_m = \sqrt{C'\Delta^{-2A} - \frac{2B}{1+2A}\Delta} \qquad \text{(See Appendix)} \qquad (37)$$

Since $A > 0$ and $B < 0$, then $C' = 0$ is required and

$$\mathcal{U}_m = \sqrt{\frac{-2B}{1+2A}\Delta} = \sqrt{\frac{-G\Phi_1\Delta}{\frac{16}{15}\Phi_1 - 3}} \qquad (38)$$

When this expression is used in equation (35) one can obtain

$$\left(8 + \frac{15}{2}\frac{G}{C_1}\right)\frac{d\Delta}{dX} + \frac{15\,\Delta^{-3/2}}{\sqrt{C_1}} = 0 \qquad (39)$$

where $$C_1 = -\frac{2B}{1+2A} = \frac{-G\Phi_1}{\frac{16}{15}\Phi_1 - 3}$$

The variables in this equation are separable and a solution can be obtained by integration.

$$\int_{\Delta_0}^{\Delta} \Delta^{3/2} d\Delta + \int_0^X \frac{15}{\left(8+\frac{15}{2}\frac{G}{C_1}\right)\sqrt{C_1}} dX = 0 \tag{40}$$

$$\Delta = \left[\Delta_0^{5/2} - \frac{75X}{(16+15G/C_1)\sqrt{C_1}}\right]^{2/5} \tag{41}$$

At the location of film dry-up X_d the film thickness will be zero, thus from (41)

$$\boxed{X_d = \frac{\Delta_0^{5/2}}{75}(16+15G/C_1)\sqrt{C_1}} \tag{42}$$

Local heat transfer rates can now be calculated by determining the wall heat flux.

$$q_w = -k\,\partial t(0)/\partial y = k(t_w - t_v)\delta \tag{43}$$

or

$$\boxed{Nu_w = 1/\Delta = \left[\Delta_0^{5/2} - \frac{75X}{(16+15G/C_1)\sqrt{C_1}}\right]^{-2/5}} \tag{44}$$

where Nu_w is the dimensionless Nusselt number $q_w \delta_i / k(t_w - t_v)$.
Note that equations (41) and (44) can also be written as

$$\Delta = \Delta_0 (1 - X/X_d)^{2/5} \tag{45}$$

$$Nu_w = \Delta_0^{-1}(1 - X/X_d)^{-2/5} \tag{46}$$

Total heat transfer rates in the evaporation region $x_o < x < x_d$ can also be determined by an over all heat balance.

$$q_{t_0} = \rho\int_0^{\delta_0} u(i_v - i)dy = \rho\int_0^{\delta_0} u[i_{\ell v} - c_p(t - t_v)]dy \tag{47}$$

or in dimensionless terms

$$Q_{T_0} = Pr\int_0^{\Delta_0} U(I - T)dY \tag{48}$$

$$= Pr\int_0^{\Delta_0} U_{m_0}(2Y/\Delta_0 - Y^2/\Delta_0^2)(I - 1 + Y/\Delta_0)dY \tag{49}$$

$$Q_{T_0} = \frac{Pr^2\Delta_0^{2/3}\sqrt{C_1}}{\Phi_1} \tag{50}$$

However, if we wish to determine the total heat transfer rate based on uniform velocity and temperature profiles at the leading edge of the plate, then

$$q_{T_i} = \rho u_i(i_v - i_i) = \rho u_i[i_{\ell v} - c_p(t_i - t_v)] \tag{51}$$

or

$$Q_{T_i} = Pr\,U_i(I - T_i) \tag{52}$$

From continuity and equations (21) and (38) we can derive

$$U_i = \frac{2}{3}\Delta_0^{3/2}\sqrt{C_1} \tag{53}$$

or $$Q_{T_i} = \frac{2}{3} P_r \Delta_o^{3/2} \sqrt{C_1}\,(I - T_i) \qquad (54)$$

giving $$\boxed{Q_{T_i}/Q_{T_o} = \frac{I - T_i}{I - \frac{3}{8}}} \qquad (55)$$

SOLUTIONS-TURBULENT FLOW

Profiles

Velocity:

The 1/5th power low profile for velocity is selected as it has been shown to approximate low Reynolds number flow on flat plates very well[1]. We can then use

$$\frac{u}{u_m} = \left(\frac{Y}{\Delta}\right)^{1/5} \quad \text{and} \quad \int_0^\Delta u^2 dY = \frac{5}{7} u_m^2 \Delta \qquad (56)$$

Wall shearing stress for a flat plate related to the velocity boundary-layer thickness has been given as (Ref. 1, pg. 275).

$$\tau_w = 0.0228 \frac{\rho u^2}{g} \left(\frac{\nu}{u \delta}\right)^{1/4} \qquad (57)$$

or for the present case

$$\frac{g \tau_w \delta_i^2}{\nu^2 \rho} = 0.0228\, u_m^2 (u_m \Delta)^{-1/4} \qquad (58)$$

Temperature:

Similarity between velocity and temperature profiles are assumed, thus

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[1] J. G. Knudsen and D. L. Katz, Fluid Dynamics and Heat Transfer, McGraw-Hill Book Co., New York, 1958, Pg. 274.

$$\frac{t-t_w}{t_v-t_w} = \left(\frac{y}{\delta}\right)^{1/5} \quad \text{or} \quad T = 1-(Y/\Delta)^{1/5} \tag{59}$$

and
$$\int_0^\Delta (T-1)U\,dY = \frac{5}{6}U_m\Delta\left(\frac{1}{7}-1\right) \tag{60}$$

The wall heat flux is expressed as

$$-\left[k\,\partial t/\partial y\right]_{y=0} = h(t_w-t_v) \quad \text{or} \quad Nu_w = -\partial T(0)/\partial Y \tag{61}$$

Colburn's analogy between heat and momentum transfer is now applied[1]

$$Nu_w = \frac{f}{2}U_m Pr^{1/3} \tag{62}$$

Since
$$\frac{f}{2} = \tau_w g/\rho u^2 \tag{63}$$

equation (62) can be reduced to

$$Nu_w = 0.0228\,U_m(U_m\Delta)^{-1/4}Pr^{1/3} \tag{64}$$

<u>Solutions</u>

If equations (56) and (58) are used in the dimensionless momentum integral equation (17) except a wall shear is substituted for $dU/dY|_{Y=0}$ we obtain

$$\frac{dU_m}{dX} + \left(\frac{U_m}{2\Delta} + \frac{7}{10}\frac{G}{U_m}\right)\frac{d\Delta}{dX} + \frac{7}{10}(0.0228)\frac{U_m^{3/4}}{\Delta^{5/4}} = 0 \tag{65}$$

Likewise, when equations (59), (60), and (64) are used in the dimensionless energy differential equation (18) the result is

$$\frac{dU_m}{dX} + \frac{U_m}{\Delta}\frac{d\Delta}{dX} - \frac{0.0228}{\frac{5}{6}\left(\frac{1}{7}-1\right)}\cdot\frac{U_m^{3/4}Pr^{4/3}}{\Delta^{5/4}} = 0 \tag{66}$$

[1]Op. cit., Knudsen, Pg. 474

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If these two equations are combined we obtain

$$\frac{dU_m}{dX}+\left[\frac{\frac{1}{2}\Phi_2+1}{\Phi_2+1}\cdot\frac{U_m}{\Delta}+\frac{\frac{7}{10}\Phi_2 G}{\Phi_2+1}\cdot\frac{1}{U_m}\right]\frac{d\Delta}{dX}=0 \qquad (67)$$

or $$\frac{dU_m}{dX}+\left[\frac{AU_m}{\Delta}+\frac{B}{U_m}\right]\frac{d\Delta}{dX}=0$$

where $$\Phi_2 = 12\,P_r^{4/3}/(1-I/7) \qquad (68)$$

$$A = (\Phi_2/2+1)/(\Phi_2+1) \qquad (69)$$

$$B = \frac{7}{10}\Phi_2 G/(\Phi_2+1) \qquad (70)$$

The solution to equation (67) is

$$U_m=\sqrt{C'\Delta^{-2A}-\frac{2B}{1+2A}\Delta} \qquad (71)$$

However, the condition $\left[U_m\right]_{\Delta=0}=0$ requires that $C' = 0$. Thus,

$$U_m=\sqrt{\frac{-2B\Delta}{1+2A}}=\sqrt{C_2\Delta} \qquad (72)$$

where $C_2 = -2B/(1+2A) = \frac{14}{10}\Phi_2 G/(\Phi_2+3)$

Using this expression in equation (66) yields

$$\Delta^{1/8}d\Delta = 0.0228\left(\frac{7}{15}\right)\left(\frac{\Phi_2}{C_2^{1/8}}\right)dX \qquad (73)$$

Integration gives

$$\Delta = \left[\Delta_0^{9/8} + \frac{21}{40} \cdot \frac{0.0228}{C_2^{1/8}} \bar{\Phi}_2 X\right]^{8/9} \tag{74}$$

The point of dry-up is established by $[\Delta]_{X=X_d} = 0$ thus

$$X_d = -\Delta_0^{9/8} \cdot \frac{40 C_2^{1/8}}{21(0.0228)\bar{\Phi}_2} \tag{75}$$

$$\Delta = \Delta_0 (1 - X/X_d)^{8/9} \tag{76}$$

By using equation (72) in (64) yields a relation for local heat transfer rates as

$$Nu_w = 0.0228\, C_2^{3/8} \Delta_0^{1/8} (1 - X/X_d)^{1/9} Pr^{1/3} \tag{77}$$

The total heat transfer rate from the region of evaporation can be found as in the laminar flow case and is given as

$$Q_{T_0} = \frac{-10 Pr^2 \sqrt{C_2}\, \Delta_0^{3/2}}{\bar{\Phi}_2} \tag{78}$$

Likewise, the total heat transfer rate compared to the quantity given in equation (77) is found to be

$$Q_{T_i}/Q_{T_0} = (I - T_i)/(I - 1/7) \tag{79}$$

DISCUSSION

Probably, the most valuable results of this analysis are predicted values for X_d, Q_{T_i}, and Nu. Some sample values of X_d and Q_{T_o} for water are given in Table I for both laminar and turbulent films. Values of Q_{T_i} can be determined from equations (55) and (79), however, ratios of Q_{T_i}/Q_{T_o} will be near unity for most practical cases. Curves of Nu and Δ vs. X/X_d are shown in Figures 4 and 5. One will probably quickly scrutinize the curves of Nu and wonder why the different trends for turbulent and laminar flow. Realize that the temperature boundary conditions at $X \rightarrow X_d$ for both laminar and turbulent flows are not valid so we should not expect the rapid changes in the heat flux there. Also, it is likely that the film will change from laminar to turbulent far from the location of "dry-up". Possibly the criteria posed by Kays[1] can be used to estimate the location of transition, i.e.

$$\delta_m = 0.664\sqrt{\nu x/u_m} \qquad (80)$$

where δ_m is the momentum thickness of the boundary-layer defined as

$$\delta_2 = \int_0^{\delta} \frac{u}{u_m}\left(1 - \frac{u}{u_m}\right) dy \qquad (81)$$

One difficulty in using the heat transfer predictions given here is estimating values of Δ_o . Unless t_i is much different from t_v, then Δ_o is near unity. To be more precise, however, some additional analysis needs to be done on the entrance region, $o < x < x_o$, to determine the location of initial evaporation and for fully-developed velocity and temperature profiles. Perhaps, some of the results developed from flat-plate, parallel-flow analyses could be applied, however, at this point this is not entirely evident.

[1] W. M. Kays, Convective Heat and Mass Transfer, McGraw-Hill Book Co., San Francisco, 1966, Pg. 93

Additional efforts on this problem should be directed to checking the analysis by experimental measurements. One simple method would be to measure only x_d and compare to predicted values. Conclusively, agreement of measured values of Nu, Q_{T_1}, and X_d with predictions would prove the theory correct.

APPENDIX A

Non-Homogenous Differential Equation Solution

Given is a differential equation of the form

$$\frac{du}{dx} + a\frac{u}{x} + \frac{b}{u} = 0 \qquad (1a)$$

from which a solution $u(a,b,x)$ is required. First the substitution $w = u^2$ is made to give

$$\frac{dw}{dx} + 2a\frac{w}{x} + 2b = 0 \qquad (2a)$$

Next let $w = u \cdot v$ to yield

$$u\frac{dv}{dx} + \left(\frac{du}{dx} + 2a\frac{u}{x}\right)v + 2b = 0 \qquad (3a)$$

To satisfy this equation let the quantity in parentheses equal to zero, i.e.

$$\frac{du}{dx} + 2a\frac{u}{x} = 0 \qquad (4a)$$

Integration yields

$$u = x^{-2a} \qquad (5a)$$

The remaining terms of equation (3a) must also be zero, thus

$$x^{-2a}\frac{dv}{dx} + 2b = 0 \qquad (6a)$$

$$v = -\frac{2b}{1+2a}x^{2a+1} + c \qquad (7a)$$

Since

$$w = u \cdot v = cx^{-2a} - \frac{2b}{1+2a}x \qquad (8a)$$

and $u^2 = w$, then

$$u = \sqrt{w} = \sqrt{cx^{-2a} - \frac{2b}{1+2a}x} \qquad (9a)$$

Substitution of this expression into equation (1a) and obtaining equality proves it is indeed a solution.

Table I - Values of X_d and Q_{T_o} for turbulent and laminar films using properties of water.

Γ	G	Δ_o	Laminar X_d	Laminar Q_{T_o}	Turbulent X_d	Turbulent Q_{T_o}
10	10^4	0.5	4.1×10^1	1.4×10^2	3.4×10^{-1}	5.5×10^1
100	10^4	0.5	8.1×10^1	7.2×10^2	1.1×10^1	1.9×10^3
10	10^4	0.9	1.8×10^2	3.4×10^2	6.6×10^{-1}	1.3×10^2
100	10^4	0.9	3.5×10^2	1.7×10^3	2.1×10^1	4.7×10^3
10	10^6	0.5	1.3×10^3	4.4×10^2	6.0×10^{-1}	5.5×10^2
100	10^6	0.5	2.6×10^3	2.3×10^3	1.9×10^1	1.9×10^4
10	10^6	0.9	5.0×10^3	1.0×10^3	1.2×10^0	1.3×10^3
100	10^6	0.9	1.1×10^4	5.5×10^3	3.7×10^1	4.7×10^4

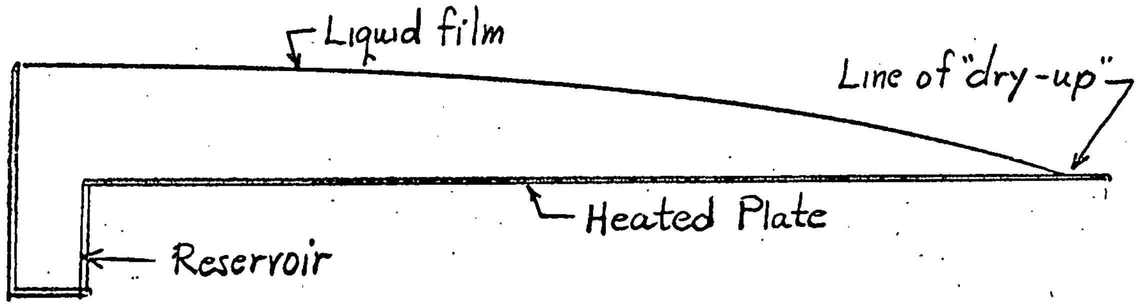

Figure 1 Thin film evaporation from a flat plate

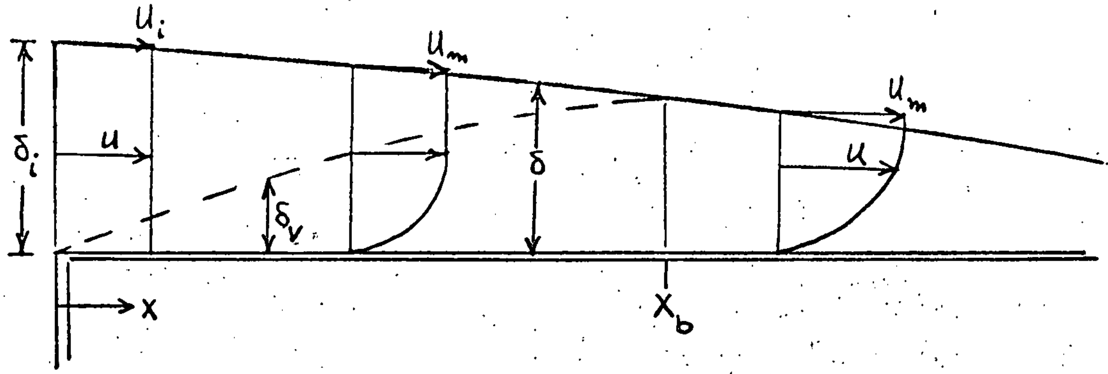

Figure 2 Velocity profile development

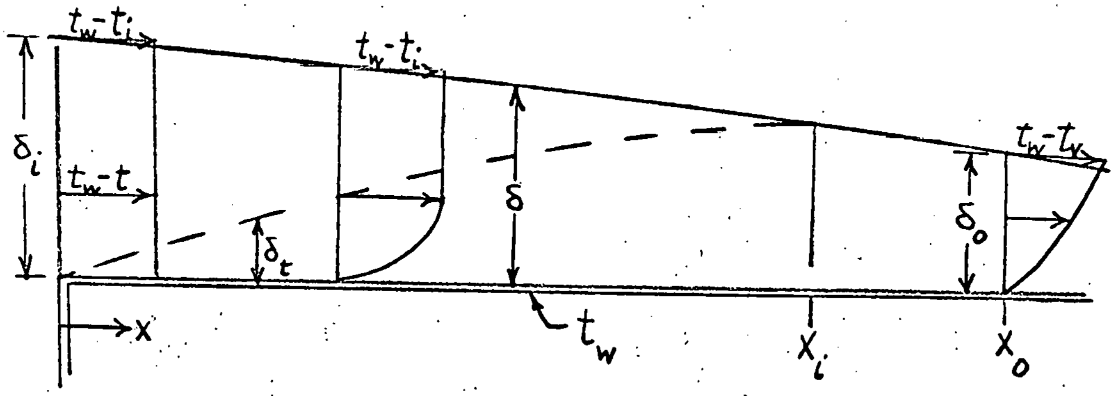

Figure 3 Temperature profile development

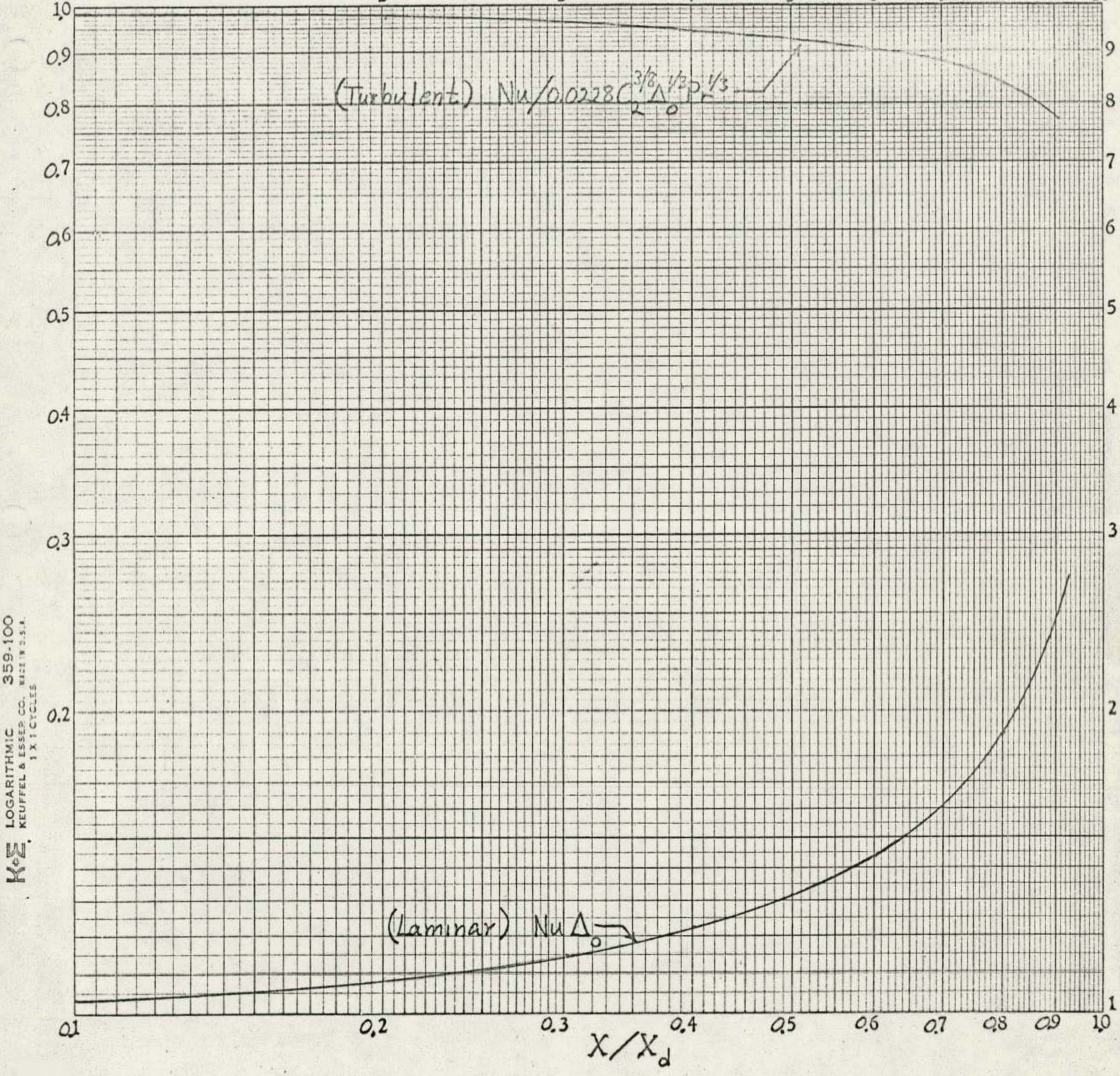

Figure 4 Local heat transfer rates for the laminar and turbulent films

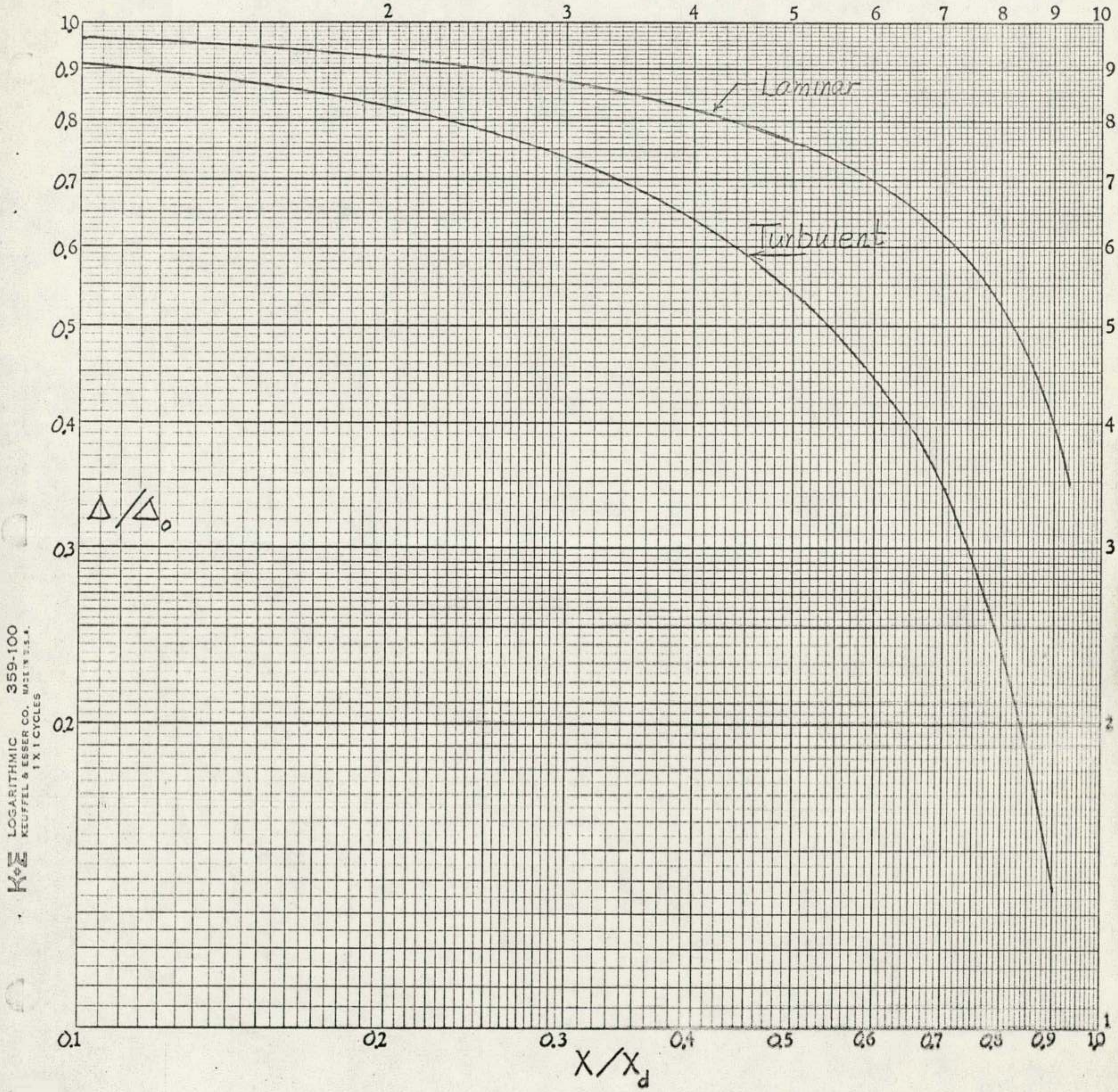

Figure 5 Film thickness for laminar and turbulent flows

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