Thinking as Communicating

This book is an attempt to change our thinking about thinking. Anna Sfard undertakes this task convinced that many long-standing, seemingly irresolvable quandaries regarding human development originate in ambiguities of the existing discourses on thinking. Standing on the shoulders of Vygotsky and Wittgenstein, the author defines thinking as a form of communication. The disappearance of the time-honored thinking-communicating dichotomy is epitomized by Sfard's term, *commognition*, which combines communication with cognition. The commognitive tenet implies that verbal communication with its distinctive property of recursive self-reference may be the primary source of humans' unique ability to accumulate the complexity of their action from one generation to another. The explanatory power of the commognitive framework and the manner in which it contributes to our understanding of human development is illustrated through commognitive analysis of mathematical discourse accompanied by vignettes from mathematics classrooms.

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To my parents, Janina and Zygmunt Bauman

Thinking as Communicating

Human Development, the Growth of Discourses, and Mathematizing

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Series Foreword

This series for Cambridge University Press is widely known as an international forum for studies of situated learning and cognition.

Innovative contributions are being made by anthropology; by cognitive, developmental, and cultural psychology; by computer science; by education; and by social theory. These contributions are providing the basis for new ways of understanding the social, historical, and contextual nature of learning, thinking, and practice that emerge from human activity. The empirical settings of these research inquiries range from the classroom to the workplace, to the high-technology office, and to learning in the streets and in other communities of practice. The situated nature of learning and remembering through activity is a central fact. It may appear obvious that human minds develop in social situations and extend their sphere of activity and communicative competencies. But cognitive theories of knowledge representation and learning alone have not provided sufficient insight into these relationships.

This series was born of the conviction that new and exciting interdisciplinary syntheses are underway as scholars and practitioners from diverse fields seek to develop theory and empirical investigations adequate for characterizing the complex relations of social and mental life, and for understanding successful learning wherever it occurs. The series invites contributions that advance our understanding of these seminal issues.

Roy Pea
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Lucy A. Suchman

Introduction

> If we see knowing not as having an essence, to be described by scientists or philosophers, but rather as a right, by current standards, to believe, then we are well on the way to seeing *conversation* as the ultimate context within which knowledge is to be understood. Our focus shifts from the relation between human beings and the objects of their inquiry to the relation between alternative standards of justification, and from there to the actual changes in those standards which make up intellectual history.
>
> Richard Rorty[1]

This book is a result of years-long attempts to change my own thinking about thinking, a task seemingly as improbable as breaking a hammer by hitting it with itself. In this unlikely undertaking, I have been inspired by Lev Vygotsky, the Byelorussian psychologist who devoted his life to "characterizing the uniquely human aspects of behavior,"[2] and by Ludwig Wittgenstein, the Austrian-British philosopher who insisted that no substantial progress can be made in this kind of endeavor unless the ways we talk, and thus think, about uniquely human "forms of life" undergo extensive revisions.

My admittedly ambitious undertaking had modest beginnings. I was initially interested in learning and teaching mathematics. Like many others before me, I was mystified by what could best be described as vagaries of the human mind: Whereas some people juggled numbers, polygons, and functions effortlessly, some others were petrified at the very mention of numbers or geometric figures. Many of those who erred in their use of mathematical terms and techniques seemed to err in systematic, surprisingly

[1] Rorty (1979, pp. 389–390).
[2] Vygotsky (1978, p. 1).

similar ways. And then there was the wonder of little children doing strange things with numbers before gradually becoming able to handle them the standard way. Above all, however, one could not but puzzle over why the persistent attempts to improve mathematics learning over many decades, if not centuries, did not seem to have any sustainable effect. After years of grappling with these and similar phenomena, I realized that one cannot crack the puzzles of mathematical thinking without taking a good look at human thinking at large. I ended up wondering with Vygotsky about how the unique human abilities "have been formed in the course of human history" and about "the way they develop over an individual's lifetime."[3]

I soon discovered that whoever forays into this exciting territory dooms herself to an uneasy life. The first predicament of the student of human development is her being torn between two conflicting wishes: the wish to be scientific, whatever this word means to her, and the desire to capture the gist of those phenomena that are unique to humans. Whenever one of these needs is taken care of, the other one appears to be inherently unsatisfiable. Indeed, across history, the tug-of-war between the two goals, that of scientific reproducibility, rigor, and cumulativeness, on the one hand, and that of doing justice to the complexity of the "uniquely human," on the other, resulted in the pendulum-like movement between the reductionist and the "gestaltist" poles. Reductionist theories, of which behaviorism is arguably the most extreme example, can boast the scientific operationality of their vocabulary, but they eventually kill their object by throwing away some of its vital parts. Socioculturally minded followers of Vygotsky, on the other hand, aware of the futility of the search conducted "under the lamp" rather than in those dark places where answers to their questions may really be hiding, fail to communicate their rich ideas clearly enough to give rise to well-defined programs of study.

Today, our sense of helplessness may well be at its most acute. New technologies afford unprecedented insights into human phenomena and produce high-resolution evidence of the utmost complexity of human forms of life. With audio and video recorders as standard ingredients of the researcher's toolkit, the fleeting human action acquires permanence and becomes researchable in ways unknown to our predecessors. When carefully documented and transcribed, even the most common of everyday conversations prove to be a complex, multifaceted phenomenon, and an inexhaustible source of wonderings. This makes us as aware as ever of the fact that our ability to analyze and explain lags behind our ability to observe

[3] Ibid.

and to see. In this respect, our current situation is comparable to that of the 17th-century scientists faced with the newly invented microscope: Powerful, high-resolution lenses that reveal what was never noticed before are yet to be matched by an equally powerful analytic apparatus.

Inadequacies of conceptual tools are what Wittgenstein had in mind when he complained, more than half a century ago, about the state of research on human thinking. "Psychological concepts are just everyday concepts," he said, whereas what we need are "concepts newly fashioned by science for its own purpose."[4] These words seem to have as much force today as they had when they were written. Lacking a designated, operationally defined vocabulary, the study of humans remains plagued by resilient dilemmas. Just look at time-honored controversies about human development that recur time and again, alas in different disguises, throughout history. Take, for example, the famous "nature versus nurture" dilemma, the "mind and body" problem, or the controversy about the "transfer of learning." All these quandaries have an appearance of disagreements about empirical facts but may, in reality, be a matter of lexical ambiguities. The blurriness of the vocabulary is the most obvious explanation for our inability to overcome the differences and build on each other's work: Unknown to ourselves, we are likely to be using the same words – *nature*, *nurture*, *mind*, *transfer* – in different ways. Similarly, our inability to capture the complexity of human phenomena may well be a matter of an inadequacy of our analytic methods, the weakness that, in the absence of explicit, operational definitions, seems incurable.

At a closer look, the lack of operationality is only the beginning of the researcher's problem. Without clear definitions, one is left at the mercy of metaphors, that is, of concepts created by transferring familiar words into unfamiliar territories. Indeed, if we are able to use words such as *nurture* or *transfer* in the context of human learning and development, it is because both these terms are known to us from everyday discourse. The services rendered by metaphors, however, are not without a price: Together with the unwritten guidelines for how to incorporate the old term into new contexts are hordes of unforeseen metaphorical entailments, some of which may interfere with the task of gaining useful insights into the observed phenomena. Whereas the use of metaphor cannot be barred – after all, this is one of the principal mechanisms of discourse building – the risks of metaphorical projections may be considerably reduced by providing the metaphorically engendered notions with operational definitions.

[4] Wittgenstein (1980, § 62).

Being explicit and operational about one's own use of words, however, is not an easy matter. Some people circumvent the challenge by turning to numbers. Precise measurement seems such an obvious antidote to the uncertainties of descriptive narratives! Rather than merely describing what the child does when grappling with mathematical problems, those who speak "numberese" would look at students' solutions, divide them into categories, and check distributions. Rather than scrutinizing the utterances of a girl executing an arithmetic operation, they would measure her IQ, consider her grades, and decide whether the numbers justify labeling her as "learning disabled." Never mind the fact that in the quantitative discourse the numbers may be originating in categorizations as underdefined as those that belong to its "qualitative" counterpart (after all, there is no reason to assume that the words signifying things to be counted, when not defined in operational terms, are more operational than any other). Forget the fact that in their zeal to produce simplicity, order, and unification, the quantitatively minded interlocutors are likely to gloss over potentially significant individual differences. It is only too tempting to believe that numbers can say it all and that when they speak, there is no need to worry about words.

I do worry about words, though, and this book is the result of this concern. In spite of my liking for numbers – after all, I am the native of mathematics – I am acutely aware of the perils of the purely numerical talk. The uneasy option of operationalizing the discourse about uniquely human forms of life seems the only alternative. On the following pages, I take a close look at basic terms such as *thinking*, *learning*, and *communication* and try to define them with the help of clear, publicly accessible criteria. If this operationalizing effort raises some brows – if some readers protest, saying that thinking and communication are natural phenomena and thus not anything that people should bother to define – let me remind them that defining relates to the ways we talk about the world, not the world as such, and it is up to us, not to nature, to decide how to match our words with phenomena. And to readers who feel that I am trying to tell them how to talk, let me explain that this, too, is not the case. All I want is to be understood the way I intend, on my own terms. For me, being explicit about my use of words is simply a matter of "conceptual accountability," of being committed to, and responsible for, the effectiveness of my communication with others.

The conceptualization I am about to propose may be regarded as an almost self-imposing entailment of what was explicitly said by Vygotsky and what was implied by Wittgenstein. The point of departure is Vygotsky's claim that historically established, collectively implemented activities

are developmentally prior to all our uniquely human skills. Being one of these skills, human thinking must also have a collective predecessor. Obviously, interpersonal communication is the only candidate. In this book, therefore, thinking is defined as the *individualized version of interpersonal communication* – as a communicative interaction in which one person plays the roles of all interlocutors. The term *commognition*, a combination of *communication* and *cognition*, stresses that interpersonal communication and individual thinking are two facets of the same phenomenon.

In the nine chapters of this book, the introduction to the commognitive perspective is accompanied by a careful examination of its theoretical consequences and its implications for research and for educational practice. The task is implemented in two steps. Part I (chapters 1 through 4) is devoted to the double project of telling a story of human thinking and creating a language in which this story may usefully be told. After presenting a number of time-honored controversies regarding human learning and problem solving (chapter 1), and after tracing the roots of these quandaries to certain linguistic ambiguities (chapter 2), the commognitive vision is introduced as a possible cure for at least some of the persistent dilemmas and uncertainties (chapter 3). Although it is repeatedly stressed that language is not the only medium in which communication, and thus thinking, can take place, it is now claimed that verbal communication may well be the primary source of the distinctively human forms of life (chapter 4). Indeed, if one were to name a single feature that would set humankind apart from all the others in the eyes of a hypothetical extraterrestrial observer, the most likely choice would be our ability to accumulate complexity of action, that is, the fact that our forms of life, unlike those of other species, evolve and grow in intricacy and sophistication from one generation to another, constantly redefining the nature and range of individual development. It may now be argued that this gradual growth is made possible by the fact that our activities are verbally mediated. More specifically, thanks to the special property of human language known as recursivity, the activity-mediating discourses and the resulting texts become the primary repository of the gradually increasing complexity. Consistent with this vision, research on human development becomes the study of the growth of discourses.

In part II I return to the questions that started me on this project: I use the commognitive lens to make sense of one special type of discourse called *mathematical*. By choosing mathematics I hope to be able to illustrate the power of the commognitive framework with a particular clarity. Mathematical thinking has been psychologists' favorite object of study since the advent of the disciplined inquiry into human cognition. Widely regarded as perhaps

the most striking instantiation of the human capacity for abstraction and complexity, mathematics is also a paragon of rigor and clarity: It is decomposable into relatively neatly delineated, hierarchically organized layers that allow for many different levels of engagement and performance. The tradition of using mathematics as a medium within which to address general questions about human thinking goes back to Jean Piaget[5] and continues with the wide variety of developmental psychologists and misconception seekers, ending up, at least for now, with the sociocultural thinkers who vowed to reclaim the place of the social within the time-honored trinity world–society–individual.[6] Throughout history, students of the human mind were often divided on questions of epistemology, methodology, and the meaning of observed phenomena, but they always agreed that mathematical thinking is a perfect setting for uncovering general truths about human development.[7]

In the four chapters devoted to mathematical thinking, I develop the commognitive vision of mathematics as a type of discourse – as a well-defined form of communication, made distinct by its vocabulary, visual mediators, routines, and the narratives it produces (chapter 5). The questions of the nature and origins of the objects of mathematical discourse are then addressed, and the claim is made that mathematics is an autopoietic system – one that spurs its own development and produces its own objects (chapter 6). I follow with a close glimpse at uniquely mathematical ways of communicating (chapter 7) and at the gains of communicating in these special ways (chapter 8). All along, particular attention is given to the question of how mathematical discourse comes into being and how and why it subsequently evolves. The vision of mathematics as a discourse, and thus as a form of human activity, makes it possible to identify mechanisms that are common to the historical development of mathematics and to its individual learning. Having stated all this, I return to the initial quandaries and ask myself whether the commognitive vision has yielded the wished-for resolution. At the same time, I wonder about a series of new puzzles, some of them already being taken care of and some others still waiting to be transformed into researchable questions (chapter 9).

Throughout the book, theoretical musings are interspersed with numerous empirical instantiations. Although the examples are mostly mathematical, they are rather elementary and easily accessible to anybody who knows

[5] E.g., Piaget (1952).
[6] E.g., Lave (1988) and Walkerdine (1988).
[7] H. J. Reed and J. Lave (1979) make a compelling case for using mathematics as a "laboratory" for studying human thinking in their article with the telltale title "Arithmetic as a tool for investigating the relation between culture and cognition."

a thing or two about basic arithmetic. The mathematical slant, therefore, should not deter nonmathematical readers, not even those who suffer from mathematical anxiety. It is also worth mentioning that the book may be read in different ways, depending on one's needs and foci. Those interested mainly in theorizing about human thinking may satisfy themselves with part I, where references to mathematics are scarce. Those who reach for this book because of their interest in mathematical thinking can head directly to part II. The glossary at the end of the volume will help them, if necessary, with concise explanations of basic terms and tenets.

Acknowledgments

> An argument . . . in which the parties express disagreement is nonetheless co-constructed.
>
> Jacoby and Ochs, 1995[1]

Once we agree that thinking is an individualized form of interpersonal communication, we must also concede that whatever one creates is a product of collective doing. Even when sitting alone at her desk and deeply immersed in thoughts, a person is engaged in a conversation with others. As is any human artifact, therefore, this book is full of "echoes and reverberations" of conversations that occurred at many different times and places, involving numerous people whom I never met, and probably many others of whom I haven't even heard. Being "filled with others' words,"[2] this text has therefore more contributors than I am aware of. When echoing other authors' words or when taking exception with what they said, I have dragged them into this conversation, sometimes intentionally and sometimes unconsciously. If their roles were revealed, not all of these involuntary contributors would probably agree to take any credit for the final product. Nevertheless, I would dearly like to acknowledge them all. Unfortunately, I can express my gratitude only to those few people of whose contribution I am aware, hoping to be forgiven by all the others.

Let me begin with Lev Semionovitch Vygotsky and Ludwig Wittgenstein, two giants whose shoulders proved wide enough to accommodate legions of followers and a wide variety of interpreters. Although libraries have already been filled with exegetic treatises, the Byelorussian psychologist and the Austrian-born philosopher continue to inspire new ideas even

[1] Jacoby and Ochs (1995, p. 171).
[2] Bakhtin (1999, p. 130).

as I am writing these lines. This, it seems, is due to one important feature their writings have in common: rather than provide information, they address the reader as a partner in thinking; rather than presenting a completed edifice with all the scaffolding removed, they extend an invitation for a guided tour of the construction site; rather than present firm convictions, they share the "doubt that comes *after* belief."[3] These two writers had a major impact on my thinking; I can only hope they had a similar effect on my ability to share it.

I am indebted to a number of people who stirred my fascination with the phenomenon called mathematics and thereby with human thinking. The list begins with the elementary school teacher, Mr. Żendara, who said, "Open, sesame" and led our group of 11- and 12-year-olds into the cave full of mathematical wonders; it continues with the high school mathematics teacher, Aniela Ehrenfeught, whose enthusiasm for mathematics was truly contagious; and it culminates with the long list of university professors, whose live shows of awe-inspiring acrobatics on the highest levels of mathematical abstraction ignited their students' interest in the miracle of human cognition.

Ellice Forman kicked the writing project off by declaring me "the person who needs a book" and alerting me to the option of Cambridge University Press (this was Ellice's gentle way of telling me that my texts were too long and too theoretical to be published as journal articles). Another friendly kick, this time by Joan Ferrini-Mundy, jettisoned me out of the box straight into transcontinental commuting. It took me to Michigan State University, where I found all I needed to complete this project: a peaceful environment and a consolidated group of colleagues and students who challenged me with difficult questions. Glenda Lappan, Betty Phillips, and George Leroi joined Joan in making my stay possible, while Nathalie Sinclair, Nick Jackiw, Helen Featherstone, and Jeanne Wald, with their sustained interest, constant encouragement, and stimulating conversation over countless coffees, lunches, and dinners, made this stay no less enjoyable than fruitful. I consider myself lucky to have the friendship of all these special individuals.

Most of the empirical studies featured in these pages were collective endeavors in which I was but one of several researchers. I am grateful to Carolyn Kieran, Liora Linchevsky, Miriam Ben-Yehuda, Anna Prusak, Irit Lavie, and Sharon Avgil for their collaboration and their multifarious contributions. A less direct, but no less important, input was that of graduate students who, on both sides of the Atlantic Ocean, mulled over intricacies of human learning and often had the courage to announce that as far as they

[3] Wittgenstein (1969, p. 23e).

were concerned, the queens and kings of educational research had been suffering from a scarcity of attire. Sincere thanks go also to those who delved into successive versions of different parts of this book to reemerge with tons of high-quality grist for my mill. Among them, let me count Jihad Al–Shwaikh, Shai Caspi, Paul Cobb, Helen Featherstone, Einat Heyd-Metzuyanim, Talli Nachlieli, Jill Newton, Andreas Ryve, Nathalie Sinclair, Michal Tabach, Eran Tavor, and Hava Tuval. The incisive, often quite critical comments of these first readers pushed the project forward, albeit not always in the directions the critics themselves considered desirable.

I was fortunate to benefit from the erudition and connoisseurship of Roy Pea, the editor of the Learning in Doing series and an expert on learning sciences (according to Roy's own definition, an expert is a person who "notice[s] features of situations and problems that escape the attention of novices").[4] Four anonymous reviewers have to be thanked for flagging pitfalls and inspiring additional searches. Eric Schwartz, Helen Wheeler, and Susan Thornton of Cambridge University Press walked me gently through the mazes of publishing processes, and for this joint trip they will always be remembered with fondness. To Jean Beland and Georgia Old from Michigan State University I am grateful for helping me through the labyrinths of manuscript preparation.

I am grateful to the publishers of the *Journal for Research in Mathematics Education* (copyright 2005 by the National Council of Teachers of Mathematics. All Rights Reserved); *For the Learning of Mathematics*; *Mind, Culture, and Activity*; and *Cognition and Instruction* for letting me use fragments of my former writings that they published.

My family is present between the covers of this volume no less than Wittgenstein and Vygotsky, even if not so explicitly. Michael Sfard and Emi Sfard made a paramount contribution by giving me the opportunity to watch two exemplary cases of human development in real time and in a close-up. Their early experience as mathematics learners inspired some of the stories told on these pages. Emi, who grew up to favor images over numbers, made yet another contribution to this volume by preparing its cover illustration. Leon Sfard, my constant interlocutor and my most helpful critic, should probably be regarded as a cocreator of this text. Indeed, even if eavesdroppers can hear only one voice, this book is a record of our ongoing dialogue. I am unspeakably grateful to this dear little crowd of mine for coaxing me into the mood for writing and for giving me the strength and emotional resilience to persevere until the completion of this project – and well beyond.

[4] Bransford et al. (2006, p. 25).