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THINNED ARRAYS USING PATTERN SEARCH ALGORITHMS

A. Razavi and K. Forooraghi

Faculty of Engineering
Department of Electrical Engineering
Tarbiat Modares University (TMU)
Iran

Abstract—In this paper, pattern search (PS) algorithms are introduced as a new tool for array thinning. It is shown that by selection of a fitness function which controls more than one parameter of the array pattern, and also by proper setting of weight factors in fitness function, one can achieve very good results. The method is tested on linear arrays of isotropic and non-isotropic elements and is shown to be useful in both cases. Mainlobe scanning to different angles is also tested and the results are success. In all cases studied in this paper, relative sidelobe level (SLL) is less than −20 dB with only small increase in mainlobe beamwidth compared to the case of a uniform array. Results of PS optimization are compared in two cases, which their starting points of optimization are different from each other. There is also a comparison made between results of array thinning using Genetic Algorithm (GA) and PS, and PS is shown to be a fast and reliable algorithm to be used in array thinning problem.

1. INTRODUCTION

In the problem of array thinning, the number of all possible combinations is large and increases exponentially with number of array elements. Hence checking every possible combination to find the optimum one is nearly impossible. One needs a faster and more reliable method to find the optimum solution. No deterministic method can be found for array thinning, instead there are probabilistic methods which focus on density of "on" elements in different parts of array and its effects on far-field pattern. Also non-gradient based optimization methods such as GA [1–5] and Particle Swarm

Optimization (PSO) [6, 7] are introduced and used in electromagnetics optimization problems as well as array thinning and are proved to be useful. Haupt [3] used GA in process of thinning a linear array of 200 elements, and resulting sidelobe levels were lower than −18 dB in all cases shown. Johnson and Rahmat Samii [4] used GA to thin a 40 elements linear array and achieved sidelobe levels of somewhat lower than −20 dB. Mahanti et al. [8] used GA to thin a large linear array of uniformly excited isotropic elements to yield SLL equal to or below a fixed level, while the percentage of thinning is equal to or above a fixed value. Also efforts are made to use pattern search algorithm in optimization of phased array antennas [9] and PS is proved to be useful in sidelobe level reduction of phased arrays. Clonal Selection [10, 11] and Bees Algorithm [12] are two other optimization tools which are used recently in different problems of array synthesis field with successful results. In this paper the pattern search algorithm has been used as a new method for array thinning problem.

Pattern search algorithm [13] is a method of optimization which does not need any information about the gradient of the objective function. It searches a set of points around the current point of parameters to find a point where the objective function has a lower value (in case of minimization). The search points are located on a mesh with specified pattern (hence the name Pattern Search) and with constant size. After a point with lower objective function value is found, the algorithm sets the point as its current point and again searches its neighboring points to find a new optimized point in next iteration. Neighboring points form a mesh around the current point where its size can be increased or decreased during the algorithm run time to respectively increase speed or accuracy. Also when there is no point on the mesh where value of objective function is lower than the current point, the algorithm will reduce the mesh size (for example by a factor of 2) to search around the current point with more resolution. This helps the algorithm to find the minimum more precisely. However there is a major difference between general case of PS and utilization of PS for array thinning. The difference is that in the latter, the parameters form a bit string, so the mesh size is always constant and summations are all in modulo-2.

The objective of this paper is to find a configuration for a thinned array which has a normalized SLL as low as possible and with a half-power beamwidth (HPBW) not so much more than the case of the uniform array which has the narrowest possible HPBW except for the case of super directivity [14]. This goal is achieved by using PS as the optimization tool. The results are excellent for arrays of 50 elements which are very difficult and time consuming if not impossible to thin with other statistical methods. The resulting arrays have low SLL as much as $-20\,\text{dB}$ and lower and HPBW increase no more than 22 percents.

2. FITNESS FUNCTION

Far field pattern of a thinned linear array positioned along z-axis is described by [14]

$$AP(\theta) = F(\theta) \sum_{n=1}^{N} a_n e^{j(n-1)(kd\cos(\theta)+\beta)} \tag{1}$$

where
N = number of array elements
a_n = amplitude of element $n = \begin{cases} 1 & \text{on} \\ 0 & \text{off} \end{cases}$
d = Element spacing
$F(\theta)$ = element pattern
β = progressive phase shift

In order to control array pattern as desired, different parameters of the far field pattern must be considered in the fitness function. The first and most important parameter is the normalized SLL that is desired to be as low as possible. Also one of main purposes of array thinning is to reduce power consumption. So there is a need to control beamwidth of sidelobes, in order to reduce power radiated in undesired directions. This makes sense of introducing the sum of pattern values at every angle as another parameter in fitness function. It is evident that this parameter decreases as the number of sidelobes and hence the number of pattern nulls increase. The last parameter is the half power beamwidth of the main beam. It is also desired to be low. In order to get a single fitness function, the three parameters are combined.

One can multiply all the parameters to form the fitness function. But the drawback of this solution is that each parameter is given a weight which is equal to multiple of the other two. Instead the fitness function is chosen as

$$FF = w_1 \times sll + w_2 \times sum + w_3 \times hpbw \tag{2}$$

where
sll = normalized sidelobe level
sum = sum of far-field pattern values at every angle
$hpbw$ =half power beamwidth
w_1, w_2, w_3 = weight of corresponding parameters

It is evident that by minimizing this fitness function all of the three parameters are minimized.

3. OPTIMALLY THINNED ARRAYS USING PS

All array discussed in this section, are assumed to be excited symmetrically. This assumption divides the number of independent parameters by two and hence make run time of algorithm shorter, but at the same time makes the domain of all possible array excitations much smaller. The optimizations in this paper are done in two categories of isotropic and non-isotropic elements, as seen below.

3.1. Isotropic Elements

The first optimization is done for a 50 elements linear array of isotropic elements. The weights in the fitness function are chosen as $w_1 = 10$, $w_2 = 1$, $w_3 = 2$ and the main beam is desired to be at broadside. In this case and all other optimizations in this paper, element spacing is chosen as $d = \lambda/4$. With these assumptions and after 25 iterations, the algorithm results in an array of 40 turned on elements (80% filled) which has a SLL of -21.25 dB. The price paid for this improvement is an increase of about 12.5% in HPBW comparing to a uniform array. The far-field pattern of this array is shown in Fig. 1.

In the next step the object is to scan main lobe to $\theta = 60°$, to see whether PS can be used for main beam scanning while thinning the array or not. This optimization is done on the same array as previous case and weights in fitness function are also chosen to have the same value as before. Progressive phase shift is applied on elements in order to scan main beam to desired angle. After 18 iterations, these assumptions result in a 76% filled array with SLL of -20.53 dB which is an acceptable result achieved in a very short time. The far-field pattern of this array is drawn in Fig. 2. This array has a 22% wider main beam than a uniform array of same element spacing and scan angle.

3.2. Non-isotropic Elements

The next optimization is done on a linear array with main beam at broadside and element spacing of $d = \lambda/4$. Pattern of array elements is chosen to be $F(\theta) = \sin(\theta)$. The weights in fitness function are chosen as before. Optimization with PS results in an 80% filled array and SLL of -20.92 dB. Fig. 3 shows the far-field pattern of this array. It is evident that PS is suitable for thinning arrays of non-isotropic

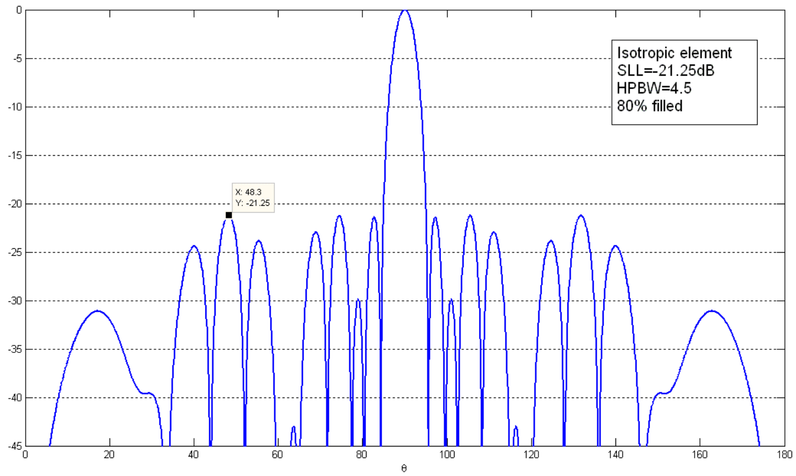

Figure 1. Far field pattern of an optimized 50 elements thinned array with isotropic elements and main beam at broadside. SLL $= -21.25$ dB, HPBW $= 4.5$ degrees, 80% filled.

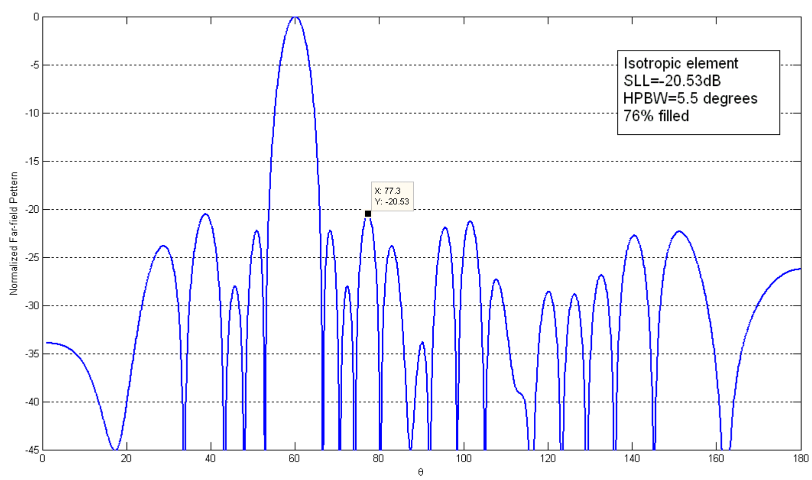

Figure 2. Far field pattern of an optimized 50 elements thinned array with isotropic elements and main beam at 60 degrees. SLL $=$ -20.53 dB, HPBW $= 5.5$ degrees, 76% filled.

elements as much as isotropic elements. Main beam of this array is 12.5% wider than a uniform array in terms of HPBW.

In the next step, mainlobe scanning of previous array to different angles than broadside is considered. This is done to investigate the

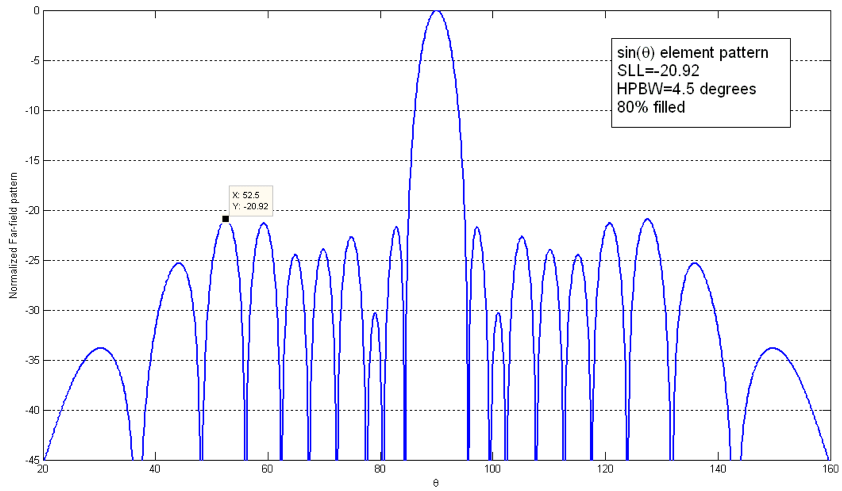

Figure 3. Far field pattern of an optimized 50 elements thinned array with $F(\theta) = \sin(\theta)$ element pattern and main beam at broadside. SLL $= -20.92$ dB, HPBW $= 4.5$ degrees, 80% filled.

effectiveness of PS in a case where element pattern has a maximum in a direction other than desired mainlobe. This optimization is done on the array of previous case but with main beam scanned to $\theta = 60°$. The SLL is expected to be higher than previous case since the element pattern has undesired influence on the overall pattern. The result of optimization with PS is an 80% filled array with SLL of -20.13 dB which is higher than previous case as expected. Far-field pattern of this array is shown in Fig. 4. The HPBW is 22% greater than a uniformly excited array.

4. PS VERSUS GA

A comparison between results of array thinning using GA and PS is done in this section. Three sets of results are shown and compared together. All three are linear arrays of 200 isotropic antennas with element spacing of $d = \lambda/4$ and main beam at broadside. First is the result of running GA to thin the above mentioned array, for 50 generations. It is a 56.5% filled array with a SLL of -21.52 dB and beamwidth of 1.2 degrees. The far-field of array pattern and configuration of elements are shown in Fig. 5. In this figure dark lines represent "on" elements while the light lines represent "off" elements in the array.

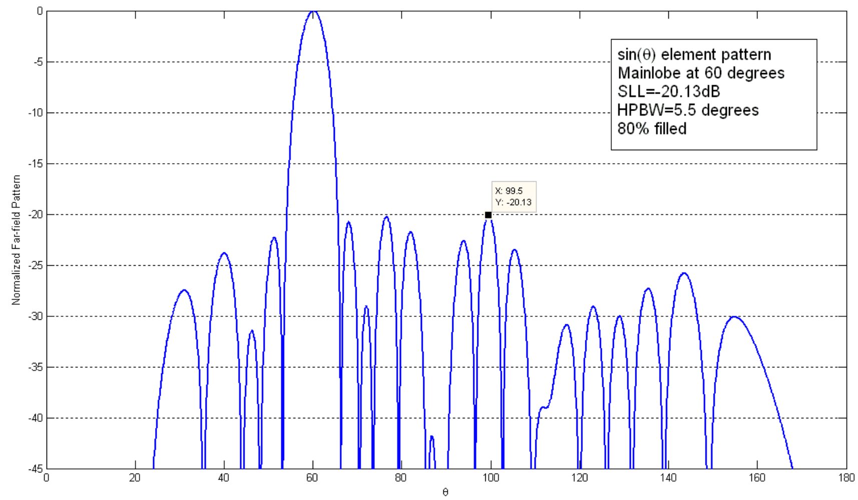

Figure 4. Far field pattern of an optimized 50 elements thinned array with $F(\theta) = \sin(\theta)$ element pattern and main beam 60 degrees. SLL $= -20.13$ dB, HPBW $= 5.5$ degrees, 80% filled.

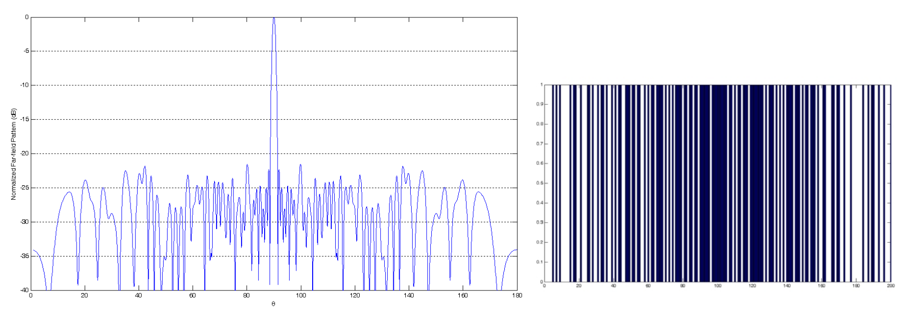

Figure 5. Far-field pattern (left) and element configuration (right) of result no. 1. "on" elements are shown with dark lines.

The second result shown discussed here is achieved using PS. In this case, initial point for PS is chosen as the state of an array with all elements turned off. After 52 iterations, the algorithm reaches at an array of 24.5% 'on' element which has sidelobe level of -14.91 dB and its beamwidth of mainlobe is 1.06 degrees. The far-field pattern of this array and its configuration of elements is shown in Fig. 6.

The last result compared in this section is again achieved using PS to thin the array. But instead of an all off state as the initial point, the initial point is selected as follows. First a hundred random states are generated. Then fitness function is calculated for these states and states are sorted according to their fitness value. In the next step the best of the random states is chosen as starting point for the PS algorithm. In this case optimization is stopped after 52 iterations to compare the results with results of the second case. Final result is a 52% filled array that has SLL as low as -22.27 dB and mainlobe beamwidth of 1.16 degrees. Fig. 7 shows the far-field pattern of this array along with configuration of its elements.

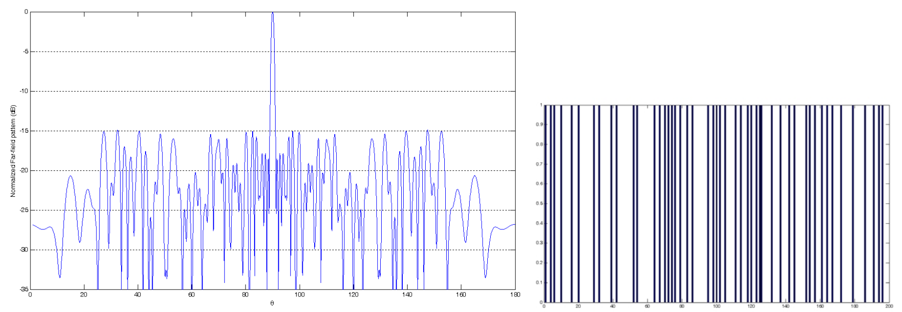

Figure 6. Far-field pattern (left) and element configuration (right) of result no. 2. "on" elements are shown with dark lines.

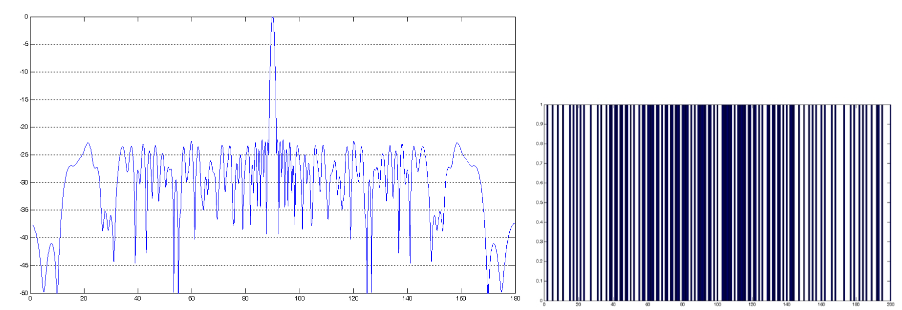

Figure 7. Far-field pattern (left) and element configuration (right) of result no. 3. "on" elements are shown with dark lines.

By comparing these results one can readily find that while using PS for optimization, choice of starting point is very important. PS's main way of escaping local minima is based on the fact that mesh size of algorithm is adjustable. And the fact that the algorithm usually starts with a large mesh size and the size is reduced during algorithm run, to increase the precision of result. PS seems to have no effective way of escaping local minima in array thinning problem. This may be caused by the fact that array thinning problem has binary nature and in this case mesh size of PS must always be constant. That is the reason, why choice of starting point is important. By choosing the starting point in proposed way, we help PS not to stick in local minima. However because of the nature of array thinning problem, there are so many local minima in fitness functions of large arrays and no method can surely find the global minimum of the fitness function, except for searching the whole domain of all possible configurations. But by proper selection of starting point, the probability of reaching an acceptable result is significantly raised. On the other side binary coding is very well suited for GA. It is also notable that PS makes minimum changes in array configuration of starting point, to reach an optimum point. This is best seen when the starting point is set to all zeros, and the resulting optimized array has much lower number of "on" elements than the other cases. Although the resulting SLL in this case is not as low as in other cases. At last it is evident that by choosing a proper configuration as the starting point of PS, resulting SLL can be lower than that of an array, thinned using GA. While the number of "on" elements in this case is also lower than GA result. Most interestingly PS takes much shorter time to converge than GA. For example while trying to thin an array of 200 isotropic elements with main lobe at broadside, and while running Matlab 7.1 on an intel core duo 2.0 GHz CPU, with 1024 MB of RAM, time required for each algorithm to reach at a point with −20 dB or lower SLL is measured. Measured time is 205.2 s for GA and 148.8 s for PS. This difference would be larger while thinning larger arrays.

5. CONCLUSION

In this paper PS is used as a method of thinning large arrays of antennas. The results are very good and have SLL of −20 dB and lower in all cases. The method is tested on arrays of both isotropic and non-isotropic elements and is shown to be useful. The price paid for achieving low sidelobes and lower number of "on" elements, is the increase in HPBW which can be controlled by introduced fitness function. By choosing different weights in this fitness function one may achieve different patterns. For example with narrower HPBW and more relaxed SLL. This can be done easily and no other modification is needed. PS provides a very fast method to reach acceptable results on large arrays which are impractical to thin using other statistical methods or by searching the whole domain.

Also a comparison is made between results of array thinning, using GA and PS. It is shown that by choosing a proper starting point for

PS, better results can be achieved than GA. All these results suggest that PS can be considered as a notable new tool for the problem of thinning large arrays of antennas.

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