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Research Article

Thinning of Concentric Circular Antenna Arrays Using Improved Binary Invasive Weed Optimization Algorithm

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This study presents a novel optimization algorithm based on invasive weed optimization (IWO) for reduction of the maximum side lobe level (SLL) with specific half power beam width (HPBW) of thinned large multiple concentric circular arrays of uniformly excited isotropic elements. IWO is a powerful optimization technique for many continuous problems. But, for discrete problems, it does not work well. In this paper, the authors propose an improved binary IWO (IBIWO) for pattern synthesis of thinned circular array. The thinning percentage of the array is kept equal to or more than 50% and the HPBW is attempted to be equal to or less than that of a fully populated, uniformly excited, and half wavelength spaced concentric circular array of the same number of elements and rings. Simulation results are compared with previous published results of DE, MPSO, and BBO to verify the effectiveness of the proposed method for concentric circular arrays.

1. Introduction

Circular antenna arrays have considerable interest in a variety of applications including radar, sonar, and mobile and commercial satellite communication systems [1]. It consists of a number of elements which are usually omnidirectional and arranged on a circle and can be employed for beamforming in the azimuth plane, for example, at the base stations of the mobile radio communications systems [2–4]. As very popular type of antenna array is the circular array that has several advantages over other array geometries such as all-azimuth scan capability (it can perform 360-degree scan around its center) and the beam pattern can be kept invariant [5]. Moreover, circular arrays are less sensitive to mutual coupling as compared with linear and rectangular arrays since these do not have edge elements [1]. Concentric circular antenna array (CCAA) that contains many concentric circular rings of different radii and number of elements have several advantages including flexibility in array pattern synthesis and design both in narrowband and broadband beamforming applications [2–4]. CCAA is also used in direction-of-arrival applications since it provides almost invariant azimuth angle converge.

Uniform concentric circular antenna array (UCCA) is one of the most important types of the CCA where the interelement spacing in individual ring is kept almost half of the wavelength and all the elements in the array are uniformly excited. Uniform antenna arrays exhibit high directivity; however, they usually suffer from high side lobe level [1]. To reduce the SLL, the array is made aperiodic by altering the positions of the antenna elements with uniform amplitude excitations. The other method to reduce SLL is to use an equally spaced array with radically tapered amplitude distribution [3, 4]. However, uniform excitation is desired to reduce the complexity in designing a feed network.

The process is known as array thinning and is widely employed to reduce the SLL of antenna array with large number of elements. Array thinning is related to the removal of some elements from a uniformly spaced or periodic array to achieve a desired radiation pattern. Thinning not only reduces SLL but also brings down the cost, weight, and power consumption by decreasing the number of radiating elements. However, synthesis of antenna array is a tough challenge, and it cannot be solved by analytical methods. But there are various global optimization algorithms for thinning such as genetic algorithms (GA) [6], particle swarm optimization [7], differential evolution (DE) [8], and biogeography-based optimization (BBO) [9]. Haupt has used GA for thinning of linear arrays and a center element fed concentric ring array antenna to reduce the SLL [10, 11]. Orthogonal GA has been proposed by Zhang et al. for pattern synthesis of thinned linear array [12]. Ghosh and Das utilized DE with global and local neighborhood (DEGL) for thinning planar circular array [13]. Thinning of a planar concentric circular array to SLL reduction using modified PSO (MPSO) algorithm is proposed by Pathak et al. [14]. Chatterjee et al. have compared the performance of MPSO and gravitational search algorithm for thinning of a scanned concentric ring array [15]. BBO has been utilized by Singh and Kamal for thinning large multiple concentric circular ring arrays [16]. Wang et al. proposed a chaotic binary PSO (CBPSO) algorithm to synthesis of thinned linear and planar antenna array [17]. Basu and Mahanti introduced the fire fly algorithm (FFA) to thinning of concentric two-ring circular array antenna [18]. Chatterjee et al. have compared the performance of binary FFA and binary PSO to thin planar concentric ring antenna array to minimize SLL in a number of predefined φ-planes [19].

In this paper, we present the method of optimization of uniformly spaced concentric circular array using an improved binary variant of recently proposed metaheuristic algorithm called invasive weed optimization. The IWO algorithm has found successful application in many electromagnetic problems like design of printed Yagi antenna [20], E-shaped MIMO antenna [21], multifeed reflector antennas [22], broadband patch antenna [23], conformal phased arrays [24], circular antenna arrays [25], and so forth. Results obtained using IWO for these problems are encouraging. However, the nature of reproduction operators in classical IWO limits its application, such as discrete optimization problems. In this paper, an improved binary version of IWO algorithm (IBIWO) has been proposed for large multiple concentric ring arrays of isotropic elements for SLL reduction and at the same time keeping the half power beam width (HPBW). The thinning percentage of the array is kept to more than 50% and the HPBW is kept equal to or less than that of the array with the same number of elements and rings. The same problem has been dealt by DEGL, MPSO, and BBO, respectively. To the best of our knowledge, IWO has not been utilized for the thinning of concentric circular array before. Simulation results obtained are compared with the above algorithm.

The rest of the paper is organized in follows. A formulation of the thinned CCAA pattern synthesis as an optimization task has been discussed in Section 2. Section 3 gives a comprehensive overview of the proposed IBIWO algorithm. Section 4 presents the simulation results and in Section 5 conclusions are presented.

2. Thinned Planar Circular Array

Thinning an array means turning off some radiating elements in a uniformly spaced or periodic array in order to generate a pattern with low side lobe levels. Typical applications for thinned array include satellite-receiving antennas that operate against a jamming environment, ground-based high frequency radars, and design of interferometer array of radio astronomy. In this work, we assumed that the positions of elements are fixed and the elements can have only two states: either "on" or "off." An antenna in an "on" state only contributes to the total array pattern. On the other hand, an antenna is in "off" state if the element is either passively terminated to a matched load or is open circuited and hence it does not contribute to the total array pattern. Thinning an array to produce low side lobes is much simpler than the more general problem of nonuniform spacing of the elements. Nonuniform spacing has an infinite number of possibilities for placement of the elements.

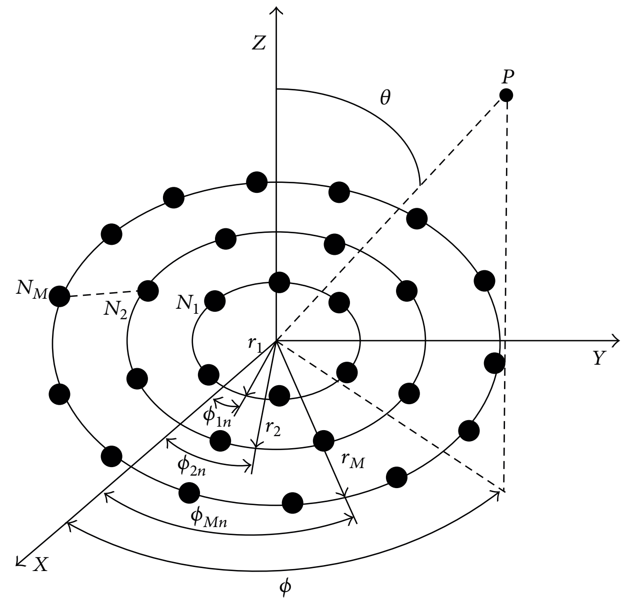


Figure 1: Multiple concentric circular rings array of isotropic antennas.

In CCAA, the elements are arranged in such a manner that all antenna elements are positioned in multiple concentric circular rings, which vary in radii and in number of elements. Figure 1 shows the general configuration of CCAA with M concentric circular rings, where the mth ($m = 1, 2, \ldots, M$) ring has a radius r_m and the corresponding number of elements is N_m. Assuming that all the elements are isotropic sources, the far-field pattern of this array can be written as

$$E(\theta, \phi) = \sum_{m=1}^{M} \sum_{n=1}^{N_m} A_{mn} \exp\left[j\left(k r_m \sin(\theta)\left(\cos\left(\phi - \phi_{mn}\right)\right)\right)\right]. \tag{1}$$

Normalized absolute power pattern $P(\theta, \phi)$ in dB can be expressed as follows:

$$\begin{aligned} P(\theta, \phi) &= 10 \log_{10} \left[\frac{|E(\theta, \phi)|}{|E(\theta, \phi)|_{\max}} \right]^2 \\ &= 20 \log_{10} \left[\frac{|E(\theta, \phi)|}{|E(\theta, \phi)|_{\max}} \right], \end{aligned} \tag{2}$$

where $k = 2 * \pi/\lambda$, r_m is the radius of the mth circle $= N_m d_m/2\pi$, d_m is the interelement arc spacing of the mth circle, $\phi_{mn} = 2n\pi/N_m$ is the angular position of nth element of the mth ring, and θ and ϕ are the zenith and azimuth angle, respectively. All the elements have the same excitation phase of zero degrees.

3. Invasive Weed Optimization Algorithms

3.1. Continuous IWO. Invasive weed optimization (IWO) is a metaheuristic algorithm that mimics the colonizing behavior of weeds. The IWO algorithm may be summarized as four steps and more details can be found in [26].

(I) Initialization: the solutions are produced in the given n-dimensional search space randomly.

(II) Reproduction: in this step the parent weed produces seeds depending on its own fitness, as well as the colony's lowest and highest fitness.

(III) Spatial distribution: the generated seeds are randomly scattered over the d-dimensional search space by perturbing them with normally distributed random numbers with zero mean and a variable variance. The standard deviation for a particular iteration can be given as in

$$\delta_{\text{cur}} = \frac{(\text{iter}_{\max} - \text{iter})^n}{\text{iter}_{\max}^n}(\delta_{\text{initial}} - \delta_{\text{final}}) + \delta_{\text{final}}. \quad (3)$$

The position of the new seed can be given as in (7):

$$x_{\text{son}} = x_{\text{parent}} + sd = x_{\text{parent}} + \text{rand}\ n(0,1) * \delta_{\text{cur}}, \quad (4)$$

where iter and $\text{iter}_{\max}$ are the current iteration and the maximum iteration, δ_{initial} and δ_{final} are the initial and final standard deviation, δ_{cur} is the standard deviation of the current iteration, and n is the nonlinear index.

(IV) Competitive exclusion: the new seeds grow to flowering weeds and are placed together with parent weeds in the colony. A competition is adopted for limiting maximum number of plants in a colony. Weeds with the worst fitness are eliminated until the maximum number of weeds $P_{\max}$ in the colony is reached. The steps 1 to 4 are repeated until the maximum number of iterations has been reached.

3.2. Binary IWO. The presentation used in IWO is a real-valued vector. We make some improvements for classical IWO to deal with discrete problems. Firstly, appropriate coding method is utilized to represent the actual problem. In general, the weed in BIWO will be replaced by the binary coding sequence; each takes value of 1 or 0. Secondly, binary space spread should be employed. In IWO algorithm, the generated seeds are randomly scattered over the d-dimensional search space by perturbing them with normally distributed random numbers with zero mean and a variable standard deviation. But this continuous spread method will not make sense in the binary coding sequence. We take a mutation mechanism to create seed in the BIWO. Each bit in the parent weed will be mutated with a probability which is calculated by the following sigmoid function:

$$P_i^k = \frac{1}{1 + e^{-D_i^k + 6}} + \frac{1}{1 + e^{D_i^k + 6}}, \quad (5)$$

where D_i^k is the spread distance generated by the normally distributed function $N(0, \sigma_{\text{cur}})$. When the mutation probability of each bit in a weed is calculated, then a uniformly distributed random numbers in the range $[0, 1]$ will be generated to specify the mutation bits through following equations:

$$\overline{D_i^k} = \begin{cases} 1, & \text{if } (\rho < f(D_i^k)), \\ 0, & \text{else}, \end{cases}$$
$$x_i^{k+1} = \text{mod}\left(x_i^k + \overline{D_i^k}, 2\right). \quad (6)$$

3.3. Improved Binary IWO. As mentioned in Section 3.1, we can know that δ_{cur} defines the exploration ability and exploitation ability of algorithm and acts as both diversification and intensification components of BIWO; it has a great effect on final solutions. A good diversification will make the final solution near to global optimum. The algorithm will use this component to identify most the potential spaces where the global optimum may lie in. A good intensification will help the algorithm to exploit the potential areas to find the global optimum. It will increase the convergence speed of the algorithm and search the better final solution. Hence, it is very important to keep an efficient balance between diversification and intensification of the algorithm. But BIWO uses a fixed δ_{cur} to produce seeds related to each weed and suffers from the lack of fine balance between exploration and exploitation.

In order to overcome the drawbacks of BIWO, adaptive dispersion mechanism is integrated in the BIWO algorithm The adaptive dispersion mechanism is that the δ_{cur} of the current generation distribute linearly among the weeds as weed with the highest fitness achieves the lowest δ_{cur} and the lowest fitness achieves the highest δ_{cur}, which can be represented by (7). j is the index of weeds in the colony which are sorted according to their fitness, σ_{cur} can be calculated by (3), P_{sum} is the sum number of weeds in the current generation, and σ_j is the δ_{cur} of jth weeds to produce seeds. Hence, the plant with lower fitness will have the chance to produce good seeds in current generation. In addition, this concept will increase the diversification of algorithm so the algorithm will explore the search space effectively:

$$\sigma_j = (\sigma_{\text{cur}}) \times \left(\frac{j}{P_{\text{sum}}}\right). \quad (7)$$

The pseudocode of the novel binary IBIWO is given as follows:

(1) Randomly produce weeds P_0 in binary coding sequence;

(2) For each iter < $\text{iter}_{\max}$ do

(3) Calculate each weed's fitness value by (8);

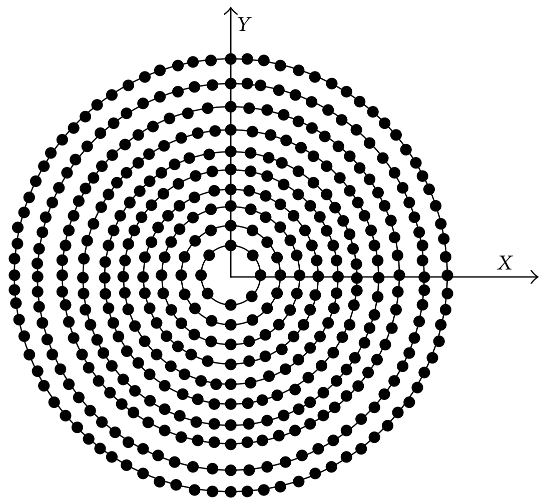


Figure 2: Ten-ring concentric circular ring array of isotropic antennas.

(4) Sort all fitness values;

(5) Record maximum and minimum value;

(6) Calculate the number of seeds;

(7) Calculate each seed's standard variance by (3) and (7);

(8) Produce seeds by (6);

(9) Add the seed to population;

(10) If (Num = $|X|$) > $P_{\max}$ then;

(11) Sort the population X of their fitness;

(12) Save population of weed with best fitness until Num = $P_{\max}$;

(13) End if

(14) End for

4. Design Examples

In this work, an array of ten-ring concentric circular is considered. Each circular has $8m$ isotropic elements spaced uniformly, where m represents the circular number counted from the innermost circular. The total number of element is 440; such a fully populated array is shown in Figure 2. Two instances are considered similar to that reported in [13, 14, 16]. The objective function is given as

$$\begin{aligned}\text{Fitness} &= \text{SLL}_{\max} + \left(\text{HPBW}_o - \text{HPBW}_d\right)^2 \\ &\quad + \left(T_o^{\text{off}} - T_d^{\text{off}}\right)^2 H(T),\end{aligned} \tag{8}$$

where $\text{SLL}_{\max}$ is the value of maximum side lobe level, HPBW_o and HPBW_d are the obtained and desired half power beam width, respectively. We assume that the values of T_o^{off}, T_d^{off} are obtained and desired value of number of switched off elements. $H(T)$ is the Heaviside step functions defined as follows:

$$\begin{aligned}H(T) &= \begin{cases}1, & \text{if } T \le 0,\\ 0, & \text{if } T > 0,\end{cases} \\ T &= \left(T_o^{\text{off}} - T_d^{\text{off}}\right).\end{aligned} \tag{9}$$

Table 1: Parameters of IBIWO and BIWO.

Parameter	Value
Initial size of plants	20
δ_{initial}	15
δ_{final}	3
n	3
$\text{iter}_{\max}$	150
$P_{\max}$	50

In this section, we use two thinned array cases to evaluate the capability and versatility of the proposed algorithm. In all the examples, the array with all elements "on" is used as initial solutions. All simulations are conducted in a Windows 7 Professional OS environment using 12-core processors with Intel Xeon (R), 3.33 GHz, 72 GB RAM and the codes are implemented in Matlab 7.10.

Both cases are optimized by IBIWO and BIWO. The stopping criterion for IBIWO and BIWO is the maximum number of iterations. Because of the randomness nature, all the experiments have been run 25 times with 150 iterations independently. The parameters for IBIWO and BIWO taken are as follows (Table 1).

Case One. In this case, the interelement arc spacing in the entire circular is fixed at 0.5λ. The objective is now to find the optimal set of on and off elements that will generate a pencil beam in the XZ plane keeping the HPBW unchanged, fixing the number of switched-off elements equal to 220 or more, and reducing the maximum SLL further.

Figure 3 shows the best pattern obtained by the IBIWO, and the results are compared with the results of fully populated (all the elements are turned on) uniform array. From Figure 3, we notice that the $\text{SLL}_{\max}$ with the full populated array is −17.37 dB and the $\text{SLL}_{\max}$ lower to −30.18 dB after thinning by IBIWO. The HBPW of the best pattern achieved by IBIWO is 4.67, which is nearly equal to that of fully populated array.

To further verify the performance of the IBIWO, the results of IBIWO are compared with the results of DEGL, MPSO, BBO, and BIWO thinned arrays which are given in Table 2. The results used for comparison are given by the references [13, 14, 16]; the maximum number of iterations of these algorithms is equal to 150. The maximum SLL achieved by IBIWO is −30.18 dB and BW is 4.6° while 231 elements are switched off. The maximum SLL obtained by MPSO, DEGL, BBO, and BIWO are −23.22 dB, −21.91 dB, −26.55 dB, and −28.59 dB, respectively. Hence, the maximum SLL achieved by IBIWO is lower by 6.96 dB, 8.27 dB, 3.63 dB, and 1.59 dB than the maximum SLL obtained by MPSO, DEGL, BBO,

Table 2: Results obtained by different algorithms with fixed $d_m = 0.5\lambda$.

Array	SLL (dB)	HPBW (deg)	d_m	Number of switched-off elements
Fully populated	−17.37	4.5	0.5λ	0
MPSO [14]	−23.22	4.6	0.5λ	231
DEGL [13]	−21.91	4.6	0.5λ	220
BBO [16]	−26.55	4.6	0.5λ	224
BIWO	−28.59	4.67	0.5λ	227
IBIWO	−30.18	4.67	0.5λ	231

Table 3: Excitation amplitude distribution using IBIWO with fixed $d_m = 0.5\lambda$.

$m \setminus n$	Case one
1	11001101
2	1111110101010010
3	101010111111101100011011
4	00111010111110010110010100000111
5	0000010110000000000000100100011100010101 0
6	111000100110111111110001000101100011111101101110
7	01110001001001101111111000000111001111000011111110001000
8	00010000100101010101111011000000001001110010010010001101111101001
9	00011000000001001101010100000000000000001011010010110101111110110110001100110
10	0011000111100111001110001000010110000001100010000001001101001111101011010000010100

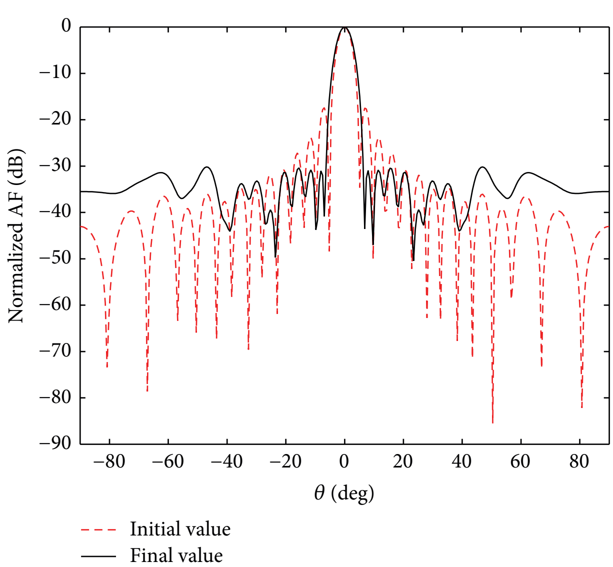


Figure 3: The pattern of the best thinned array obtained by IBIWO algorithm and fully population array (fixed $d_m = 0.5\lambda$).

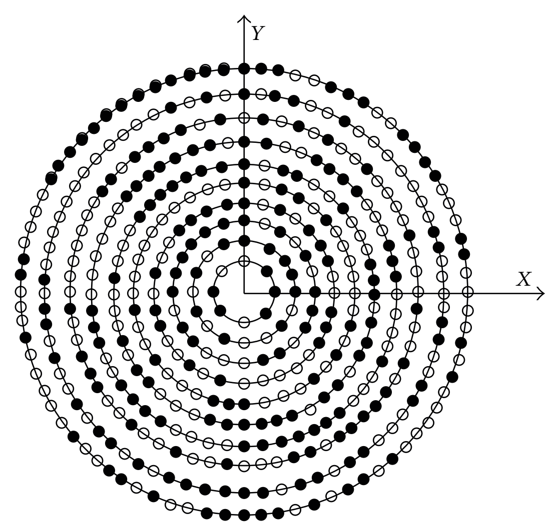


Figure 4: Thinned array obtained by IBIWO (fixed $d_m = 0.5\lambda$).

and BIWO thinned arrays, respectively. Evidently, the IBIWO provides better SLL. The number of switched-off elements achieved by IBIWO is more than that obtained by DEGL, BBO, and BIWO, expect MPSO. The thinned antenna array obtained by IBIWO is shown in Figure 4.

The comparison radiation patterns of MPSO, BBO, BIWO, and IBIWO are also drawn in Figure 5. The optimal amplitude excitations obtained by IBIWO are shown in Table 3.

The convergence characteristics of IBIWO and BIWO are shown in Figure 6. From Figure 6 we can know that the convergence speed of the proposed algorithm is faster than BIWO.

Case Two. In this case, the interelement arc spacing is made uniform and the same but not fixed at 0.5λ. The objective is now to find the optimal set of on and off elements that will generate a pencil beam in the XZ plane keeping the HPBW unchanged, fixing the number of switched-off elements equal to 220 or more, and reducing the maximum SLL further. The interelement arc spacing is allowed to vary between $[0.5\lambda, \lambda]$.

Table 4: Results obtained by different algorithms with optimized d_m.

Array	SLL (dB)	HPBW (deg)	d_m	Number of switched-off elements
Fully populated	−17.37	4.5	0.5λ	0
MPSO [14]	−23.85	4.0	0.6266λ	227
DEGL [13]	−24.81	4.5	0.5285λ	220
BBO [16]	−26.6	4.4	0.5296λ	230
BIWO	−28.75	4.5	0.5445λ	232
IBIWO	−31.17	4.5	0.53λ	238

Table 5: Excitation amplitude distribution using IBIWO with optimized d_m.

$m \setminus n$	Case two
1	11011000
2	1111100011111010
3	100111110111011011011000
4	10010000111011110110000101001010
5	0101110101101101100110000110001001010000
6	000010101101001111110110000010011111101110100011
7	10100001101011111101010110011000010010010010101001001010
8	0100000110111101110000111100000110010011010000100110011010001000
9	100000001101000110101110100101000001100001000000000101101100010010100000
10	001001110011010011101010001100001000100010000001001110001111001011111100110010001

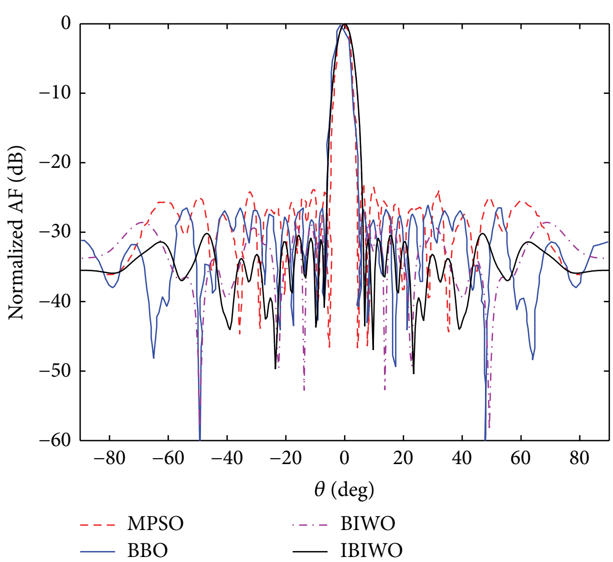


Figure 5: Radiation pattern for ten-ring antenna array obtained using IBIWO results as compared with the results achieved by MPSO, BBO, and BIWO (fixed $d_m = 0.5\lambda$).

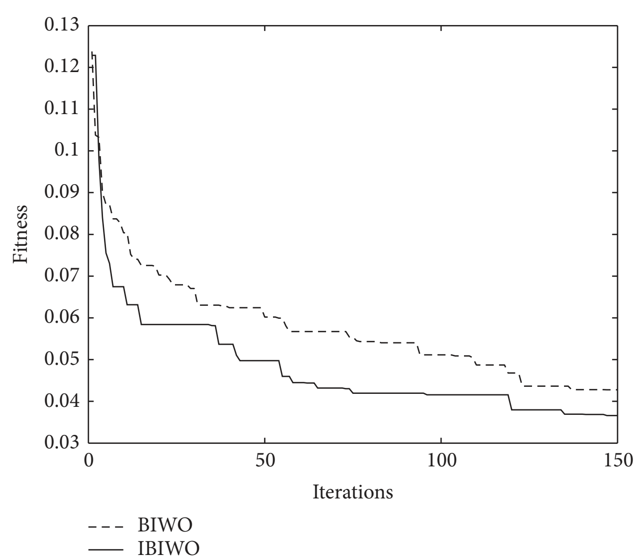


Figure 6: Comparison of convergence characteristics between BIWO and IBIWO.

The parameters for BIWO and IBIWO are also the same as in the previous example. The best result obtained by IBIWO is listed in Table 4.

Figure 7 shows the best pattern with optimized d_m obtained by the IBIWO, and the results are compared with the results of fully populated (all the elements are turned on) uniform array.

To further verify the performance of the IBIWO, the results of IBIWO are compared with the results of fully populated uniform array and DEGL, MPSO, BBO, and BIWO thinned arrays which are given in Table 4. The results used for comparison are given by the references; the maximum number of iterations of these algorithms is equal to 150. In this case, the maximum SLL achieved by IBIWO is −31.17 dB and BW is 4.5° while 238 elements are switched off. The maximum SLL obtained by uniform array, DEGL, MPSO, BBO, and BIWO are −17.37 dB, −24.81 dB, −23.85 dB, −26.6 dB, and −28.75 dB, respectively. Hence, the maximum SLL achieved by IBIWO is lower by 13.8 dB, 6.36 dB, 7.32 dB, 4.57 dB, and

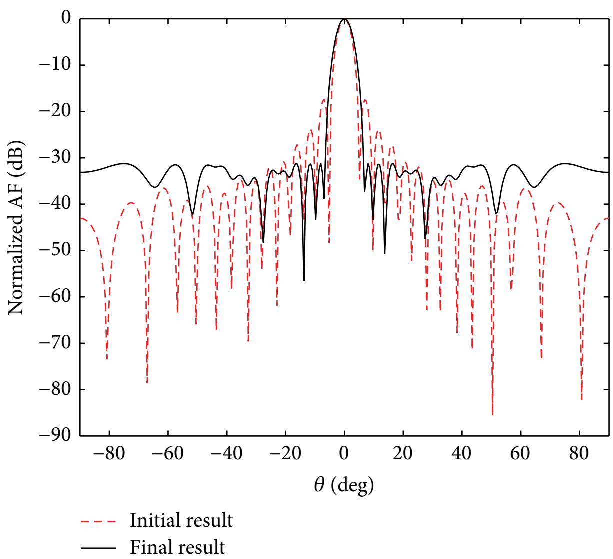


FIGURE 7: The pattern of the best thinned array obtained by IBIWO algorithm and fully population array (optimized d_m).

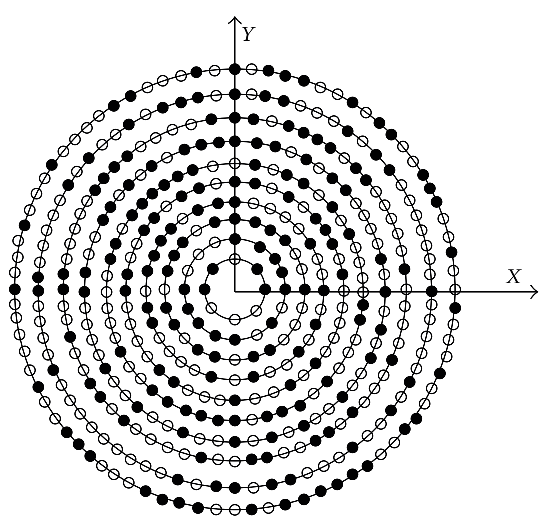


FIGURE 8: Thinned array obtained by IBIWO (optimized d_m).

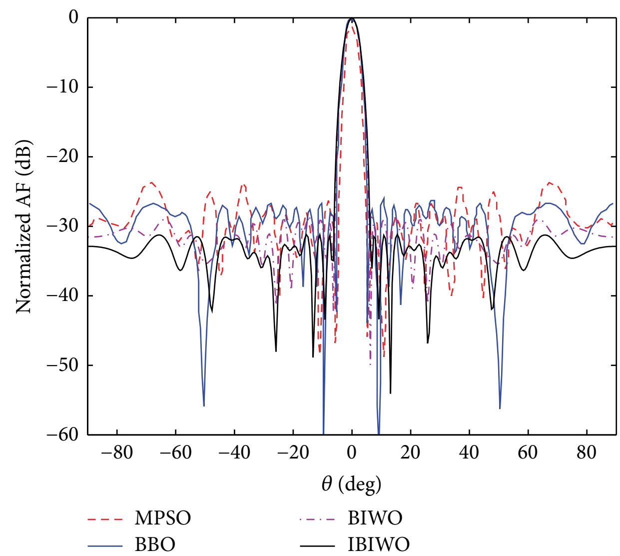


FIGURE 9: Radiation pattern for ten-ring antenna array obtained using IBIWO results as compared with the results achieved by MPSO, BBO, and BIWO (optimized d_m).

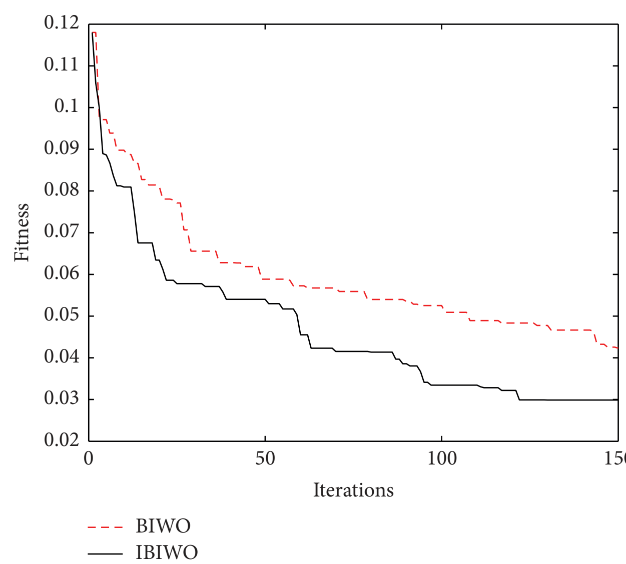


FIGURE 10: Comparison of convergence characteristics between BIWO and IBIWO (optimized d_m).

2.42 dB than the maximum SLL obtained by fully populated, DEGL, MPSO, BBO, and BIWO thinned arrays, respectively. The number of switched-off elements achieved by IBIWO is more than that of the other four algorithms. The thinned antenna array obtained by IBIWO is shown in Figure 8.

The comparison radiation patterns of MPSO, BBO, BIWO, and IBIWO are also drawn in Figure 9. The optimal amplitude excitations obtained by IBIWO are shown in Table 5.

The convergence characteristics of IBIWO and BIWO are shown in Figure 10. From Figure 10 we can know that the convergence speed of the proposed algorithm is faster than BIWO.

From the above results, in the synthesis of thinned concentric multiple ring antenna arrays, it can be observed clearly that the proposed algorithm with adaptive dispersion can take a good balance between the local search and global exploration. In both cases, the IBIWO can achieve better solutions compared with the above algorithms.

5. Conclusions

In this paper, the improved binary IWO is proposed for the thinning large concentric ring arrays of isotropic elements to generate a pencil beam in the vertical plane with maximum SLL reduction. The obtained pattern has half power beam width which is very close to the value of a fully populated array of the same size and shape and yet has a lower SLL.

Two cases are discussed in this paper. One is to generate a thinned array with fixed interelement array spacing and the second with optimized interelement array spacing. Both cases are to obtain a thinned array with minimum SLL and 50% or more percentage of thinning. Comparisons of the IBIWO and other techniques (DEGL, MPSO, BBO, and BIWO) show the efficiency of the proposed technique.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] R. S. Elliot, *Antenna Theory and Design*, John Wiley & Sons, 2006.

[2] M. Dessouky, H. Sharshar, and Y. Albagory, "Efficient sidelobe reduction technique for small-sized concentric circular arrays," *Progress in Electromagnetics Research*, vol. 65, pp. 187–200, 2006.

[3] M. Dessouky, H. Sharshar, and Y. Albagory, "Optimum normalized-Gaussian tapering window for side lobe reduction in uniform concentric circular arrays," *Progress in Electromagnetics Research*, vol. 69, pp. 35–46, 2007.

[4] M. Dessouky, H. Sharshar, and Y. Albagory, "A novel tapered beamforming window for uniform concentric circular arrays," *Journal of Electromagnetic Waves and Applications*, vol. 20, no. 14, pp. 2077–2089, 2006.

[5] N. Pathak, P. Nanda, and G. K. Mahanti, "Synthesis of thinned multiple concentric circular ring array antennas using particle swarm optimization," *Journal of Infrared, Millimeter, and Terahertz Waves*, vol. 30, no. 7, pp. 709–716, 2009.

[6] D. E. Goldberg, *Enetic Algorithms in Search, Optimization, and Machine Learning*, vol. 412, Addison-Wesley, Reading, Mass, USA, 1989.

[7] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *Proceedings of the IEEE International Conference on Neural Networks*, pp. 1942–1948, December 1995.

[8] R. Storn and K. Price, "Differential evolution—a simple and efficient heuristic for global optimization over continuous spaces," *Journal of Global Optimization*, vol. 11, no. 4, pp. 341–359, 1997.

[9] D. Simon, "Biogeography-based optimization," *IEEE Transactions on Evolutionary Computation*, vol. 12, no. 6, pp. 702–713, 2008.

[10] R. L. Haupt, "Thinned arrays using genetic algorithms," *IEEE Transactions on Antennas and Propagation*, vol. 42, no. 7, pp. 993–999, 1994.

[11] R. L. Haupt, "Thinned concentric ring arrays," in *Proceedings of the IEEE International Symposium on Antennas and Propagation and USNC/URSI National Radio Science Meeting (APSURSI '08)*, pp. 1–4, San Diego, Calif, USA, July 2008.

[12] L. Zhang, Y.-C. Jiao, F.-S. Zhang, and H. Li, "Linear array thinning based on orthogonal genetic algorithm," in *Proceedings of the International Conference on Microwave and Millimeter Wave Technology (ICMMT '10)*, pp. 1057–1059, Chengdu, China, May 2010.

[13] P. Ghosh and S. Das, "Synthesis of thinned planar concentric circular antenna arrays-a differential evolutionary approach," *Progress In Electromagnetics Research B*, no. 29, pp. 63–82, 2011.

[14] N. Pathak, G. K. Mahanti, S. K. Singh, J. K. Mishra, and A. Chakraborty, "Synthesis of thinned planar circular array antennas using modified particle swarm optimization," *Progress in Electromagnetics Research Letters*, vol. 12, pp. 87–97, 2009.

[15] A. Chatterjee, G. K. Mahanti, and N. N. Pathak, "Comparative performance of gravitational search algorithm and modified particle Swarm optimization algorithm for synthesis of thinned scanned concentric ring array antenna," *Progress in Electromagnetics Research B*, vol. 25, pp. 331–348, 2010.

[16] U. Singh and T. S. Kamal, "Synthesis of thinned planar concentric circular antenna arrays using biogeography-based optimisation," *IET Microwaves, Antennas and Propagation*, vol. 6, no. 7, pp. 822–829, 2012.

[17] W.-B. Wang, Q.-Y. Feng, and D. Liu, "Synthesis of thinned linear and planar antenna arrays using binary PSO algorithm," *Progress in Electromagnetics Research*, vol. 127, pp. 371–387, 2012.

[18] B. Basu and G. K. Mahanti, "Thinning of concentric two-ring circular array antenna using fire fly algorithm," *Scientia Iranica*, vol. 19, no. 6, pp. 1802–1809, 2012.

[19] A. Chatterjee, G. K. Mahanti, and A. Mahanti, "Synthesis of thinned concentric ring array antenna in predefined phi-planes using binary firefly and binary particle swarm optimization algorithm," *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, 2014.

[20] S. H. Sedighy, A. R. Mallahzadeh, M. Soleimani, and J. Rashed-Mohassel, "Optimization of printed yagi antenna using invasive weed optimization (IWO)," *IEEE Antennas and Wireless Propagation Letters*, vol. 9, pp. 1275–1278, 2010.

[21] S. Karimkashi, A. A. Kishk, and D. Kajfez, "Antenna array optimization using dipole models for MIMO applications," *IEEE Transactions on Antennas and Propagation*, vol. 59, no. 8, pp. 3112–3116, 2011.

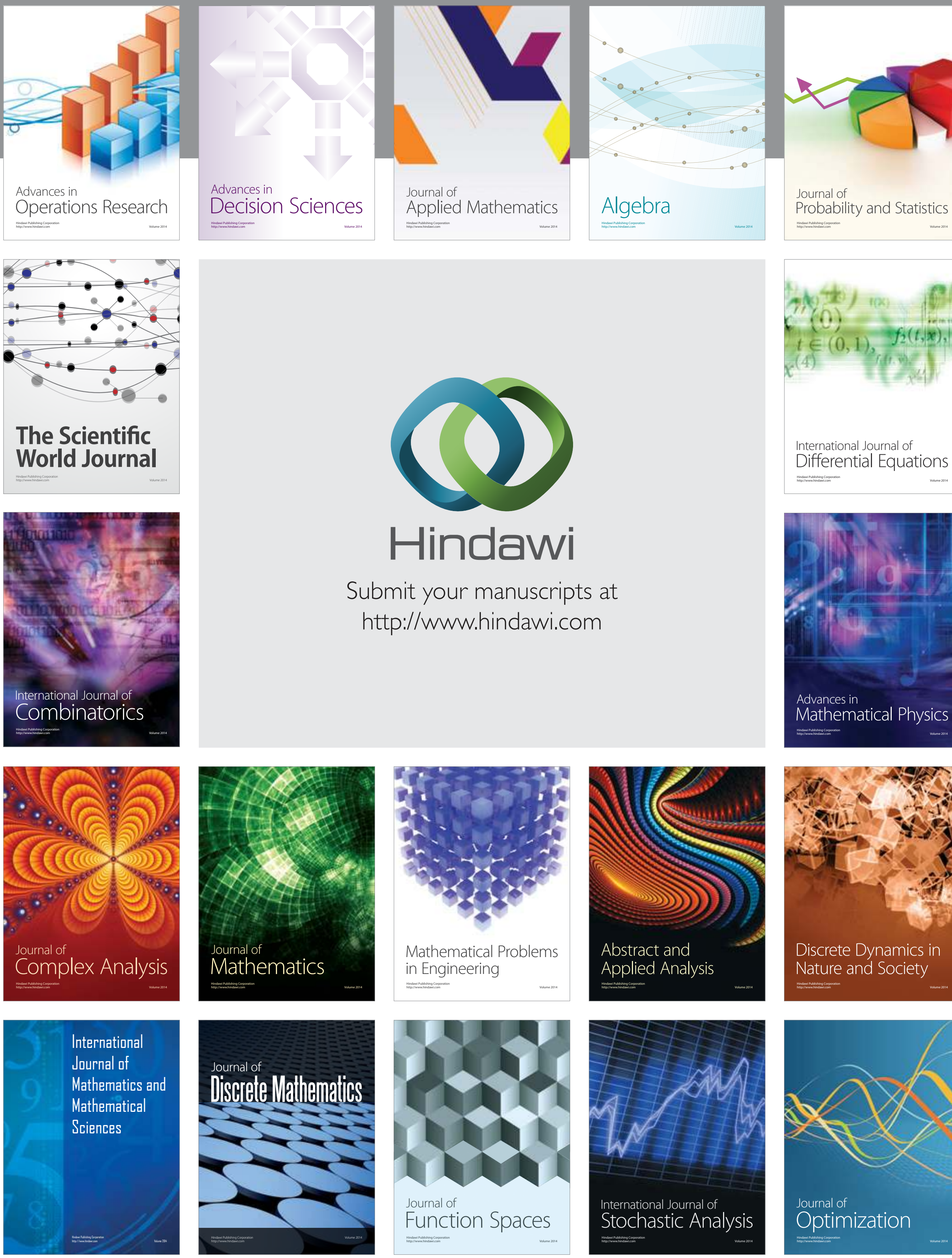
[22] A. Foudazi and A. R. Mallahzadeh, "Pattern synthesis for multi-feed reflector antennas using invasive weed optimisation," *IET Microwaves, Antennas and Propagation*, vol. 6, no. 14, pp. 1583–1589, 2012.

[23] F. M. Monavar, N. Komjani, and P. Mousavi, "Application of invasive weed optimization to design a broadband patch antenna with symmetric radiation pattern," *IEEE Antennas and Wireless Propagation Letters*, vol. 10, pp. 1369–1372, 2011.

[24] Y.-Y. Bai, S. Xiao, C. Liu, and B.-Z. Wang, "A hybrid IWO/PSO algorithm for pattern synthesis of conformal phased arrays," *IEEE Transactions on Antennas and Propagation*, vol. 61, no. 4, pp. 2328–2332, 2013.

[25] G. G. Roy, S. Das, P. Chakraborty, and P. N. Suganthan, "Design of non-uniform circular antenna arrays using a modified invasive weed optimization algorithm," *IEEE Transactions on Antennas and Propagation*, vol. 59, no. 1, pp. 110–118, 2011.

[26] A. R. Mehrabian and C. Lucas, "A novel numerical optimization algorithm inspired from weed colonization," *Ecological Informatics*, vol. 1, no. 4, pp. 355–366, 2006.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