Third-harmonic generation from Mie-type resonances of isolated all-dielectric nanoparticles

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Abstract

Subwavelength silicon nanoparticles are known to possess a set of strongly localized modes, including those with resonant electric and magnetic dipolar polarizabilities. Here we com-

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pare experimentally the third-harmonic generation efficiency from individual and uncoupled silicon nanodisks for resonant excitation at the Mie-type dipolar resonances. Using nonlinear spectroscopy, we observe that the fundamental magnetic dipolar mode yields efficient third-harmonic radiation in contrast to the electric dipolar mode. This is further supported by full-wave numerical simulations, where the volume-integrated local fields and the directly simulated nonlinear response are shown to be negligible at the electric dipolar resonance compared to the magnetic one.

Introduction

In nanoplasmonics, noble metal nanoparticles allow for shaping local electromagnetic fields on demand, an ability that has spawned many applications as well as inspired a vast amount of research in photonics.¹ Having control over the local fields is crucial for nonlinear-optical applications where frequency conversion, self-action and all-optical switching are highly sensitive to the structure geometry and the excited electromagnetic modes.² Apart from the electric dipolar (ED) modes in simple geometries like spheres^{3,4} or dipole antennas,^{5,6} the modes with considerable magnetic dipolar (MD) moment^{7–9} were extensively studied in the nonlinear-optical regime.¹⁰ For instance, efficient second- and third-harmonic (TH) response was found in split-ring resonators,¹¹ fishnet metamaterials,¹² split-hole cavities,¹³ and nanosandwiches.¹⁴ A special contribution of MD modes to the nonlinear multipolar response of metasurfaces was indicated.^{14,15}

Observation of MD modes¹⁶ in dielectric resonators¹⁷ and in particular in silicon nanoparticles^{18,19} was an important step towards low-loss nanophotonics.²⁰ Having extremely low absorption in the near-infrared spectral region, silicon is even more attractive in the nonlinear regime.^{21,22} Previously, we have shown that MD modes can substantially improve the nonlinear response of silicon nanodisks^{23,24} and their oligomers.²⁵ Apart from the fact that the MD mode is often the lowest-order mode of a high-index subwavelength spherical or disklike nanoparticle, it remained unclear whether there is any further advantage of utilizing this mode for nonlinear conversion processes as compared to other, higher-order modes.

In this paper, we compare the nonlinearities of optically isolated silicon nanoparticles when pumped at their electric and magnetic dipolar resonances by employing third-harmonic generation (THG) spectroscopy technique. As depicted schematically in Fig. 1(a), we observe that exciting the magnetic dipolar resonance leads to a large THG yield compared to the case of electric dipolar resonance because of the stronger field confinement within the nanodisk for the MD resonance. This is supported by rigorous numerical calculations of the electric field distributions that show strong enhancement of the overall local field for the magnetic dipole mode and by rigorous numerical simulations of the nonlinear-optical response.

Methods

Samples. Nanodisk arrays are fabricated starting with a silicon-on-insulator (SOI) wafer, produced by *Soitec*, using electron-beam lithography and reactive-ion etching processes. The top (100)-cut Si layer of the wafer is 260-nm-thick. We consider two square 400x400 μ m² arrays distinguished by the disk diameter values of d = 345 and 360 nm, respectively. The nanodisk lattice spacing is $p = 2.85 \ \mu$ m for all the diameters. As opposed to our previous results on nonlinear spectroscopy of Si metasurfaces,²³ such a period value allows for studying isolated nanodisks due to negligible optical coupling ($d \ll p$ and $\lambda < p$). The choice of the nanodisk diameter and height specifies the MD and ED resonance wavelengths.²⁶ A scanning electronic microscopy image of a 360-nm disk array part is shown in Fig. 1(b).

Nonlinear spectroscopy. In order to study the nonlinear-optical response of the nanodisks and to clarify the roles played by ED and MD resonances in the harmonic generation process, we conduct nonlinear THG spectroscopy measurements. An optical parametric oscillator (Angewandte Physik & Electronik GmbH OPO PP Automatic, 200 fs pulses) is pumped by a Ti:Sapphire oscillator (Coherent Chameleon Ultra II, repetition rate 76 MHz) and tuned in the range of 1.0-1.5 μ m. The beam polarization is conditioned by a pair of laser Glan prisms. The waist diameter is set at 11 μ m by an aspherical lens. The beam is focused from the back of the sample, and the transmitted third-harmonic beam is measured. The full thickness of the SOI wafer (500 μ m) is comparable to the waist depth. The resulting peak intensity reached the values of about 1 GW/cm² in the sample plane. For the preliminary focusing at the area of interest, a CCD-camera and a halogen lamp radiation are used. The transmitted and collimated THG signal is selected by a set of blue filters and detected by a photomultiplier tube connected to a lock-in amplifier. The periodic arrangement formed a square diffraction pattern; we deliberately collect only the zeroth transmitted order in order to measure the forward-scattered signal only. This signal is proven to be of the TH origin by checking its cubic dependence on the pump power and by direct measurements of its spectrum by a spectrometer. It has been also verified that the THG beam is polarized in parallel to the orientation of the pump beam polarization. It should be pointed out that the penetration depth of the THG into silicon does not exceed the nanodisk height.

The key feature of our approach is the possibility to cancel out all the sample-irrelevant contributions to the measured spectra, such as pulse duration and power variations, along with spectral features of the setup itself, as well as the dispersion of $\chi^{(3)}$ for bulk silicon. Our nonlinear spectroscopy setup has an option for measuring the linear transmittance spectra by focusing white light to a 20- μ m-wide spot on the sample plane and consecutively measuring its spectrum using an IR-spectrometer. All measurements are carried out under normal incidence and with all the beams polarized along the side of the sample periodicity cell. The numerical aperture of our excitation train was equal to 0.15.

Estimation of third-harmonic output via local fields integration. The fact of the higher TH yield upon pumping the MD resonances if compared with the ED resonance is supported by estimating the overall nonlinear polarization within the nanodisks. This has been done by integrating the local fields over the volume of the nanodisk. To simulate the nanodisks response to external radiation we implement a model using *FDTD Solutions* software by Lumerical Solutions, Inc. This model includes a plane wave pulse with the wide spectral range $(0.8-1.4 \ \mu m)$ as an external radiation source located inside the silica substrate. For a Si nanodisk, dimensions are set equal to those of the experiment; namely, the height is 260 nm, and the diameter is 360 nm. After calculating the electromagnetic field components outside and inside the disk, we integrate the Cartesian components of the time-averaged electric field:

$$|E_{\text{int},i}(\lambda)|^2 = \int_{V_{\text{disk}}} |E_i(\mathbf{r},\lambda)|^2 d\mathbf{r},$$
(1)

and the absolute value of the squared total field:

$$|E_{\text{int}}(\lambda)|^2 = \int_{V_{\text{disk}}} (|E_x(\mathbf{r},\lambda)|^2 + |E_y(\mathbf{r},\lambda)|^2 + |E_z(\mathbf{r},\lambda)|^2) d\mathbf{r}, \qquad (2)$$

over the disk volume V_{disk} . The spectral features of the source are taken into account by a normalizaton procedure. The disk is considered isolated due to the perfectly matched layers set as boundary conditions.

Numerical simulations of third-harmonic output. Our nonlinear spectroscopy result is compared with direct numerical calculations of THG response. We consider the same model of the nanodisk on the silica substrate using *FDTD Solutions* software by *Lumerical Solutions, Inc.* The material of the disk was augmented by a constant isotropic nonlinear susceptibility $\chi^{(3)}$. For each wavelength, pump was simulated to be a plane wave pulse with a relatively narrow spectral width of 5 nm, which was close to the experimental value. Nonlinear signal was recorded in the spectral vicinity of the respective tripled frequency as detected by a two-dimensional transmission monitor and integrated as a function of the two coordinates and frequency.

Results and Discussion

The main experimental results of our study are shown in Fig. 2. For the linear optical response characterization, we normalize the white-light transmittance spectrum by the spectrum of the nearby substrate area measured under the same conditions. We observe two dips in the normalized transmittance spectra. The dips correspond to the electric dipolar resonance found at shorter wavelengths, and magnetic dipolar resonance found at longer wavelengths; this interpretation is supported by the corresponding numerical calculations summarized below. A difference in the position of the magnetic resonance wavelength observed for the two samples is about 30 nm, and this is more substantial for the magnetic rather than for electric resonance due to the sensitivity of the former to the disk diameter.²⁶

The measured THG spectra are shown in Figs. 2(a,b) with black dots and corresponding error bars. First, despite the simultaneous illumination of both the nanodisk arrays and the bulk silicon SOI substrate, we observe a significant THG enhancement from the nanodisks compared to the bulk silicon THG. The enhancement peak corresponds to the wavelength of the MD resonance. Interestingly, the THG enhancement is highly pronounced at the MD resonance wavelength (this was observed in our previous work for optically coupled nanodisks too²³) whereas at the ED resonance there is no any sufficient enhancement. The THG line is expectedly narrower than that of the transmittance spectrum. A small, yet persistent, red shift of the central THG resonance wavelength is observed with respect to the transmittance dip. The maximum enhancement defined as a ratio of the normalized THG signal to the inverse value of the relative nanodisk area is around 30, which is consistent with our previous findings for sparsely arranged oligomers.²⁵

In order to elucidate the observed asymmetry in efficiencies between the third-harmonic generation at the magnetic resonance versus that at the electric resonance we calculate numerically the volume-integrated local electromagnetic fields within the disk. It is known that the nonlinear polarization depends strongly on the strength of the local electromagnetic field; in the case of the scalar nonlinear susceptibility $\chi^{(3)}$ (see, e. g., Ref.²⁷):

$$\tilde{P}(t) = \chi^{(3)} \tilde{E}^3_{\text{local}}(t).$$
(3)

Here, P(t) is the induced third-order nonlinear polarization that results in TH radiation in the far field. Although the mechanism that leads to the resulting far-field THG pattern is complicated, here we rely on the assumption that the local field value E_{local} is the dominant mechanism for the overall TH response. First, using finite-difference time-domain (FDTD) approach we calculate $\mathbf{E}_{local}(\mathbf{r}, \lambda)$ maps within the single silicon nanodisk situated on a silica substrate. The dimensions of the nanodisks are taken from the experimental values (h =260 nm, d = 360 nm). Then, the time-averaged fields are integrated over the volume of the nanodisk; the integrated values $\mathbf{E}_{int}(\lambda)$ give a direct estimate of how much TH should be detected at a particular pump wavelength λ . The spectra of the volume-integrated local fields are provided in Fig. 3. Here, the components and the absolute field value squared are plotted in the spectral range of three low-order Mie-type resonances that carry MD, ED and magnetic quadrupolar^{18,19} (MQ) moments. From the spectra of the integrated local fields, it can be seen that the ED resonance has little contribution to the overall local field enhancement. The strongest nonlinear response is, therefore, expected at the MD and MQ resonances. This is also supported by the corresponding local field maps that are provided in Fig. 3 for MD at $\lambda = 1.296 \ \mu m$, for ED at $\lambda = 1.03 \ \mu m$ and for MQ at $\lambda = 0.896 \ \mu m$. The largest local electric field value is obtained at the MQ resonance. If one compares the ED and MD modes, the peak local field values within the nanodisk are in favor of the MD resonance: the normalized E-field peaks at 2.71 at the ED resonance versus 3.20 at the MD resonance. Also, the overall volume covered by hotspots is visually larger for the MD mode. Therefore, the experimentally observed enhanced THG at the MD resonance as compared to the THG output at the ED resonance is can be validated by the simple local-field hypothesis. This observation makes MD resonances of high-index nanoparticles very relevant to nonlinear-optical applications in nanophotonics.

Moreover, direct numerical simulations show that the THG response takes place at the MD resonance of an isolated silicon disk, while there is little nonlinear signal at the ED resonance pump frequency (Fig. 4). A significant enhancement of nonlinear processes efficiency is also predicted at the MQ resonance; an experimental verification of this fact could be interesting for ultraviolet photonics applications. An apparent red spectral shift of the TH signal with respect to the spectral position of the MD mode-associated dip is consistent with the integrated electric field in Fig. 3.

Conclusion

In summary, we have studied the third-harmonic response of silicon nanodisks at their electric and magnetic Mie-type dipolar resonances. The enhanced up-conversion efficiency at the magnetic dipolar resonance of the nanodisks has been observed due to the strong field confinement, whereas the electric dipolar resonance yielded almost no nonlinear conversion. The experimental findings have been supported by rigorous nonlinear numerical calculations, where considerably lower average local field density and overall THG has been detected at the electric resonance, as compared with the magnetic resonance. Our results provide an important step towards the better understanding of artificial resonant all-dielectric nanostructures and their nonlinear-optical response at the nanoscale.

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Authors' Contributions. All the authors contributed to conceptualizing this manuscript. EVM carried out numerical simulations and experiments. EVM and MRS wrote the first draft. EVM, MRS and ASS chose the methods and constructed the experimental setup. IS and IB fabricated the sample. IS, DNN, IB, YSK and AAF revised the manuscript. All the authors gave final approval for publication.

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Figure 1: (a) Third-harmonic generation from electric dipolar (d) and magnetic dipolar (m) resonances of individual silicon nanodisks. (b) A scanning electron micrograph of a sample under study. (c) Schematic representation of the structure under study with dimensions and axes defined. (d) Maps of the local electric field at the ED resonance and magnetic field at the MD resonance proving the multipolar moments of the resonances. The maps are given in the central xz section of the disk for h = 260 nm, d = 360 nm and $\lambda = 1.030$ and 1.296 μ m for the ED and MD resonances, respectively.



Figure 2: Experimental results. Third-harmonic generation spectra (dots and error bars) and corresponding linear transmittance spectra (red curves) for nanodisks 345 nm (a) and 360 nm (b) in diameter.



Figure 3: Results of the FDTD numerical calculations. (Left panel) Absolute squared electric field integrated over the volume of the nanodisk; height is 260 nm, diameter is 360 nm. Different field components are indicated with differently colored curves: E_x is given in green, E_y in blue, E_z in red, and total field in black. The gray shaded area denotes the calculated transmittance spectrum. (Right panels) Field distributions in the central sections of the nanodisk at the MD resonance ($\lambda = 1.296 \ \mu m$), ED resonance ($\lambda = 1.03 \ \mu m$) and MQ resonance ($\lambda = 0.896 \ \mu m$).



Figure 4: FDTD numerical calculation results for the isolated nanodisk THG response. Nanodisk height and diameter are 260 nm and 360 nm, respectively.