Progress of Theoretical Physics, Vol. 32, No. 5, November 1964

Three-Body Leptonic Decays of *K*-Mesons and Hyperons

Takao KANEKO

Department of Physics, Faculty of Science and Technology
Meijō University, Nagoya

(Received June 26, 1964)

A method of treating weak interaction processes involving composite particles is formulated on the basis of the theory of bound states of quantum field theory. The method is applied to the analysis of the decay $K^+ \to \pi^0 + \bar{l} + \nu$. And it is shown that under a possible set of assumptions ξ-value of this decay must lie between -1 and $+1$ irrespective of the structure of π and K-meson.

Then the method is applied to the leptonic decays of hyperons and it is shown that if we take $\bar{K}$-baryon model for hyperons there is a possibility of understanding the smallness of the decay rates compared to the ones expected from the universal coupling theory.

§ 1. Introduction

At the time of the proposal of composite model of elementary particles by S. Sakata, it was pointed out that present status of the theory of elementary particles should be compared to the early days of the theory of atomic nuclei.[1]

After that, many investigations have been done along this line of thought. In 1956, K. Matumoto[2] proposed the mass formula of composite particles, which corresponds to Weizsäcker's empirical mass formula for nucleus, and Z. Maki[3] examined the possibility of the formation of π-meson with a nucleon and an anti-nucleon from field theoretical viewpoints, which corresponds to the deuteron problem in the nuclear theory. In 1959, Ikeda, Ogawa and Ohnuki[4] have developed group theoretical investigations based on the assumption of the symmetry among a proton, a neutron and a Λ-particle in the strong interaction,[5] which corresponds to the group theoretical investigations of nucleus. Furthermore, the analysis of various weak processes have been done from the viewpoints of composite particles by Z. Maki, Y. Ohnuki, M. Nakagawa and other authors.[6]

Some of these investigations, especially the search for mass formula and group theoretical investigations based on some symmetries were inherited by a great many authors,[7] although from somewhat different viewpoints.

But if we take more seriously the above mentioned analogy between present status of the theory of elementary particles and early days of the nuclear theory, we should remember the various aspects of the developments of the nuclear theory. When we review the developments of the nuclear theory, we will immediately be aware of the roles played by quantum mechanics as the dynamics of nucleons, and also the roles played by weak interaction phenomena (β-decay).

Here we want to notice on these points, that is, weak interactions may play important roles in revealing the structures of elementary particles, and also the current quantum field theory may play the role of the dynamics of fundamental particles.

From these points we believe that the investigations of weak interactions based on composite model, with the use of quantum field theory, might have very important meaning in the present days. Several authors have made some works on weak interactions from such a viewpoint, for example, the above mentioned works by Z. Maki, Y. Ohnuki, M. Nakagawa and other authors.

But in these works, methods of treating composite particles were not so general ones, and so, processes treated there were very limited ones. So we intend in this paper to formulate more general method of treating composite particles in weak interactions with the use of the quantum field theory and to apply the method to the investigations of the real processes.

In § 2, we take up the decay $K^+ \to \pi^0 + \bar{l} + \nu$ as an example, and try to formulate such a method on the basis of the theory of bound states of quantized field.[8] The method we formulate here is general enough to treat various weak processes involving composite particles. Also in § 2, we show that our analysis based on the composite model leads in some approximations and under a possible set of assumptions to the results which are essentially equivalent to the ones derived from the conventional local interactions. In § 3, we treat the leptonic decays of hyperons with the use of the method formulated in § 2, and show that if we take recently proposed $\bar{K}$-baryon model for hyperons[9] we might be able to understand the smallness of the hyperon leptonic decay rates ($\Delta S = 0$ and 1) compared to the ones expected from universal coupling theory. In the Appendix we will derive the recursion formula for spin 1/2 composite particles.

§ 2. On K_{l3}-decays

In this section we treat $K^+ \to \pi^0 + \bar{l} + \nu$ decays from the viewpoints of composite model.

In the lowest order of weak interactions, S-matrix of this decay is given as

$$S = -i(2\pi)^4 \delta^4(P - Q - p_l - p_\nu)(\bar{\nu} v_\lambda l) \frac{G}{\sqrt{2}} \langle \pi^0 : Q | \bar{\Lambda}(0) v_\lambda P(0) | K^+ : P \rangle, \quad (1)$$

where P, Q, p_l and p_ν are four-momenta of K^+, π^0, $\bar{l}$ and ν respectively. Λ, P, l and ν are the field operators of Λ-particle, proton, lepton (electron or μ-meson) and neutrino respectively. G is the universal weak coupling constant, and $v_\lambda = \gamma_\lambda (1 + \gamma_5)$. π^0 and K^+-meson are considered to be composite particles of the following configurations:

$$\pi^0 = \frac{1}{\sqrt{2}}[P\bar{P} - N\bar{N}],$$

$$K^+ = [\bar{\Lambda}P],$$

where P and N denote proton and neutron respectively.

With the use of the recursion formula for spin 0 composite particle,[10] the matrix element $\langle \pi^0 : Q | \bar{\Lambda}(0) v_\lambda P(0) | K^+ : P \rangle$ in Eq. (1) can be expressed as

$$\langle \pi^0 : Q | \bar{\Lambda}(0) v_\lambda P(0) | K^+ : P \rangle$$

$$= (-i)^2 \int \cdots \int dx_1\, dx_2\, dy_1\, dy_2\, f_Q^{\pi^0}(X : \xi)\ K_X^\pi\, K_Y^K$$

$$\times \langle 0 | T : \frac{1}{\sqrt{2}} \{P(x_1)\bar{P}(x_2) - N(x_1)\bar{N}(x_2)\} \bar{\Lambda}(0) v_\lambda P(0) \cdot \Lambda(y_1)\bar{P}(y_2) | 0 \rangle$$

$$\times f_P^{K^+}(Y : \eta), \tag{2}$$

where

$$K_X^\pi = \Box_X - m_\pi^2, \qquad K_Y^K = \Box_Y - m_K^2, \qquad X = \frac{1}{2}(x_1 + x_2),$$

$$\xi = x_1 - x_2, \qquad Y = \frac{1}{2}(y_1 + y_2), \qquad \eta = y_1 - y_2,$$

and $f_Q^{\pi^0}(X : \xi)$, $f_P^{K^+}(Y : \eta)$ are quite arbitrary except for the normalization condition,

$$\int d\xi f_Q^{\pi^0}(X : \xi) \langle 0 | T : \frac{1}{\sqrt{2}} \{P(x_1)\bar{P}(x_2) - N(x_1)\bar{N}(x_2)\} | \pi^0 : Q \rangle = \frac{1}{2(2\pi)^3}, \tag{3}$$

$$\int d\eta \langle K^+ : P | T : \Lambda(y_1)\bar{P}(y_2) | 0 \rangle f_P^{K^+}(Y : \eta) = \frac{1}{2(2\pi)^3}. \tag{4}$$

For such $f_Q^{\pi^0}(X : \xi)$ and $f_P^{K^+}(Y : \eta)$, we can take as follows:

$$f_Q^{\pi^0}(X : \xi) = \frac{1}{2(2\pi)^3} \frac{1}{C_{\pi^0}(Q)} \langle \pi^0 : Q | T : \frac{1}{\sqrt{2}} \{P(x_1)\bar{P}(x_2) - N(x_1)\bar{N}(x_2)\} | 0 \rangle, \tag{5}$$

$$f_P^{K^+}(Y : \eta) = \frac{1}{2(2\pi)^3} \frac{1}{C_{K^+}(P)} \langle 0 | T : \bar{\Lambda}(y_1) P(y_2) | K^+ : P \rangle, \tag{6}$$

where $C_{\pi^0}(Q)$ and $C_{K^+}(P)$ are defined by

$$C_{\pi^0}(Q) = \int d\xi \langle 0 | T : \frac{1}{\sqrt{2}} \left\{ \bar{P}\left(\frac{\xi}{2}\right) P\left(-\frac{\xi}{2}\right) - \bar{N}\left(\frac{\xi}{2}\right) N\left(-\frac{\xi}{2}\right) \right\} | \pi^0 : Q \rangle$$

$$\times \langle \pi^0 : Q | T : \frac{1}{\sqrt{2}} \left\{ P\left(\frac{\xi}{2}\right) \bar{P}\left(-\frac{\xi}{2}\right) - N\left(\frac{\xi}{2}\right) \bar{N}\left(-\frac{\xi}{2}\right) \right\} | 0 \rangle, \tag{7}$$

$$C_{K^+}(P) = \int d\eta \langle 0 | T : \bar{\Lambda}\left(\frac{\eta}{2}\right) P\left(-\frac{\eta}{2}\right) | K^+ : P \rangle \langle K^+ : P | T : \Lambda\left(\frac{\eta}{2}\right) \bar{P}\left(-\frac{\eta}{2}\right) | 0 \rangle. \tag{8}$$

With the use of Eqs. (5) and (6), Eq. (2) is expressed as

$$\langle\pi^0 : Q|\bar{\Lambda}(0)v_\lambda P(0)|K^+ : P\rangle$$

$$=\left[\frac{-i}{2(2\pi)^3}\right]^2\frac{1}{C_{\pi^0}(Q)C_{K^+}(P)}\int\cdots\int dx_1\,dx_2\,dy_1\,dy_2$$

$$\times\langle\pi^0 : Q|T : \frac{1}{\sqrt{2}}\{P(x_1)\bar{P}(x_2)-N(x_1)\bar{N}(x_2)\}|0\rangle$$

$$\times K_X{}^\pi K_Y{}^K\langle 0|T : \frac{1}{\sqrt{2}}\{P(x_1)\bar{P}(x_2)-N(x_1)\bar{N}(x_2)\}\bar{\Lambda}(0)v_\lambda P(0)\cdot\Lambda(y_1)\bar{P}(y_2)|0\rangle$$

$$\times\langle 0|T : \bar{\Lambda}(y_1)P(y_2)|K^+ : P\rangle. \tag{9}$$

In Eq. (9), we can interpret the factors $\langle\pi^0 : Q|T : \frac{1}{\sqrt{2}}\{P(x_1)\bar{P}(x_2)-N(x_1)\bar{N}(x_2)\}|0\rangle$ and $\langle 0|T : \bar{\Lambda}(y_1)P(y_2)|K^+ : P\rangle$ as the wave functions of π^0 and K^+-meson respectively. The factor

$$K_X{}^\pi K_Y{}^K\langle 0|T : \frac{1}{\sqrt{2}}\{P(x_1)\bar{P}(x_2)-N(x_1)\bar{N}(x_2)\}\bar{\Lambda}(0)v_\lambda P(0)\cdot\Lambda(y_1)\bar{P}(y_2)|0\rangle$$

can be interpreted as the contribution from weak vertex part involving the corrections of the strong interactions.

Now we proceed to calculate the right-hand side of Eq. (9) approximately. First we consider the vacuum expectation value

$$\langle 0|T : \frac{1}{\sqrt{2}}\{P(x_1)\bar{P}(x_2)-N(x_1)\bar{N}(x_2)\}\bar{\Lambda}(0)v_\lambda P(0)\cdot\Lambda(y_1)\bar{P}(y_2)|0\rangle .$$

We decompose T-products into N-products and pick up only the terms with the simplest configurations. Then we get

$$\langle 0|T : \frac{1}{\sqrt{2}}\{P(x_1)\bar{P}(x_2)-N(x_1)\bar{N}(x_2)\}\bar{\Lambda}(0)v_\lambda P(0)\cdot\Lambda(y_1)\bar{P}(y_2)|0\rangle$$

$$\approx\frac{6}{\sqrt{2}}\{S_F{}'(x_1-x_2 : m_P)[S_F{}'(y_1-0 : m_\Lambda)v_\lambda S_F{}'(0-y_2 : m_P)]$$

$$-S_F{}'(x_1-y_2 : m_P)[S_F{}'(y_1-0 : m_\Lambda)v_\lambda S_F{}'(0-x_2 : m_P)]$$

$$-S_F{}'(x_1-x_2 : m_N)[S_F{}'(y_1-0 : m_\Lambda)v_\lambda S_F{}'(0-y_2 : m_P)]\}, \tag{10}$$

where m_P, m_N and m_Λ are masses of proton, neutron and Λ-particle respectively. $S_F{}'(x)$ is a total propagator for spin 1/2 particle.

In Eq. (10), the first and the third terms do not contribute to the matrix element $\langle\pi^0:Q|\bar{\Lambda}(0)v_\lambda P(0)|K^+:P\rangle$ by virtue of the parity conservation in strong interactions.

From the transformation properties under Lorentz transformations, Eq. (10) can be expressed as follows:

$$\langle 0|T:\frac{1}{\sqrt{2}}\{P(x_1)\bar{P}(x_2)-N(x_1)\bar{N}(x_2)\}\bar{\Lambda}(0)v_\lambda P(0)\cdot\Lambda(y_1)\bar{P}(y_2)|0\rangle$$

$$\approx\frac{-6}{\sqrt{2}}[\gamma(x_1-y_2)\Sigma_P^{(1)}((x_1-y_2)^2)+\Sigma_P^{(2)}((x_1-y_2)^2)]$$

$$\times\{[\gamma y_1\Sigma_\Lambda^{(1)}(y_1^2)+\Sigma_\Lambda^{(2)}(y_1^2)]v_\lambda[-\gamma x_2\Sigma_P^{(1)}(x_2^2)+\Sigma_P^{(2)}(x_2^2)]\},\tag{11}$$

where $\gamma(x_1-y_2)$, γy_1 and γx_2 are scalar products of two four-vectors, for example γ_μ and $(x_1-y_2)_\mu$. $\Sigma_P^{(1)}(x^2)$, $\Sigma_P^{(2)}(x^2)$, $\Sigma_\Lambda^{(1)}(x^2)$ and $\Sigma_\Lambda^{(2)}(x^2)$ are defined by

$$S_F'(x:m_P)=\gamma x\Sigma_P^{(1)}(x^2)+\Sigma_P^{(2)}(x^2),\tag{12}$$

$$S_F'(x:m_\Lambda)=\gamma x\Sigma_\Lambda^{(1)}(x^2)+\Sigma_\Lambda^{(2)}(x^2).\tag{13}$$

Here we assume that i) *$\Sigma_P^{(i)}(x^2)$ and $\Sigma_\Lambda^{(i)}(x^2)$ have non-zero finite value at $x=0$ (for $i=1$ and 2)*,*) ii) *the space time region in which the decay process $K^+\to\pi^0+\bar{l}+\nu$ occurs is very small, and therefore, in Eq. (11), terms proportional to x_1, x_2, y_1 and y_2 give negligibly small contributions to the matrix element compared to the terms $\Sigma_P^{(2)}$ and $\Sigma_\Lambda^{(2)}$.*

If we accept these assumptions, Eq. (11) can be approximated as

$$\langle 0|T:\frac{1}{\sqrt{2}}\{P(x_1)\bar{P}(x_2)-N(x_1)\bar{N}(x_2)\}\bar{\Lambda}(0)v_\lambda P(0)\cdot\Lambda(y_1)\bar{P}(y_2)|0\rangle$$

$$\approx\frac{-6}{\sqrt{2}}\Sigma_P^{(2)}((x_1-y_2)^2)\Sigma_\Lambda^{(2)}(y_1^2)v_\lambda\Sigma_P^{(2)}(x_2^2).\tag{14}$$

Next we consider the wave functions $\langle\pi^0:Q|T:\frac{1}{\sqrt{2}}\{P(x_1)\bar{P}(x_2)-N(x_1)\times\bar{N}(x_2)\}|0\rangle$ and $\langle 0|T:\bar{\Lambda}(y_1)P(y_2)|K^+:P\rangle$ in Eq. (9). From the transformation properties under Lorentz transformations, T-invariance and C-invariance in strong interactions, these can be expressed in the following form:[11]

$$\langle\pi^0:Q|T:\frac{1}{\sqrt{2}}\{P(x_1)\bar{P}(x_2)-N(x_1)\bar{N}(x_2)\}|0\rangle$$

$$=\exp(-iQX)\gamma_5\{f_1^\pi-\gamma Q f_2^\pi-\gamma\xi\cdot Q\xi f_3^\pi-(\gamma\xi\cdot\gamma Q-\gamma Q\cdot\gamma\xi)f_4^\pi\},\tag{15}$$

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*) Apparently the assumption i) in the text is not valid in the field theory, but we suppose here that the field theory will be modified violently in the very small space time region and we should like to take the above assumption as the working hypothesis.

$$\langle 0|T:\bar{\Lambda}(y_1)P(y_2)|K^+:P\rangle$$
$$=\exp(iPY)\gamma_5\{f_1^K+\gamma P f_2^K+\gamma\eta\cdot P\eta f_3^K+(\gamma\eta\cdot\gamma P-\gamma P\cdot\gamma\eta)f_4^K\}, \tag{16}$$

where f_i^π's are functions of ξ^2 and $(Q\xi)^2$, f_i^K's are functions of η^2 and $(P\eta)^2$. Here we assume that K-meson is a pseudoscalar particle and the relative parity of N and Λ is even.

Then we consider the case in which the extensions of the wave functions of π^0 and K^+-meson are negligibly small. In the conventional theory, these particles are always considered to be point particles. But, in our theory, wave functions of these particles include relative coordinates of the constituents. This fact gives us a possibility to investigate the structure of these particles, but here, as the first step, we consider only the case where the terms proportional to the relative coordinate can be neglected compared to the other terms.

In this approximation, Eqs. (15) and (16) give

$$\langle\pi^0:Q|T:\frac{1}{\sqrt{2}}\{P(x_1)\bar{P}(x_2)-N(x_1)\bar{N}(x_2)\}|0\rangle$$
$$\approx\exp(-iQX)\gamma_5\{f_1^\pi-f_2^\pi\cdot\gamma Q\}, \tag{17}$$
$$\langle 0|T:\bar{\Lambda}(y_1)P(y_2)|K^+:P\rangle$$
$$\approx\exp(iPY)\gamma_5\{f_1^K+f_2^K\cdot\gamma P\}. \tag{18}$$

It should be noted here that even in the above approximation f_i^π's and f_i^K's are not constants but functions of relative coordinate and center of mass momentum.

With the use of Eqs. (14), (17) and (18), Eq. (9) can be approximated as

$$\langle\pi^0:Q|\bar{\Lambda}(0)v_\lambda P(0)|K^+:P\rangle$$
$$\approx\left[\frac{-i}{2(2\pi)^3}\right]^2\frac{-24}{\sqrt{2}C_{\pi^0}(Q)C_{K^+}(P)}\int\cdots\int dx_1dx_2dy_1dy_2\exp(iPX-iQX)$$
$$\times[f_1^\pi f_2^K P_\lambda+f_2^\pi f_1^K Q_\lambda]K_X^\pi K_Y^K\Sigma_P^{(2)}((x_1-y_2)^2)\Sigma_\Lambda^{(2)}(y_1^2)\Sigma_P^{(2)}(x_2^2). \tag{19}$$

Here we define the weak form factors $f_+(s)$ and $f_-(s)$ in the usual way, that is

$$\langle\pi^0:Q|\bar{\Lambda}(0)v_\lambda P(0)|K^+:P\rangle=f_+(s)(P+Q)_\lambda+f_-(s)(P-Q)_\lambda, \tag{20}$$
$$s=-(P-Q)^2.$$

Then we get from Eq. (19)

$$f_\pm(s)\approx\frac{1}{2}\left[\frac{-i}{2(2\pi)^3}\right]^2\frac{-24}{\sqrt{2}C_{\pi^0}(Q)C_{K^+}(P)}\int\cdots\int dx_1dx_2dy_1dy_2$$
$$\times\exp(iPY-iQX)[f_1^\pi f_2^K\pm f_2^\pi f_1^K]K_X^\pi K_Y^K\Sigma_P^{(2)}((x_1-y_2)^2)\Sigma_\Lambda^{(2)}(y_1^2)\Sigma_P^{(2)}(x_2^2). \tag{21}$$

Furthermore, in order to see the magnitude of s-dependence of these weak form factors, we take derivatives of $f_\pm(s)$ with respect to s, then we get

$$\frac{d}{ds}f_\pm(s)\approx\frac{1}{2}\left[\frac{-i}{2(2\pi)^3}\right]^2\frac{-24}{\sqrt{2}C_{\pi^0}(Q)C_{K^+}(P)}\frac{1}{2(P-Q)_\mu}\int\cdots\int dx_1dx_2dy_1dy_2$$
$$\times\exp(iPY-iQX)\{2\xi_\mu\cdot Q\xi[\bar{f}_1^\pi f_2^K\pm\bar{f}_2^\pi f_1^K]-2\eta_\mu\cdot P\eta[f_1^\pi\bar{f}_2^K\pm f_2^\pi\bar{f}_1^K]$$
$$-i(X+Y)_\mu[f_1^\pi f_2^K\pm f_2^\pi f_1^K]\}K_X{}^\pi K_Y{}^K\Sigma_P{}^{(2)}((x_1-y_2)^2)\Sigma_A{}^{(2)}(y_1^2)\Sigma_P{}^{(2)}(x_2^2), \tag{22}$$

where $\bar{f}_i^\pi$'s and $\bar{f}_i^K$'s are derivatives of f_i^π's and f_i^K's with respect to $(Q\xi)^2$ and $(P\eta)^2$, respectively.

From these results we can derive the following conclusions:

i) If we assume that pion and K-meson have similar structures and we can put $f_i^\pi\approx f_i^K$, then, in Eq. (21) we have $f_1^\pi f_2^K-f_2^\pi f_1^K\approx0$, and therefore $f_-/f_+\approx0$.

ii) The integrands of Eq. (22) are all proportional to ξ or η or $(X+Y)$, therefore we can infer that $df_\pm(s)/ds\ll f_\pm(s)$, according to the assumptions stated below Eqs. (12) and (13). In other words, we get the result

$$\frac{df_\pm(s)}{ds}/f_\pm\approx0\,.$$

iii) From i) and ii), we get[12]

$$\frac{\Gamma(K_{\mu3})}{\Gamma(K_{e3})}\approx0.5\sim1\,.$$

These three results are quite analogous to the ones where we consider K^+ and π^0-meson as elementary particles, and introduce $|\varDelta S|=1$ weak current

$$J_\lambda=i\left\{\frac{\partial\pi^0}{\partial x_\lambda}K^*-\pi^0\frac{\partial K^*}{\partial x_\lambda}\right\}\,.$$

In this case we get the following results:

i) $f_-/f_+=0.$

ii) $f_\pm$ are constants.

iii) $\Gamma(K_{\mu3})/\Gamma(K_{e3})\approx0.7.$

From these similarity, we may infer that physical meaning of the approximations hitherto we adopted are essentially to reduce the composite particles to the elementary particles.

On the other hand, experiments show the following results:[13]

i) $f_-/f_+\approx0\sim2$ or $-6\sim-9.$

ii) When we express $f_\pm(s)$ as $f_\pm(s)=f_\pm(0)\left[1+\lambda_\pm\frac{s}{m_\pi^2}\right]$, $\lambda_\pm=0.036\pm0.045.$

iii) $$\frac{\Gamma(K_{\mu3})}{\Gamma(K_{e3})}=0.79\pm0.19\,.$$

If we make the following approximations:

$$f_i^\pi(\xi^2,\ (Q\xi)^2)=f_i^\pi(\xi=0),$$
$$f_i^K(\eta^2,\ (P\eta)^2)=f_i^K(\eta=0),$$

then we get the following expression for the ξ-value:

$$\xi=\frac{f_-}{f_+}=\frac{f_1^\pi f_2^K-f_2^\pi f_1^K}{f_1^\pi f_2^K+f_2^\pi f_1^K}=\frac{1-ab}{1+ab}, \tag{23}$$

where $a=f_2^\pi/f_2^K$ and $b=f_1^K/f_1^\pi$.

In Eq. (23) we can expect that both a and b are positive. Because f_1^π and f_1^K can be interpreted as the renormalization Z-factor of π and K-meson[14] and therefore they are both positive.[15] As to the value a, Z. Maki have already estimated[14] this value in the chain approximation and have got $a^2\approx 0.8\sim 0.9$, and $a=1$ in the full symmetry limit. So negative value for a seems unreasonable.

On these arguments, here we assume that both a and b are positive, then we get immediately the following limitation on the ξ-value:

$$-1\leq\xi\leq 1.$$

In other words, in our approximations, the ξ-value should lie in the region -1 to $+1$ irrespective of the structures of π and K-meson, so this limitation seems to be very stringent one. Therefore according to our arguments, $\xi\approx -6\sim -9$ is very hard to understand. The value $\xi\approx 0.66$ (D. Luers et al., Phys. Rev. **133** (1964), B1276) seems to accord with our limitations on ξ-value.

The problem of what the effects of compositness of the particles are is very interesting, but it is not so easy to derive clear-cut conclusions even in our method of treating composite particles.

§ 3. On leptonic decays of hyperons

In this section we discuss the leptonic decay of hyperons. From the point of view of Sakata model, Σ-particle is considered to be three body composite particle and Λ-particle is an elementary particle. But, we may consider these particles as effective two-body composite particles, that is, Σ-particle may be considered as $[\Lambda\pi]$ or $[\bar{K}N]$ bound state, and Λ-particle as $[\bar{K}N]$ bound state. From these viewpoints, in this section, first we discuss $\Sigma^+\to\Lambda+\bar{l}+\nu$ decay on the basis of $\bar{K}$-baryon model for hyperons. This model was adopted recently by Z. Maki and M. Nakagawa in the analysis of nonleptonic decays of hyperons.[9] They have shown that this model can offer a successful prospect of explaining the decay of hyperons. Hence we also adopt this model[16] to discuss $\Sigma^+\to\Lambda+\bar{l}+\nu$ process, and consider nucleon as elementary particle.

In this model Σ and Λ-particle are composite particles with the following configurations:

$$\Sigma^+ = [\overline{K}_0 P], \tag{24}$$

$$\Lambda = \frac{1}{\sqrt{2}}[\overline{K}P + \overline{K}_0 N]. \tag{25}$$

We consider S-matrix of the above process in the lowest order of weak interaction. This is given as follows:

$$S = -i(2\pi)^4 \delta^4 (P - Q - p_l - p_\nu)(\bar{\nu} v_\lambda l) \frac{G}{\sqrt{2}} \langle \Lambda : Q | J_\lambda(0) | \Sigma^+ : P \rangle. \tag{26}$$

We apply the recursion formulas for spin 1/2 composite particles composed of two particles (see the Appendix) to the matrix element $\langle \Lambda : Q | J_\lambda(0) | \Sigma^+ : P \rangle$ in Eq. (26). Then we get

$$\begin{aligned} &\langle \Lambda : Q | J_\lambda(0) | \Sigma^+ : P \rangle \\ &\quad = \int \cdots \int dx_1 dx_2 dy_1 dy_2 \tilde{h}_Q{}^\Lambda (X : \xi) [i\gamma Q - m_\Lambda] D_\Lambda(X) \\ &\times \langle 0 | T : \Lambda(X : \xi) J_\lambda(0) \overline{\Sigma}(Y : \eta) | 0 \rangle \overleftarrow{D}_\Sigma(Y) [i\gamma P - m_\Sigma] h_P{}^\Sigma (Y : \eta), \end{aligned} \tag{27}$$

where $\tilde{h}_Q{}^\Lambda(X : \xi)$ and $h_P{}^\Sigma(Y : \eta)$ are quite arbitrary except for the normalization condition.

$$\int d\xi \tilde{h}_Q{}^\Lambda (X : \xi) \langle 0 | T : \frac{1}{\sqrt{2}} \{K^*(x_1) P(x_2) + K_0{}^*(x_1) N(x_2)\} | \Lambda : Q \rangle = \frac{1}{2(2\pi)^3}, \tag{28}$$

$$\int d\xi \langle \Sigma^+ : P | T : K_0(x_1) \overline{P}(x_2) | 0 \rangle h_P{}^\Sigma (X : \xi) = \frac{1}{2(2\pi)^3}. \tag{29}$$

We take as $\tilde{h}_Q{}^\Lambda(X : \xi)$ and $h_P{}^\Sigma(X : \xi)$ which satisfy Eqs. (28) and (29) the following:

$$\tilde{h}_Q{}^\Lambda (X : \xi) = \frac{1}{2(2\pi)^3} \frac{1}{C_\Lambda(Q)} \langle \Lambda : Q | T : \frac{1}{\sqrt{2}} \{K(x_1) \overline{P}(x_2) + K_0(x_1) \overline{N}(x_2)\} | 0 \rangle, \tag{30}$$

$$h_P{}^\Sigma (X : \xi) = \frac{1}{2(2\pi)^3} \frac{1}{C_\Sigma(P)} \langle 0 | T : K_0{}^*(x_1) P(x_2) | \Sigma^+ : P \rangle, \tag{31}$$

where we define $C_\Lambda(Q)$ and $C_\Sigma(P)$ as

$$\begin{aligned} C_\Lambda(Q) &= \int d\xi \langle \Lambda : Q | T : \frac{1}{\sqrt{2}} \left\{ K\left(\frac{\xi}{2}\right) \overline{P}\left(-\frac{\xi}{2}\right) + K_0\left(\frac{\xi}{2}\right) \overline{N}\left(-\frac{\xi}{2}\right) \right\} | 0 \rangle \\ &\times \langle 0 | T : \frac{1}{\sqrt{2}} \left\{ K^*\left(\frac{\xi}{2}\right) P\left(-\frac{\xi}{2}\right) + K_0{}^*\left(\frac{\xi}{2}\right) N\left(-\frac{\xi}{2}\right) \right\} | \Lambda : Q \rangle, \end{aligned} \tag{32}$$

$$C_\Sigma(P) = \int d\xi \langle \Sigma^+ : P | T : K_0\left(\frac{\xi}{2}\right) \overline{P}\left(-\frac{\xi}{2}\right) | 0 \rangle \langle 0 | T : K_0{}^*\left(\frac{\xi}{2}\right) P\left(-\frac{\xi}{2}\right) | \Sigma^+ : P \rangle. \tag{33}$$

With the use of Eqs. (30) and (31), Eq. (27) is rewritten as

$$\langle \Lambda : Q | J_\lambda(0) | \Sigma^+ : P \rangle$$
$$= \int \cdots \int dx_1 dx_2 dy_1 dy_2 \left[\frac{1}{2(2\pi)^3}\right]^2 \frac{1}{C_\Lambda(Q) C_\Sigma(P)}$$
$$\times \langle \Lambda : Q | T : \frac{1}{\sqrt{2}} \{K(x_1) \bar{P}(x_2) + K_0(x_1) \bar{N}(x_2)\} | 0 \rangle$$
$$\times [i\gamma Q - m_\Lambda] D_\Lambda(X) \langle 0 | T : \frac{1}{\sqrt{2}} \{K^*(x_1) P(x_2) + K_0^*(x_1) N(x_2)\} J_\lambda(0) K_0(y_1) \bar{P}(y_2) | 0 \rangle$$
$$\times \overleftarrow{D}_\Sigma(Y) [i\gamma P - m_\Sigma] \langle 0 | T : K_0^*(y_1) P(y_2) | \Sigma^+ : P \rangle , \tag{34}$$

where

$$X = \frac{1}{2}(x_1 + x_2), \quad Y = \frac{1}{2}(y_1 + y_2).$$

As in § 2, the wave functions of Σ^+ and Λ-particles in Eq. (34) can be expressed in the following form:

$$\langle 0 | T : K_0^*(y_1) P(y_2) | \Sigma^+ : P \rangle$$
$$= \exp(iPY) \gamma_5 \{f_1^\Sigma + \gamma P f_2^\Sigma + \gamma\eta \cdot P\eta f_3^\Sigma + (\gamma\eta \cdot \gamma P - \gamma P \cdot \gamma\eta) f_4^\Sigma\} u_\Sigma(P)$$
$$= \exp(iPY) \gamma_5 \{f_1^\Sigma + i m_\Sigma f_2^\Sigma + \gamma\eta \cdot P\eta f_3^\Sigma + 2(i m_\Sigma \gamma\eta - P\eta) f_4^\Sigma\} u_\Sigma(P), \tag{35}$$

$$\langle \Lambda : Q | T : \frac{1}{\sqrt{2}} \{K(x_1) \bar{P}(x_2) + K_0(x_1) \bar{N}(x_2)\} | 0 \rangle$$
$$= \exp(-iQX) \bar{u}_\Lambda(Q) \gamma_5 \{f_1^\Lambda - \gamma Q f_2^\Lambda - \gamma\xi \cdot Q\xi f_3^\Lambda - (\gamma\xi \cdot \gamma Q - \gamma Q \cdot \gamma\xi) f_4^\Lambda\}$$
$$= \exp(-iQX) \bar{u}_\Lambda(Q) \gamma_5 \{f_1^\Lambda + i m_\Lambda f_2^\Lambda - \gamma\xi \cdot Q\xi f_3^\Lambda - 2(i m_\Lambda \gamma\xi + Q\xi) f_4^\Lambda\}. \tag{36}$$

Here we assumed that relative parity of Λ and nucleon is even, and relative parity of Σ and Λ is also even.[17]

In Eqs. (35) and (36) we can consider that the last two terms (proportional to ξ or η) give negligibly small contributions in the integral in Eq. (34) compared to the first two terms, according to the assumptions in § 2. Therefore we may use the following approximations:

$$\langle \Lambda : Q | T : \frac{1}{\sqrt{2}} \{K(x_1) \bar{P}(x_2) + K_0(x_1) \bar{N}(x_2)\} | 0 \rangle [i\gamma Q - m_\Lambda]$$
$$\approx \exp(-iQX) \bar{u}_\Lambda(Q) \gamma_5 \{f_1^\Lambda + i m_\Lambda f_2^\Lambda\} [i\gamma Q - m_\Lambda] = 0, \tag{37}$$

$$[i\gamma P - m_\Sigma] \langle 0 | T : K_0^*(y_1) P(y_2) | \Sigma^+ : P \rangle$$
$$\approx [i\gamma P - m_\Sigma] \exp(iPY) \gamma_5 \{f_1^\Sigma + i m_\Sigma f_2^\Sigma\} u_\Sigma(P) = 0, \tag{38}$$

where the last equality is derived from the relation

$$[i\gamma P - m_\Sigma] \gamma_5 u_\Sigma(P) = 0, \tag{39}$$

$$\bar{u}_\Lambda(Q) \gamma_5 [i\gamma Q - m_\Lambda] = 0. \tag{40}$$

From Eqs. (37) and (38), we can conclude that, in our approximations, matrix element $\langle \Lambda:Q|J_\lambda|\Sigma^+:P\rangle$ is equal to zero. This result is due to the factor $(i\gamma P-m_\Sigma)$ and $(i\gamma Q-m_\Lambda)$ in Eq. (34). If we consider Σ^+ and Λ-particle to be elementary particles, these factors do not appear in Eq. (34), therefore we may consider that the origin of the fact that main parts of the above matrix element vanish is the compositeness of Σ^+ and Λ-particle.*)

This fact suggests that, if we assume the universal coupling theory, the matrix element of the decay $\Sigma^+\to\Lambda+\bar{l}+\nu$ is considerably small compared to that of nucleon beta decay, in the composite model adopted here, because in the ordinary beta decay there do not appear any composite particles. Furthermore, we can show that even if we take another model for Σ^+ and Λ-particle (we may consider the model in which Λ-particle is elementary and Σ^+ is $[\Lambda\pi^+]$ composite particle and also the model in which Λ is elementary and Σ^+ is $[\bar{K}_0P]$ composite particle) the main parts of the matrix element $\langle\Lambda:Q|J_\lambda(0)|\Sigma^+:P\rangle$ vanish as far as we assume $P_{\Lambda N}=P_{\Lambda\Sigma}=+1$ with the use of the method formulated in this paper.

On the other hand, in the usually accepted theory of $\Sigma^+\to\Lambda+\bar{l}+\nu$ decay,[18] where contributions from vector current are negligibly small because of the conserved vector current hypothesis and contributions from axial vector current are estimated with the use of Goldberger-Treiman relation, decay rates of the above process is about factor 3 smaller than that expected from universal coupling theory without corrections of strong interactions.

Next we should like to point out that this viewpoint may be extended to the other leptonic decays of hyperons.

Experimentally, rates of leptonic decays of hyperons seem to be about one order small compared to the value expected from the universal coupling theory.[19] In order to understand this discrepancy, it is believed in general that strangeness changing weak interaction coupling constants are about one order smaller than the one which does not change strangeness.

Recently several authors[20] succeeded in constructing such a theory of weak interactions. But here we show, in quite a different way, that if we take an appropriate composite model for hyperons, we might be able to understand above mentioned discrepancy between universal coupling theory and experiments.

Here we consider $\bar{K}$-baryon model for hyperons. According to this model, nucleon can be considered to be elementary, Λ-particle is $[\bar{K}N]_{I=0}$ composite particle, Σ is $[\bar{K}N]_{I=1}$ composite particle and Ξ-particle is the superposition of $[\bar{K}\Sigma]_{I=1/2}$ bound state and $[\bar{K}\Lambda]_{I=1/2}$ bound state, that is

$$\Lambda=[\bar{K}N]_{I=0}\,,\quad \Sigma=[\bar{K}N]_{I=1}\,,$$
$$\Xi=a[\bar{K}\Sigma]_{I=1/2}+b[\bar{K}\Lambda]_{I=1/2}\,, \tag{41}$$

*) It will be necessary to make clear the physical meaning of these points. We shall now study this point.

where we suppress the various factors which emerge from the isotopic spin space and represent the configurations symbolically.

In this model, matrix elements of the leptonic decays of Λ, Σ and Ξ-particles necessarily contain the following factors respectively:

$$[i\gamma P-m_\Lambda]\langle 0|\Lambda(Y:\eta)|\Lambda:P\rangle$$
$$=[i\gamma P-m_\Lambda]\langle 0|T:K^*(y_1)N(y_2)|\Lambda:P\rangle$$
$$\text{for the decays of } \Lambda\text{-particles,} \tag{42}$$

$$[i\gamma P-m_\Sigma]\langle 0|\Sigma(Y:\eta)|\Sigma:P\rangle$$
$$=[i\gamma P-m_\Sigma]\langle 0|T:K^*(y_1)N(y_2)|\Sigma:P\rangle$$
$$\text{for the decays of } \Sigma\text{-particles,} \tag{43}$$

$$[i\gamma P-m_\Xi]\langle 0|\Xi(Y:\eta)|\Xi:P\rangle$$
$$=[i\gamma P-m_\Xi]\langle 0|T:\ \{aK^*(y_1)\Sigma(y_2)+bK^*(y_1)\Lambda(y_2)\}|\Xi:P\rangle$$
$$\text{for the decays of } \Xi\text{-particles.} \tag{44}$$

It should be noted here that an ordinary nucleon beta-decay matrix element does not contain the factor $[i\gamma P-m_N]$.

If we assume that $P_{\Lambda N}$ (relative parity of N and Λ), $P_{\Sigma\Lambda}$ and $P_{\Xi\Lambda}$ are all even, wave functions of Λ, Σ and Ξ-particles in Eqs. (42), (43) and (44) can be expressed in the following way:

$$\langle 0|T:K^*(y_1)N(y_2)|\Lambda:P\rangle$$
$$=\exp(iPY)\gamma_5\{f_1^\Lambda+im_\Lambda f_2^\Lambda+\gamma\eta\cdot P\eta f_3^\Lambda+(\gamma\eta\cdot\gamma P-\gamma P\cdot\gamma\eta)f_4^\Lambda\}u_\Lambda(P), \tag{45}$$

$$\langle 0|T:K^*(y_1)N(y_2)|\Sigma:P\rangle$$
$$=\exp(iPY)\gamma_5\{f_1^\Sigma+im_\Sigma f_2^\Sigma+\gamma\eta\cdot P\eta f_3^\Sigma+(\gamma\eta\cdot\gamma P-\gamma P\cdot\gamma\eta)f_4^\Sigma\}u_\Sigma(P), \tag{46}$$

$$\langle 0|T:\{aK^*(y_1)\Sigma(y_2)+bK^*(y_1)\Lambda(y_2)\}|\Xi:P\rangle$$
$$=\exp(iPY)\gamma_5\{f_1^\Xi+im_\Xi f_2^\Xi+\gamma\eta\cdot P\eta f_3^\Xi+(\gamma\eta\cdot\gamma P-\gamma P\cdot\gamma\eta)f_4^\Xi\}u_\Xi(P). \tag{47}$$

In these equations, we may consider that the first two terms form the main terms according to the assumptions in § 2. Then Eqs. (45), (46) and (47) are approximated as

$$[i\gamma P-m_\Lambda]\exp(iPY)\gamma_5\{f_1^\Lambda+im_\Lambda f_2^\Lambda\}u_\Lambda(P)=0, \tag{48}$$

$$[i\gamma P-m_\Sigma]\exp(iPY)\gamma_5\{f_1^\Sigma+im_\Sigma f_2^\Sigma\}u_\Sigma(P)=0, \tag{49}$$

$$[i\gamma P-m_\Xi]\exp(iPY)\gamma_5\{f_1^\Xi+im_\Xi f_2^\Xi\}u_\Xi(P)=0. \tag{50}$$

Therefore we may conclude that the main parts of the matrix elements of leptonic decays of these hyperons vanish in $\bar{K}$-baryon model for hyperons.

Combining with these results we may infer that matrix elements of the leptonic decays of baryons ($\varDelta S=0$ and $|\varDelta S|=1$) are much smaller than that expected from the universal coupling theory, with the only exception of the ordinary beta-decay of nucleon.

Since the numerical estimations of these decay rates cannot be made in this stage of our theory, in some sense the above statements might be only a conjecture.

§ 4. Conclusions and discussions

In this paper we have formulated a method of treating weak processes where composite particles are involved. Our method is based on the bound state theory of quantum field theory and therefore it can be constructed rigorously on the axioms of quantum field theory. (See the Appendix.)

In § 2 we took up $K^+\rightarrow\pi^0+\bar{l}+\nu$ decay as an example and made approximations that can be considered to diminish the properties of compositeness of the particles, and we get the results that are essentially equivalent to the conventional elementary particle theory. This result seems to give a support to our treatment of composite particles. But our method of approximations are not well-examined ones and further improvements are hoped.

In § 3 we discussed the problem of leptonic decays of hyperons. The method of treatment is essentially equivalent to the one adopted in § 2. And we got promising results to understand these experiments. It should be noted here that the results of § 3 are independent of the method of calculation of contributions from weak vertex part. Therefore we may consider that these results are the more reliable ones.

On the other hand, the analogous results to ours in § 3 were derived by various authors, as was mentioned in § 3, in quite different ways. Therefore it is very interesting to explore the relations between these methods and ours, but up to now this has not been done. We, however, should like to note here the following point.

In § 2 we adopted the Sakata model and in § 3 we adopted the $\bar{K}$-baryon model for hyperons. Therefore it might be objected the inconsistency with our arguments. But we should like to remember that in the history of nuclear theory α-particle model for nucleus played important roles as well as the independent particle model. We believe that such a flexible viewpoint will be needed also in the present stage of the elementary particle theory. The relations between the Sakata model and the $\bar{K}$-baryon model should be searched for in the course of such investigations.

Acknowledgements

The author would like to express his sincere thanks to the members of Institute of Elementary Particles in Nagoya University, especially to Prof. Z.

Maki for many helpful discussions and his continuous encouragement.

Appendix

Derivation of recursion formula for spin 1/2 composite particle

Recursion formula for spin 1/2 composite particle can be derived in a quite analogous way to that of spin 0 composite particle (see reference 9), especially K. Nishijima: Phys. Rev. **111** (1958), 995).

First we assume asymptotic conditions in the following way (in the sense of weak convergence):

$$\Lambda^{\text{in}}(X:\xi)=\Lambda(X:\xi)+\int dX' S^R(X-X':m_\Lambda)D_\Lambda(X')\Lambda(X':\xi), \qquad (\text{A}\cdot 1)$$

$$\Lambda^{\text{out}}(X:\xi)=\Lambda(X:\xi)+\int dX' S^A(X-X':m_\Lambda)D_\Lambda(X')\Lambda(X':\xi), \qquad (\text{A}\cdot 2)$$

$$\bar{\Sigma}^{\text{in}}(X:\xi)=\bar{\Sigma}(X:\xi)-\int dx' \bar{\Sigma}(X':\xi)\overleftarrow{\bar{D}}_\Sigma(X')S^A(X'-X:m_\Sigma), \qquad (\text{A}\cdot 3)$$

$$\bar{\Sigma}^{\text{out}}(X:\xi)=\bar{\Sigma}(X:\xi)-\int dX' \bar{\Sigma}(X':\xi)\overleftarrow{\bar{D}}_\Sigma(X')S^R(X'-X:m_\Sigma), \qquad (\text{A}\cdot 4)$$

where X is coordinate of center of mass of composite particle and ξ is relative coordinate, S^R and S^A are retarded and advanced functions for spin 1/2 particles, and

$$D_\Lambda(X)=\gamma_\mu\frac{\partial}{\partial X_\mu}+m_\Lambda\,,\quad \bar{D}_\Sigma(X)=\gamma_\mu\frac{\partial}{\partial X_\mu}-m_\Sigma\,.$$

From Eqs. (A·1) and (A·2) we get

$$\{T:\Lambda(X:\xi)\psi(Z)\cdots\}$$
$$=\{T:\Lambda^{\text{in}}(X:\xi)\psi(Z)\cdots\}-\int dX' S^R(X-X':m_\Lambda)D_\Lambda(X')\{T:\Lambda(X':\xi)\psi(Z)\cdots\}, \qquad (\text{A}\cdot 5)$$

$$\{T:\Lambda(X:\xi)\psi(Z)\cdots\}$$
$$=\{T:\Lambda^{\text{out}}(X:\xi)\psi(Z)\cdots\}-\int dX' S^A(X-X':m_\Lambda)D_\Lambda(X')\{T:\Lambda(X':\xi)\psi(Z)\cdots\}, \qquad (\text{A}\cdot 6)$$

where $\psi(Z)\cdots$ is the product of arbitrary kinds of field operators including fermion fields of even number, and $\{T:\cdots\}$ means T-product of field operators.

Then we subtract Eq. (A·5) from Eq. (A·6) and get

$$\Lambda^{\text{out}}(X:\xi)\{T:\psi(Z)\cdots\}-\{T:\psi(Z)\cdots\}\Lambda^{\text{in}}(X:\xi)$$
$$=\int dX'[S^A(X-X':m_\Lambda)-S^R(X-X':m_\Lambda)]D_\Lambda(X')\{T:\Lambda(X':\xi)\psi(Z)\cdots\} \qquad (\text{A}\cdot 7)$$

$$=\int dX' S(X-X':m_A)D_A(X')\{T:A(X':\xi)\psi(Z)\cdots\},$$

where

$$S(X-X':m_A)=S^A(X-X':m_A)-S^R(X-X':m_A).$$

We take the vacuum expectation value of Eq. (A·7) and devide it into the positive and the negative frequency parts, then we get

$$\sum_B\langle 0|A(X:\xi)|B\rangle\langle B|\{T:\psi(Z)\cdots\}|0\rangle$$

$$=\int dX' S^{(+)}(X-X':m_A)D_A(X')\langle 0|\{T:A(X':\xi)\psi(Z)\cdots\}|0\rangle, \qquad (A\cdot 8)$$

where $|B\rangle$ represents physical one particle state corresponding to the field operator $A(X:\xi)$, and Σ_B means the summation over these states if there exist many such states. Furthermore, in deriving Eq. (A·8), we have used the equation for one particle states

$$\langle 0|A^{\text{out}}(X:\xi)|B\rangle=\langle 0|A(X:\xi)|B\rangle.$$

For $S^{(+)}(X-X':m_A)$ in Eq. (A·8), it is convenient to use the following expressions:

$$S^{(+)}(X-X':m_A)=\left[\gamma_\mu\frac{\partial}{\partial(X-X')_\mu}-m_A\right]\Delta^{(+)}(X-X':m_A)$$

$$=\left[\gamma_\mu\frac{\partial}{\partial(X-X')_\mu}-m_A\right]\frac{-i}{2(2\pi)^3}\int\frac{d\boldsymbol{P}}{P_0}\exp(iP(X-X')) \qquad (P^2+m^2{}_A=0)$$

$$=\left[\gamma_\mu\frac{\partial}{\partial(X-X')_\mu}-m_A\right]\frac{-i}{2(2\pi)^3}\Sigma_B\exp(iP_B(X-X')) \qquad (P_B{}^2+m_A{}^2=0)$$

$$=\frac{-i}{2(2\pi)^3}\Sigma_B[i\gamma P_B-m_A]\exp(iP_B(X-X')). \qquad (P_B{}^2+m_A{}^2=0)$$

With the use of this expression, from Eq. (A·8) we get the following relation:

$$\langle 0|A(X:\xi)|A:Q\rangle\langle A:Q|\{T:\psi(Z)\cdots\}|0\rangle$$

$$=\frac{-i}{2(2\pi)^3}\int dX'[i\gamma Q-m_A]\exp(iQ(X-X'))D_A(X')\langle 0|\{T:A(X':\xi)\psi(Z)\cdots\}|0\rangle, \qquad (A\cdot 9)$$

where $|A:Q\rangle$ means the physical one particle state of A-particle with four-momentum Q. Next we define $\tilde{h}_Q{}^A(X:\xi)$ which satisfies the following normalization condition:

$$\int d\xi\,\tilde{h}_Q{}^A(X:\xi)\langle 0|A(X:\xi)|A:Q\rangle=\frac{1}{2(2\pi)^3}. \qquad (A\cdot 10)$$

From the translation invariance of the theory, we can proof that $\tilde{h}_Q{}^A(X:\xi)$ which satisfies condition (A·10) has the following properties:

$$\tilde{h}_Q{}^{\Lambda}(X:\xi)\exp(iQ(X-X'))=\tilde{h}_Q{}^{\Lambda}(X':\xi). \tag{A·11}$$

Here we multiply Eq. (A·9) with $\tilde{h}_Q{}^{\Lambda}(X:\xi)$ from the left-hand side and integrate with ξ, and use the properties (A·11), then we get the following recursion formula:

$$\begin{aligned}&\langle \Lambda:Q|\{T:\psi(Z)\cdots\}|0\rangle\\&=-i\int dXd\xi\,\tilde{h}_Q{}^{\Lambda}(X:\xi)\,[i\gamma Q-m_\Lambda]D_\Lambda(X)\langle 0|\{T:\Lambda(X:\xi)\psi(Z)\cdots\}|0\rangle\,.\end{aligned} \tag{A·12}$$

In quite analogous way, from Eqs. (A·3) and (A·4) we obtain the recursion formula of the following form:

$$\begin{aligned}&\langle 0|\{T:\psi(Z)\cdots\}|\Sigma:P\rangle\\&=i\int dXd\xi\langle 0|\{T:\bar{\Sigma}(X:\xi)\psi(Z)\cdots\}|0\rangle\overleftarrow{D}_\Sigma(X)\,[i\gamma P-m_\Sigma]h_P{}^{\Sigma}(X:\xi),\end{aligned} \tag{A·13}$$

where $h_P{}^{\Sigma}(X:\xi)$ satisfies the following normalization condition:

$$\int d\xi\langle\Sigma:P|\bar{\Sigma}(X:\xi)|0\rangle h_P{}^{\Sigma}(X:\xi)=\frac{1}{2(2\pi)^3} \tag{A·14}$$

and has the property

$$h_P{}^{\Sigma}(X:\xi)\exp(iP(X'-X))=h_P{}^{\Sigma}(X':\xi). \tag{A·15}$$

Equations (A·12) and (A·13) are desired recursion formula for spin 1/2 composite particles composed of two fundamental particles.

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