Three Decades of Development in DOA Estimation Technology

Zeeshan Ahmad*[1], Iftikhar Ali[2]
[1]School of Communication Engineering, Chongqing University, P.R.China
[2]Military College of Signals, National University of Science & Technology, Pakistan
*Corresponding author, e-mail: engr.zeeshan@hotmail.com[1], iayousafzai@hotmail.com[2]

Abstract
This paper presents a brief overview of narrowband direction of arrival (DOA) estimation algorithms and techniques. A comprehensive study is carried out in this paper to investigate and evaluate the performance of variety of algorithms for DOA estimation. Two categories of DOA estimation algorithms are considered for discussion which are Classical methods and Subspace based techniques. Classical methods include Sum-and-Delay method and Capon's Minimum Variance Distortionless Response (MVDR) while Subspace based techniques are multiple signal classification (MUSIC) and The Minimum Norm Technique. Also ESPIRIT technique is evaluated. Inefficiencies are pointed out and solutions are suggested to overcome these shortfalls. Simulation results shows that the MUSIC algorithm is able to better represent the DOAs of signals with more prominent peaks. The Min-Norm algorithm also identifies the DOAs of signals similar to the MUSIC algorithm, but produces spurious peaks at other locations. The MVDR method identifies the DOAs of signals, but the locations are not represented by sharp peaks, due to spectral leakage. The classical beamformer also produces several spurious peaks. MUSIC show higher accuracy and resolution than the other algorithms. It should be noted that MUSIC is more applicable because it can be used for different array geometries.

Keywords: narrowband DOA estimation, array signal processing, music, ESPRIT

1. Introduction

According to the definition of IEEE "Antenna is a transmitting or receiving system that is designed to radiate or receive electromagnetic waves" It has been long debated in electromagnetic systems literature whether antenna arrays play a significant role in satellite, RADAR, G.P.S and long distance communication. Findings of some recent empirical literature show that properly designed antenna array system, operating autonomously, along with optimized and robust algorithm plays an instrumental role in uplifting the performance of satellite navigation and communication systems. While engineers have generally reached a consensus on the central role of antenna arrays in Satellite navigation systems, G.P.S, RADARS and communication systems growth, theoretical and empirical work supporting this concept is still very much in progress.

The antenna array refers to a set of microphones or antennas connected and arranged in a regular structure to form a single antenna that is able to produce a required directional radiation pattern, which we cannot achieve through individual antennas. For some applications single element antennas are unable to meet the gain or radiation pattern requirements. Combining several single antenna elements in an array can be a possible solution [1]. In GPS and satellite navigation system we often require very high directivity and the single-element antenna fails to achieve this requirement because the radiation pattern of single-element antenna is comparatively wide and has low directivity (gain). Though high directivity can be achieved by enlarging the dimensions of single element antenna but it may leads to the appearance of multiple side lobes and technologically inconvenient shapes and dimensions [2]. Another approach is to increase the electrical size of an antenna by constructing an assembly of radiating elements in a proper electrical and geometrical configuration – antenna array. Not necessarily but for sake of simplicity and convenience, the array elements are mostly assumed to be identical. The individual elements may be of any type like wire dipoles or loops, apertures, etc [2].

During the twentieth century the world has become increasingly dependent on electromagnetic systems. Satellites orbiting the earth provide communications links vital to commerce and government. Radar systems help to navigate aircraft and ships as well as to control the traffic of these vehicles in very crowded skies and harbors. In wartime, the effective coordination and control of land, sea, and air forces require reliable communications. Radar systems are used to locate and track enemy forces, guide friendly forces to their targets, and direct shell and missile fire [3].

Estimation of parameters is one of the major applications of array signal processing when signals are impinged on the array. Number of signals, magnitudes, frequencies, direction of arrival (DOA), distances and speeds of signals are some common parameters that are to be identified by the antenna array system. Of all these parameters, the DOA estimation is very important and attracts most attention, especially in far-field signal applications, in which case the wave front of the signal may be treated planar, indicating that the distance is irrelevant. Thus, this paper presents the detailed investigation of DOA estimation and advancement in it with time in the past three decades.

2. DOA Estimation

DOA estimation is the prominent figure in the field of array signal processing applied in radars, sonar's, seismic and communication systems. Various types of information can be extracted from an incoming wave impinged on antenna array which are the coupled signals at different points in space [4]. There are two types of data involved, one is the training data from which the adaptive weights are calculated and the other is primary data from which various type of information can be extracted like detection and parameter estimation (angle, range, Doppler estimation), including their direction of arrival (DOA) [5].

There are many applications where accurate estimation of a signals direction of arrival (DOA) is of particular interest. Radar, sonar, and mobile communication are but a few examples of the many possible applications. DOA methods can be used to design and adapt the directivity of array antennas; for example, an antenna array can be designed to accept signals from some specific direction, while rejecting signals from all other directions and declared as interference [6].

The main reason for choosing aspects of DOA estimation for research is that majority of systems nowadays solely rely on this unique technology for its successful operations, like the US Global Positioning System (GPS), Russian GLONASS etc and Europe, China, Japan and India are in process of developing navigation satellite systems [7].

3. Signal Model for Narrowband Antenna Array

In this section we briefly introduce the basic signal model for narrowband antenna arrays which will be used throughout the paper. Structure of delay propagation, forming spatial covariance matrix and its spectral decomposition are the main contents of this signal model. For simplicity we will use the uniform linear array only for discussion. Subspaces are formed by considering associations of eigenvalues and eigenvectors with the signal and noise components of the signal.

3.1. Propagation Delays in Uniform Linear Arrays

Consider a system of M elements uniform linear array, numbered 0, 1, …, M - 1. Considering half-a-wavelength spacing between the adjacent array elements, it can be assumed that signals received by the array elements are correlated. Half-a-wavelength ($d/\lambda=1/2$) is often referred to as the design wavelength of the array since it characterize compromise between a narrow beamwidth and grating lobes. A baseband signal $s(t)$ is received by each array element at a different time instant. The phase of the baseband signal, $s(t)$, received at element 0 is taken as zero and the phase of $s(t)$ received at other elements will be calculated with respect to this. To measure the phase difference, it is necessary to measure the difference in the time the signal $s(t)$ arrives at element 0 and the time it arrives at element k. From Figure 1 the time delay between the 0th element and k element using basic trigonometry can be computed as [6]:

$$\Delta t_k = \frac{kD \sin\theta}{C} \tag{1}$$

Where C is the speed of light.

If we Suppose $s(t)$ to be a narrowband digitally modulated signal with lowpass equivalent $s_l(t)$, carrier frequency fc, and symbol period T. It can be written as:

$$s(t) = Re\{s_l(t)e^{j2\pi f_c t}\} \tag{2}$$

The signal received by the kth element is given by:

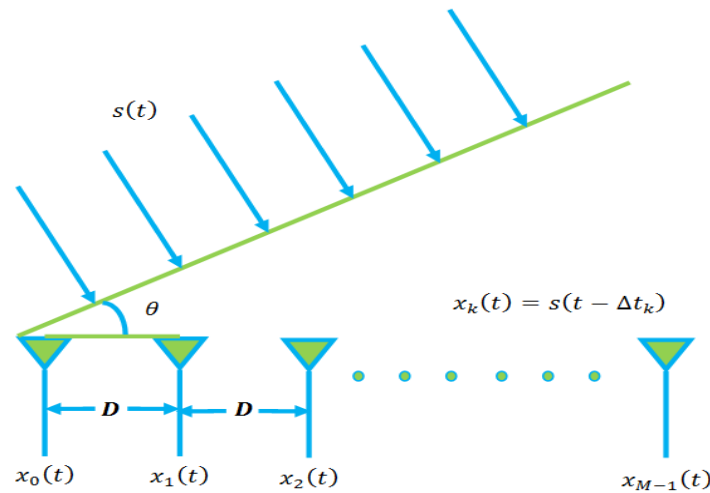$$x_k(t) = Re\{s_l(t-\Delta t_k)e^{j2\pi f_c(t-\Delta t_k)}\} \tag{3}$$

Figure 1. Uniform Linear Array

Now suppose that the received signal at the kth element is downconverted to the baseband. In that case, the baseband received signal is:

$$x_k(t) = s_l(t-\Delta t_k)e^{-j2\pi f_c \Delta t_k} \tag{4}$$

Now sample the received baseband signal with symbol period T seconds i.e.,

$$x_k(nT) = s_l(nT-\Delta t_k)e^{-j2\pi f_c \Delta t_k} \tag{5}$$

In practice,

$$T \gg \Delta t_k \ , \ k = 0,1,2,3,\ldots\ldots,M-1 \tag{6}$$

So Equation (5) can be rewritten as:

$$x_k(nT) \approx s_l(nT)e^{-j2\pi f_c \Delta t_k} \tag{7}$$

Where $C = \lambda f_c$, where λ is the wavelength of thepropagating wave. The element spacing can be computed in wavelengths as $d = D/\lambda$. Using these Equation, (7) can be written as:

$$x_k(nT) \approx s_l(nT)e^{-j2\pi nd \sin\theta} \tag{8}$$

To avoid aliasing in space, $D \le \lambda/2$. Equation (8) is simplified to:

$$x_k(nT) \approx s_l(nT)e^{-j\pi k \sin\theta} \tag{9}$$

Three Decades of Development in DOA Estimation Technology (Zeeshan Ahmad)

In discrete time notation with time index *n* Equation (9) can be written as:

$$x_k[n] \approx s[n]e^{-j\pi k \sin \theta} = s[n]a_k(\theta) \tag{10}$$

Let the *n*th sample of the baseband signal at the *k*th element be denoted as $x_k[n]$. When there are *r* signals present, the *n*th symbol of the *i*th signal will be denoted $s_i[n]$ for *i* = 0, 1, …, *r* − 1. The baseband, sampled signal at the *k*th element can be expressed as:

$$x_k[n] \approx \sum_{i=0}^{r-1} s_i[n]\, a(\theta_i) \tag{11}$$

If the propagating signal is not digitally modulated and is narrowband, the approximation shown in (8) is still valid.

Equation (11) can be written in matrix form as follows:

$$
\begin{bmatrix} x_0[n] \\ x_1[n] \\ . \\ . \\ x_{N-1}[n] \end{bmatrix}
=
\begin{bmatrix} a_0(\theta_0) & a_0(\theta_1) & . & . & a_0(\theta_{r-1}) \\ a_1(\theta_0) & & . & . & . \\ . & & . & . & . \\ . & & . & . & . \\ a_{N-1}(\theta_0) & . & & . & a_{N-1}(\theta_{r-1}) \end{bmatrix}
\begin{bmatrix} s_0[n] \\ s_1[n] \\ . \\ . \\ s_{r-1}[n] \end{bmatrix}
+
\begin{bmatrix} v_0[n] \\ v_1[n] \\ . \\ . \\ v_{r-1}[n] \end{bmatrix}
\tag{12}
$$

Where additive noise, $v_k[n]$, is considered at each element. Equation (12) can be written in compact matrix notation, as follows:

$$x_n = [\mathbf{a}(\theta_0) \quad \mathbf{a}(\theta_1) \quad . \quad . \quad . \quad \mathbf{a}(\theta_{r-1})]\mathbf{s}_n + \mathbf{v}_n = \mathbf{A}\mathbf{s}_n + \mathbf{v}_n \tag{13}$$

Where;

$\mathbf{x}_n$ = $M \times 1$ vector
$\mathbf{A}$ = $M \times r$ matrix
$\mathbf{s}_n$ = signal vectors, and
$\mathbf{v}_n$ = noise vector.

The matrix **A** composed of columns $\mathbf{a}(\theta_i)$, are called the steering vectors (direction vectors) of the signals $s_i(t)$. The set of all possible steering vectors is known as the *array manifold*. The array manifold can be computed in two ways that is analytically and experimentally. Mostly for linear, planar, or circular array configurations, it is computed analytically, while it can be computed experimentally for more complex antenna array geometries. In the absence of noise, the signal received by each element of the array can be written as:

$$x_n = \mathbf{A}\mathbf{s}_n \tag{14}$$

From above equation it is clear that linear combination of the columns of **A** forms the data vector $\mathbf{x}_n$. These elements span the *signal subspace*. In the absence of noise, one can obtain observations of several vectors **x**$_n$ and once we estimate *r* linearly independent vectors, a basis for the signal subspace can be calculated.

Next we will compute the spatial covariance matrix of the antenna array. Assume that the signal and noise vectors are uncorrelated and zero mean. Also the noise vector *is* a vector of Gaussian, white noise samples with zero mean and correlation matrix $\sigma^2 \mathbf{I}$. Let $\mathbf{R}_{ss} = E[\mathbf{s}_n\mathbf{s}_n^H]$. Then we can write the spatial covariance matrix as:

$$\mathbf{R}_{xx} = E[\mathbf{x}_n\mathbf{x}_n^H] = E[(\mathbf{A}\mathbf{s}_n + \mathbf{v}_n)(\mathbf{A}\mathbf{s}_n + \mathbf{v}_n)^H] = \mathbf{A}E[\mathbf{s}_n\mathbf{s}_n^H]\mathbf{A}^H + E[\mathbf{v}_n\mathbf{v}_n^H] = \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma^2\mathbf{I}_{N \times N} \tag{15}$$

Since the matrix $\mathbf{R}_{xx}$ **can be** unitarily decomposed and has real eigenvalues because it is Hermitian (complex conjugate transpose). Using the data matrix **X,** we can find the eigenvectors of the autocorrelation matrix by an alternative method. The rows of the matrix **X** are complex conjugate transpose of the data vectors obtained from the array of sensors.

Suppose that the data matrix **X** contains K snapshots of data obtained from N sensors in a linear array. The matrix **X** is $K \times N$ and can be written as:

$$X = UDV^H \tag{16}$$

Where;

        U is a $K \times K$ matrix whose columns are orthonormal,

        D is a diagonal $K \times N$ matrix, and

        V is an $N \times N$ matrix whose columns are also orthonormal.

This decomposition is known as the singular value decomposition (SVD). The SVD of **X** is related to the spectral decomposition (eigen decomposition) of the spatial covariance matrix R_{xx}. The columns of the matrix **V** will be eigenvectors of R_{xx} and the diagonal elements of the matrix **D** will be square roots of the eigen values of R_{xx}. In practice, the $N - r$ smallest eigenvalues will not be precisely σ^2; rather, they will all have small values compared to the signal Eigen values. This is because the matrix R_{xx} is not known perfectly, but must be estimated from the data. A common estimator for the spatial covariance matrix is the sample spatial covariance matrix, which is obtained by averaging rank-one data matrices of the form $(\mathbf{x}_n \mathbf{x}_n^H)$, i.e.

$$R_{xx} = \frac{1}{K} \sum_{i=0}^{K-1} \mathbf{x}_n \mathbf{x}_n^H \tag{17}$$

Where K is the total number of snapshots of data available from the sensors. Although the discussion so far has focused on the uniform linear array, the principles of signal and noise subspaces also apply to other array geometries such as the uniform planar and the semispherical arrays.

4. Classification of DOA

      There are many ways to classify the DOA estimation methods. Here we have broadly categorized Direction of Arrival (DOA) estimation into four groups that are [8]:

        a) Conventional Techniques

        b) Subspace Based Techniques

        c) Maximum Likelihood Techniques

        d) Integrated Techniques (Combine Property Restoral Techniques and Subspace Based Techniques)

      A large number of elements are required to achieve high resolution in case of Conventional Techniques since they are based on classical beamforming techniques. Subspace based methods are high resolution sub-optimal techniques which exploit the Eigen structure of the input data matrix. Maximum likelihood techniques are the optimal techniques which show tremendous performance under low SNR conditions even but are computationally intensive. The integrated approach use property restoral based techniques to separate multiple signals and estimate their spatial signatures from which their direction of arrival (DOA) can be estimated using subspace techniques [6-9].

      DOA estimation is one of the main focusing content and area of research in array signal processing, and expansively applied in the field of radar, sonar, GPS and was extended to communication in the last decade. There are two types of techniques available to do DOA estimation, which are currently attracting focus of the researchers towards this technology.

4.1. Non-Subspace Techniques

      These methods depend on spatial spectrum, and locations of peaks in the spectrum determine the DOAs of signals. These methods are conceptually simple but offer modest or poor performance in terms of resolution. One of the main advantages of these techniques is that can be used in situations where we lack of information about properties of signal [10].

4.2. Subspace Techniques

      There are certain limitations in resolution which is hindering the growth of non-subspace or classical methods of DOA estimation. They do not exploit the structure of narrowband input

data model of the measurements which give rise to certain limitations. Subspace-based methods depend on observations concerning the Eigen decomposition of the covariance matrix into a signal subspace and a noise subspace. Two of these methods MUSIC and ESPRIT were applied here to determine DOA [10-11].

5. DOA Estimation Based on Classical Method

Classical direction of arrival (DOA) methods are essentially based on beamforming. The two classical techniques for DOA are the delay-and-sum method and the minimum variance distortionless response (MVDR) method. The basic idea behind the classical methods is to scan a beam through space and measure the power received from each direction. Directions from which the largest amount of power is received are taken to be the DOAs [9-12].

5.1. Delay and Sum Method

Delay-and-Sum method is the simplest classical method based on beam forming for estimation of DOA. Figure 2 shows classical narrowband beamformer structure where the output signal y(k) is given by a linearly weighted sum of the sensor elements output [13]. That is:
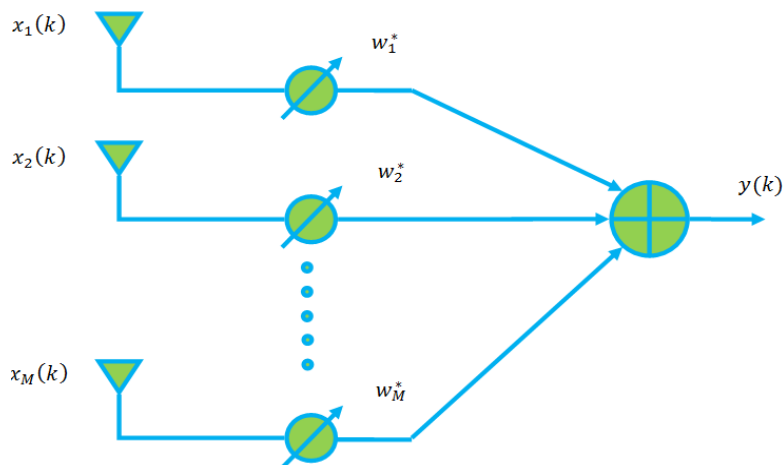
$$y(k) = \boldsymbol{w}^H \boldsymbol{x}(k) \tag{18}$$

Figure 2. Delay-and-Sum Method

The total output power of the above conventional beamformer can be expressed as:

$$P_{cbf} = E[|y(k)|^2] = E[|\boldsymbol{w}^H \boldsymbol{x}(k)|^2] = \boldsymbol{w}^H E[\boldsymbol{x}(k)\boldsymbol{x}^H(k)]\boldsymbol{w} = \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w} \tag{19}$$

Where $\boldsymbol{R}_{xx}$ is the auto correlation matrix of the array input data and contains useful information about both the array response vectors and the signal themselves, and by careful interpretation of $\boldsymbol{R}_{xx}$ we can estimate signal parameters. This equation plays an important role in the in all the conventional DOA estimation algorithms.

Consider a signal $s(k)$ impinging on the array at an angle θ_0. Using the narrowband input data model, the power at the beamformer output can be expressed as:

$$\begin{aligned} P_{cbf}(\theta_0) &= E[|y(k)|^2] = E[|\boldsymbol{w}^H \boldsymbol{x}(k)|^2] = E\left[\left|\boldsymbol{w}^H\big(\boldsymbol{a}(\theta_0)s(k) + \boldsymbol{n}(k)\big)\right|^2\right] \\ &= \ (|\boldsymbol{w}^H \boldsymbol{a}(\theta_0)|^2(\sigma_s^2 + \sigma_n^2)) \end{aligned} \tag{20}$$

Where:

$\boldsymbol{a}(\theta_0)$ = Steering vector associated with the DOA angle θ_0

$n(k)$ = Noise vector at the array input and

$\sigma_s = E[s(k)^2]$ and $\sigma_n = E[n(k)^2]$ are the signal power and noise power respectively.
The above equation clearly demonstrates that output power is maximized when $w = a(\theta_0)$.

Therefore all of the possible weight vectors, the receiver antenna has the highest gain in the direction θ_0, when $w = a(\theta_0)$. This is because $w = a(\theta_0)$ aligns the phases of the signal components arriving from θ_0 at the sensors, causing them to add constructively.

In classical beamforming approach to DOA estimation, the beam is scanned over the angular region of interest in discrete steps by forming weights $w = a(\theta)$ for different θ, and the output power is measured. Using Equation (19), the output power at the classical beamformer as a function of the angle of arrival is given by:

$$P_{cbf}(\theta) = w^H R_{xx} w = a^H(\theta) R_{xx} a(\theta) \tag{21}$$

Therefore if we have an estimate of autocorrelation matrix and know the steering vector $a(\theta)$ for all θ's of interest (either through calibration or analytical computation), it is possible to estimate the output power as a function of the angle of arrival θ. The output power as a function of the angle of arrival is often termed as the spatial spectrum. Clearly the direction of arrival can be estimated by locating peaks in the spatial spectrum defined in Equation (21).

The delay and sum method has many disadvantages. The width of the beam and the height of the sidelobes limit the effectiveness when signal arriving from multiple directions and/or sources are present because the signal over a wide angular region contribute to the measured average power at each look direction. Hence this technique has poor resolution [14]. Although it is possible to increase the resolution by adding more sensor elements, increasing the number of sensors increase the number of receivers and the amount of storage required for the calibration data i.e. $a(\theta)$.

5.2. Capon's Minimum Variance Method

This method has a similarity with the previously described delay-and-sum technique in which the power of the received signal is measured in all possible directions. In simple words in forming the beam in the desired look direction, all the degrees of freedom accessible to the array were utilized. This work goes very well when the single signal is available else contribution from both desired and undesired signals is contained by the array output power. Capon's Method contributes in solving the poor resolution problem by using the idea to utilize some of the degrees of freedom to form a beam in the desired look direction and at the same time using the remaining degrees of freedom to form nulls in the direction of interfering signal [15-16].

To measure the power from DOA, θ, the gain of beamformer is constrained to be 1 in that direction and contribution to the output power from the signals approaching from all other directions is minimized by using the remaining degrees of freedom. Mathematically this problem is known as a constrained minimization process [15]. For every probable angle, θ, the power of the signal is minimized pertaining to w subject to the constraint that $w^H a(\theta) = 1$, this is the basic idea behind the constrained minimization process.

$$min_w E[|y(k)|^2] = min_w w^H R_{xx} w \tag{22}$$

After solving this equation the weight vector which is obtained, is termed as Minimum Variance Distortionless Response (MVDR) beamformer weight, since for a specific look direction, it minimize the variance (average power) of the output signal while passing the signal coming in the look direction without distortion (unity gain and zero phase shift).

The above equation (22) has peaks for the certain angles, represents the estimates of the angles of arrival of the signals.

Using Lagrange multiplier, its weights are given by [17]:

$$w = \frac{R_{xx}^{-1} a(\theta)}{a^H(\theta) R_{xx}^{-1} a(\theta)} \tag{23}$$

Now using the Capon's beam forming method as the output power of the array as an angle of arrival's function, given by the Capon's spatial spectrum is as follows:

$$P_{Capon}(\theta) = \frac{1}{a^H(\theta)R_{xx}^{-1}a(\theta)} \tag{24}$$

The DOA's can be assessed by computing and plotting the Capon's spectrum over the whole range of θ and detecting the peaks in the spectrum.

The drawback of this method includes the requirement of an inverse matrix computation which may become ill-conditioned if highly correlated signals are present and expensive for large arrays. As compared to the delay-and-sum beam former this method provides higher and better resolution.

Suppose if other signals that are correlated with the signal of interest are present because it inadvertently uses that correlation to reduce the processor output power without spatially nulling it. In other words, we can say that the correlated components may be united detrimentally in the process of minimizing the output power.

6. Subspace Methods For DOA Estimation

Low resolution is the major limiting factor, in spite of broader use of classical beam-forming based methods due to the less computational complexity, affecting the development of non-subspace based techniques for the DOA estimation. The efforts of the researchers become more on the subspace based DOA estimators to achieve and attain the high resolution, by making use of the signal subspace. These methods termed as the signal subspace methods are originated during the research on spectral estimation where the estimation of autocorrelation of a signal and the noise model is made and then used to achieve a matrix whose Eigen structure produces the signal and the noise subspaces. By functioning the spatial covariance matrix, this similar technique can also be used in array antenna DOA estimation [18].

6.1. Music Algorithm

In lots of DOA estimation algorithms with excellent performance, one of the earliest proposed algorithm is the multiple signal classification (MUSIC) based on eigenvalue decomposition of the signal covariance matrix [19]. MUSIC stands for Multiple Signal Classification. MUSIC gives the estimation of number of sources and hence their direction of arrival. MUSIC is a technique based on exploiting the Eigen structure of input covariance matrix. Using Singular Value Decomposition (SVD) of the data matrix or Eigen decomposition of sample covariance matrix, we can obtain Eigen vectors easily.

Due to orthogonality between the signal subspace and noise subspace as shown previously, the MUSIC try to find all the possible steering vectors of the incoming signals lie in the signal subspace that are orthogonal to the noise subspace [19-20]. If $a(\theta)$ is the steering vector corresponding to one of the incoming signals, then $a(\theta)^H Q_n = 0$. The function for MUSIC spectrum can be written as:

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta)Q_n Q_n^H a(\theta)} \tag{25}$$

The above function will assume a very large value when θ is equal to the DOA of one of the signals. The MUSIC algorithm was proposed in 1979 by Schmidt [21]. In first phase MUSIC estimates a basis for the noise subspace, Q_n, afterwards, determines the r peaks in (25); the associated angles provide the DOA estimates.

The MUSIC algorithm may be summarized as [21]:

Step 1: Estimate the input covariance matrix R_{xx} in accordance to $\{x(k), k = 1,2,3 \dots \dots M\}$.

$$\widehat{R}_{xx} = \frac{1}{M}\sum_{k=1}^{M} xx^H$$

Step 2: Perform eigen decomposition on $\widehat{R}_{xx}$

$$\widehat{R}_{xx}Q = Q\Lambda$$

Where $\Lambda = diag\{\lambda_1, \lambda_2, \ldots\ldots, \lambda_M\}$, $\lambda_1 \geq \lambda_2 \geq, \ldots\ldots, \geq \lambda_M$ are the eigen values and $Q = \{q_1, q_2, \ldots\ldots, q_M\}$ are the corresponding eigen vectors of $\widehat{R}_{xx}$.

Step 3: Estimate the number of signals *D,* from the multiplicity *K,* of the smallest eigen value λ_{min} as:

$$D = M - K$$

Step 4: The MUSIC spectrum can be obtained as follow:

$$P_{MUSIC}(\theta) = \frac{a^H(\theta)\, a(\theta)}{a^H(\theta)Q_n Q_n^H a(\theta)} = \frac{1}{a^H(\theta)Q_n Q_n^H a(\theta)}$$

Where $Q_n = \{q_{D+1}, \ldots\ldots, q_M\}$

Step 5: *D* is the largest peak of $P_{MUSIC}(\theta)$ which correspond to estimates of the Direction-Of-Arrival.

6.1.1. Disadvantages of MUSIC Algorithm

The MUSIC algorithm has good performance and is widely used till now because of its super resolution capability. Although there are many positives in MUSIC algorithm, there are numerous barriers and competing existing solutions that are hindering the growth of MUSIC algorithm. Inefficiencies from which MUSIC algorithm is suffering are given below:

Its performance degraded when the signals are correlated and so is not able to identify DOAs of correlated signals.

MUSIC algorithm is also computationally complex and expensive because it involves a search over the function P_{MUSIC} for the peaks.

If the numbers of sources are overestimated, it is possible that MUSIC algorithm gives spurious peaks and this happened usually when the steering vector is not in the signal subspace and is perpendicular to some of the noise eigenvectors [20].

6.1.2. Proposed Solutions

The above mentioned inefficiencies pose challenge for sustaining the growth of MUSIC algorithm. Many innovative techniques were proposed in the past to make the MUSIC algorithm more robust and efficient. These innovations include Prime MUSIC, Root MUSIC etc. [22].

There are numerous techniques available in the literature to overcome these deficiencies. One such techniques is known as Spatial smoothing, which is an essential technique in multipath propagation environment and can be applied to overcome this problem. To perform spatial smoothing, the array must be divided up into smaller, possibly overlapping subarrays and the spatial covariance matrix of each subarray is averaged to form a single, spatially smoothed covariance matrix. The MUSIC algorithm is then applied on the spatially smoothed matrix [23].

6.2. The Minimum Norm Method

Kumaresan and Tufts, proposed a method called the Minimum Norm Method, which is applied in a manner similar to MUSIC algorithm over the DOA estimation problem and is defined as "the vector lying in the noise subspace whose first element is one having minimum norm" [24-25].

The vector is given as:

$$g = \begin{bmatrix} 1 \\ \hat{g} \end{bmatrix} \tag{26}$$

As soon as the minimum norm vector is known, the DOAs are specified by the largest peaks of the function as follows [25]:

$$P_{MN}(\theta) = \frac{1}{\left| a^H(\theta) \begin{bmatrix} 1 \\ \hat{g} \end{bmatrix} \right|} \tag{27}$$

Now the objective is to determine and establish the minimum norm vector "**g**". So for that purpose let $\boldsymbol{Q}_s$ be the matrix whose columns develop the basis for the signal subspace. $\boldsymbol{Q}_s$, can be divided as [25]:

$$\boldsymbol{Q}_s = \begin{bmatrix} \alpha^* \\ \overline{\boldsymbol{Q}}_s \end{bmatrix} \tag{28}$$

In the meantime, the vector **g** lies in the subspace of noise and will be orthogonal to the $\boldsymbol{Q}_s$ (signal subspace), so we come up with the equation as follows [25]:

$$\boldsymbol{Q}_s^H \begin{bmatrix} 1 \\ \hat{g} \end{bmatrix} = 0 \tag{29}$$

The system of equations above will be underdetermined; therefore we are going to use the minimum Frobenius norm solution enlightened below:

$$\hat{g} = -\overline{\boldsymbol{Q}}_s (\overline{\boldsymbol{Q}}_s^H \overline{\boldsymbol{Q}}_s)^{-1} \alpha \tag{30}$$

From Equation (29), we can write as:

$$\boldsymbol{I} = \boldsymbol{Q}_s^H \boldsymbol{Q}_s = \alpha \alpha^* - \overline{\boldsymbol{Q}}_s^H \overline{\boldsymbol{Q}}_s \tag{31}$$

From this equation, we can have:

$$\boldsymbol{I} = (\overline{\boldsymbol{Q}}_s^H \overline{\boldsymbol{Q}}_s)^{-1} \alpha = (\boldsymbol{I} - \alpha \alpha^*)^{-1} \alpha = \alpha / (1 - \|\alpha\|^2) \tag{32}$$

By using the above Equation (32), we can eliminate the calculation of the inverse matrix in Equation (30). Now we can compute **g** based only on the signal subspace orthonormal basis, given below:

$$\hat{g} = -\overline{\boldsymbol{Q}}_s \alpha / (1 - \|\alpha\|^2) \tag{33}$$

As soon as **g** has been computed, the evaluation of Min-Norm function given above is done and the angles of arrival are also specified by the *r* peaks. This technique called the Min-Norm technique is commonly reflected as a high-resolution method although it is inferior to both MUSIC and ESPRIT algorithms.

6.3. ESPRIT Algorithm

A novel and a vital approach for the signal parameter estimation problem was proposed and then termed as "ESPRIT". ESPRIT is alike the MUSIC algorithm that works by exploiting the underlying data model, then generates estimates that are effective, effective and asymptotically unbiased. In addition to this, it has numerous advantages over MUSIC.

Roy and Kailath proposed this method for the DOA estimation called the ESPRIT which stands for "Estimation of Signal Parameter via Rotational Invariance Technique" [26].

It is observed that in terms of array imperfections this algorithm in more vigorous and robust as compared with the MUSIC algorithm. Other than that its storage constraints and computation complexity are lesser than MUSIC. This is because this algorithm does not take in extensive search throughout all the probable steering vectors. Nonetheless it investigates the rotational invariance property generated by the two sub-arrays in the signal subspace, resulted from the original array with a translation invariance structure. ESPRIT does not need the exact knowledge of the array manifold vectors unlike the MUSIC algorithm so the array adjustment requirements are not strict, so with the two sub array's corresponding elements, it is decomposed into equal sized two sub-arrays, expatriate from each other by a static translational distance [26-27].

The TLS ESPRIT algorithm is sum up below [26-27]:

Step 1: Obtain an estimate of $\hat{\boldsymbol{R}}_{xx}$ of $\boldsymbol{R}_{xx}$ *from measurement.*

Step 2: Perform Eigen decomposition on $\hat{\boldsymbol{R}}_{xx}$, i.e.

$$\widehat{\boldsymbol{R}}_{xx} = \boldsymbol{V}\Lambda\boldsymbol{V}$$

Where $\Lambda = diag\{\lambda_1, \lambda_2, \ldots\ldots, \lambda_M\}$, and $\boldsymbol{V} = \{V_1, V_2, \ldots\ldots\ldots, V_M\}$ are the Eigen values and Eigen vectors respectively.

Step 3: Estimate the number of signals $\widehat{D}$, from the multiplicity *K,* of the smallest Eigen value λ_{min} as:

$$\widehat{D} = M - K$$

Step 4: obtain the signal subspace estimate $\widehat{\boldsymbol{V}}_s = [\widehat{\boldsymbol{V}}_1, \ldots\ldots\ldots, \widehat{\boldsymbol{V}}_D]$ and decompose it into subarray matrices.

$$\widehat{\boldsymbol{V}}_s = \begin{bmatrix} \widehat{\boldsymbol{V}}_0 \\ \widehat{\boldsymbol{V}}_1 \end{bmatrix}$$

Step 5: Compute the Eigen decomposition $(\lambda_1 > \lambda_2 >, \ldots\ldots, > \lambda_{2D})$

$$\widehat{\boldsymbol{V}}_{01}{}^H \widehat{\boldsymbol{V}}_{01} = \begin{bmatrix} \widehat{\boldsymbol{V}}_0{}^H \\ \widehat{\boldsymbol{V}}_1{}^H \end{bmatrix} [\widehat{\boldsymbol{V}}_0 \quad \widehat{\boldsymbol{V}}_1] = \boldsymbol{V}\Lambda\boldsymbol{V}$$

And partition *V* into $\check{D}$ x $\check{D}$ Sub matrices

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{V}_{11} & \boldsymbol{V}_{12} \\ \boldsymbol{V}_{21} & \boldsymbol{V}_{22} \end{bmatrix}$$

Step 6: Calculate the Eigen values of $\Psi = -\boldsymbol{V}_{12}\boldsymbol{V}_{22}^{-1}$

$\widehat{\Phi}_k$ = eigen values of $(-\boldsymbol{V}_{12}\boldsymbol{V}_{22}^{-1}), \ \forall k = 1, \ldots\ldots\ldots, \widehat{D}$

Step 7: Estimate the angle of arrival as:

$$\hat{\theta}_k = cos^{-1}[c\frac{(\arg(\widehat{\Phi}_k))}{w_0\Delta}]$$

As comprehended from the above argument, ESPRIT eliminates the search procedure which is inherent in most of the DOA estimation methods. In terms of Eigen values ESPRIT produces the DOA estimates directly.

7.   Simulation Results & Discussions

To analyze the performance of these algorithms, a 10-element uniform linear array having half-wavelength inter-element spacing with two signals of equal power impinged on the array is simulated in Matlab. The data vectors are generated using (12) and $\boldsymbol{R}_{xx}$ is computed using (2.20).

a)   Sum-and-Delay Method and Capon's Minimum Variance method:

For two signals arriving on array at at (a) $-30°$ and *30°* and (b) *30°* and *40*, Sum-and-Delay method and Capon's Minimum Variance Method (MVDR) are plotted for angles between -90° and +90° using (21) and (22). Figure 3 and Figure 4 shows the corresponding plots.
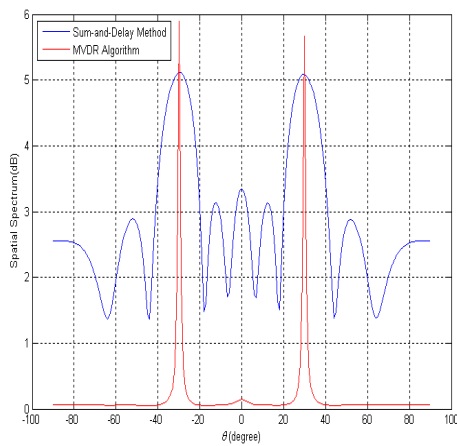
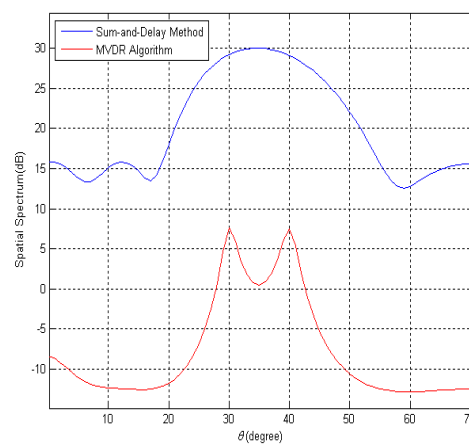Figure 3. Comparison of Sum-and-Delay Method and Capon's Minimum Variance Method

Figure 4. Comparison of Resolution Performance of Sum-and-Delay Method and Capon's Minimum Variance Method

In the first scenario with maximum angle separation ($\theta_1 = -30°$ and $\theta_2 = 30°$), both the methods estimate the DOA accurately but the problem with Sum-and-Delay method is that it produce spurious peaks. Additionally the width of the beam and the height of the sidelobes in Sum-and-Delay method limit the effectiveness. The resolution of MVDR method is better than Classical Sum-and-Delay method.

From the second scenario it is obvious that when the DOAs of the signals impinged on the array are close ($\theta_1 = 30°$ and $\theta_2 = 40°$), then Sum-and-Delay method fails to differentiate between the two signals while the MVDR method can estimate the DOAs of two signal by giving two different peaks in its spectrum for the corresponded DOA.

It is clear from the above discussion that the MVDR method offers superior performance and high resolution than Sum-and-Delay method.

b) MUSIC Algorithm and The Minimum Norm method:

Assuming space has two signals, the angle of incidence $\theta_1 = 30°$ and $\theta_2 = -30°$, Figure 5 is the space that corresponds to the signal spectrum for MUSIC and the Min Norm technique.
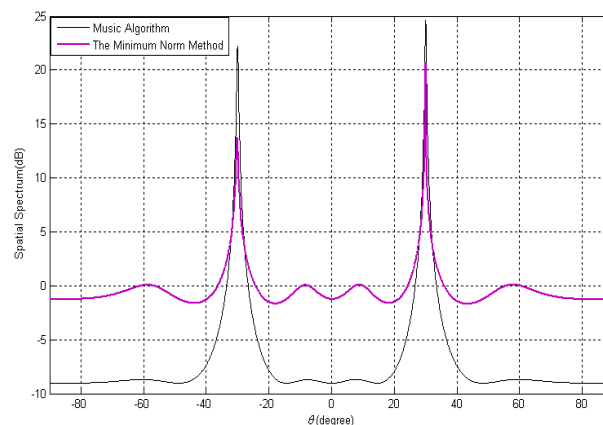

Figure 5. Comparison of MUSIC and Min Norm Technique

From the above Figure 5 it is clear that both MUSIC and Min Norm technique estimate the DOAs of impinged signal accurately. Both MUSIC and Min Norm technique are generally considered to be high resolution techniques but from the figure it is clear that Min Norm

technique produce spurious peaks at other locations, so it is still inferior to MUSIC and other high resolution techniques.

      c)   MUSIC Algorithm and Capon's Minimum Variance method

A simulation is performed with 10 sensors uniform linear array tracking two signals (−30° and 30°), each with an SNR of 10dB. Figure 6 and 7 shows the comparative performance results using the MUSIC algorithm and the Capon algorithm (MVDR).
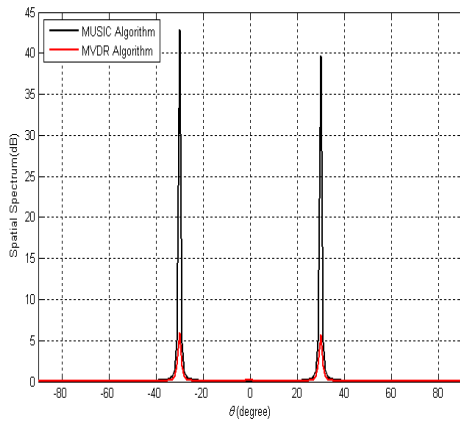

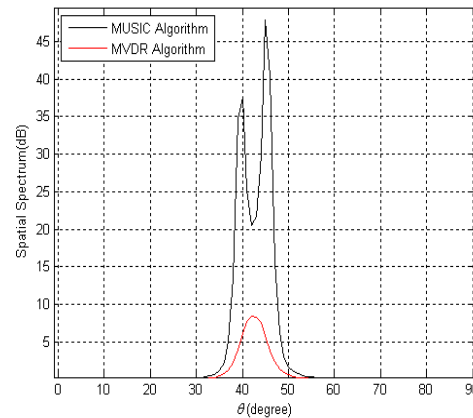Figure 6. Comparison of Resolution performance of MUSIC and Capon's Minimum Variance Technique

Figure 7. Comparison of Resolution performance of MUSIC and Capon's Minimum Variance Technique

It can be seen that in first scenario (Fig.6) the DOAs of the two signals are correctly estimated by the two algorithms. The peaks of the MUSIC algorithm are more prominent comparatively to that of MVDR method. The resolution of MUSIC algorithm is high than that of MVDR method that's why the peaks of MVDR is not as much prominent as that of MUSIC.

In Figure 7 it is clear that when the angle separation is less between the two signals then MVDR algorithm is unable to track the DOAs of signals while the MUSIC algorithm accurately estimate the DOAs of two signal by producing two distinct peaks. Though MVDR algorithm is simpler but the problem is that it cannot distinguish between two signals when their DOAs lie very close to each other and also the resolution of MVDR method is very low that's why it cannot produce very prominent peaks for the estimated DOA as compared to MUSIC algorithm.

      d)   The Minimum Norm method and Classical methods

Under the same array condition, simulations were performed to analyze the performance of Min Norm technique against the classical Sum-and-Delay method and MVDR algorithm.

Figure 8 represents the plot for the Min Norm technique and MVDR algorithm. It is clear that the DOAs estimated by both the methods are accurate. The peaks for Min Norm Technique is more prominent compared to MVDR algorithm which means that the resolution of Min Norm technique is higher than that of MVDR algorithm. But the Min Norm technique produces spurious peaks at other locations which can limit the performance of this technique, while MVDR algorithm does not produce spurious peaks.

Figure 9 is the corresponding plot for the Min Norm technique and classical Sum-and-Delay method. The resolution of Min Norm technique is higher and Classical Sum-and-Delay method produces much higher spurious peaks.

So Min Norm technique shows superior performance over the Sum-and-Delay method but shows comparatively equivalent performance as that of MVDR method. The resolution of Min Norm is high but it produces spurious peaks at other locations while the resolution of MVDR is low but it does not produce spurious peaks.
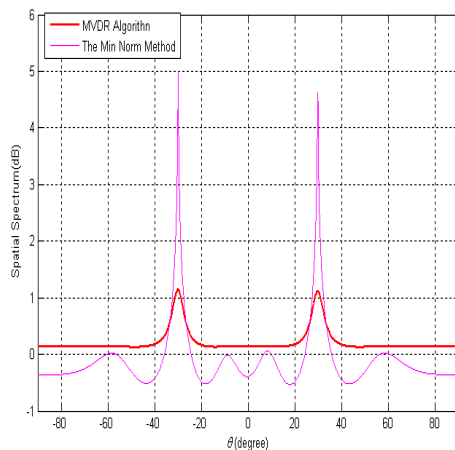
Figure 8. Comparison of Resolution Performance of The Min Norm and Capon's Minimum Variance Technique
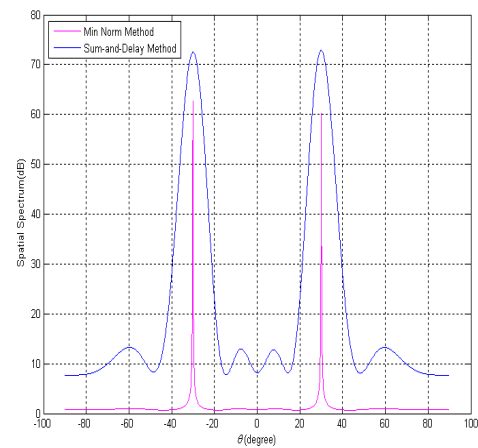
Figure 9. Comparison of Resolution Performance of The Min Norm Technique and Sum-and-Delay Method

Figure 10 depicts the comparative spectrums of all the previously discussed algorithms to estimate DOA of the signals impinged on uniform linear array. These algorithms are Classical Methods (Sum-and-Delay method and Capon's Minimum Variance method), Subspace based techniques (The Minimum Norm method and MUSIC).

It can be seen in figure above that the MUSIC algorithm and the MVDR method identify the two signals and have no other spurious components. Of the two, the MUSIC algorithm is able to better represent the locations with more prominent peaks. The Min-Norm algorithm also identifies the signals similar to the MUSIC algorithm, but produces spurious peaks at other locations. The low-resolution classical beamformer identifies the two signals, but the locations are not represented by sharp peaks, due to spectral leakage. The classical beamformer also produces several spurious peaks.
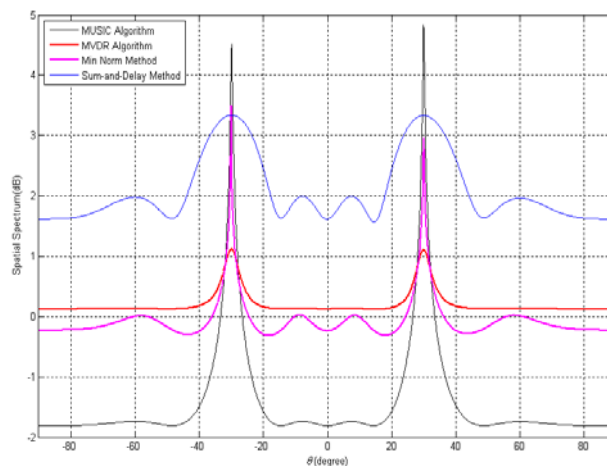

Figure 10. Spatial Spectrum for MUSIC, Capon's Minimum Variance, Min Norm and Sum-and-Delay Method

8. Conclusion
DOA estimation is an important content of array signal processing. There are many Positioning and timing systems such as GPS, radars, seismic communication and other satellite navigation systems which are widely used in today's human life, requires accurate estimation of

a signals direction of arrival (DOA). There are two popular groups of algorithm used for estimation of DOA that are classical methods (Sum-and-Delay method and MVDR method) and Subspace based techniques (Min Norm technique and MUSIC algorithm). A detailed analysis is carried out in this paper to show the performance of these algorithms.

From simulation results it is clear that the classical methods are very simple and have less compute load but these methods suffers from low resolution. In Sum-and-Delay method the width of the beam and the height of the sidelobes limit the effectiveness Hence this technique has poor resolution. Although it is possible to increase the resolution by adding more sensor elements, increasing the number of sensors increase the number of receivers and the amount of storage required for the calibration data i.e. $a(\theta)$.

To improve the resolution of classical Sum-and-Delay method there is another algorithm belongs to the same category known as Capon's Minimum Variance Distortionless Response method. The resolution problem in Sum-and-Delay method was addressed and solved in this algorithm. But the drawback of this method includes the requirement of an inverse matrix computation which may become ill-conditioned if highly correlated signals are present and expensive for large arrays.

To have much better performance, there are algorithms proposed which are known as subspace based techniques (MUSIC, Min Norm, and ESPRIT). The MUSIC algorithm has good performance and is widely used till now because of its super resolution capability. Although there are many positives in MUSIC algorithm, there are Inefficiencies from which MUSIC algorithm is suffering. The performance of MUSIC algorithm degraded when the signals are correlated and so is not able to identify DOAs of correlated signals. MUSIC algorithm is also computationally complex and expensive because it involves a search over the function P_{MUSIC} for the peaks. If the numbers of sources are overestimated, it is possible that MUSIC algorithm gives spurious peaks and this happened usually when the steering vector is not in the signal subspace and is perpendicular to some of the noise eigenvectors.

The Min Norm technique address the problem of high computational complexity but the problem it faces is that this technique produces spurious peaks at other locations which limits the performance of algorithm.

ESPRIT algorithms solve the problem and came up with the solution to both computational complexity and maintaining the resolution performance. ESPRIT eliminates the search procedure which is inherent in most of the DOA estimation methods. In terms of Eigen values ESPRIT produces the DOA estimates directly. Also there are variants of MUSIC algorithm which addresses the problem of high computational complexity. These proposed solution were root MUSIC, and spatial smoothing is the most notable technique to address the problem.

References

[1] Igor V Minin, Oleg V Minin. 2Basic Principles of Fresnel Antenna Arrays. *Springer*. 2008; 31
[2] Antenna Theory: Analysis Design, Third Edition, by Constantine A. Balanis ISBN 0-471-66782-X Copyright © 2005 John Wiley & Sons, Inc.
[3] William C Cummings. An Adaptive Nulling Antenna for Military Satellite Communications. *The Lincoln Laboratory Journal*. 1992; 5(2).
[4] Ahmed Magdy, KR Mahmoud, SG Abdel-Gawad, II Ibrahim. Direction of Arrival Estimation Based on Maximum Likelihood Criteria Using Gravitational Search Algorithm, PIERS Proceedings, Taipei. 2013; 25(28).
[5] Nickel U. Principles of Adaptive Array Processing. In Advanced Radar Signal and Data Processing (pp. 5-1 – 5-20). Educational Notes RTO-EN-SET-086, Paper 5. Neuilly-sur-Seine, France: RTO. Available from: http://www.rto.nato.int/abstracts.asp. 2006.
[6] Jeffrey Foutz, Andreas Spanias, Mahesh K Banavar. Narrowband Direction of Arrival Estimation for Antenna Arrays, Morgan & Claypool Publishers. 2008.

[7] Liu W, Zhai CR, Zhan XQ, Zhang YH. Assessment and analysis of radio frequency compatibility among several global navigation satellite systems. *Radar, Sonar & Navigation, IET*. 2011; 5(2): 128,136.

[8] Suchita W Varade, KD Kulat. Robust Algorithms for DOA Estimation and Adaptive Beamforming for Smart Antenna Application. *Second International Conference on Emerging Trends in Engineering and Technology*, ICETET-09. 2009.

[9] Schell SV, Gardner WA. High Resolution Direction Finding. Chapter 17, Handbook of Statistics 10 Signal Processing and Applications, ed. Bose, N.K., and Rao, C.R., 1993

[10] Shahriar Shirvani-Moghaddam, Sakineh Almasi-Monfared. A Comprehensive Performance Study of Narrowband DOA Estimation Algorithms. *International Journal on Communications Antenna and Propagation (IRECAP)*. 2011; 1(4).

[11] Santiago EA, Saquib M. Noise Subspace-Based Iterative Technique for Direction Finding. *Aerospace and Electronic Systems, IEEE Transactions on*. 2013; 49(4): 2281-2295.

[12] H Krim, M Viberg. Two decades of array signal processing research: the parametric approach. *IEEE Signal Processing Magazine*. 1996; 13: 67–94. doi:10.1109/79.526899

[13] LC Godara. *Application of antenna arrays to mobile communications. Part II: Beamforming and direction of arrival considerations.* Proceedings of the IEEE. 1997; 85: 1195-1245. doi:10.1109/5.622504

[14] TS Rappaport, JC Liberti Jr. Smart Antennas for Wireless Communications: IS-95 and Third Generation CDMA Applications, Upper Saddle River, NJ: Prentice Hall, 1999.

[15] A Alexiou, M Haardt. Smart antenna technologies for future wireless systems: trends and challenges. *IEEE Communications Magazine*. 2004; 42: 90–97. doi:10v.1109/MCOM.2004.1336725

[16] J Capon. High resolution frequency-wavenumber spectral analysis. *Proceedings of the IEEE*. 1969; 57: 1408–1518.

[17] S Haykin. Adaptive Filter Theory (3rd ed.), Prentice Hall, Inc., Upper Saddle River, NJ. 1996.

[18] Villemin et al.: Spatio-temporal-based joint range and angle estimation for wideband signals. *EURASIP Journal on Advances in Signal Processing*. 2013: 131

[19] Zhang Xiaofei, Lv Wen, Shi Ying, Zhao Ruina, Xu Dazhuan. A Novel DOA estimation Algorithm Based on Eigen Space. *Microwave, Antenna, Propagation and EMC Technologies for Wireless Communications, International Symposium on*. 2007: 551,554. doi: 10.1109/MAPE.2007.4393677

[20] Ying Zhang, Boon Poh Ng, MUSIC-Like DOA Estimation without Estimating the Number of Sources. *Signal Processing, IEEE Transactions on*. 2010; 58(3): 1668,1676. doi: 10.1109/TSP.2009.2037074

[21] RO Schmidt. Multiple emitter location and signal parameter estimation. *IEEE Transactions on Antennas and Propagation*. 1986; AP-34: 276–280. doi:10.1109/TAP.1986.1143830

[22] Barabell AJ. Improving the Resolution Performance of Eigenstructure-based Direction Finding Algorithms. *Proceedings of the ICASSP-83*. 1983; 336-339.

[23] Evans JE, Johnson JR, Sun DF. High Resolution Angular Spectrum Estimation Techniques for Terrain Scattering Analysis and Angle of Arrival Estimation in ATC Navigation and Surveillance System. *M.I.T Lincoln Lab.*, Lexington, MA, Rep. 582, 1982.

[24] R Kumaresan, DW Tufts. Estimating the angles of arrival of multiple plane waves. *IEEE Transactions on Aerospace and Electronic Systems*. 1983; AES-19: 134–138.

[25] P Stoica, R Moses. *Introduction to Spectral Analysis.* Upper Saddle River, NJ: Prentice Hall. 1997.

[26] Roy R, Kailath T. ESPRIT- Estimation of Signal Parameter via Rotational Invariance Technique. *IEEE Transactions on Acoustics, Speech and Signal Processing*. 1989; 29(4): 984-995.

[27] Roy R, Kailath T. ESPRIT- Estimation of Signal Parameter via Rotational Invariance Technique. *Optical Engineering*. 1990; 29(4): 296-313.