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Three-dimensional analysis of infiltration from the disc infiltrometer
2. Physically based infiltration equation

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Abstract. In situ measurement of soil hydraulic properties may be achieved by analyzing the unconfined efflux from disc tension infiltrometers, once consistent infiltration equations can be derived. In this paper an analytical, three-dimensional infiltration equation is developed, based on the use of parameters with sound physical meaning and adjustable for varying initial and boundary conditions. The equation is valid over the entire time range. For practical purposes, a simplified solution is also derived. The full and simplified equations give excellent agreement with published experimental results and are particularly useful for determining soil hydraulic properties through application of inverse procedures.

Introduction

In the first paper of this series [*Smettem et al.*, this issue] dealing with infiltration from a disc infiltrometer, three-dimensional cumulative efflux of water from the disc source was analyzed, and the following equation was derived:

$$i_{3D} = i_{1D} + \pi r_d (0.3)^{1/2} \hat{S}_0^2 t / (\theta_0 - \theta_n) \tag{1}$$

where the subscripts 3D and 1D refer to the three-dimensional unconfined cumulative infiltration (L^3) and the one-dimensional cumulative infiltration flow (L^3), respectively; r_d is the disc radius (L); θ_0 is the volumetric water content (L^3L^{-3}) corresponding with the supply potential h_0, (L); θ_n is the initial volumetric water content; and $\hat{S}_0$ is an approximation to the sorptivity ($LT^{-1/2}$) given by the equation

$$\hat{S}_0^2 = 2(\theta_0 - \theta_n) \int_{\theta_n}^{\theta_0} D(\theta)\, d\theta \tag{2}$$

where $D = K dh/d\theta$ is the diffusivity (L^2T^{-1}), K is the hydraulic conductivity (LT^{-1}), and h is the soil water pressure head (L).

A more precise expression of sorptivity S_0 can be given by [*Parlange*, 1975]

$$S_0^2 = \int_{\theta_n}^{\theta_0} (\theta_0 + \theta - 2\theta_n) D(\theta)\, d\theta \tag{3}$$

The obvious and practically important feature of (1) is that the difference between three-dimensional and one-dimensional flow is linear in time t and independent of gravity. Although the linearity with time was confirmed for short times by the experiments described by *Smettem et al.* [this issue], the proportionality constant, $(0.3)^{1/2}$, was shown to be too small. A more favorable comparison between experimental and analytical results was obtained for a proportionality constant of 0.75. This underprediction of the analytical solution can be explained partly by the fact that the effect of disc curvature was neglected for the three-dimensional analysis and partly by the fact that a simple, first-order wetting front assumption was used for (1), together with the simplified estimation of sorptivity by $\hat{S}_0$.

The aim of this paper is to extend the theory developed by *Smettem et al.* [this issue] in order to derive a physically based equation for infiltration from the disc infiltrometer that is valid for all times and accounts for corrections of the proportionality constant $(0.3)^{1/2}$. In addition, we derive a simplified infiltration equation for practical use.

Theory

Writing (1) in terms of infiltration per unit area, we obtain

$$I_{3D} = I_{1D} + \frac{\gamma}{r_d} \frac{S_0^2}{(\theta_0 - \theta_n)} t \tag{4}$$

where I is the cumulative infiltration (L^3L^{-2}), S_0 is the sorptivity given by (3), and γ is the proportionality constant corrected for the use of simplified wetting front, sorptivity, and gravity assumptions. Note that (1) predicts a minimum value of $\gamma = (0.3)^{1/2}$ when $S_0^2 = \hat{S}_0^2$.

For unsaturated conditions, the one-dimensional infiltration can be expressed in the quasi-exact analytical form [*Parlange et al.*, 1982]:

$$\frac{2(K_0 - K_n)^2}{S_0^2} t = \frac{2}{1-\beta} \frac{(K_0 - K_n)(I_{1D} - K_n t)}{S_0^2} - \frac{1}{1-\beta} \cdot \ln \{[\exp(2\beta(K_0 - K_n)(I_{1D} - K_n t)/S_0^2) + \beta - 1]\}(\beta)^{-1} \tag{5}$$

where K_0 and K_n are the hydraulic conductivity values corresponding to θ_0 and θ_n, respectively, and β is a shape

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constant introduced through the expression [*Haverkamp et al.*, 1990]

$$\frac{K - K_n}{K_0 - K_n} \cong \frac{\theta - \theta_n}{\theta_0 - \theta_n}\left[1 - \frac{2\beta(\theta_0 - \theta_n)}{S_0^2}\int_\theta^{\theta_0} D(\bar{\theta})\, d\bar{\theta}\right] \quad (6)$$

where $\bar{\theta}$ is a variable of integration.

Although (6) is exact only if $dK/D\, d\theta$ is a constant equal to $2\beta(K_0 - K_n)(\theta_0 - \theta_n)/S_0^2$ (i.e., when the conductivity behaves like an exponential function of pressure head [*Gardner*, 1958]), it is claimed to represent K very well over the pressure head ranges of interest in soil water studies [*Philip*, 1969].

If β is considered as an integral shape constant and calculated through a cumulative infiltration identification analysis, rather than from (6), then the argument as to how well (6) represents the hydraulic conductivity function is less important. Referring to a recent paper by *Fuentes et al.* [1992], which derived an analytical expression of the second term of the infiltration time series solution independent of any particular choice for the soil characteristic equations, the shape parameter β can be calculated by

$$\beta = 2 - 2 \cdot \frac{\int_{\theta_n}^{\theta_0} (K - K_n/K_0 - K_n)(\theta_0 - \theta_n/\theta - \theta_n)\, D(\theta)\, d\theta}{\int_{\theta_n}^{\theta_0} D(\theta)\, d\theta} \quad (7)$$

substitution of (5) into (4) yields a three-dimensional infiltration equation for a disc in implicit form:

$$2\,\frac{(K_0 - K_n)^2}{S_0^2}\, t = \frac{2}{1-\beta}\,\frac{K_0 - K_n}{S_0^2} \cdot \{I_{3D} - K_n t - [\gamma S_0^2/r_d(\theta_0 - \theta_n)]t\}$$
$$- \frac{1}{1-\beta} \ln \{\{\exp [2\beta(K_0 - K_n)/S_0^2] \cdot [I_{3D} - K_n t - (\gamma S_0^2/r_d(\theta_0 - \theta_n))t]] + \beta - 1\}\{\beta\}^{-1}\} \quad (8)$$

In spite of its relative complexity, (8) has the advantage of being valid for the entire time range from $t = 0$ to $t \to \infty$. However, since disc infiltrometer experiments do not require very long time ranges of application, we now consider a simplified but highly accurate expansion of (8). This expansion is of particular interest when we consider inverse procedures for characterizing soil hydrodynamic properties.

Simplified Time Expansion

In a first stage we differentiate (8) in time to obtain the following flux dependent equation:

$$I_{3D} - K_n t - \frac{\gamma S_0^2 t}{r_d(\theta_0 - \theta_n)} = \frac{S_0^2}{2\beta(K_0 - K_n)} \cdot \ln\left\{1 + \frac{\beta(K_0 - K_n)}{q_{3D} - K_0 - [\gamma S_0^2/r_d(\theta_0 - \theta_n)]}\right\} \quad (9)$$

with the flux (LT^{-1}) at the soil surface defined by

$$q_{3D} = (d/dt)(I_{3D}) \quad (10)$$

Substituting a series expansion for the term in ln into (9) and simplifying, (9) can be expressed in a time series form (see the appendix):

$$I_{3D} = S_0 t^{1/2} + \left[K_n + \frac{\gamma S_0^2}{r_d(\theta_0 - \theta_n)} + \tfrac{1}{3}(K_0 - K_n)(2 - \beta)\right]t \quad (11)$$

which is identical to one-dimensional infiltration with a supplementary term proportional to time t, accounting for the geometrical three-dimensional effects. Even though this equation loses precision at longer times, it has the advantage of being a simple, explicit three-dimensional infiltration equation.

Short Time Behavior

As time $t \to 0$, (11) takes the form

$$I_{3D} = S_0 t^{1/2} \quad (12)$$

Since this short time behavior is often used in field experiments for estimating sorptivity S_0 from the slope of I_{3D} versus $t^{1/2}$ [*White et al.*, 1992], it can be important to estimate the time over which the short time relation is valid. Expressing the second term of (11) as a small percentage (ε%) of the square root of time term, the time t^* for which $S_0 t^{1/2}$ is $(100 - \varepsilon)$% of the three-dimensional infiltration is given approximately ($K_n \approx 0$) by

$$t^* = \frac{\varepsilon^2 S_0^2}{\{[\gamma S_0^2/r_d(\theta_0 - \theta_n)] + (1/3)K_0(2 - \beta)\}^2 10^4} \quad (13)$$

As the sorptive term in the denomination of (13) is bigger than the gravity term, it is evident that the short time behavior $I_{3D} = S_0 t^{1/2}$ may break down far more quickly than for the case of one-dimensional flow. It also becomes questionable whether the routine method of estimating S_0 may still be applied without generating errors in the value of S_0 as found experimentally by *Thony et al.* [1991].

Taking as an example the soil and infiltration case for *Smettem et al.* [this issue], $S_0 t^{1/2}$ represents only 90% of I_{3D} after 58 s and only 80% after 230 s.

Long Time Expansion

The derivation of the long time behavior of the three-dimensional infiltration equation (8) is straightforward. Considering $(\beta - 1)$ as negligible when compared to the term in exp for times tending towards infinity, the term in ln can be simplified considerably, yielding the expression

$$I_{3D} = \left[K_0 + \frac{\gamma S_0^2}{r_d(\theta_0 - \theta_n)}\right]t + \frac{S_0^2}{2(K_0 - K_n)(1 - \beta)} \ln\left(\frac{1}{\beta}\right) \quad (14)$$

This equation shows that $I_{3D} - [K_0 + \gamma^2 S_0^2/r_d(\theta_0 - \theta_n)]t$ approaches a constant as $t \to \infty$, which is perfectly consistent with *Wooding*'s [1968] steady state analysis, which indicates at the same time the existence of a water content profile at infinite time.

Differentiating (14) in time gives the steady state infiltration flux as $t \to \infty$:

$$q_{3D} = K_0 + \frac{\gamma S_0^2}{r_d(\theta_0 - \theta_n)} \tag{15}$$

which contains the as yet unknown constant γ.

Proportionality Constant γ

The proportionality constant γ was introduced to correct the earlier value $(0.3)^{1/2}$ (equation (1)) for possible gravity effects on the profile edges, as well as for the somewhat crude approximations for the water content profile and sorptivity used in the initial derivation of the equation.

From experimental measurement an optimal value of $\gamma = 0.75$ was found. This result is compared with some values obtained for particular soil conditions.

The most evident γ value can be derived for the case of steep water content profiles and a diffusivity with delta function behavior [see *Smettem et al.*, this issue]. For this case we obtain

$$\gamma = (0.3)^{1/2}\hat{S}_0^2/S_0^2 \tag{16}$$

Depending on the type of soil the fraction, $\hat{S}_0^2/S_0^2$ is approximately 1.06–1.12, which increases the value of γ to $\gamma \cong 0.6$. However, this correction is only for the sorptivity estimation; gravity corrections may give a further increase in the value of γ. Note that the ratio $\hat{S}_0^2/S_0^2$ is twice the capillary scale parameter b, discussed by *White and Sully* [1987], who gave a reasonable estimate of $b = 0.55$ for four different soils tested. However, it should be remembered that the ratio (and thus b) is a function not only of soil type, but also of initial (θ_n) and boundary (θ_0) conditions.

Once the lower limit of γ has been fixed, the upper limit can be approached by considering the behavior of a "linear" soil. Neglecting gravity effects in (11), we have

$$I_{3D} = S_0 t^{1/2} + \frac{\gamma S_0^2}{r_d(\theta_0 - \theta_n)} t \tag{17}$$

This can be compared with the particular short time solution given by *Chu et al.* [1975] which is relevant for two-dimensional edge effects in linear heat diffusion. Using a constant diffusion coefficient D^*, an equivalent flux expression was developed by *Warrick* [1992]:

$$q_{3D} \cong 0.5 S_0 t^{-1/2} + 0.885 S_0 (D^*)^{1/2}/r_d \tag{18}$$

Comparing (17) and (18) together with the sorptivity equation for constant linear diffusivity [*Philip*, 1969]

$$D^* = \pi S_0^2/4(\theta_0 - \theta_n)^2 \tag{19}$$

gives a value of $\gamma = 0.784$. Although this value is remarkably close to the optimal value obtained from the experimental results, we have still not corrected for gravity effects on the profile edges. Thus the upper limit of γ may be slightly greater than 0.78. An alternative possibility for estimating the value of γ is through the long time steady state flow equation (equation (15)).

Wooding [1968] presented a full solution for steady state flow from a shallow circular pond for soils with exponential conductivity behavior [*Gardner*, 1958]:

$$K = K_S \exp(\alpha h) \tag{20}$$

with α a constant > 0. The simplified form of the *Wooding* [1968] solution is given by

$$q_{3D} = K_0 + 4\phi_0/\pi r_d \tag{21}$$

with

$$\phi_0 = \int_{h_n}^{h_0} K(h)\, dh = \int_{\theta_n}^{\theta_0} D(\theta)\, d\theta \tag{22}$$

or, after combining (2), (20), and (22)

$$\phi_0 = \frac{K_0 - K_n}{\alpha} = \frac{\hat{S}_0^2}{2(\theta_0 - \theta_n)} \tag{23}$$

Comparison of (15) and (23) gives

$$\gamma = \frac{2}{\pi}\frac{\hat{S}_0^2}{S_0^2} \tag{24}$$

which is close to $\gamma = 0.7$, depending on the ratio of $\hat{S}_0^2/S_0^2$ (see (16)).

Wooding [1968] expressed his steady state flux in a dimensionless form by use of the scale parameter:

$$a = \alpha r_d/2 \tag{25}$$

which can be expressed in a more general form, independent of the α parameter as

$$a = \frac{r_d(K_0 - K_n)(\theta_0 - \theta_n)}{\hat{S}_0^2} \tag{26}$$

On careful examination, *Weir* [1987] found the simple approximation of (21) to be somewhat imprecise; he proposed an alternative approximation valid for the realistic range of values: $a < 0.4$, which increases the steady state flux of (21) by about 6%. This was

$$q_{3D} = (K_0 - K_n)\{\{2\sin^2(a)\}\{a^2\pi \sin(a)\cos(a) + 2a^2\sin^2(a)\ln(a) - 1.073a^4\}^{-1}\} \tag{27}$$

The corresponding γ becomes

$$\gamma \cong \frac{\hat{S}_0^2}{S_0^2}\{\{2\sin^2(a)\}\{a\pi\sin(a)\cos(a) + 2a\sin^2(a)\ln(a) - 1.073a^3\}^{-1} - a\} \tag{28}$$

which makes the value of γ vary between

$$0.751 < \gamma < 0.792 \tag{29}$$

for the range of scale factors $0.1 < a < 0.4$ and $\hat{S}_0^2/S_0^2 \cong 1.1$. Moreover, combination of (26) and (28) clearly shows that for increasing radius, but identical initial and boundary conditions, the value of γ increases slightly.

As a result, we may conclude that from both "limited time" and "long time" analyses, the value of γ for normal working conditions is reasonably bounded by

$$0.6 < \gamma < 0.8 \tag{30}$$

Furthermore, this value agrees with the experimental results reported in the first paper [*Smettem et al.*, this issue].

A more detailed analysis of the estimation of γ in relation to soil, initial, and boundary conditions will be presented in a subsequent paper of this series.

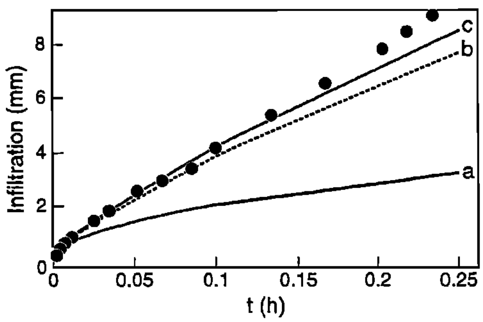

Figure 1. Progressive contribution of each component to the full three-dimensional infiltration equation as compared with experimental data (circles) of *Quadri et al.* [1994]: Line a, equation (12); line b, equation (17); line c, equation (11).

Results

Simulations were performed using the experimental results of *Quadri et al.* [1994] for the C horizon of Manawatu fine sandy loam. These experiments were conducted with a box of repacked sandy soil using a disk of 60 mm radius at a supply potential of −50 mm H_2O. The initial water content θ_n was 0.09 m^3 m^{-3}, and the water content at the supply potential θ_0 was 0.375 m^3 m^{-3}. Additional measured hydraulic parameters were sorptivity, $S_0 = 63.2$ mm $h^{-1/2}$, hydraulic conductivity at the supply potential, $K_0 = 72$ mm h^{-1}, and hydraulic conductivity at the initial water content, K_n, assumed to be $K_n = 0$. The value of β was calculated through (7) as 0.563 and the value of γ was obtained from *Smettem et al.* [this issue] as 0.75. The experimental I_{3D} data were obtained from Figure 6 of *Quadri et al.* [1994]. In viewing these data the reader should bear in mind that from Figure 4 of *Quadri et al.* [1994], the experimental data appeared high relative to both the numerical simulations and *Wooding*'s [1968] equation, presumably due to experimental errors.

Figure 1 compares the behavior of the full infiltration equation (8) and the simplified time equation (16) with experimental data over the experimental time range of $t <$ 900 s. Agreement between experimental and simulated results shows that (8) gives an excellent description of three-dimensional infiltration from a disc infiltrometer. The simplified equation (11) is as accurate as the full equation over the time period of this experiment.

The fact that the parameters entering the three-dimensional infiltration equation have a sound physical meaning and can be adjusted (where necessary) for varying initial and boundary conditions makes (11) particularly suitable for predictive purposes and also for possible characterization of hydraulic soil properties through inverse procedures.

In Figure 1 we also show the progressive contribution of each infiltration component entering (11), namely, (1) the short time expansion (equation (12)), (2) the influence of sorptive flow as given by (17), and (3) the full equation as given by (11).

Comparing points 2 and 3 above, the relative importance of the sorptive term proportional in time t with respect to the gravity term is clearly demonstrated. Comparison of points 1 and 2 illustrates the danger of using only the short time expansion (equation (12)) for determination of sorptivity for three-dimensional flow experiments.

To compare the validity of the different infiltration equations for longer times, we extended the simulation to time $t = t_{grav}$ with

$$t_{grav} = (S_0/K_0)^2 \tag{31}$$

being an estimation of the time at which gravity begins to dominate the flow process [*Philip*, 1969]. It can often be considered as a reasonable time limit beyond which disc-infiltration experiments should not be carried out, since the assumptions of uniform initial conditions and homogeneous soil properties are likely to be invalid.

Solid line a of Figure 2 represents the relative contribution of the short time term $S_0 t^{1/2}$ to the total three-dimensional infiltration flow as a function of time. Again, we note the rapid decline in the influence of $S_0 t^{1/2}$ over times that may often be used to obtain estimates of S_0 from the slope of I_{3D} at early times.

Dashed line b in Figure 2 shows the validity of the limited time expansion (equation (11)) as compared with the full three-dimensional infiltration equation (8) over long time periods. It clearly shows that the simplified equation can be used for three-dimensional disc infiltrometer experiments without serious loss of precision.

Conclusions

A three-dimensional infiltration equation for disc infiltrometers has been developed, together with a simplified, limited time solution. Both equations give excellent agreement with experimental results, showing that the equations are appropriate for describing three-dimensional flow from disc infiltrometers.

The parameters entering the equations have a sound physical meaning and can be adjusted (where necessary) for varying initial and boundary conditions. This presents the possibility of using the equations to obtain soil hydraulic properties through application of inverse procedures. This approach will be explored in a subsequent paper. A correction constant, γ, is introduced to account for gravity effects at the edges of the water content profiles. A theoretical estimate of the correction constant has been presented and in a following paper will be the subject of further analysis for

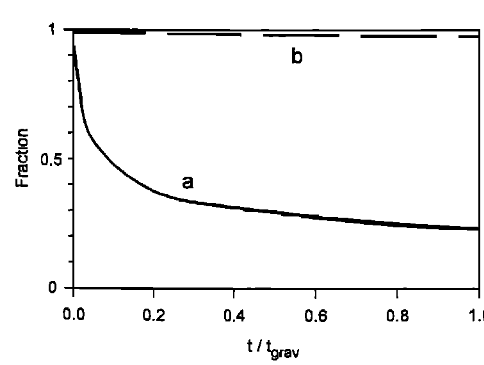

Figure 2. Relative contribution of the short time $St^{1/2}$ (line a) and of the limited time expansion (equation (11)) to total three-dimensional infiltration flow as a function of time (line b).

different soils and/or changing initial and boundary conditions disc radii and times.

It is shown that the field determination of sorptivity from the slope of cumulative infiltration (I_{3D}) versus square root of time ($t^{1/2}$) is questionable, since the validity of the short time expansion $I_{3D} = S_0 t^{1/2}$ may only be maintained for very short times, much shorter than for one-dimensional infiltration flow.

Appendix

Differentiating (9) with respect to time t we obtain

$$q_{3D} - K_n - g = -[S_0^2][2(q_{3D} - K_0 - g) \cdot [q_{3D} - K_0 - g + \beta(K_0 - K_n)]]^{-1} \, dq_{3D}/dt \quad \text{(A1)}$$

where for convenience, $g = \gamma S_0^2 / r_d(\theta_0 - \theta_n)$.

Writing cumulative three-dimensional infiltration as a power series in $t^{1/2}$.

$$I_{3D} = A_1 t^{1/2} + A_2 t + A_3 t^{3/2} + \cdots \quad \text{(A2)}$$

we find the series for q_{3D} and dq_{3D}/dt by differentiation. From (A1),

$$2(q_{3D} - K_n - g)(q_{3D} - K_0 - g)[q_{3D} - K_0 - g + \beta(K_0 - K_n)] = -S_0^2 \frac{dq_{3D}}{dt} \quad \text{(A3)}$$

and by expanding and equating coefficients of $t^{-3/2}$, t^{-1}, $t^{-1/2}$ etc. we obtain A_1, A_2, A_3, etc. Equating coefficients of $t^{-3/2}$, we find $A_1^{3/4} = S_0^2 A_1/4$, i.e., $A_1 = S_0$.

Equating coefficients of t^{-1}, we find

$$(A_1/2)(A_1/2)[A_2 - K_0 - g + \beta(K_0 - K_n)] + (A_1/2)(A_2 - K_0 - g)(A_1/2) + (A_2 - K_n - g)(A_1/2)(A_1/2) = 0 \quad \text{(A4)}$$

which gives $A_2 = K_n + g + (2 - \beta)(K_0 - K_n)/3$.

Equation (A2) then yields the simple, explicit three-dimensional infiltration equation

$$I_{3D} = S_0 t^{1/2} + \left[K_n + \frac{\gamma S_0^2}{r_d(\theta_0 - \theta_n)} + \tfrac{1}{3}(2 - \beta)(K_0 - K_n) \right] t \quad \text{(A5)}$$

which is only valid for limited times. To increase the precision, additional terms of the expansion could be included by identifying A_3, and A_4 etc. However, as is shown in this paper, use of a time series with only two terms seems to be sufficiently precise for disc infiltrometer experiments under field conditions.

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