Three-Dimensional Geometry and Topology

VOLUME 1

William P. Thurston

EDITED BY SILVIO LEVY

PRINCETON UNIVERSITY PRESS
PRINCETON, NEW JERSEY
1997

Contents

Preface					
Reader's Advisory					
1	Wh	at Is a Manifold?	3		
	1.1	Polygons and Surfaces	4		
	1.2	Hyperbolic Surfaces	7		
	1.3	The Totality of Surfaces	17		
	1.4	Some Three-Manifolds	31		
2	Нур	perbolic Geometry and Its Friends	43		
	2.1	Negatively Curved Surfaces in Space	45		
	2.2	The Inversive Models	53		
	2.3	The Hyperboloid Model and the Klein Model	64		
	2.4	Some Computations in Hyperbolic Space	74		
	2.5	Hyperbolic Isometries	86		
	2.6	Complex Coordinates for Hyperbolic Three-Space	98		
	2.7	The Geometry of the Three-Sphere	103		
3	Geo	metric Manifolds	109		
	3.1	Basic Definitions	109		
•	3.2	Triangulations and Gluings	118		
	3.3	Geometric Structures on Manifolds	125		
	3.4	The Developing Map and Completeness	139		
	3.5	Discrete Groups	153		
	3.6	Bundles and Connections	158		
	3.7	Contact Structures	168		
	3.8	The Eight Model Geometries	179		
	3.9	Piecewise Linear Manifolds	190		
	3.10	Smoothings	193		

4	The	Structure of Discrete Groups	2	09	
	4.1	Groups Generated by Small Elements	. 2	209	
	4.2	Euclidean Manifolds and Crystallographic Groups .	. 2	221	
	4.3	Three-Dimensional Euclidean Manifolds	. 2	231	
	4.4	Elliptic Three-Manifolds	. 2	42	
	4.5	The Thick-Thin Decomposition	. 2	253	
	4.6	Teichmüller Space	. 2	258	
	4.7	Three-Manifolds Modeled on Fibered Geometries $\ .$.	. 2	277	
Glossary				89	
Bibliography				95	
Tn	Indov				