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Three-dimensional numerical simulation of gaseous flow structure in semidetached binaries

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ABSTRACT

The results of numerical simulation of mass transfer in semidetached non-magnetic binaries are presented. We investigate the morphology of gaseous flows on the basis of three-dimensional gas-dynamical calculations of interacting binaries of different types (cataclysmic variables and low-mass X-ray binaries).

We find that taking into account a circumbinary envelope leads to significant changes in the stream–disc morphology. In particular, the obtained steady-state self-consistent solutions show an absence of impact between the gas stream from the inner Lagrangian point L_1 and the forming accretion disc. The stream deviates under the action of the gas of the circumbinary envelope, and does not cause the shock perturbation of the disc boundary (traditional hotspot). At the same time, the gas of the circumbinary envelope interacts with the stream and causes the formation of an extended shock wave, located on the stream edge. We discuss the implication of this model without hotspot (but with a shock wave located outside the disc) for interpretation of the observations. The comparison of synthetic light curves with observations proves the validity of the discussed gas-dynamical model without hotspot.

We have also considered the influence of the circumbinary envelope on the mass transfer rate in semidetached binaries. The obtained features of flow structure in the vicinity of L_1 show that the gas of the circumbinary envelope plays an important role in the flow dynamics, and that it leads to significant (in order of magnitude) increase of the mass transfer rate. The most important contribution to this increase is from the stripping of the mass-losing star atmosphere by interstellar gas flows.

The parameters of the formed accretion disc are also given in the paper. We discuss the details of the obtained gaseous flow structure for different boundary conditions on the surface of mass-losing star, and show that the main features of this structure in semidetached binaries are the same for different cases.

The comparison of gaseous flow structure obtained in two- and three-dimensional approaches is presented. We discuss the common features of the flow structures and the possible reasons for revealed differences.

Key words: accretion, accretion discs – hydrodynamics – methods: numerical – binaries: close – stars: mass-loss.

1 INTRODUCTION

The semidetached binaries belong to the class of interacting stars, where one component fills its critical surface, which causes mass transfer between the components of the system. In general, the form of the critical surface may be complex (Kruszewski 1963) and special mathematical models are required to describe the process of mass transfer in such a system (see the review of these models in Lubow 1993). However, in the standard treatment, semidetached binaries are considered under the assumption that orbits of the components are circular and their rotation is synchronous with the orbital movement. In this case the critical surface can be identified with internal surface (Roche surface) in the restricted three-body problem and it is assumed that mass transfer between the

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components of the system occurs through the vicinity of the inner Lagrangian point L_1, where pressure gradient is not balanced by gravitational force.

The gas dynamics of mass transfer through the inner Lagrangian point L_1 has been investigated by many authors. The detailed analysis of matter flow in the vicinity of L_1 was carried out by Lubow & Shu (1975). Using a perturbation method they evaluated main characteristics of the flow. In another approach, based on the analysis of the Bernoulli integral, the stream parameters were specified as well and the dependence of the mass transfer rate upon the degree of Roche lobe overfilling was obtained (Paczyński & Sienkiewicz 1972; Savonije 1978).

For adequate description of the the mass transfer process in a binary system, besides determination of stream parameters it is also necessary to consider the further behaviour of flow lines during movement of matter from L_1. It is the process of mass that generates the general flow structure and, accordingly, determines the basic observational evidence; therefore most attention was paid to study this question. For the first time movement of particles leaving L_1 and moving in the gravitational field of the binary system was considered by Warner & Peters (1972), Lubow & Shu (1975) and Flannery (1975). These results were obtained using a simplified ballistic approach for analysis of the gas movement without taking into account gas-dynamical effects. To study the influence of the circumbinary envelope on gas movement and, accordingly, for a correct description of the flow, the solving of full system of gas-dynamical equations is required. This is possible only in the framework of rather complex mathematical models.

The use of numerical methods for investigation of gas dynamics of mass transfer in semidetached binaries was limited by the computer power for a long time, so two-dimensional (2D) models were used for the analysis of the flow structure. Despite the restrictions of the 2D approach, it allowed one to consider some details of the flow structure correctly and to obtain a set of interesting results (see, e.g., Sawada, Matsuda & Hachisu 1986; Sawada et al. 1987; Taam, Fu & Fryxell 1991; Blondin, Richards & Malinowski 1995; Murray 1996). More recently the possibility of numerical gas-dynamical simulation of mass transfer in the framework of more realistic three-dimensional (3D) models (Nagasawa, Matsuda & Kuwahara 1991; Hirose, Osaki & Minishige 1991; Molteni, Belvedere & Lanzafame 1991; Sawada & Matsuda 1992; Lanzafame, Belvedere & Molteni 1992, 1994; Belvedere, Lanzafame & Molteni 1993; Meglicki, Wickramasinghe & Bicknell 1993; Armitage & Livio 1996) has appeared. In particular, these authors considered the formation of an accreting disc in semidetached binaries (Nagasawa et al. 1991; Sawada & Matsuda 1992) and investigated the interaction of a stream of matter leaving L_1 with the disc (Hirose et al. 1991; Armitage & Livio 1996).

Unfortunately, many 3D investigations were carried out during a rather short time-scale and this fact has not allowed one to consider the real flow morphology, accordingly, to evaluate the influence of forming circumbinary envelope on the flow structure. Some progress in the investigation of general flow structure in semidetached binaries was achieved (Molteni et al. 1991; Lanzafame et al. 1992, 1994; Belvedere et al. 1993), where 3D numerical simulations were carried out on the sufficiently large time intervals. A set of interesting results was obtained in these works. However using the SPH (smoothed particle hydrodynamics) method has not allowed one to consider the influence of the circumbinary envelope on the flow structure as the computational restrictions of SPH method did not permit one to investigate flows with large density gradients, and, accordingly, the account of the influence of circumbinary envelope on the mass transfer was not quite correct. For the first time the morphology of gaseous flows in binaries was accurately considered by Bisikalo et al. (1997a,b).

In this work we present the results of 3D numerical study of the flow structure in semidetached non-magnetic binaries. The TVD (total variation diminishing) method of solving of gas-dynamical equations used in this paper has allowed us to investigate the morphology of gaseous flows in the system and to consider the influence of forming circumbinary envelope, despite the presence of significant density gradients. The numerical simulations of mass transfer in semidetached binaries have been conducted on large time intervals that allowed us to consider the main features of flow structure in steady-state regime. The earlier conclusions on the flow structure for low-mass X-ray binaries (Bisikalo et al. 1997a,b) are generalized in the present work for the wider class of objects.

In Section 2, the properties of used physical, mathematical and numerical models are described. Section 3 contains the results of numerical simulations. In this Section the stream–disc interaction, comparison of synthetic light curves with observations, flow structure in the vicinity of L_1, influence of accepted boundary conditions, and comparison of the results obtained in 2D and 3D models are discussed. Our conclusions follow in Section 4.

2 THE MODEL

2.1 Physical model

The semidetached binaries such as cataclysmic variables (CVs), low-mass X-ray binaries (LMXBs) and supersoft X-ray sources (SSS) show a lot of interesting observational evidence. The observations of CVs light curves and X-ray light curves of LMXBs offer strong evidence of a complex flow structure in these systems and they allow one to make assumptions on the structure of gaseous flows. In particular, in some cataclysmic binaries, the most well studied of which is Z Cha, the complex picture of eclipse ('double eclipse') is observed (see, e.g., Hack & La Dous 1993; Cherepashchuk et al. 1996). For its explanation the hypothesis of a hotspot in interaction zone between the stream and the disc outer edge was suggested (Smak 1970). For a number of X-ray sources such as X1822–371 ('dipping' sources), a significant decrease of the radiation flux at orbital phase 0.8 is observed. It is explained by a bulge in the accretion disc created by impact of the stream with the disc (White & Holt 1982; Mason 1989; Armitage & Livio 1996).

Doubtless, the presence of such appreciable observational evidence of the complex flow structure in semidetached binaries requires a detailed study of the gas flow. In this work the investigations of the flow structure are carried out for two types of semidetached binaries: (1) with parameters typical for LMXB parameters, where the main-sequence dwarf fills its Roche lobe and transfers matter to the neutron star, and (2) with parameters typical for CVs, where the accreting star is a white dwarf. It is assumed in our model that magnetic field is negligible and does not influence the gas flow in the systems considered.

For the numerical simulations we use systems with parameters similar to those for X1822–371 (Armitage & Livio 1996), and for Z Cha (Goncharskij, Cherepashchuk & Yagola 1985). For the first system it was assumed that the mass-losing component has mass M_1 equal to 0.28 $M_\odot$; the gas temperature on the surface is $T = 10^4$ K; the mass of the compact star is $M_2 = 1.4$ $M_\odot$; the orbital period of the system is $P_{orb} = 1.78$ d; and the distance between centres of components is $A = 7.35$ $R_\odot$. For Z Cha the following parameters are adopted: the mass of mass-losing red dwarf is $M_1 = 0.19$ $M_\odot$;

the temperature of gas on the surface is $T = 5 \times 10^3$ K; the mass of the compact star (white dwarf) is $M_2 = 0.94\ \mathrm{M}_\odot$; the orbital period of the system is $P_{\mathrm{orb}} = 0.074$ d; and $A = 0.78\ \mathrm{R}_\odot$. We suppose that mass-losing stars in both models fill their Roche lobes. The accretor radius for Z Cha is adopted as $R_2 = 0.009\ \mathrm{R}_\odot$, that is equal to the radius of a typical white dwarf. For X1822–371 where the accretor is a neutron star, the computer facilities do not permit us to resolve the real star size, therefore we adopt the accretor radius equal to $R_2 = 0.05\ \mathrm{R}_\odot$. It should be noted that the larger accretor radius adopted for the LMXB case does not influence the flow structure at distances larger than R_2, therefore the obtained results are correct for all the calculated region except for a small zone around the neutron star ($r \le R_2$).

For the adequate description of the flow structure it is necessary to take into account the influence of radiative processes on gas dynamics. The accurate consideration of non-adiabatic processes in the numerical model increases the time of calculations significantly. Therefore, taking into account our computing environment and the necessity of carrying out calculations at large time-scales for getting a the steady-state solution, we simplify the model by using the adiabatic gas-dynamics and an ideal gas equation of state $P = (\gamma - 1)\rho\varepsilon$ (P – pressure, ρ – density, ε – specific internal energy). To consider the energy losses the ratio of specific heats was assumed to be close to 1, $\gamma = 1.01$, which corresponds to the near-isothermal case (Landau & Lifshitz 1959). Such a technique for taking into account the radiative losses is well known and is used in practice repeatedly (see, e.g., Sawada et al. 1986, 1987; Spruit et al. 1987; Molteni et al. 1991; Matsuda et al. 1992; Bisikalo et al. 1995).

2.2 Mathematical model

To describe the gas flow, the system of 3D gas-dynamical equations in integrated form and an ideal gas equation of state with the ratio of specific heats $\gamma = 1.01$ is used. The system of equations for the fluid element with volume V and surface Σ has the following form:

$$\frac{\partial}{\partial t}\int_V \rho \mathrm{d}V + \int_\Sigma \rho(\boldsymbol{v}\cdot\boldsymbol{n})\mathrm{d}\Sigma = 0,$$

$$\frac{\partial}{\partial t}\int_V \rho\boldsymbol{v} \mathrm{d}V + \int_\Sigma \rho\boldsymbol{v}(\boldsymbol{v}\cdot\boldsymbol{n})\mathrm{d}\Sigma + \int_\Sigma P\boldsymbol{n}\mathrm{d}\Sigma = \int_V \rho\boldsymbol{F}\mathrm{d}V,$$

$$\frac{\partial}{\partial t}\int_V \rho E \mathrm{d}V + \int_\Sigma \rho h(\boldsymbol{v}\cdot\boldsymbol{n})\mathrm{d}\Sigma = \int_V \rho(\boldsymbol{F}\cdot\boldsymbol{v})\mathrm{d}V\,.$$

The specific external force $\boldsymbol{F}$ includes the Coriolis force and the gravitational force of the two masses and centrifugal force described by the Roche potential, and has the form:

$$\boldsymbol{F} = -\,\mathrm{grad}\,\Phi + 2[\boldsymbol{v}\times\boldsymbol{\Omega}],$$

where the Roche potential Φ can be written in the form:

$$\Phi(\boldsymbol{r}) = -\frac{GM_1}{|\boldsymbol{r}-\boldsymbol{r}_1|} - \frac{GM_2}{|\boldsymbol{r}-\boldsymbol{r}_2|} - \frac{1}{2}\Omega^2\left(\boldsymbol{r}-\boldsymbol{r}_\mathrm{c}\right)^2.$$

Here $\boldsymbol{v}$ – velocity; h – specific full enthalpy: $h = \varepsilon + |\boldsymbol{v}|^2/2 + P/\rho$; E – specific full energy: $E = \varepsilon + |\boldsymbol{v}|^2/2$; $\boldsymbol{\Omega} = (0, 0, \Omega)$; $\Omega = 2\pi/P_{\mathrm{orb}}$; $\boldsymbol{r}_1$, $\boldsymbol{r}_2$ – the centres of components of the system; and $\boldsymbol{r}_\mathrm{c}$ – the centre of mass of the system.

The boundary conditions on the surface of the mass-losing star are determined under the assumption that this star fills its Roche lobe. To build the boundary conditions on the Roche lobe we use a standard procedure of solving the Riemann problem between two regions – one on the surface of the star and the other corresponding to the nearest computational cell (see, e.g., Sawada et al. 1986; Sawada & Matsuda 1992). At the surface of the mass-losing star we choose the value of density $\rho = \rho_0$. We also guess that everywhere on the surface of this component the gas velocity is directed along the surface normal, and its value is fixed to be equal to the local sonic velocity $v = c_0$. For Z Cha we also consider the case when the gas velocity is fixed to be zero for all the surface of mass-losing star.

It should be noted that the boundary value of density on the surface of mass-losing star has no influence on the solution, because of the scaling of the system of equations with respect to ρ (with simultaneous scaling of P). So an arbitrary value of ρ_0 can be accepted in calculations. However, when considering the particular system with known mass-loss rate, to determine the real values it is necessary to increase the calculated values of density in accordance with a scale defined by the ratio of real value of density on the surface of the mass-losing star to the model one.

On the surface of the compact star and the outer numerical boundary the free outflow conditions are used.

As initial conditions low-density ambient matter ($\rho \sim 10^{-6}\rho_0$) in the rest in the rotational frame is accepted. Subsequently, this matter is forced out from the system by the gas injecting from mass-losing star.

2.3 Numerical model

The main question for numerical simulation of the gas-dynamical models is choosing the solving method and appropriate scheme for the system of equations. Among the large variety of finite-difference schemes the so-called Godunov-type schemes (Godunov 1959) are considered to be the most exact ones.

In the present work, we use the modification of explicit TVD Roe scheme (Roe 1986) for numerical solving of the system of gas-dynamical equations. The original scheme (first order of spatial approximation) is modified by monotonic flux limiters in Osher's form (Chakravarthy & Osher 1985), which makes the scheme of third order of approximation. The special model simulations show that the given scheme permits us to describe adequately the flow structure including shock waves and tangential discontinuities and does not result in artificial fluctuations and smearing of features of flow. Moreover the used scheme permits us to consider the flows with large density gradients, which are of special importance for consideration of the influence of the gas of the circumbinary envelope on the flow structure.

The system of gas-dynamical equations is solved in a Cartesian coordinate system, which is predetermined as follows:

(i) the zero of coordinate system is located in the centre of mass-losing star;
(ii) X axis is directed from the centre of mass-losing star to the accretor;
(iii) Z axis is directed along the axis of orbital rotation;
(iv) Y is determined to get a right-handed coordinate system.

The computation region is a parallelepipedon $(-A, 2A) \times (-A, A) \times (0, A)$ (owing to symmetry about the equatorial plane calculations were conducted only in the top half-space). Non-uniform difference grids (more fine near the accretor) containing $78 \times 60 \times 35$ grid points for the system X1822–371 and $84 \times 65 \times 33$ grid points for the system Z Cha is used.

The solution of the system of equations was carried out from initial conditions up to the steady-state regime. To check the establishment of the steady-state regime we have numerically monitored flow parameters (density and pressure) as a function of time. We have checked these parameters inside spheres around the accretor (with different radii) and inside the sphere closed to the outer boundary. When the flow patterns do not depend on the time we suppose that steady state is reached. To assure that the obtained solution is steady we have continued the calculations additionally during 3–5 orbital periods. The runs have been stopped at 12 orbital periods for X1822–371, and at 20 orbital periods for Z Cha. Characteristic time step in both runs is approximately 10^{-4} orbital period, so the total number of steps is $\sim 1.2 \times 10^5$ and $\sim 2 \times 10^5$ respectively. The runs were conducted on NEC ALPHA computer (AlphaStation 250 4/266), and CPU time per grid point was approximately equal to 8×10^{-5} s. The total CPU time for both runs is approximately 1 month.

3 RESULTS AND DISCUSSION

Let us consider the characteristic features of the flow structure in semidetached binaries, obtained in the framework of the 3D gas-dynamical model described in Section 2. As it was mentioned above, the calculations have been carried out for typical representatives LMXBs and CVs. The obtained results testify to the qualitatively similar nature of the flow in considered systems, which, in turn, permits us to establish the general character of the steady flow structures for semidetached non-magnetic binaries.

Taking into account the qualitative similarity of results, the general properties of flow structure will be described below for the X1822–371 system. The results obtained for the system Z Cha will be used to discuss the quantitative characteristics.

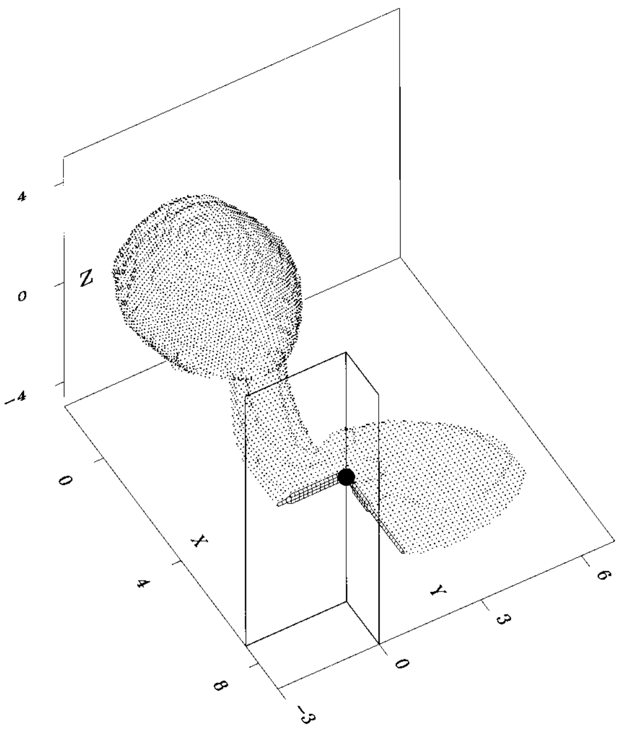

Figure 1. 3D view of density isosurface at the level $\rho = 0.005\rho_0$. The values of coordinates *X*, *Y* and *Z* are in units $R_\odot$. Accretor is marked by the filled circle. Cross-sections of density isosurface by planes *XZ* and *YZ* passing through the accretor are also shown.

3.1 Stream–disc interactions

The general structure of the gaseous flows, illustrating the morphology of mass transfer in the system X1822–371, is presented in Fig. 1, where 3D view of density isosurface at the level $0.005\rho_0$ is shown. The cross-sections of density isosurface by planes *XZ* and *YZ* passing through the accretor are also shown. The flow structure presented in Fig. 1 is steady-state and corresponds to a time exceeding 10 orbital periods. The analysis of presented results allows us to reveal the following features of the flow structure.

(i) The matter of the stream is redistributed into three parts: the first part forms a quasi-elliptic accretion disc; the second part moves around the accretor beyond the disc; the third part of the stream moves towards the external Lagrangian point L_2, then a fraction of this matter leaves the system, while a considerable amount of the gas changes its direction of motion through the action of the Coriolis force and comes back to the system.

(ii) The interaction between the stream and the disc is not a shock one.

(iii) The stream of matter moving from the vicinity of L_1 changes size as it spreads towards the accretor: the thickness of the stream decreases, and its width in the orbital plane increases.

(iv) The thickness of the accretion disc is smaller than the stream thickness.

A more detailed analysis of the structure of gaseous flows in the system and evaluation of linear sizes of the disc can be carried out for the flow structure in the equatorial plane. In Fig. 2 density isolines and velocity vectors in this plane for the region with dimensions from 2 to 10 $R_\odot$ on axis *X* and from −3 to 3 $R_\odot$ on axis *Y* are presented. In Fig. 2 four flow lines are shown as well, labelled by markers 'a', 'b', 'c' and 'd'. These flow lines illustrate the directions of matter flows in the system.

The analysis of results presented in Fig. 2 verifies the above conclusion that part of the matter in the stream falls into the disc at once (flow line 'd'), and then loses angular momentum under the action of numerical viscosity and accretes. The obtained

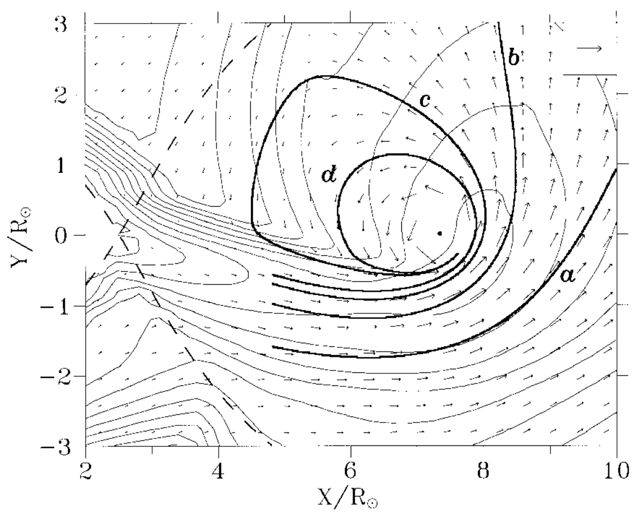

Figure 2. Density isolines and velocity vectors in the equatorial plane of the system. Roche equipotentials are shown by dashed lines. Four flow lines, labelled by markers 'a', 'b', 'c' and 'd' are also presented in this figure. The accretor is marked by the filled circle. Vector in the upper right corner corresponds to the value of velocity of 800 km s^{-1}.

quantitative evaluations show that in steady-state regime the fraction of accreted matter, for the given semidetached binary, is approximately equal to 75 per cent of the total amount of matter injected into the system.

The part of the matter that remains in the system and influences the flow structure (flow lines 'a', 'b' and 'c' in Fig. 2) is of special interest. Hereafter we shall name this part of the matter the *circumbinary envelope*. It should be noted that a significant part of the gas in the circumbinary envelope (see flow lines 'a' and 'b') interacts with the matter ejected from the surface of the mass-losing star. The influence of this part of the circumbinary envelope on the flow structure results in considerable change of the mass transfer regime. The detailed analysis of this effect will be presented in Section 3.3 of this paper. Another part of the circumbinary envelope (see flow line 'c') makes a revolution around the accretor and shocks the stream edge, facing the orbital movement. This interaction results in a significant change of the general flow structure in the system and, in particular, in absence of a hotspot in the disc, as well as to the formation of an extended shock wave, located along the stream edge. The detailed description of the influence of this part of circumbinary envelope on the morphology of gas flows in the system is presented below.

To estimate linear sizes of the disc it is necessary to find the marginal ('last') flow line along which the matter falls directly into the disc. Flow line 'd' in Fig. 2 is the marginal one and it is easy to determine sizes of the calculated quasi-elliptic accretion disc: $2.3 \times 2.0\ R_\odot$ ($0.31 \times 0.27A$). The thickness of the disc increases with increasing distance from the accretor and changes from $\sim$0.05 to $\sim$0.27 $R_\odot$ (or from 0.9 to 5.2 accretor radii). Similar evaluations of the parameters for quasi-elliptical disc using the marginal flow line is obtained for Z Cha system as well. For this system the forming steady-state disc has the size $0.25 \times 0.22\ R_\odot$ ($0.32 \times 0.28A$), and its thickness varies in the range from $\sim$0.006 to $\sim$0.04 $R_\odot$ (or from 0.7 to 4.5 accretor radii). It should be noted that for the considered types of systems, which have components with approximately equal mass ratios, but considerably different various characteristic parameters (separation A and orbital period), radii and disc thickness (adimensionalized using A) are approximately identical. Moreover, locations of discs in the relation to the accretor Roche lobe also coincide for different systems.

The conducted analysis shows that in all flow lines, belonging to the disc, up to the marginal flow line 'd' the flow is smooth. The absence of breaks indicates the shock-free interaction between the stream and matter of the disc. There is no a hotspot on the disc edge.

The presented flow structure shows that the stream deflects under the action of the gas of circumbinary envelope (flow line 'c' in Fig. 2), approaches the disc along a tangent line and does not cause any shock perturbation of the disc edge.

At the same time the analysis of results shows that the interaction between stream and circumbinary envelope results in formation of an extended shock wave, located along the stream edge turned towards orbital movement. The parameters of this shock wave, as well as the total energy production in it can be evaluated from Fig. 3. Here the normalized distribution of the energy production specific rate δE (erg s^{-1} cm^{-2}) along the shock wave in the equatorial plane is presented. In Fig. 3 the boundaries of the shock are shown by dashed lines: the line on the left shows the starting point of the stream, i.e. the point in the vicinity of L_1, the one on the right – the ending point of the stream, i.e. the contact between the stream and the disc. From the analysis of Fig. 3 we can see that the main energy production in the system takes place in a compact region of the shock wave, located near the accretion disc. This fact is of great importance for observational data interpretation as the compactness of the energy production area permits us to explain the phase dependences of light-curve features practically in the same manner as it was made earlier under the assumption of the existence of a hotspot.

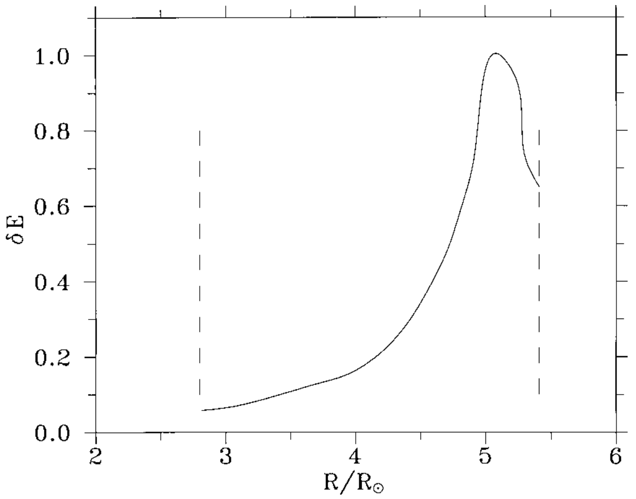

Figure 3. Normalized distribution ofspecific rate of energy release δE (erg s^{-1} cm^{-2}) along the shock wave in the equatorial plane of the system. The boundaries of the shock wave are marked by dashed lines.

The idea of hotspot was suggested to explain the observed light curves of CVs revealing the additional source of luminosity. It is evident that any new model has to explain this source of luminosity as well. In the considered model the energy release occurs in the extended shock wave, located on the stream boundary. To be convinced of the adequacy of the substitution of hypothetical hotspot by the shock along the stream edge let us first compare an energy release in the hypothetical hotspot and in the shock wave.

In the model with hotspot the energy release arising from interactions between the stream and the disc can be estimated as (see, e.g., Pringle & Wade 1985):

$$\Delta E_{\rm spot} = \frac{1}{4}\frac{GM_2\dot{M}}{R_{\rm out}} \qquad ({\rm erg\ s^{-1}}),$$

where $\dot{M}$ is the rate of mass loss by mass-losing star and $R_{\rm out}$ is the distance between the hypothetical hotspot and the accretor. In the standard model the mass transfer rate $\dot{M}$ is defined as:

$$\dot{M} = \rho_0 c_0 S,$$

where S is the stream cross-section in the vicinity of L_1 (Lubow & Shu 1975). Supposing that $R_{\rm out}$ is equal to the radius of the accretion disc obtained from the gas-dynamical calculations presented above it is possible to estimate $\Delta E_{\rm spot}$. The comparison of the energy release rate in the shock wave $\Delta E_{\rm shock}$ with $\Delta E_{\rm spot}$ shows that $\Delta E_{\rm shock}$ exceeds $\Delta E_{\rm spot}$ by a factor of 2. The obtained evaluation of energy release in cataclysmic binary Z Cha indicates that for this type of semidetached binary $\Delta E_{\rm shock}$ is approximately equal to $\Delta E_{\rm spot}$ and the main part of the energy is also released in a part of the shock close to the disc. It means that in the considered model without hotspot the value of additional energy release is enough to explain the observable excess of luminosity. Of course, it is not sufficient proof of the adequacy of the considered model. The real proof of the model adequacy can be obtained only from the direct comparison of the synthetic light curves with observations.

3.2 Comparison of synthetic light curves with observations

At the present time the maximum information on the flow structure can be obtained from the analysis of light curves of cataclysmic variables. The well-known humps on the light curves are observed for cataclysmic variables in quiescent state (see, e.g., Hack & La Dous 1993). These humps repeat regularly on the orbital period and have an amplitude up to 1 mag (Wood et al. 1986). Moreover, for five cataclysmic binaries in quiescent state (Z Cha, OY Car, V2051 Oph, HT Cas, IP Peg) the so-called double eclipse is observed.

To explain these light curves the hypothesis of hotspot is widely used. According to this hypothesis the hotspot is formed as a result of the shock interaction of stream of matter leaving L_1 with the outer edge of the accretion disc (see, e.g., Smak 1970; Hack & La Dous 1993; Shore, Livio & van den Heuvel 1994). However, as it follows from the numerical studies of steady-state self-consistent gaseous flows structure the stream of matter from L_1 does not cause the shock perturbation of the disc. In the above gas-dynamical model the zone of energy release is located outside the disc. This zone is formed as the result of shock interaction between the gas of the circumbinary envelope revolving around the accretor and the stream of matter from L_1.

To be convinced of the adequacy of the considered model we have built the synthetic light curve for cataclysmic variable Z Cha and have compared it with observations. To obtain the synthetic light curves we used the methods described in papers by Khruzina (1992) and by Khruzina & Cherepashchuk (1994). The detailed description of this method adapted for the considered gas-dynamical model is given in Bisikalo et al. (1998).

Observable and synthetic light curves are presented on Fig. 4. The comparison of these curves shows good qualitative agreement. Practically all characteristic features of the observable light curve are repeated on the theoretical one. It should be also noted that we have built synthetic light curves for the different types of cataclysmic variables. The comparison of obtained curves (Bisikalo et al. 1998) with observation shows that in the framework of the considered model with energy release zone located outside the disc it is possible to explain practically all types of observable light curves.

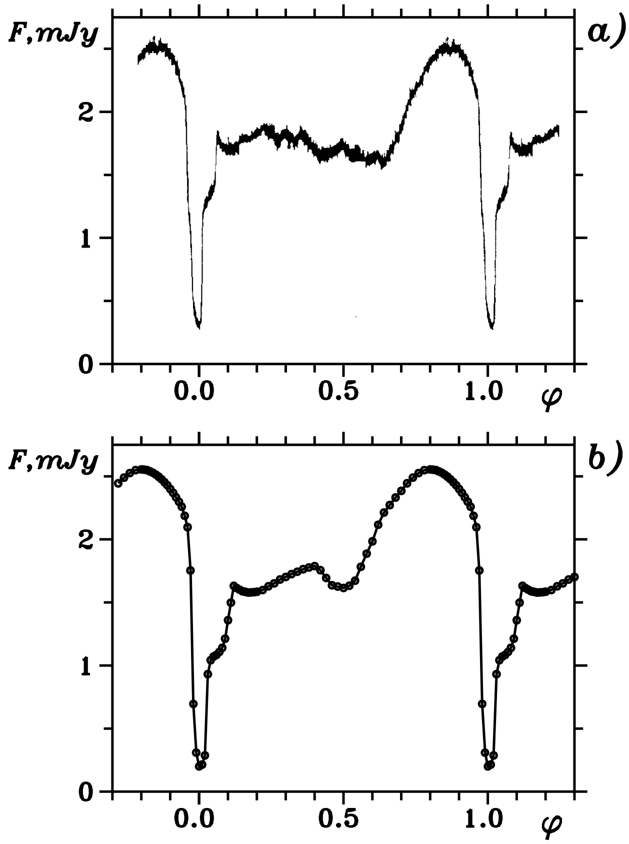

Figure 4. (a) Optical light curve of cataclysmic variable Z Cha (Wood et al. 1986); (b) synthetic light curve of Z Cha obtained for the gas-dynamical model without hotspot (Bisikalo et al. 1998).

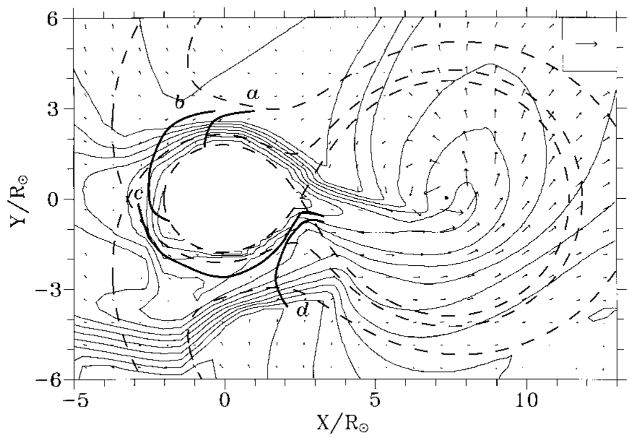

Figure 5. The same as Fig. 2 for extended region of the system. Four flow lines labelled by markers ‘a’, ‘b’, ‘c’ and ‘d’ show the directions of gas movement near the mass-losing star.

3.3 Flow structure in the vicinity of L_1

The parameters of the stream in the vicinity of L_1 were specified both in analytical (Paczyński & Sienkiewicz 1972; Lubow & Shu 1975; Savonije 1978) and numerical (see, e.g., Nazarenko 1993) works. The main characteristics of matter stream obtained in various works differ only in details, and at present these expressions are widely used as standard for the mass transfer analysis in semi-detached binaries (see, e.g., Pringle & Wade 1985; Shore et al. 1994). Unfortunately, in all these works the influence of forming circumbinary envelope of the system on the flow structure in the vicinity of the mass-losing star was not taken into account. In this work we can consider the contribution of circumbinary envelope on the basis of 3D numerical simulations.

The general flow structure in the equatorial plane of considered system is presented on Fig. 5, where density isolines and velocity vectors are presented. This figure is similar to Fig. 2 but the results are presented for larger region with sizes from −5 to 13 $R_\odot$ on *X*-axis and from −6 to 6 $R_\odot$ on *Y*-axis. On Fig. 5 four flow lines (labelled by markers ‘a’, ‘b’, ‘c’, ‘d’), illustrating the directions of flows in the system, are shown as well.

The analysis of results presented on Fig. 5 shows that a significant part of the gas of the circumbinary envelope (flow lines ‘a’ and ‘b’) reaches the surface of the mass-losing star (Roche lobe) and prevents the gas from escaping from the star surface. A part of the matter of the envelope (flow lines ‘c’ and ‘d’) blows away the gas from the surface of the mass-losing star and becomes involved in the process of stream formation.

The details of the flow structure near the inner Lagrangian point are shown in Fig. 6, where the same flow parameters as in Fig. 5 are presented in a small region of the equatorial plane with sizes from 1 to 4 $R_\odot$ for *X*-direction and from −1.5 to 1.5 $R_\odot$ for *Y*-direction. Fig. 6 shows that the gas of the circumbinary envelope considerably

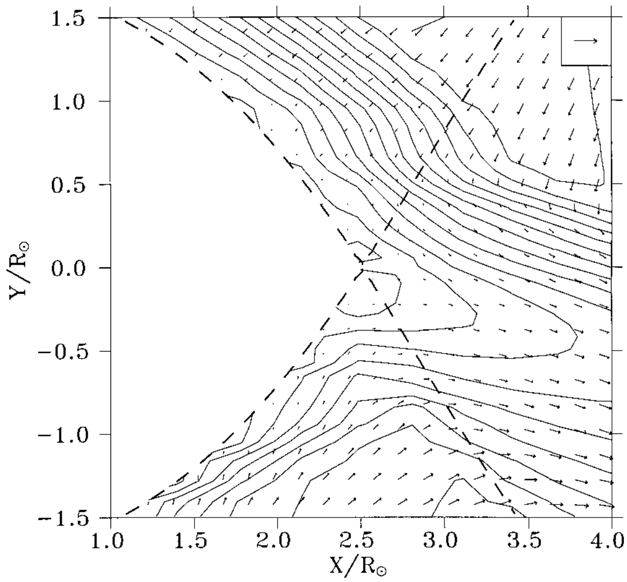

Figure 6. Density isolines and velocity vectors in the vicinity of L_1 (equatorial plane). Roche equipotentials are shown by dashed lines. Vector in the upper right corner corresponds to the value of velocity of 300 km s^{-1}.

changes the flow structure near L_1 and, in particular, strips off part of the matter of the star atmosphere. In this figure we can see also the asymmetry of the influence of circumbinary envelope on the stream. The gas moving from above – on its orbital movement – is accreted by the mass-losing star and only in a small area in the proximity of L_1 does it blow out matter from the surface and transfer it into the stream. The gas of the envelope, coming to L_1 from below, strips off the matter from a significant part of the surface.

The considered effect of 'stripping off' the matter from the surface of the mass-losing star considerably changes the usual point of view on the mechanism of stream formation and on the mass transfer parameters. According to the standard model, the atmosphere structure near the surface of a mass-losing star is determined by the equation of hydrostatic equilibrium (see, e.g., Lubow & Shu 1975; Pringle & Wade 1985). For the adopted parameters of the system and the temperature (sonic velocity) of the gas on the surface of the mass-losing star, the energy of the gas is not sufficient for the direct escape from the star surface. Therefore the matter flow connected with thermal escape will be negligible in comparison with the flow through the vicinity of L_1. The gas of surface layer can flow along the surface of the star; however in this case the total mass flow to the system increases slightly (Lubow & Shu 1975). The situation changes drastically in the case of taking into account the circumbinary envelope of the system, as in this case the gas of the surface layer can be 'stripped off' and blown away into the system.

On a significant part of the star surface the rarefied gas of circumbinary envelope has sufficiently large impulse component tangential to the star surface to capture matter from the surface layer of the star. Subsequently this matter, together with the gas leaving the vicinity of L_1, forms the stream and determines the value of the total mass transfer. The analysis of results shows that in the considered systems X1822−371 and Z Cha the mass transfer rate is one order of magnitude higher than one predicted by theoretical estimations, calculated without taking into account the influence of circumbinary envelope and with the same values of gas parameters on the surface of the mass-losing star.

3.4 Influence of the accepted boundary conditions on the flow structure

The results of calculations for semidetached binaries of various types show the qualitative similarity of the flow structure. Formally, this fact allows the assertion that the change of parameters of the system does not result in considerable changes of the flow structure. However, before making a conclusion about the universal character of the proposed model, let us consider possible variations of the flow structure caused by changes of boundary parameters of the gas, injected into the system.

The study of this problem is stimulated first of all by the fact that the adopted boundary conditions at the surface of a mass-losing star determine the flow structure. Moreover, in view of limited clarification of the behaviour of the surface layers of star when it fills Roche lobe, there is some arbitrariness in specifying of the appropriate boundary conditions.

As it was mentioned above, to specify the boundary conditions on the surface of a mass-losing star it is necessary to estimate the temperature, the density and gas velocity. The first parameter – temperature – is determined from the observations and its value is known with the satisfactory accuracy. The second parameter – density – does not influence the solution because of the scaling of the system of equations upon ρ. Therefore, the only boundary condition that is not well defined and that can influence the flow structure is the gas velocity.

The results of calculations for low-mass X-ray binary X1822–371 and for cataclysmic binary Z Cha have been obtained under the assumption that the gas on all the surface of the mass-losing star has velocity that is equal to the local sonic velocity and directed along the normal to the surface, i.e. the Mach number $\mathcal{M}$ is equal to 1 and it is the limiting case of maximum possible value of the boundary. Another limiting case corresponds to zero gas velocity. So, calculations for Z Cha have been reconducted under the assumption that the boundary gas velocity is equal to zero, in order to study the influence of the boundary conditions on the flow structure.

The results obtained under this assumption show that there are some insignificant quantitative changes in the flow structure. In particular: (i) the mass transfer rate, in comparison with the first model ($\mathcal{M} = 1$), has decreased now by 25 per cent; (ii) the total energy production in the shock wave has decreased by 15 per cent. At the same time, the common flow structure has preserved all characteristic features. In runs conducted with different boundary parameters we have obtained the formation of circumbinary envelope which deflect the stream and lead to the shock-free stream–disc interaction. This fact allows us to conclude that the considered model of the flow structure for semidetached non-magnetic binaries is an universal one.

3.5 Comparison of results obtained in 3D and 2D models

For a long time the 2D approach was the main one in numerical simulations of mass transfer in binaries. Increasing computational facilities allows one to include the third dimension into the gas-dynamical models and as a consequence to make them more realistic. Unfortunately the 3D models are very time-consuming and accordingly are not very refined, so the question of the applicability of the 2D approach is not just academic.

To evaluate the validity of the 2D approach we have made the 2D

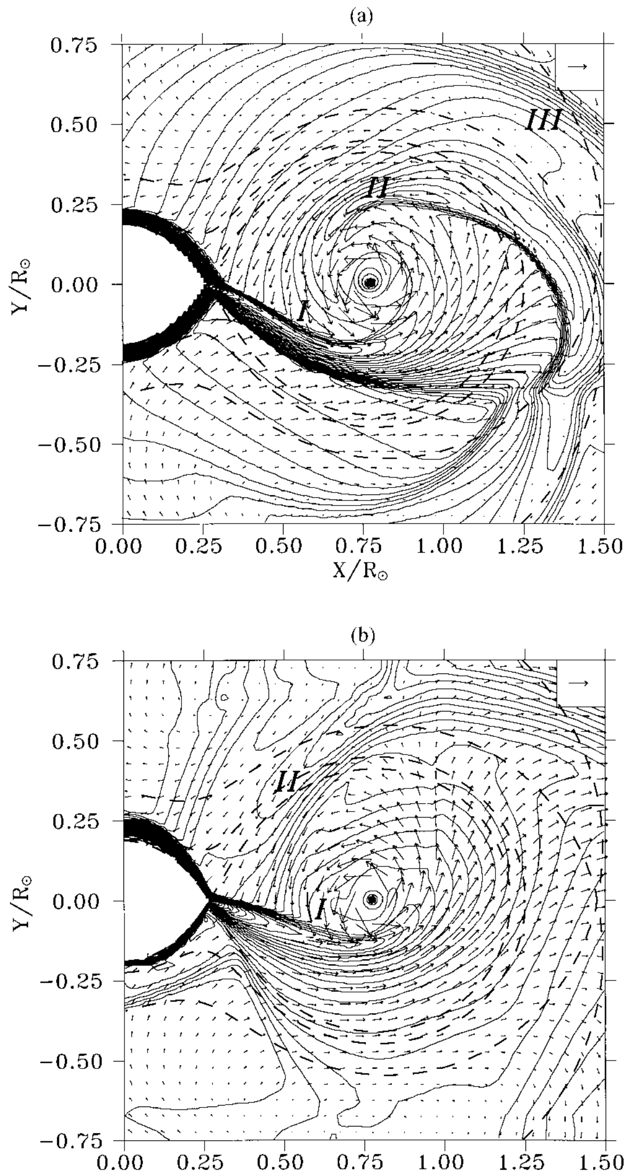

Figure 7. (a) Density isolines and velocity vectors for Z Cha for 2D run (equatorial plane). Roche equipotentials are shown by dashed lines. Shock waves are denoted by markers *I*, *II*, *III*. Vector in the upper right corresponds to the value of velocity of 1000 km s^{-1}. (b) Density isolines and velocity vectors for Z Cha for 3D run (equatorial plane). Roche equipotentials are shown by dashed lines. Shock waves are denoted by markers *I*, *II*. Vector in the upper right corner corresponds to the value of velocity of 1000 km s^{-1}.

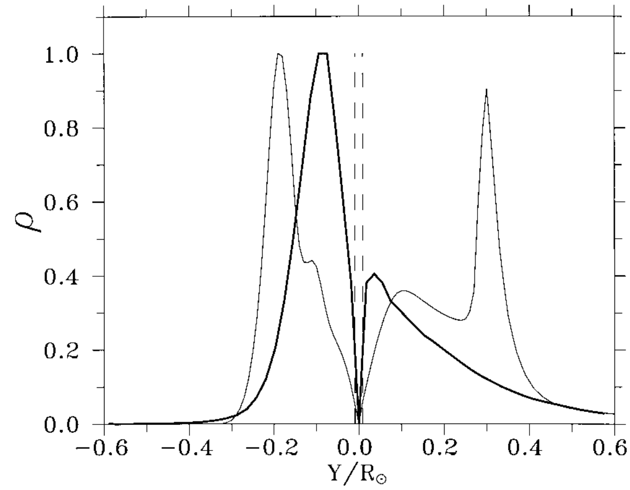

Figure 8. Normalized density distributions along *Y*-axis passing through the accretor for 2D (thin line) and 3D (thick line) cases. Accretor is placed at the point $Y = 0$, and vertical dashed lines around this point correspond to the accretor size.

simulation of the flow structure for Z Cha binary system. To make the comparison accurately we have used the same parameters and boundary conditions as in the 3D model described above. The results of calculations for 2D and 3D models are presented on Figs 7(a) and (b). On these figures density isolines and velocity vectors in the region of the equatorial plane with sizes from 0.0 to 1.5$R_\odot$ for *X*-direction and from -0.75 to 0.75 $R_\odot$ for *Y*-direction are shown.

Comparison of these two figures shows that steady-state flow structures obtained in 2D and 3D models have a set of common features:

(i) the accretion disc is formed;

(ii) the circumbinary envelope plays an important role in the formation of the gaseous flows structure;

(iii) the stream–disc interactions are shock-free, and the hotspot does not exist in both models;

(iv) part of the stream revolves around the accretor and interacts with itself causing the formation of shock wave *I* on the outer edge of the stream;

(v) the interaction of gas of the stream with circumbinary envelope causes the formation of the shock waves labelled by marker *II*.

Summarizing the above points we may conclude that the qualitative characteristics of the flow structure in the inner region of the considered binary are similar. In turn, it means that 2D model give a rather correct qualitative description of the gas flow structure in this region.

As it follows from a comparison of results presented in Fig. 7 there are also some quantitative differences of the flow structure in the vicinity of the accretor. In particular, in 3D case the accretion disc has an elliptic form and the stream of matter leaving L_1 goes close to the accretor. In 2D case the disc is more circular and the angle of deviation of the stream is greater than in 3D case, therefore the stream goes far from the accretor. This fact may be confirmed by the results presented in Fig. 8, where the density distributions (normalized to unity) along the line passing through the accretor parallel to *Y*-axis is shown. From the physical point of view the close moving of the stream in 3D case is rather evident, because in this case we consider all three dimensions and gas of the disc can expand in *Z*-direction, which, in turn, allows the stream to go closer to the accretor. The distance between the stream and the accretor defines the ratio between the gas flow leaving the system through the vicinity of L_2 and gas flow revolving around the accretor. This difference in the gas fluxes leads to the flow structure changes in the outer regions of the system. In the 3D case (where the stream moves close to the accretor) most of the stream is involved in movement around the accretor, while in 2D case the gas outflow through the vicinity of L_2 is more effective. In particular, for 2D case we see that the dominant gas flux leaving the system through L_2 moves around the centre of mass of the binary system in clockwise direction and causes the formation of typical bow shock labelled by *III*, while in 3D case this bow shock does not arise.

It should be noted that the obtained 3D solution was a steady-state one. The run have been conducted up to 20 orbital periods and we have not found changes in the flow structure. For 2D case the solution is quasi-steady-state and even for large evolution time (few orbital periods) quasi-periodic changes of the flow structure is observed.

Resuming the comparison between obtained 2D and 3D solutions, we may say that for the considered case of $\gamma = 1.01$:

(i) the 2D model gives a rather good qualitative description of the flow structure in the inner regions of the binary, while at the outer regions the 2D results are not reliable;

(ii) the 2D model does not give an adequate quantitative description.

4 CONCLUSION

This work deals with the results of 3D numerical simulation of the gaseous flow structure in semidetached binaries of various types. The analysis of results shows the significant influence of rarefied gas of the circumbinary envelope on the flow patterns in these systems. The gas of the circumbinary envelope interacts with the stream of matter and deflects it. This leads, in particular, to the shock-free (tangential) interaction between the stream and the outer edge of forming accretion disc, and, as the consequence, to the absence of hotspot in the disc.

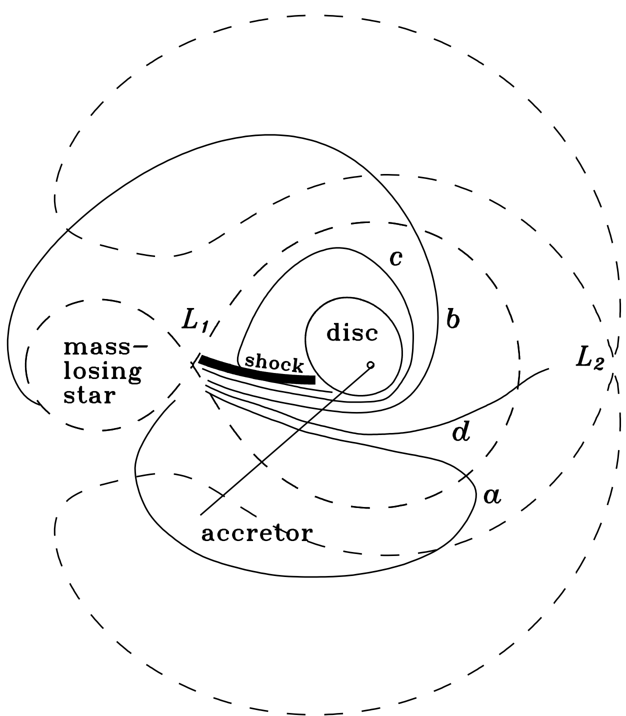

Figure 9. Schematic presentation of the main features of flow structure in semidetached binaries. Roche equipotentials are shown by dashed lines. Lagrangian points L_1 and L_2, location of the accretor, and quasi-elliptic accretion disc are marked on the figure. Shock wave located on the stream boundary is presented by bold line. Flows labelled by markers 'a', 'b', 'c' and 'd' are also shown on this figure. Flow 'd' corresponds to matter leaving the system through the vicinity of outer Lagrangian point L_2. Flow 'c' belongs to circumbinary envelope and interacts with the gas stream resulting in formation of the shock wave. Flows 'a' and 'b' belong to circumbinary envelope as well and strip out gas of the atmosphere of mass-losing star, increasing the rate of mass transfer.

At the same time it is shown that the interaction of the gas of the circumbinary envelope with the stream results in the formation of an extended shock wave located along the boundary of the stream. The comparison of synthetic light curves with observations proves the validity of discussed gas-dynamical model without hotspot.

It should be mentioned that taking into account the circumbinary envelope also leads to a drastic change of the mass transfer parameters in the system. The calculated mass transfer rate increases in order of magnitude as compared with values from the standard model. Moreover, the gas of the circumbinary envelope changes the flow structure near the surface of mass-losing component, which eventually influences the common structure of gas flows in the system and consequently affects the interpretation of observational data.

The qualitative similarity of the obtained solutions for the various types of semidetached systems permits one to speak about some universal character of the considered gas-dynamical model for non-magnetic semidetached binaries. The main features of the obtained flow structure are summarized on Fig. 9, where gaseous flows in systems, location of the shock wave, as well as forming quasi-elliptical accretion disc are presented schematically.

It should be noted that the presented results are obtained for the steady-state case. For the non-stationary (transient) case when morphology of the flow is determined by external factors and is not self-consistent, more features of the flow may appear, in particular, arising of the zone of the disc-stream shock interaction of the disc with the stream is possible. For example, if the disc was formed before the mass-losing star had filled its Roche lobe then, after the beginning of mass exchange, the occurrence of hotspot would be possible. Nevertheless, when the flow structure will become steady-state we will get a solution without hotspot. Therefore the lifetime of such formation is of importance. As the characteristic lifetime of hotspot it is natural to take the interval during which the quantity of matter transferred to the system by the stream becomes comparable with the mass of the accretion disc, as after the total replacement of disc matter the solution will be self-consistent. For the mass transfer rate and accretion disc parameters typical for semidetached binaries it should be expected that on time intervals of the order of tens of orbital periods the solution will become steady-state. It means that, for most of the time the gaseous flows structure is described by the model without hotspot as presented above.

Summarizing, we may conclude that the correct consideration of circumbinary envelope in the numerical model of the mass transfer in semidetached binaries makes it possible to find the new features of the flow structure and thus changes our view on the morphology of gaseous flows. We should note that despite the similarity of observational evidence of the hypothetical hotspot and the shock wave obtained in calculations, the calculated morphology of matter flow in semidetached binaries differs drastically from the standard model, which, in turn, requires the revision of some standard concepts.

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REFERENCES

Armitage P. J., Livio M., 1996, ApJ, 470, 1024
Belvedere G., Lanzafame G., Molteni D., 1993, A&A, 280, 525
Bisikalo D. V., Boyarchuk A. A., Kuznetsov O. A., Popov Yu. P., Chechetkin V. M., 1995, Astron. Rep., 39, 325
Bisikalo D. V., Boyarchuk A. A., Kuznetsov O. A., Chechetkin V. M., 1997a, Astron. Rep., 41, 786, preprint astro-ph/9802004
Bisikalo D. V., Boyarchuk A. A., Kuznetsov O. A., Chechetkin V. M., 1997b, Astron. Rep., 41, 794, preprint astro-ph/9802039
Bisikalo D. V., Boyarchuk A. A., Kuznetsov O. A., Khruzina T. S., Cherepashchuk A. M., Chechetkin V. M., 1998, Astron. Rep., 42, 33, preprint astro-ph/9802134
Blondin J. M., Richards M. T., Malinowski M. L., 1995, ApJ, 445, 939
Chakravarthy S., Osher S., 1985, AIAA Paper, No. 85–0363
Cherepashchuk A. M., Katysheva N. A., Khruzina T. S., Shugarov S. Yu., 1996, Highly Evolved Close Binary Stars: Catalog. Gordon and Breach, London
Flannery B., 1975, MNRAS, 170, 325
Godunov S. K., 1959, Math. Sbornik, 47, 271
Goncharskij A. V., Cherepashchuk A. M., Yagola A. G., 1985, Ill-Posed Problems of Astrophysics. Nauka, Moscow (in Russian)
Hack M., La Dous C., 1993, Cataclysmic Variables and Related Objects. NASA SP-507, Monograph Series on Nonthermal Phenomena in Stellar Atmospheres
Hirose M., Osaki Y., Mineshige S., 1991, PASJ, 43, 809
Khruzina T. S., 1992, SvA, 36, 29
Khruzina T. S., Cherepashchuk A. M., 1994, Astron. Rep., 38, 386
Kruszewski A., 1963, Acta Astron., 13, 106
Landau L. D., Lifshitz E. M., 1959, Fluid Mechanics. Pergamon, Elmsford
Lanzafame G., Belvedere G., Molteni D., 1992, MNRAS, 258, 152
Lanzafame G., Belvedere G., Molteni D., 1994, MNRAS, 267, 312
Lubow S. H., 1993, in Sahade J., McCluskey J. Jr, Kondo Y., eds, The Realm of Interacting Binary Stars. Kluwer, Dordrecht, p. 25
Lubow S. H., Shu F. H., 1975, ApJ, 198, 383
Mason K. O., 1989, in Proc. 23rd ESLAB Symp. on Two-Topics in X-ray Astronomy. ESA SP-296 p. 113
Matsuda T., Ishii T., Sekino N., Sawada K., Shima E., Livio M., Anzer U., 1992, MNRAS, 255, 183
Meglicki Z., Wickramasinghe D., Bicknell G. V., 1993, MNRAS, 264, 691
Molteni D., Belvedere G., Lanzafame G., 1991, MNRAS, 249, 748
Murray J. R., 1996, MNRAS, 279, 402
Nagasawa M., Matsuda T., Kuwahara K., 1991, Numer. Astrophys. Jpn., 2, 27
Nazarenko V. V., 1993, Astron. Reports, 37, 55
Paczyński B., Sienkiewicz R., 1972, Acta Astron., 22, 73
Pringle J. E., Wade R. A., eds, 1985, Interacting Binary Stars. Cambridge Univ. Press, Cambridge
Roe P. L., 1986, Annu. Rev. Fluid Mech., 18, 337
Savonije G. L., 1978, A&A, 62, 317
Sawada K., Matsuda T., 1992, MNRAS, 255, 17
Sawada K., Matsuda T., Hachisu I., 1986, MNRAS, 219, 75
Sawada K., Matsuda T., Inoue M., Hachisu I., 1987, MNRAS, 224, 307
Shore S., Livio M., van den Heuvel E. P. J., 1994, Interacting Binaries. Springer-Verlag, Berlin
Smak J., 1970, Acta Astron., 20, 312
Spruit H. C., Matsuda T., Inoue M., Sawada K., 1987, MNRAS, 229, 517
Taam R. E., Fu A., Fryxell B. A., 1991, ApJ, 371, 696
Warner B., Peters W. L., 1972, MNRAS, 160, 15
White N. E., Holt S. S., 1982, ApJ, 257, 318
Wood J., Horne K., Berriman G., Wade R., O'Donoghue D., Warner B., 1986, MNRAS, 219, 629

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