Three-Dimensional Spiraling of Interacting Spatial Solitons

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We report the observation of three-dimensional spiraling collision of interacting two-dimensional spatial solitons. The solitons are photorefractive screening solitons and are phase incoherent to each other at all times. The collision provides the solitons with angular momentum which is manifested in a centrifugal repulsion force. When it is balanced by attraction, the solitons spiral about each other in a DNA-like structure. [S0031-9007(97)02853-6]

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Optical spatial solitons [1] have attracted a substantial research interest in the last three decades. Several types of one-dimensional (1D) or two-dimensional (2D) bright or dark (vortex in two dimensions) spatial solitons have been demonstrated experimentally, including Kerr-type solitons [2], photorefractive solitons [3], $\chi^{(2)}$ quadratic solitons [4], and solitons in a saturable medium [5]. All these solitons occur when the diffraction of a light beam is exactly balanced by the nonlinear self-focusing effect (bright solitons) or self-defocusing effect (dark solitons).

Collisions between solitons are perhaps the most fascinating features of self-trapped beams, since, in many aspects, solitons interact like particles: being able to maintain their separate identities (in some cases), fuse (in others), or generate entirely new soliton beams [6]. Each possibility is fully determined by the initial trajectories of the colliding solitons and the interaction force they exert on each other (resulting from the nonlinear change in the refractive index induced by both solitons). For example, if two bright spatial solitons are mutually coherent and in phase, they constructively interfere, giving rise to an increase in the optical intensity in the region between them. This leads to an increase in the refractive index in their central region. As a result, more light is attracted toward the central region and is self-guided there. The net result is that the solitons appear to attract each other during their propagation in the nonlinear medium. On the other hand, when the initial relative phase between the colliding solitons is equal to π, the solitons destructively interfere in the central region and they appear to repel each other. However, if the solitons are mutually phase incoherent (i.e., the relative phase between the soliton beams varies much faster than the response time of the medium) [7,8], their intensities, rather than their amplitudes, are superimposed and this makes the interaction phase insensitive. Obviously, the total intensity in the central region cannot be lowered now; thus mutually incoherent bright solitons always attract each other. In spite of the complexity regarding soliton interactions, most of the soliton collision properties can be described using linear waveguide theory [9,10]. Whether 1D or 2D bright spatial solitons are involved in a collision, the attraction and repulsion forces are key factors determining the result of the soliton interaction.

Before stable 2D bright spatial solitons were observed, the study of soliton interaction was limited to the unconstrained transverse dimension of a 1D waveguide and the longitudinal dimension [2,11]. The recent observations of stable 2D spatial solitons has enabled observations of full (2 + 1)D (two transverse plus one longitudinal dimensions) soliton interactions. In particular, collisions of 2D solitons were observed in photorefractive media [7] and in saturable nonlinear atomic media [5]. The latter has also reported 3D spiraling of bright spatial solitons, when the solitons were generated from the breakup of an input vortex beam. When this input "bright ring" was launched into a self-focusing medium, it exhibited instability and fragmented into two 2D solitonlike beams. Since the input vortex had carried initial angular momentum, the bright solitonlike beams were forced (by conservation of angular momentum) to spiral while moving away from each other [5].

However, in principle, two 2D solitons should be able to spiral about each other even when they individually do not carry initial angular momentum, as predicted by Snyder's group in 1991 [12]. This should occur when two 2D solitons collide with trajectories that are not lying in a single plane, and at the same time, they attract each other just enough to "capture" each other [12]. Then, the solitons orbit about each other in a DNA-like structure, as illustrated in Fig. 1. In this Letter, we demonstrate just that: collision of two 2D mutually incoherent photorefractive solitons that are fully controllable in three dimensions. The solitons fuse, spiral about each other, or bypass each other depending on the distance between them and their trajectories. When each input beam is individually launched (the other beam is absent) it possesses no angular momentum. Nevertheless,

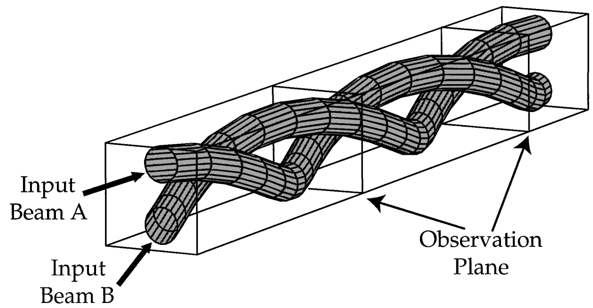

FIG. 1. An illustration of the soliton spiraling process. The arrows indicate the initial direction of the two soliton beams.

the collision process provides the soliton pair with angular momentum as the simultaneously launched solitons form a two-body system, and this drives the solitons to spiral about each other. The angular momentum is manifested in a mutual repulsion (centrifugal) force. When repulsion is exactly balanced by attraction due to the soliton interaction, the solitons capture each other (as celestial objects do) and spiral about each other in a DNA-like structure. We find that this process is most easily realized when the solitons are mutually incoherent with a very small angular separation between their initial propagation directions. When the initial distance between the two solitons is too large providing not enough attraction force, they move away from each other. When the distance is too small, the solitons fuse into one beam.

The choice of two mutually incoherent solitons merits further discussion. In principle, soliton spiraling should be realizable with either mutually coherent or mutually incoherent solitons. A necessary condition is, of course, that the mutual repulsion due to the centrifugal force will be balanced by attraction. However, while the force between mutually incoherent solitons is always attraction, the force between mutually coherent solitons depends on their relative phase and can be manifested in either attraction or repulsion. We find it much simpler, therefore, to realize a system of spiraling solitons with mutually incoherent solitons, for which the mutual force is not subject to phase variations. Furthermore, it is known that the soliton parameters, such as the propagation constant, the beam shape, and the maximum amplitude, are all interrelated. To make the delicate spiraling soliton pair stable and accessible, the attraction force between them throughout the entire propagation distance should be maintained as constant as possible. For a coherent soliton pair, this implies that the propagation constants must be indentical (in some nonlinear medium, this also implies that the two solitons must be identical). Otherwise, after a certain propagation length, the soliton pair becomes out of phase (due to their different propagation constants) and then the solitons start to repel each other. Once the solitons move away from each other, the interaction force (which decays with the transverse distance between the solitons) can no longer balance the centrifugal force and the solitons move further apart. Thus, using a mutually incoherent pair to observe the spiraling process can avoid the stability problem since the interaction force between the solitons is always attraction.

To observe soliton spiraling, the most critical requirement is to find the condition for which the attraction force can compensate exactly the centrifugal force caused by the acquired angular momentum. In previous experiments [13,14], we have studied the behaviors of waveguides induced by photorefractive screening solitons. For 1D photorefractive screening spatial solitons, the refractive index profile of the soliton-induced waveguide is controlled by the so-called intensity ratio, which is the ratio between the peak soliton intensity and the sum of the dark and background irradiances [14]. It is found that, at a large intensity ratio, the soliton-induced waveguide is multimode and its index profile is wider and deeper than at low intensity ratios (where the soliton resembles a Kerr soliton and the waveguide it induces is a single mode waveguide). At the same time, the index of the soliton-induced waveguide drops more dramatically at the boundary of the soliton at high intensity ratio, while at low intensity ratio (around intensity ratio 3), the refractive index profile varies more smoothly across the solitons. A similar trend has also been found in 2D bright screening solitons [13]. As pointed out in a recent theoretical Letter [15], the interaction force between two solitons is proportional to the gradient of the index perturbation induced by the solitons. In our spiraling experiment, we find the most suitable intensity ratio for observation of the spiraling process is around 4 to 6. If the intensity ratio is smaller, the attraction force is too weak (the gradient of the refractive index change is too small) to compensate the centrifugal force. On the other hand, at high intensity ratios, the soliton-induced waveguide is multimode, which means that nonfundamental guided modes can be excited in the collision process [7], and this breaks the 2D symmetry and deteriorates the solitons [7,14]. In summary, to observe spiraling solitons, it is (1) necessary to have a saturable nonlinearity (such as the photorefractive nonlinearity) that stabilizes 2D solitons, and it is desirable to have (2) mutually incoherent solitons to ensure a phase-insensitive attraction force, and (3) the nonlinearity should be operated at maximum saturation that still gives rise only to single-mode soliton-induced waveguides. Only after these conditions are satisfied, one can resort to the delicate work of adjusting the initial trajectories of the colliding solitons (that should not lie in the same plane) and the distance between them.

The experiment setup is similar to that of Ref. [7] except that the input beams are now launched with their trajectories skewed with respect to each other, as illustrated by the arrows in Fig. 1. We use a $14 \times 13 \times 6.5$ mm^3 SBN:60 ($Ba_{0.4}Sr_{0.6}Nb_2O_6$) photorefractive crystal. The two 12 μm wide (FWHM) beams are first launched into the crystal with their minimum waists on the input face, marked A and B in Fig. 2(a). The angular separation and distance between the solitons are 7×10^{-3} rad and

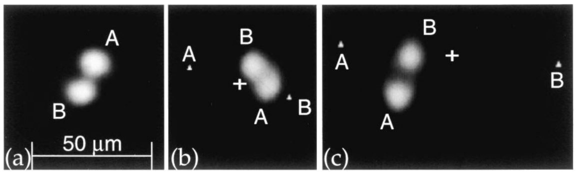

FIG. 2. (a) Beams A and B at the input face of the crystal, (b) the spiraling soliton pair after 6.5 mm of propagation, and (c) after 13 mm of propagation. The centers of diffracting A and B are marked by white triangles. The white cross indicates the center of mass of the diffracting beams A and B in (b) and (c).

14 μm, respectively. After 6.5 mm of propagation, the beams diffract to about 55 μm and increase their separation, their centers being marked by the triangles in Fig. 2(b). The diffracted beams were sampled immediately (0.1 s) after launching each individual input beam, that is, long before fanning evolves. We define the center between the diffracting beams as the "center of mass" and mark it by a white cross shown in Figs. 2–4. After we apply 6.1 kV between electrodes separated by 14 mm, two solitons form and, at the same time, their relative position rotates [16] by roughly 270° [Fig. 2(b)]. We distinguish between the output beams by blocking (or modulating) one of them at the input for a time "window" much shorter than the response time of the photorefractive material and thus being able to identify the modulation in one of the output beams. The photorefractive nonlinearity is not affected by such fast modulation as the nonlinear index change does not have time to adjust; thus the beams are easily distinguishable from the other (this technique is used in [17]). Then, we flip the crystal over and let the solitons propagate along 13 mm. We find that the solitons now rotate an additional 270° [Fig. 2(c)]. We also observe roughly 19% of energy exchange [7] between A and B after 6.5 mm of propagation and 30% of energy exchange after 13 mm of propagation because the soliton-induced waveguides are so close to each other, that energy from each soliton beam can be coupled into the waveguide induced by the other. However, since both solitons induce single-mode waveguides, this energy exchange does not break the symmetry and does not affect the interaction [7].

As we increase the initial distance between A and B to 22 μm [Fig. 3(a)] while keeping their initial angular separation unchanged, the attraction force decreases and cannot balance the centrifugal force anymore. As a result, the solitons now bypass and move away from each other. The separation becomes 28 μm [Fig. 3(b)] after 6.5 mm of propagation, although some attraction is still observed when we compare the distance between solitons A and B with the distance between diffracting beams A and B. Finally, as we reduce the initial distance between the solitons to 7 μm, [Fig. 4(a)] and also adjust the initial angular separation to 6×10^{-3} rad, we observe that A and B fuse into one beam [Figs. 4(b) and 4(c)] because the attraction force is now larger than needed for spiraling and the solitons coalesce into their common center of mass. A similar fusion result has also been observed in the previous experiments of *planar* incoherent collisions (that is, when the trajectories are in a single plane) between bright screening solitons [7,14].

A careful look at Fig. 2 reveals an interesting observation: the two identical solitons spiral about each other in elliptical (rather than circular) orbits. The reason for that is twofold. First, the trajectories of identical interacting "particles" in an effective two-body problem are in general always elliptical (circular trajectories can be obtained only under special conditions) [18]. Second, the photorefractive nonlinearity is anisotropic, and only under specific conditions is one able to obtain even individual circular 2D solitons [19]. It is, therefore, expected that the interaction force between solitons will depend on the plane of collision: whether the space charge field in the center region between the solitons is parallel (or perpendicular) to the crystalline c axis, thus maximizing (minimizing) the influence of the large r_{33} electro-optic coefficient of SBN. For both these reasons we expect that the spiraling solitons will follow elliptical orbits. Indeed, Figs. 2(b) and 2(c) show that the solitons move closer and then break apart periodically.

In addition to the nonlinearity that gives rise to screening solitons, photorefractive solitons also self-bend toward the c axis as a result of asymmetric diffusion fields [20]. Self-bending of individual and colliding solitons was observed in Refs. [7], [14], and [18] respectively. Here, the diffusion field acts as an additional force exerted on both solitons. It is, therefore, expected that the center of mass will bend toward the c axis. When the solitons

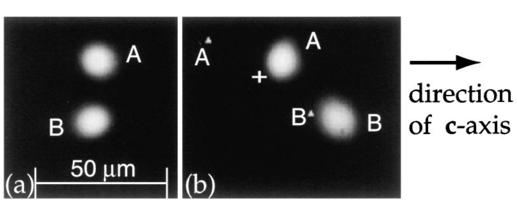

FIG. 3. (a) Beams A and B at the input face of the crystal, (b) the bypassing soliton pair after 6.5 mm of propagation. The centers of diffracting A and B are marked by white triangles. The white cross indicates the center of mass of the diffracting beam A and B in (b).

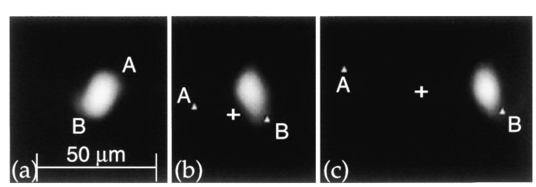

FIG. 4. (a) Beams A and B at the input face of the crystal, (b) the fused soliton pair after 6.5 mm of propagation, and (c) after 13 mm of propagation. The centers of diffracting A and B are marked by white triangles. The white cross indicates the center of mass of the diffracting beams A and B in (b) and (c).

fuse or bypass (Figs. 3 and 4, respectively), their center of mass is indeed always bent (shifted) toward the c axis as shown in Figs. 3(b), 4(b), and 4(c). When the beams spiral about each other, however, the center of mass seems to wobble, as observed in Fig. 2: after 6.5 mm it appears to shift toward the c axis [Fig. 2(b)], whereas after 13 mm it shifts in the *opposite* direction [Fig. 2(c)]. This feature of the spiraling soliton dynamics is certainly worthy of further study in the future.

We emphasize that we have reproduced all the delicate experiments described in this Letter, including the spiraling, fusion, elliptic orbits, and wobbling, in two different SBN crystals of different length and strength of nonlinearity (r_{33}). It thus confirms that these observations stem from the generic nature of spatial solitons and depend very little on the specific material properties.

In conclusion, we have observed full three-dimensional spiraling of a 2D bright soliton pair. For proper initial conditions the solitons spiral about each other in elliptical orbits. A deviation from these conditions leads to soliton fusion ("impact") or to their escape from mutual orbiting. These experiments reveal the deep similarity between the solitons in nature (not only in nonlinear optics) and particles.

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