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Three-Dimensional Topological Field theories and Strings

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Abstract

We discuss the status of the program to reformulate string theory as a theory of topological fields and gravity in three dimensions.

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1. Introduction

In its most recent incarnation, string theory may be viewed as the quantum theory of two-dimensional conformal fields on random surfaces. The two elements of such a description, the random surface and the field theory, are already evident in the simplest bosonic string model. As it moves through space, a one-dimensional closed string traces out a two-dimensional surface S, the world sheet, which in the Polyakov formulation is characterized by an intrinsic metric g_{ij}. This surface is "random" in the sense that g_{ij} is not fixed a priori, but is summed over in the string path integral. The conformal fields for the bosonic string are its coordinates $X^\mu(\sigma^1, \sigma^2)$; these are maps from S to spacetime, and may thus be interpreted as "spacetime-valued fields" on the world sheet. The action

$$S_X = \int_S d^2\sigma \sqrt{g} g^{ij} \partial_i X^\mu \partial_j X_\mu \tag{1.1}$$

is invariant under Weyl ("conformal") transformations $g_{ij} \rightarrow e^{2\lambda} g_{ij}$, and the demand that this symmetry not be anomalous determines the well-known critical dimension of spacetime. Gauge-fixing the Weyl and diffeomorphism symmetries reduces the path integral over g_{ij} to a finite-dimensional integral over moduli space,† and interactions are naturally incorporated by including surfaces of more complicated topologies to represent the splitting and joining of strings.

Of course, most models in string theory are not so simple. Some of the coordinates X^μ may be replaced by other conformal fields — world sheet spinors, for instance, in superstring theory — that can be interpreted as internal degrees of freedom of the string. Vertex operators may be used in place of world sheet boundaries to represent initial and final states. The requirement of a vanishing conformal anomaly may, perhaps, be dropped, and in the new random matrix approach, the smooth world sheet may essentially be replaced by a piecewise flat polyhedron. But the basic picture of the string world sheet as a surface carrying conformal fields has thus far remained intact.

The success of this two-dimensional picture of string theory naturally raises the question of whether there are higher dimensional analogs. In general, such "membrane theories" have

† The Weyl equivalence class of a metric on S determines a conformal structure, or, if S is oriented, a complex structure; moduli space is the space of such structures modulo diffeomorphisms.

proven to be much less interesting than string theory. It has recently become evident, however, that one particular membrane theory may essentially reproduce string theory, recasting it in a new language that may provide further insights.

The basic idea of this "topological membrane theory" [1] is to fill in the string world sheet and view it as the boundary of a three-manifold. At first sight, it would seem improbable that such a picture could reproduce string theory. A constant time cross-section of a three-dimensional "world tube" is two dimensional, and has vastly more degrees of freedom than a string; we will recover string theory only if we can suppress these extra degrees of freedom.

The possibility of doing so was first suggested by Witten [2], who showed in an important and unexpected paper that that many two-dimensional conformal field theories can be derived from three-dimensional Chern-Simons theories. The next step, deriving the world sheet metric g_{ij} from three dimensions, was suggested by Witten's work on three-dimensional gravity [3] and carried further by the authors [4]. The purpose of this brief review is to summarize these constructions.

2. Conformal Field Theory from Three Dimensions

To construct string theory from three dimensions, we must first understand how to obtain two-dimensional conformal field theories. The key to doing so is the introduction of topological field theory. A topological field theory on a three-manifold M has a huge gauge group, the entire group of diffeomorphisms of M, and consequently has relatively few physical degrees of freedom. The theories first considered by Witten have, in fact, no propagating degrees of freedom in the interior of M. The dynamics is thus restricted to the two-dimensional boundary ∂M, and we have some hope of recovering the two-dimensional structures of string theory.

In particular, let $A = A^\alpha{}_a T_\alpha dx^a$ be a gauge field for a gauge group G generated by the $\{T_\alpha\}$. The Chern-Simons action is

$$S_1 = \frac{k}{4\pi} \int_M \mathrm{Tr}(A \wedge dA + \frac{2}{3} A \wedge A \wedge A) \ , \tag{2.1}$$

which is manifestly invariant under diffeomorphisms of M, and thus "topological." Let us first take M to have the topology $[0,1] \times \Sigma$, where Σ is a closed surface. The canonical quantization of (2.1) is then reasonably straightforward: writing

$$S_1 = -\frac{k}{4\pi}\int dt \int_\Sigma \mathrm{Tr}\left[\epsilon^{ij}(A_i\partial_t A_j - A_t F_{ij})\right] \ , \tag{2.2}$$

we see that the A_i's are canonically conjugate,

$$[A_i^\alpha(x), A_j^\beta(x')] = \frac{4\pi i}{k}\epsilon_{ij}\delta^{\alpha\beta}\delta^2(x-x') \ , \tag{2.3}$$

and that physical states must obey the constraint $F_{ij}^\alpha = 0$. This constraint implies that the A_i's are flat connections on Σ, and since $\epsilon^{ij}F_{ij}$ also generates the gauge transformations of A, we must further identify gauge-equivalent connections. The physical phase space is thus the (finite dimensional) moduli space of flat connections on Σ, and our field theory has been reduced to quantum mechanics; in particular, there are not enough physical degrees of freedom to describe propagating physical excitations in the interior of M.

It can be shown [2,5–7] that the space of states of such a Chern-Simons theory is precisely the space of conformal blocks of an associated conformal field theory. To make the connection to conformal field theory more transparent, however, it is useful to consider a different topology, allowing Σ to have a boundary. The resulting picture will be closer to our intuition of a filled-in string; as Σ moves, its boundary will trace out a surface that can be identified with the string world sheet.

For simplicity, let us take $G = \mathrm{U}(1)$, and let Σ be a disk. The constraints can be solved by letting $A_i = \partial_i U$; inserting this expression into the action (2.2) and integrating by parts, we find

$$S_{\text{eff}} = -\frac{k}{4\pi}\int dt \int_\Sigma \epsilon^{ij}\partial_i U \partial_t \partial_j U = \frac{k}{4\pi}\int_C dt d\theta\, \partial_\theta U \partial_t U \ . \tag{2.4}$$

But this is simply the bosonic string action (1.1) on the cylinder $C = [0,1]\times\partial\Sigma$, with U identified with X and with the off-diagonal metric

$$g_{\theta t} = 1, \qquad g_{\theta\theta} = g_{tt} = 0 \ . \tag{2.5}$$

Although the derivation of (2.4) described here was classical, a parallel procedure may be applied in the path integral formulation of the quantum theory. There is one subtlety,

however, arising from the off-diagonal form of the metric and the choice of initial conditions [1]. A classical solution of the field equations derived from (2.4) is $U(\theta,t) = A(\theta) + B(t)$; in coordinates in which the metric is diagonal, A and B are left- and right-moving excitations. But fixing boundary conditions at $t=0$ completely determines $A(\theta)$, which thus cannot be viewed as a propagating physical field. Rather than obtaining the full bosonic string action for a cylinder, we obtain only the right-moving piece: U is a *chiral* conformal field on ∂M.

To reproduce bosonic string theory, of course, we need both left- and right-movers. We can accomplish this by replacing the disk Σ with an annulus, so the "world tube" has both an inner and an outer boundary. The resulting conformal field theory includes left-moving modes from one boundary and right-movers from the other [6,1].

These results can be generalized in several directions [5,6,8]. By allowing the gauge group G to be non-Abelian, we obtain the level k chiral Wess-Zumino-Witten action for G, where k is the coupling constant in the action (2.1). Vertex functions appear naturally as end points of open Wilson lines which start and end on ∂M. Coset constructions of conformal field theories may be obtained by starting with the difference between two Chern-Simons actions and imposing suitable boundary conditions; orbifold constructions arise from semidirect product gauge groups $P\ltimes G$, where P is a discrete automorphism group of G. By means of these and similar techniques, a very large class of rational conformal field theories can be build from Chern-Simons theories.

3. Conformal Structures from Three Dimensions

While Witten's construction represents a major step towards the three-dimensional description of string theory, a vital element is still missing. String amplitudes are described by conformal field theories on *random* surfaces; the metric g_{ij} — or after gauge-fixing, the complex structure on the world sheet — must be integrated over. So far, however, we have only obtained field theories on surfaces with fixed conformal structures; the integration is still missing.

Before we can fill in this gap, we must address an apparent paradox. Chern-Simons theories are topological, and contain no metric. But the conformal field theories we have derived involve a metric, or at least a complex structure, in their basic definition. Where has this added structure come from?

The answer lies in the commutation relations (2.3), which imply that the A_i's play the role of both positions and momenta. To canonically quantize such a theory, we must distinguish positions and momenta; in the language of geometric quantization, we must choose a polarization. This is most naturally done by introducing a complex structure on the initial surface, and selecting the $A_{\bar z}$ to be positions and the A_z to be momenta. More generally, we can choose two directions m^i and n^i at each point on the initial surface, with corresponding positions $A_i m^i$ and momenta $A_i n^i$; a conformal structure is then determined by the statement that m^i and n^i are orthogonal. If we instead choose to define amplitudes in terms of path integrals, we must select appropriate boundary conditions at $t = 0$; this again requires picking out one component of A whose boundary value we prescribe. Hints of this dependence were already present in our Abelian model on the cylinder: the chiral nature of the amplitudes could be seen only after we understood the boundary conditions.

It can be shown that the Hilbert spaces arising from different choices of polarization are unitarily equivalent, and that expectation values of operators such as Wilson lines in *closed* three-manifolds are independent of any such choices [6,9]. To derive string theory amplitudes, however, it is necessary to allow three-manifolds with boundary, and we must somehow introduce an integration over the complex structure on the boundary, promoting the dependence on polarization to something of physical significance.

In pure Chern-Simons theory, the polarization arises as a part of the formalism of quantization, and is not easily treated as a physical variable. Observe, however, that to define off-shell amplitudes in perturbative Chern-Simons theory we need a gauge-invariant regularization of the action. One natural way to regulate S_1 is to add a higher derivative term [10–12]

$$S_2 = -\frac{1}{4\gamma}\int_M F\wedge *F = -\frac{1}{4\gamma}\int_M \sqrt{-g}\,g^{ac}g^{bd}F_{ab}F_{cd} \ , \tag{3.1}$$

where we eventually take the coupling constant $\gamma\to\infty$ to recover the pure Chern-Simons action. S_2 depends explicitly on a metric g_{ab}, and we may hope that some residual dependence remains even as $\gamma\to\infty$. Indeed, the action $S_1 + S_2$ is second order in time derivatives, and we can no longer take A_z and $A_{\bar z}$ to be canonically conjugate; generic wave functions will depend on both components of A. But we know that in the limit $\gamma\to\infty$, the dependence on one component must disappear, and it seems natural to expect this component to be picked out by the metric g_{ab}.

To see that this is the case, let us again choose M to have the topology $[0,1]\times\Sigma$, and let us write g_{ab} in the Arnowitt-Deser-Misner form

$$ds^2 = -(Ndt)^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \ , \tag{3.2}$$

where h_{ij} is a fixed metric on Σ. The standard canonical quantization of $S_1 + S_2$ then leads to a Hilbert space of functions $\Psi(A_z, A_{\bar z})$, and a Hamiltonian

$$H = \gamma N\left(-\left(\frac{\delta}{\delta A_{\bar z}} - \frac{k}{4\pi}A_z\right)\left(\frac{\delta}{\delta A_z} + \frac{k}{4\pi}A_{\bar z}\right) + \frac{1}{4\gamma^2}(\epsilon^{ij}F_{ij})^2\right) \ , \tag{3.3}$$

where the complex structure in (3.3) is the one determined by the metric h_{ij}. In the limit $\gamma\to\infty$, the behavior of amplitudes will be determined by the lowest energy eigenstates, $H\Psi = 0$. This functional differential equation can be solved,* yielding

$$\Psi[A] = e^{-\frac{k}{4\pi}\int A_z A_{\bar z}}\Phi(A_{\bar z}) \ . \tag{3.4}$$

Equation (3.4) is precisely the proper form for a wave function in a pure Chern-Simons theory, but now with a particular conformal structure fixed by h_{ij}. The exponent determines the correct inner product in holomorphic quantization [6], while the dependence of Φ on $A_{\bar z}$ alone corresponds to the polarization determined by the complex structure z. Of course, states depending on A_z also exist in the regulated theory, but their contributions to any amplitude will be suppressed by powers of γ. We can thus control the polarization in the pure Chern-Simons theory by varying the metric g_{ab} in S_2.

Having encoded the two-dimensional complex structure in a three-dimensional metric, we can now integrate over complex structures by promoting this metric to the status of a dynamical variable in the path integral. In other words, we can attempt to obtain string theory from three-dimensional topological theory by coupling it to three-dimensional geometrodynamics. To do so, we first need an action for g_{ab}. The obvious candidate is the

* This problem has been discussed by Wen [13], who emphasizes the conformal invariance of the Hamiltonian; in a different context, a special case was considered by Dubrovin and Novikov [14].

three-dimensional Einstein action,

$$S_3 = \frac{1}{\kappa^2}\int_M R\sqrt{g}\ , \tag{3.5}$$

a choice made attractive by Witten's observation [3] that the physical phase space for (2+1)-dimensional Einstein gravity is closely related to the moduli space of two-dimensional complex structures (see also [15].) Moreover, if it is not already present, S_3 will be induced from the action $S_1 + S_2$ by quantum corrections.

To analyze the path integral for the action (3.5), let us again take M to have the topology $[0,1]\times\Sigma$. Einstein gravity is a constrained system, and in three dimensions the action takes the form [16]

$$S_3 = \int dt \int_\Sigma (\pi^{ij}\dot{g}_{ij} - N^i\mathcal{H}_i - N\mathcal{H}) \tag{3.6}$$

with constraints

$$\begin{aligned} \mathcal{H} &= \frac{1}{\sqrt{h}}(\pi^{ij}\pi_{ij} - \pi^2) - \sqrt{h}\,{}^{(2)}R = 0 \\ \mathcal{H}^i &= -2\pi^{ij}{}_{|j} = 0 \end{aligned} \tag{3.7}$$

These constraints have been analyzed in detail by Moncrief [17], who shows that their solutions are parameterized by the cotangent bundle to the moduli space of Σ. A sketch of his proof is as follows. If Σ is a surface of genus greater than one, then any metric on Σ is conformal to a metric $\tilde{h}$ of constant curvature -1. Writing $h_{ij} = e^{2\lambda}\tilde{h}_{ij}$, we find that the "Hamiltonian constraint" $\mathcal{H} = 0$ determines λ, which is no longer an independent variable, while the "momentum constraints" $\mathcal{H}^i$ generate diffeomorphisms of Σ. The physical degrees of freedom coming from the metric thus determine a constant negative curvature surface modulo diffeomorphisms. But by the uniformization theorem of Riemann surface theory [18], the space of such surfaces is precisely moduli space. A similar analysis of the constraints on the momenta shows that the physical degrees of freedom of π^{ij} at fixed $\tilde{h}_{ij}$ determine a cotangent vector to this moduli space.

In particular, the slice

$$\pi_{ij} - \frac{1}{2}h_{ij}h^{kl}\pi_{kl} = 0 \tag{3.8}$$

determines a section of this cotangent bundle homeomorphic to the moduli space of Σ. To define the gravitational path integral, we must specify boundary conditions that fix half the

boundary data on each boundary component. The conditions (3.8) clearly do this, and the path integral with these boundary conditions reduces to an integral over the moduli space of Σ, just as needed to recover string theory. Note that although $[0,1]\times\Sigma$ has two boundary components, only one copy of moduli space appears; the equations of motion for the metric with boundary conditions (3.8) force h_{ij} to be the same on each boundary. Moreover, the gauge-fixed path integral with these boundary conditions can be computed exactly. One finds that the *only* contribution is the integration over moduli space, with the integration measure (the Weil-Petersson measure) that naturally occurs in string theory.

4. Anomalies and the Central Charge

We are still missing an important feature of string theory, the Weyl anomaly (or the condition that it be absent). Three-dimensional gravity is not Weyl invariant; rather, the Weyl factor in the three-metric is determined by the constraints. So it is not obvious that there is any symmetry to be anomalous in the topological membrane formulation.

The solution to this problem lies in the existence of a local Lorentz anomaly in three dimensions. In general, quantum corrections to gravitationally coupled matter will induce a gravitational Chern-Simons term [19] in the action,

$$S_4 = \frac{k'}{8\pi}\int_M \epsilon^{abc}(R_{ab\alpha\beta}\omega_c{}^{\alpha\beta} + \frac{2}{3}\omega_{a\alpha}{}^{\beta}\omega_{b\beta}{}^{\gamma}\omega_{c\gamma}{}^{\alpha})\ . \tag{4.1}$$

Here $\omega_{a\alpha}{}^{\beta}$ is the spin connection for g_{ab}, and S_4 can be interpreted as a Chern-Simons term for an SO(2,1) gauge theory with connection ω. The coefficient k' induced by matter is finite, and can be calculated directly in perturbation theory [20] or by path integral methods [21]. Alternatively, one can obtain k' by means of some rather elegant index theorem arguments [2,22]. For a $U(1)^n$ Chern-Simons theory, one finds a value $k' = \frac{n}{24}$. For a non-Abelian theory, general considerations [23] suggest that $k' = \frac{c}{24}$, where c is the central charge of the corresponding WZW model; it would be interesting to check this result directly in perturbation theory.

Now, under a "large" gauge transformation (one not deformable to the identity), the Chern-Simons action (2.2) is not actually invariant, but shifts by $2\pi nk$, where n is the winding number of the transformation. The path integral will be invariant only if $\exp\{iS\}$

is unchanged, i.e., only if k is an integer. If we were working in Euclidean space, the gravitational Chern-Simons action (4.1) would be an action for the group SO(3), and this invariance condition would require k' to be quantized.

For Lorentzian metrics, this is not the case: $\pi_3(SO(2,1)) = 0$, and there are no gauge transformations with nonzero winding numbers. But SO(2,1) contains a U(1) subgroup, and it is known that the coefficient of a U(1) Chern-Simons theory must also be quantized [5,24].* The two-dimensional Weyl anomaly of string theory thus reappears as a local Lorentz anomaly in the corresponding three-dimensional theory.

A similar conclusion may be reached by investigating the central extension of the mapping class group in Chern-Simons theory [23]. The careful definition of the path integral for the Chern-Simons action requires the specification of not only a three-manifold M, but also a framing of M. Under a change of framing, the partition function changes by a phase proportional to the central charge of the associated conformal field theory. Although this phenomenon seems to be independent of the local Lorentz anomaly discussed above, the two are actually related by an index theorem [2]. A similar analysis may be applied to vertex functions [1]: vertices correspond to three-dimensional Wilson lines, and the on-shell condition for vertex functions can be understood as a requirement of invariance under diffeomorphisms of M which braid sets of Wilson lines.

It is interesting to note that the absence of the local Lorentz anomaly does not require $c = 0$, but only $c = 0 \bmod 24$. We do not yet understand the significance of this result.

5. Fermions and Ghosts

One more step, as yet uncompleted, is necessary if we are to obtain standard string amplitudes from three dimensions. In the ordinary Polyakov string, one begins with a path integral over world sheet metrics g_{ij}, and gauge-fixes the diffeomorphisms

$$\delta g_{ij} = \nabla_i\xi_j + \nabla_j\xi_i - g_{ij}\nabla_k\xi^k = (P\xi)_{ij} \ . \tag{5.1}$$

This gauge-fixing produces a Faddeev-Popov determinant $\Delta = (\det' P^\dagger P)^{1/2}$, which can be

* There is some controversy over this quantization [25], but there seems to be clear evidence that it is necessary if sufficiently complicated operators are to be consistently defined on topologically nontrivial manifolds.

rewritten as a path integral for an anticommuting b-c ghost system

$$S_{bc} = \int b_{ij}(Pc)^{ij} \ . \tag{5.2}$$

No such determinant appears in the three-dimensional gravitational path integral. It is true that one must still gauge-fix diffeomorphisms. But the momentum constraints appear in the action (3.6) in the form $-2N_i\pi^{ij}{}_{|j} = N_i(P^\dagger\pi)^i$, and it is easy to see that the resulting path integral produces a determinant that exactly cancels Δ. This result might have been anticipated from Witten's analysis of the gravitational path integral [26]: he shows that the determinants combine to form a topological invariant, the Ray-Singer analytic torsion, and for our topologies and boundary conditions the determinants in this torsion all cancel.

Of course, one may still argue that the ghost action is needed to ensure cancellation of the Weyl anomaly, and we have seen that this requirement can be derived from three dimensions. But this still leaves open the question of how to obtain the b-c action from three dimensions. We do not yet know how to do this. However, some intriguing hints come from looking at three-dimensional supersymmetry.

Consider, for instance, the supersymmetric extension of the U(1) Chern-Simons action. The superpartner of the gauge field A is a Majorana spinor λ, with an action [27]

$$S_5 = -\frac{k}{4\pi}\int_M \bar\lambda\lambda \tag{5.3}$$

and a combined supersymmetry and gauge transformation

$$\begin{aligned} \delta A_\mu &= -i\bar\eta\gamma_\mu\lambda + \partial_\mu\Lambda \\ \delta\lambda &= \frac{i}{2}\epsilon^{\mu\nu\rho}F_{\nu\rho}\gamma_\mu\eta \end{aligned} \ . \tag{5.4}$$

At first sight, it seems unlikely that this additional action can affect anything: the fermion has no kinetic term, and its field equations are simply $\lambda = 0$. On a manifold with boundary, however, we must be more careful. The action $S_1 + S_5$ is actually invariant only up to surface terms,

$$\delta S = -\frac{k}{4\pi}\int_{\partial M} n_\mu\epsilon^{\mu\nu\rho}\left(\partial_\nu\Lambda + i\bar\eta\gamma_\nu\lambda\right)A_\rho \ , \tag{5.5}$$

and we shall have to add additional terms to cancel this variation.

In particular, let Σ again be a disk, with the off-diagonal metric (2.5) for M. We have seen that the bosonic field U is chiral, so we can take $A_\theta = \partial_\theta U = 0$ on ∂M, and can choose the gauge transformation Λ to preserve this condition. If we also restrict the parameter η to satisfy $\partial_\theta \eta = 0$, we can write $\Lambda = i\bar{\eta}\gamma_\theta\psi$, where the field ψ on ∂M is defined by the condition

$$\lambda = \partial_\theta \psi \ . \tag{5.6}$$

Inserting this definition into (5.4), we find that $\gamma_\theta\delta\psi = -A_t\gamma_\theta\eta$, and the surface variation (5.5) becomes

$$\delta S = -\frac{k}{2\pi}\int_C dt d\theta A_t \partial_\theta (i\bar{\eta}\gamma_\theta\psi) = \delta\left[\frac{ik}{4\pi}\int_C dt d\theta\, \bar{\psi}\gamma_\theta\partial_\theta\psi\right] \ . \tag{5.7}$$

To maintain supersymmetry, we must therefore supplement the bosonic effective action (2.4) with a surface term of the form

$$S'_{\text{eff}} = -\frac{ik}{4\pi}\int dt d\theta\, \bar{\psi}\gamma_\theta\partial_\theta\psi \ , \tag{5.8}$$

which may be recognized as the standard fermionic action in superstring theory. Note that ψ is again chiral, and that γ_θ is a projection, ${\gamma_\theta}^2 = g_{\theta\theta} = 0$, as it should be. Sakai and Tanii have discussed the generalization of this procedure to arbitrary Chern-Simons theories, showing that one obtains the usual super-WZW models on the boundary [27].

As Witten has pointed out [28], one can also calculate the fermionic contribution to the Lorentz anomaly for this super-Chern-Simons action. To maintain supersymmetry when the action is regulated by the higher derivative term (3.1), we must use the supersymmetric extension of the regulator, which includes a kinetic term for λ. The pure super-Chern-Simons term (5.3) then acts as a mass, and the known local Lorentz anomaly for a three-dimensional massive fermion [20–22] can be used to calculate k' in (4.1). This fermionic anomaly can easily be shown to contribute the correct central charge.

For the b-c action (5.2), of course, there is no supersymmetry, and the preceding arguments do not apply directly. They strongly suggest, however, that just as a two-derivative bosonic action on the boundary comes from a one-derivative action on M, a one-derivative action on the boundary should come from a zero-derivative action on M with appropriate boundary terms. The search for such an action is in progress.

6. Conclusion

String theory is fundamentally a theory of two-dimensional random surfaces. As we have now seen, however, much of the theory can be reformulated one dimension higher, in terms of topological field theory and three-dimensional gravity. While some elements are still missing — most notably, a three-dimensional description of b-c systems — the problems do not seem insurmountable.

Whether this recasting of string theory in three dimensional terms has fundamental significance remains to be seen. Chern-Simons theories have proven useful in sorting out rational conformal field theories; a Chern-Simons action is conceptually simpler than the conformal field theory action it generates. On the other hand, much of the recent work in string theory has centered on the use of random matrix models to represent surfaces, and a three-dimensional formulation of these models seems difficult.

It is worth noting that the topological membrane theory we have discussed here may also be viewed as a low energy limit of a nontopological theory, for which string theory would arise only as an approximation. We introduced the F^2 term (3.1) as a regulator for the Chern-Simons action. But it may be useful to instead take S_2 to be a genuine physical contribution to the action. Massive excitations in the resulting model would not be restricted to the boundary of the "world tube," but would propagate in its interior, restoring dynamics to the two-dimensional membrane; although their contribution to amplitudes would be suppressed by powers of γ, their effects might sometimes be significant. It is likely, although not yet certain, that the resulting theory (including the gravitational contributions (3.5) and (4.1)) would be renormalizable [29]. Such a picture would offer a nice physical interpretation of Chern-Simons states in terms of Landau levels for particles in a magnetic field [30], and could be helpful in understanding off-shell strings [1] and the Hagedorn transition [31].

We do not yet know how powerful this point of view will prove to be. Ultimately, an evaluation of the importance of the topological membrane will have to await a much deeper understanding of string theory.

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