THREE DIMENSIONS OF THE SURVIVAL CURVE: HORIZONTALIZATION, VERTICALIZATION, AND LONGEVITY EXTENSION*

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Three dimensions of the survival curve have been developed: (1) "horizontalization," which corresponds to how long a cohort and how many survivors can live before aging-related deaths significantly decrease the proportion of survivors; (2) "verticalization," which corresponds to how concentrated aging-related ("normal") deaths are around the modal age at death (M); and (3) "longevity extension," which corresponds to how far the highest normal life durations can exceed M. Our study shows that the degree of horizontalization increased relatively less than the degree of verticalization in Hong Kong from 1976 to 2001. After age normalization, the highest normal life durations moved closer to M, implying that the increase in human longevity is meeting some resistance.

The rectangularization of the survival curve and its links to potential limits on increases in life expectancy in low-mortality countries have been debated over several decades. The shapes of survival curves were first studied by biologists (Deevey 1947; Pearl and Doering 1923; Pearl and Miner 1935). Comfort (1956, 1964) introduced the concept of the rectangularization of the survival curve, and the concept was subsequently discussed by others (Gordon 1980; Strehler 1975; Vaupel et al. 1979) before Fries popularized it in 1980. According to Fries (1980; see also Hayflick 1981), the human survival curve tends to become rectangular as mortality declines because an upper boundary to human life expectancy at 85 years is determined by fixed genetic limits. Although heated debate about this proposition continued (Fries 1984; Myers and Manton 1984a, 1984b; Schneider and Brody 1983), many demographers who have studied the transformation of survival curves in different countries have found evidence of rectangularization during the epidemiological transition (Cheung 2001; Eakin and Witten 1995; Go et al. 1995; Hill 1993; Levy 1996; Manton and Stallard 1996; Martel and Bourbeau 2003; Nagnur 1986; Nusselder and Mackenbach 1996, 1997; Paccaud et al. 1998; Pelletier, Legare, and Bourbeau 1997; Robine 2001; Rothenberg, Lentzner, and Parker 1991; Wilmoth and Horiuchi 1999).

However, because life expectancy at birth is now approaching 85 years for females in some countries and regions, such as Japan and Hong Kong, without any signs of slowing down, the links between the rectangularization of the survival curve and the limits to human life expectancy have become more controversial (Barbi, Caselli, and Vallin 2003; Oeppen and Vaupel 2002; Olshansky, Carnes, and Désesquelles 2001). Some researchers

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have argued that the belief that life expectancy is currently approaching a ceiling is wrong. The rectangularization trend that has been observed in France, Japan, Sweden, and the United States was gradually replaced by an almost-parallel shift of the survival curve to the right, which may imply that a slight tendency toward the "derectangularization" of the survival curve is emerging (Yashin et al. 2001). Indeed, human survival curves will never become totally rectangular, since it would imply zero variability in the age at death (Wilmoth and Horiuchi 1999).

Many demographers who analyze mortality changes over time in low-mortality countries share two common views: that most of the gain in life expectancy before the first half of the twentieth century was due to large reductions in mortality early in life and that recent and future gains in human life durations are and will be due mainly to mortality decline among the elderly. These two successive stages illustrate the transformations of the survival curve (Kannisto et al. 1994; Robine 2001; Wilmoth 1998, 2000, 2002).

The concept of rectangularization of the survival curve has been poorly defined and subjectively judged in visual terms on occasion. For example, Manton and Tolley (1991) proposed distinguishing between "hard" and "soft" rectangularization to distinguish the degree of rectangularization. Coale (1996) described the survival curve in low-mortality populations as being nearly flat from age 5 to 55, then starting to fall, later becoming steeper, and even following a vertical fall as an innate maximum age is approached. Another problem arises from researchers using different starting ages to measure the rectangularization of the survival curve or the compression of mortality without clearly distinguishing the two concepts (Hill 1993; Manton and Singer 1994; Manton and Stallard 1996; Paccaud et al. 1998). There is an obvious relationship between the rectangularization of the survival curve and the decrease in variability of ages at death. However, according to Wilmoth and Horiuchi (1999), the limits to life expectancy and the rectangularization of the survival curve are two independent phenomena that need to be studied separately. Some 20 years after Fries's reintroduction of the concept of rectangularization of the survival curve, there appears to be no consensus on a definition of this phenomenon and of how to assess it.

ASSESSING THE EXISTING INDICATORS

Although many indicators have been used to monitor the rectangularization of the survival curve or the compression of mortality and to study their links with limits to human longevity, the indicators of rectangularization, compression, and human longevity have scarcely been differentiated owing to the lack of clear definitions of these terms. Before we discuss the development of new indicators, we review 24 existing indicators (see Table 1). After assessing all the collected materials, concepts, and indicators, we classified these indicators according to seven headings: 3 central longevity indicators, no horizontalization indicators, 10 concentration or verticalization indicators, 4 rectangularization indicators, 3 maximum longevity indicators, 1 mapping indicator, and 3 other indicators.

It is evident that not all these indicators are required to monitor the transformations of the survival curve and their links to the limits of human longevity. Using all of them would produce redundant information. The majority of them aim to examine the steepness of the survival curve, either by measuring the concentration of ages at death around a central value or by directly measuring the verticalization of the survival curve. Examples of the former are the interquartile range (IQR) of deaths (Wilmoth and Horiuchi 1999) and C50 (the shortest age interval that is necessary to concentrate 50% of life durations; Kannisto 2000b) for directly measuring the age concentration. An example of the latter is the prolate rectangularity index κ (Eakin and Witten 1995) for directly measuring the verticalization of the survival curve. These indicators use all deaths, and their calculations are thus differently affected by any overlapping of "premature" deaths (that is, deaths before aging-related mortality becomes substantial).

Table 1. Classification of All the Existing Indicators Proposed to Monitor the Rectangularization of the Survival Curve and/or the Compression of Mortality and Its Links to the Limits to Human Longevity

- Central Longevity Indicators
  - Life expectancy (or mean)
  - Median
  - Mode
- Horizontalization Indicator
  - No indicator found
- Concentration and/or Verticalization Indicators
  - Standard deviation of life spans or of ages at death
  - Standard deviation above the mode (*SD*+)
  - Standard deviation above the third quartile
  - Interquartile range (IQR)
  - C-family (C10, C50, and C90)
  - $_{10}C_{50}$
  - Prolate index
  - Entropy Keyfitz's *H*
  - Life expectancy at median life span and third quartile
  - Fastest decline and/or highest proportion of deaths
- Rectangularization Indicators
  - Fixed rectangle
  - Moving rectangle and/or index of rectangularity (R)
  - Person-years differential (PD)
  - Person-years ratio (PR)
- Maximum Longevity Indicators
  - Life endurancy
  - Maximum life span (MLS)
  - Length of the outer tail of longevity
- Mapping Indicators
  - Percentiles
- Other Indicators
  - Coefficient of variation (CV)
  - Numerator of Keyfitz's *H* (NH)
  - Gini coefficient

On the other hand, we did not find any indicators for measuring the horizontalization of the survival curve or for indicating when the survival curve starts to fall as a result of aging-related mortality. The term *horizontalization* of the survival curve was introduced by Robine (2001) to describe the rectangularization of the survival curve owing to a decline in infant and premature mortality, which occurred in the first phase of the epidemiological transition. Intuitively, the degree of horizontalization can be measured by the age reached by some high percentile of survivors in a life table (i.e., the age reached by 90%, 95%, or 99% of the survivors), but this approach is limited to a situation in which infant mortality is low and is undermined by the arbitrary nature of the percentiles.

The tail of the survival curve has received little systematic study (Smith 1994). A few indicators have been proposed to assess the longevity extension, such as the maximum life span (MLS; Finch and Pike 1996) or life endurancy (i.e., the age reached by 1 per 10,000 or 1 per 100,000 individuals; Bell, Wade, and Goss 1992; Faber 1982; Faber and Wade 1983), but they are strongly affected by population size. The indicator proposed by Vaupel (2002), the relative length of the tail of longevity (defined as the maximum attained age minus the age attained by 10% of survivors and divided by this age), is still linked to the MLS. According to Kannisto (1999, 2000a, 2000b, 2001), indicators should be free from any fixed age or percentile determinations, which is not the case for most of them.

In this article, therefore, we introduce three complementary and coherent measures, which are free from any fixed-age or percentile determinations, to disentangle the complexity of the "rectangularization of the survival curve" that accompanies the epidemiological transition: (1) the "horizontalization" corresponds to how long a cohort can live and how many cohort members survive before aging-related deaths significantly decrease the proportion of survivors; (2) the "verticalization" corresponds to how concentrated aging-related deaths are around the modal age at death; and (3) the "longevity extension" corresponds to how far the right-hand tail, representing the highest normal life durations, can exceed the modal age at death.

MATERIALS AND METHODS

Defining the Transformations of the Survival Curve and Developing New Indicators

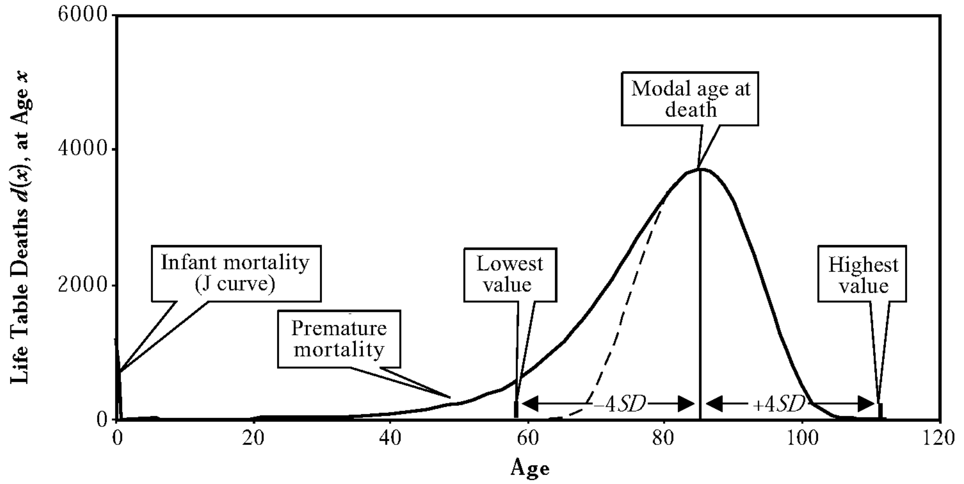
The rationale of our work is the concept of normal life durations, introduced by Lexis in 1878. Lexis combined Quetelet's (1991[1835], 1848, 1871) model of *l'homme moyen*, or "average man," and the law of the normal distribution of errors formulated by Laplace (1812) to study common human longevity. According to Quetelet, the concept of average man is not an arithmetic mean, but a typical, central value along a normal curve that expresses the deep nature of things. The modal age at death (M) accounts for the central value of the frequency distribution representing the most common individual lifetime, limited by minimum and maximum values that follow the laws of nature (see Véron and Rohrbasser 2003 for more details on Lexis's approach).

According to Lexis (1878), mortality is conceived as consisting of three parts: (1) a J-shaped curve right after birth corresponding to infant deaths, (2) the "normal" deaths around the late modal length of life that obey the law of accidental errors reflecting the natural lifetime, and (3) the deaths occurring in a transitional age range in which premature deaths and normal deaths overlap. Infant and premature deaths are associated with the early mode, while aging-related deaths are associated with M.

Lexis (1878) considered that M, the most frequent value of the distribution, represents the most central and natural characteristic of human longevity and that deaths occurring at and above it are regarded as "normal" deaths, accounting for the right-hand side of the distribution (see Figure 1). By symmetry with the right-hand side, the left-hand side of the normal distribution can be found by disentangling "normal" deaths from premature deaths (i.e., related to reproduction, risk-related behavior, accidents, and infectious diseases) before M (the dashed line in the figure).

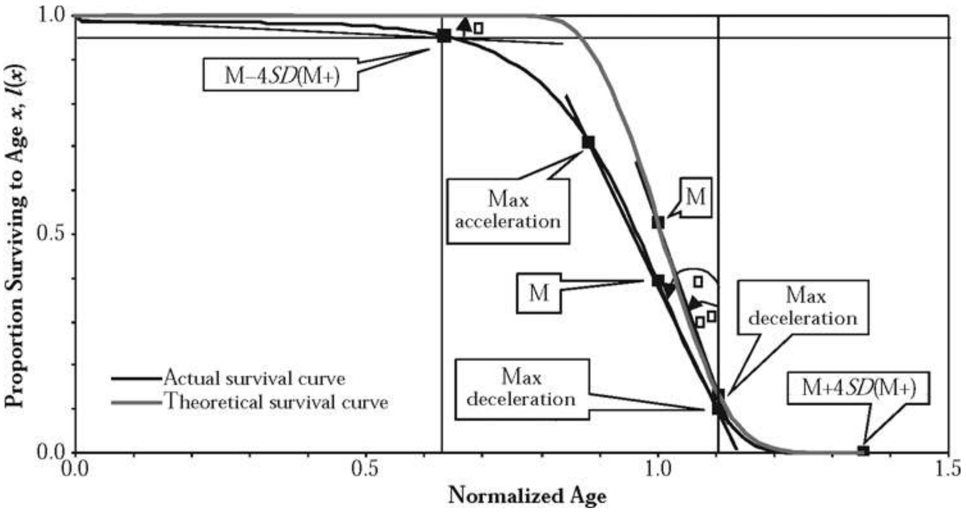
M is determined solely by deaths at old ages. These aging-related deaths are assumed to be normally distributed around M; thus, Lexis's "normal" deaths are referred to as "aging-related" deaths. Following the determination of M by Kannisto (2001), we used four standard deviations from M (M ± 4*SD*(M+)) for indicating the shortest and longest aging-related life durations (Cheung 2003; Cheung et al. 2005) (see Appendix A). The choice of four standard deviations from M (M ± 4*SD*(M+)) is sensible, since it covers about 99.99% of the total normal life durations and corresponds well to empirical observations of the longest life durations.

Figure 1. Normal Longevity Indicators: Modal Age at Death and 4 Standard Deviations From the Mode When Life Expectancy at Birth is 80 Years



Source: Adapted from Cheung (2003) and Cheung et al. (2005).

Figure 2. Three Dimensions of Transformations of the Survival Curve and Its Indicators, Actual and Theoretical Survival Curves After Age Normalization



On the basis of this framework and these normal indicators, Figure 2 illustrates the formation of the proposed indicators of the three dimensions of the survival curve, describing the probability of surviving.

The Degree of Horizontalization (β)

To construct the angle β, we used M–4*SD*(M+), indicating the lowest value within the distribution of normal life durations, to distinguish the first part of the survival curve in which almost all deaths are attributed to infant and premature mortality (see Figure 2). As infant and premature deaths are reduced, the survival curve becomes more horizontal until the age corresponding to M–4*SD*(M+). The actual degree of horizontalization is based on two points on the survival curve: (1) the point at birth where the probability of surviving is equal to 1 and (2) the point corresponding to M–4*SD*(M+). The approximately horizontal segment of the survival curve is the diagonal line connecting 1 at birth to x (where $0 < x \leq 1$) at the age corresponding to M–4*SD*(M+) (see Appendix B). The angle β lies between 0 degrees, when the diagonal line is totally horizontal (i.e., nobody dies before the age corresponding to the shortest normal life durations), and 90 degrees, when the diagonal line is vertical (i.e., everybody dies at birth). As the survival curve becomes more horizontal with the decline in infant and premature mortality, β becomes smaller.

The Degree of Verticalization (θ and θ^*)

Verticalization, the second dimension in the transformations of the survival curve, accounts for the steepness of the survival curve in the region of M. This steepness depends on the concentration of the ages at death around M. The prolate rectangularity index proposed by Eakin and Witten (1995) is based on the angle θ located on the diagonal line, connecting the point of maximum acceleration in attrition to the point of maximum deceleration on the actual survival curve (see Figure 2). Since the distribution of life durations consists of a mixture of infant, premature, and aging-related deaths before M where the point of maximum acceleration in attrition is located, it is difficult to interpret the angle θ.

However, the angle θ^* is based on aging-related deaths after the elimination of all infant and premature deaths on the theoretical survival curve (see Figure 2). In this framework, the diagonal line connects M and the point of maximum deceleration (see Appendix C). The angle θ^* lies between 0 degrees, when the diagonal line is totally vertical (i.e., everybody dies at M), and 90 degrees, when the diagonal line is totally horizontal (i.e., nobody dies). As the theoretical survival curve becomes more vertical, with increasing concentration of aging-related deaths around M, θ becomes smaller. The comparison of θ on the actual survival curve and θ^* on the theoretical curve allows one to measure how vertical the survival curve would be after all infant and premature mortality were eliminated.

M+4*SD*(M+)

The last dimension of the transformations of the survival curve, as indicated by M+4*SD*(M+), corresponds to changes in the right-hand tail of the survival curve and describes how far the highest normal life durations can exceed the modal age at death.

To allow for space, time, population group, and even species comparisons, all ages are normalized by M before the various angles and indicators are computed. On the x-axis, M becomes 1, and all other ages are expressed as a proportion of it. Only the age at birth keeps its initial value, 0 (see Figure 2). On the y-axis, the probability of surviving decreases from 1 at birth to 0 at the age at the last death. Total or perfect rectangularization corresponds to the probability of surviving remaining at 1 until the normalized age 1 when everybody dies. After normalization, the scales of the x- and y-axes are set equally for graphical display.

Application to Hong Kong

Hong Kong, which has almost 7 million inhabitants, has experienced an accelerated demographic transition. Since 1961, life expectancy at birth has increased by more than 14 years, reaching 78.6 years for males and 84.5 years for females in 2002, one of the highest values in the world. In this study, we used six complete period life tables fitted with a logistic model from age 85 to age 120, comparable for each sex, from 1976 to 2001 (Cheung et al. 2005). The data were smoothed by the graduation coefficients based on Beer's modified formula (Hong Kong Census and Statistics Department 2002).

RESULTS

Change in Horizontalization of the Survival Curve

Table 2 shows that the angle β slowly decreased from 6.3 degrees and 6.2 degrees in 1976 to 4.4 degrees and 4.7 degrees in 2001, for males and females, respectively, indicating that the survival curves became more horizontal over time. The degree of horizontalization of the survival curve indicates how a decline in infant and premature mortality affects the proportion of survivors at the age at which the first aging-related deaths occur, that is, M–4*SD*(M+), and is governed by two factors: the degree of attrition by age M–4*SD*(M+) and the value of M–4*SD*(M+) itself. For males, the attrition in survivors decreased by 0.7 percentage points, from 5.5% to 4.8%, while the age at the first aging-related deaths increased from 38.6 years to 53.3 years from 1976 to 2001. For females, the attrition in survivors decreased by 0.9 percentage point, from 6.6% to 5.7%, while the age at the first aging-related deaths increased from 51.4 years to 63.6 years during the 25-year period. The proportion of survivors at the age corresponding to M–4*SD*(M+) is similar for males and females, but the age at the first aging-related death is much higher for the latter.

Change in Verticalization of the Survival Curve

Table 2 also shows that the angle θ decreased from 27.1 degrees to 21.6 degrees for males and from 22.5 degrees to 17.5 degrees for females in 1976 and 2001, respectively. The survival curves became more vertical over time. These results also suggest that the survival curve is more vertical for females than for males, indicating that the ages at death are more concentrated around M for females.

In a what-if scenario, in which all infant and premature deaths are eliminated on the theoretical survival curves, the angle θ^* decreased from 21.7 degrees to 16.5 degrees for males and from 16.5 degrees to 12.8 degrees for females in 1976 and 2001, respectively (see Table 2). The angles θ and θ^* decreased by about 5 degrees during the 25-year period. The gap between these two angles is about 5.8 degrees, on average, and narrows over time, suggesting a decreasing impact of infant mortality on the verticalization of the survival curve.

The longest normal life durations, indicated by M+4*SD*(M+), increased only slightly, by 1 year for males, from 116.5 years to 117.5 years, and remained nearly constant for females, from 117.5 years to 117.7 years, in 1976 and 2001, respectively (see Table 2).

Compression of Normal Life Durations

Age normalization enables us to see whether the normal life durations become more compressed and whether M moves relatively closer to the longest normal life durations.

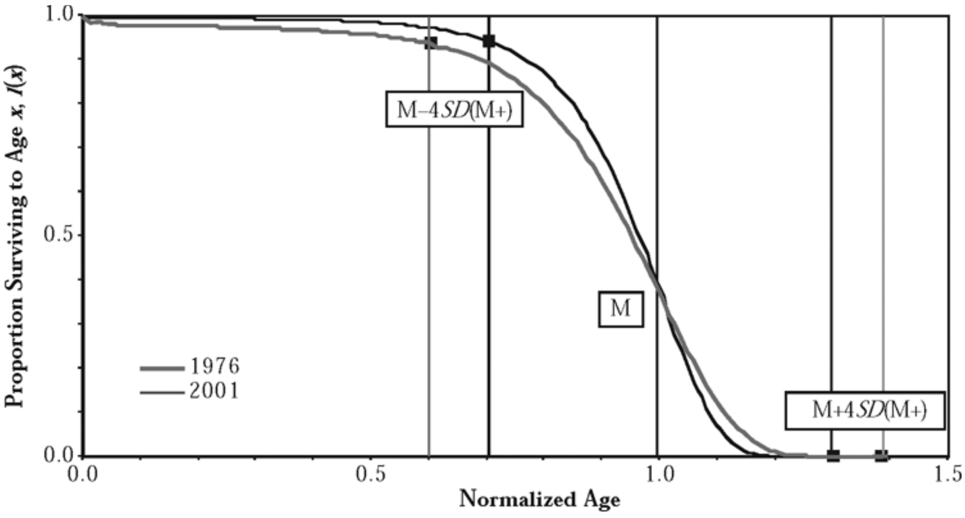
Figure 3 shows that the normalized age at M–4*SD*(M+) increased from 0.60 to 0.71, indicating that the shortest life durations corresponding to aging-related deaths moved relatively closer to M (M is expressed as value of 1) for females from 1976 to 2001. The normalized age at M+4*SD*(M+), indicating the longest life durations corresponding to aging-related deaths, decreased slightly from 1.39 to 1.30 times that of M, showing that

Table 2. Degree of Horizontalization and Verticalization of the Actual Survival Curves and the Theoretical Survival Curves Without Infant and Premature Mortality, for Males and Females, Hong Kong, 1976–2001

	1976	1981	1986	1991	1996	2001
Males						
Actual survival curves						
Modal age at death (M)	77.6	79	80.5	82	83.7	85.4
SD(M+)	9.74	9.44	9.13	8.79	8.43	8.03
M–4*SD*(M+)	38.6	41.3	44.0	46.9	50.0	53.3
M+4*SD*(M+)	116.5	116.8	117.0	117.2	117.4	117.5
l(*x*) at M–4*SD*(M+)	0.9453	0.9497	0.9511	0.9514	0.9516	0.9517
β	6.3	5.5	5.1	4.9	4.6	4.4
Age at max acceleration	63	63	63	63	72	74
Age at max deceleration	89	90	91	92	94	95
l(*x*) at max acceleration	0.7557	0.7825	0.8079	0.8336	0.7095	0.7142
l(*x*) at max deceleration	0.0997	0.1	0.1011	0.1033	0.0877	0.0929
θ	27.1	26.6	26.2	25.8	22.9	21.6
Theoretical survival curves without infant and premature mortality						
Sv_{M}	0.6921	0.4974	0.5202	0.5008	0.5217	0.4929
$Sv_{\mathrm{max\ d}}$	0.0997	0.1275	0.1351	0.1328	0.1424	0.1151
θ^*	21.7	19.9	19.6	17.8	18.1	16.5
$\Delta\theta$	5.4	6.7	6.6	8.0	4.8	5.1
Females						
Actual survival curves						
Modal age at death (M)	84.4	85.5	86.6	87.9	89.2	90.7
SD(M+)	8.26	8.01	7.75	7.46	7.14	6.77
M–4*SD*(M+)	51.4	53.4	55.6	58	60.6	63.6
M+4*SD*(M+)	117.5	117.6	117.6	117.7	117.7	117.7
l(*x*) at M–4*SD*(M+)	0.9339	0.936	0.9368	0.9371	0.94	0.9426
β	6.2	5.9	5.6	5.4	5.0	4.7
Age at max acceleration	72	73	75	76	78	80
Age at max deceleration	93	94	95	95	96	97
l(*x*) at max acceleration	0.7174	0.7267	0.7167	0.7314	0.7300	0.7337
l(*x*) at max deceleration	0.1178	0.1137	0.1104	0.1331	0.1341	0.1385
θ	22.5	21.8	20.8	19.9	18.7	17.5
Theoretical survival curves without infant and premature mortality						
Sv_{M}	0.6273	0.5031	0.5000	0.4939	0.4836	0.5170
$Sv_{\mathrm{max\ d}}$	0.1178	0.1609	0.1542	0.1478	0.1746	0.1875
θ^*	16.5	16.0	15.6	14.8	13.1	12.8
$\Delta\theta$	6.0	5.8	5.2	5.1	5.6	4.7

Notes: Sv_{m} and $Sv_{\mathrm{max\ d}}$ refer to Appendix C. $\Delta\theta$ shows the difference between the angles θ with all deaths, and θ^* shows the angle with only the aging-related deaths, indicating the deficit of the verticalization of the survival curve owing to the infant and premature deaths.

Figure 3. The Changes in M–4*SD*(M+) and M+4*SD*(M+) on Survival Curves, When Ages Are Expressed as a Proportion of the Modal Age at Death for Females, Hong Kong, 1976 and 2001



the tail of survival curves came relatively closer to M during the 25-year period. The results for males were similar over the same period.

DISCUSSION

After some 20 years of work on the rectangularization of the survival curve, there is consensus neither on the definition of this concept nor on the means to monitor the transformations of the survival curve. Appropriate measures of this rectangularization could make evidence of the proposed compression of morbidity easier to recognize (Fries 2002; Hubert et al. 2002; Vita et al. 1998). In this article, the introduction of three dimensions of the transformation of the survival curve and their indicators helped us to evaluate various competing hypotheses between the lengthening of life and the improvement of health.

Our approach was built mainly on Lexis's (1878) concept of normal life durations. M, which was the key element of this study, represents the most common or typical length of life (i.e., the largest number or the highest percentage of observations) under a given mortality regime (Kannisto 2001). As such, it is useful in studies of senescence and longevity (Horiuchi 2003). First, the use of Lexis's approach gave us a mechanical procedure for disentangling aging-related deaths from premature deaths before M at a population level without examining specific causes of deaths. Further research can consider how to adjust Lexis's approach to account for possible underestimation or overestimation of some non-aging-related deaths after M and some aging-related deaths before M. Second, normality in the distribution of deaths above the modal length of life is not a strictly necessary condition. What is necessary is the use of a symmetrical property to disentangle aging-related deaths from premature deaths that are located before M, as was suggested by Benjamin (1959). Another good example of aging-related events that are expected to show some natural symmetry and are well fitted by a normal distribution is the distribution of ages at menopause (Boldsen and Jeune 1990; Frommer 1964; MacMahon and Worcester 1966).

Although Greenwood and Irwin (1939) showed that neither the Gompertz nor the normal approach has an overwhelming advantage over the other and that neither has a significant impact on a natural "law," sensitivity tests with the latest available data in Hong Kong in 2001 showed that a direct normal fitting on the deaths by age $d(x)$ function of the life table has the best fit with the data ($R^2 = 0.998$), followed by a logistic fitting ($R^2 = 0.992$) and a Gompertz fitting ($R^2 = 0.976$). This result is congruent with our hypothesis that the deaths at and above M are normally distributed. The Gompertz-associated $d(x)$ series is far from the unfitted life table $d(x)$ series in the region of the modal age at death. The age reached by 1 out of 10,000 people in the logistic-associated $d(x)$ series is too high when compared with empirical durations of life gathered at the International Database on Longevity (Robine and Vaupel 2002). Moreover, these tests show that our longevity indicator, M+4*SD*(M+), corresponds well to the age reached by 1 out of 10,000 people according to the normal distribution.

Another methodological concern is whether the logistic model that we used to close the Hong Kong period life tables (Cheung et al. 2005) has a significant impact on the shape of the survival curve and its transformations over time. Future research with large empirical data sets, without any age censoring (such as the French, Japanese, or Swedish data sets) and without any extrapolative technique, are needed to investigate this question.

On the basis of the conceptual framework, the angle β is intended to measure the degree of horizontalization of the survival curve that corresponds to the first part of the survival curve where almost all deaths are unrelated to aging. Then the angle θ^* is computed to measure the degree of verticalization of the survival curve in the absence of infant and premature mortality. Both angles rely on M and are free from any arbitrary age limit or percentile.

Comparison of the angles θ with all kinds of deaths on the actual survival curves and θ^* with only aging-related deaths on the theoretical survival curves allows for an assessment of the underestimation of verticalization that is due to infant and premature mortality. When infant and premature mortality is not eliminated on the actual survival curve, the point corresponding to the maximum acceleration in attrition is difficult to locate because the survival attritions around this point are affected by overlapping premature and aging-related deaths. Several empirical points can be seen as the point of maximum acceleration in attrition, a situation that is more pronounced for males. We did not apply the last transformation used by Eakin and Witten (1995), taking the cosine of the angle θ leading to a unit scale from 0 to 1, because this transformation retains neither the absolute nor relative changes. The angles β, θ, and θ^* that we proposed, on a scale from 0 degrees to 90 degrees, explicitly measure changes in the degree of the horizontalization and verticalization of the survival curve without introducing any distortion. Last, with regard to our research focus on aging-related deaths and normal life durations, M is a natural choice for age normalization instead of life expectancy at birth, as Eakin and Witten (1995) used, because, by definition, infant and premature deaths are included in human life expectancy at birth.

The angle θ^*, a direct measure of the verticalization of the survival curve in the absence of infant and premature mortality, provides an indication of the concentration of the distribution of deaths around the mode. On the other hand, *SD*(M+), which is a good measure of old-age mortality compression (for the $d(x)$ series), provides only an indirect indicator of verticalization of the survival curve. Therefore, *SD*(M+) and θ^* have different functions and purposes.

M+4*SD*(M+) indicates the values of the highest normal (i.e., aging-related) life durations, putting aside potential outliers such as maximum life span. The normalized longest aging-related life durations clearly illustrate whether the tail of the survival curve is relatively compressed or expanded as M increases, independently of the size of the population and free of any percentile. While only the first two dimensions (horizontalization and verticalization) are strictly needed to assess the rectangularization of the survival curve, the third dimension (longevity extension) gives direct information on the most common, as well as the longest, normal life durations.

In terms of results, our study first showed that there was a slow upward trend in the degree of horizontalization of the survival curve in Hong Kong from 1976 to 2001. Although the age at which the first aging-related deaths occurred, as indicated by M–4*SD*(M+), increased by more than 10 years during the 25-year period, the proportion of survivors at this age did not change substantially. Further progress in horizontalization requires a significant decline in adult mortality within the age range of 50 to 70, a decline that may be difficult to achieve. The likelihood of the continuation of horizontalization is slim.

Second, our study showed that the survival curves became more vertical from 1976 to 2001. Approximately one quarter of the underestimation of verticalization, which reduced and narrowed over time, was due to infant and premature mortality. Once all ages have been normalized and expressed as a proportion of M, the age corresponding to the shortest aging-related life durations moved closer to M, the distribution of aging-related life durations overall were more concentrated around M, and the age corresponding to the longest aging-related life durations also moved closer to M. Male values approximated female values for 25 years earlier.

Although the results suggest that increases in longevity were reaching some limit in Hong Kong, many recent demographic studies have documented a substantial fall in mortality among the oldest-old and a significant increase in the number of centenarians in Europe and Japan, findings that are more in favor of an acceleration in the increase in longevity than of a slowing down (Robine, Caselli, and Saito 2003; Robine and Paccaud 2005; Robine and Saito 2003; Robine, Saito, and Jagger 2003; Robine and Vaupel 2001; Thatcher 1999, 2001; Vaupel and Jeune 1995; Wilmoth et al. 2000; Wilmoth and Lundström 1996). The changes in the three dimensions of the survival curves in these countries are not known. We do not know whether these countries have experienced a trend similar to the one observed in Hong Kong, where the survival curve continued its horizontalization and verticalization and where its tail came relatively closer to M, suggesting some limit to increased longevity.

Since Hong Kong is a city-state and our data set covers only the past 25 years, it is important to study larger and longer mortality data sets to examine whether the transformations of the three dimensions of the survival curve follow a universal pattern or whether they vary in different regions and through different stages of the epidemiological transition. Further study is needed to examine whether some Asian populations with long life expectancies or who are facing a rapid epidemiological transition, such as Japan, are experiencing specific transformations of the survival curve.

What will happen to the three dimensions in Hong Kong in the future? The survival curve will never become totally rectangular, since doing so would require that everybody would die at M without any survivors in the tail of survival curve. Many studies have suggested that human life durations depend on many factors, such as heritability, genetic heterogeneity, system redundancy, plasticity to environmental modifications, and public policies (Gavrilov and Gavrilova 2001; Herskind et al. 1996; McGue et al. 1993; Robine 2003; Vaupel, Carey, and Christensen 2003). Therefore, the fundamental questions for future transformations of the three dimensions of the survival curve will be how far the variability in human longevity can be reduced and whether the tail of the survival curve will move much further from M.

APPENDIX A

To obtain M±4*SD*(M+), the standard deviation of individual life durations above the mode (*SD*(M+)) must be calculated, that is, the root-mean-square of the positive deviations from M (Kannisto 2001:163), under the assumption that deaths are evenly distributed within each year of age.

The root-mean-square deviation from M is denoted by $RMSD_x$:

$$RMSD_x = \sqrt{\frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}{n}}.$$

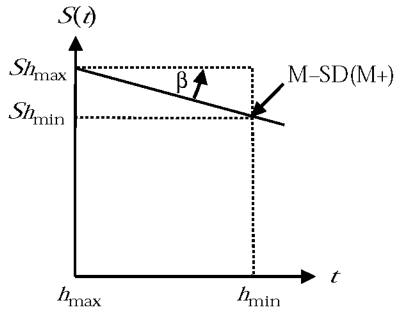
To understand this definition, one must be able to carry out the indicated operations. After the exact mode has been determined, the first step in calculating the $RMSD_x$ is to compute the deviation from the exact late mode (the mean of the normal life durations) for each observation. Each of these deviations is then squared. The n-squared deviations are then added together, and the sum is divided by n to yield the mean squared deviation. Taking the square root yields the root-mean-squared deviation (Mirer 1988:42).

APPENDIX B

In the calculation of β, the slope of the diagonal line, $\left(Sh_{\min} - Sh_{\max}\right)/\left(h_{\min} - h_{\max}\right)$, yields a negative value. Therefore, the absolute value sign is applied.

$$\beta \equiv \arctan\left|\frac{Sh_{\min} - Sh_{\max}}{h_{\min} - h_{\max}}\right|,$$

with $Sh_{\max} = 1$, the probability of survival at age 0; $Sh_{\min}$ = the probability of survival at the age corresponding to the shortest normal life durations; $h_{\min}$ = the age corresponding to the shortest normal life durations; and $h_{\max} = 0$, the starting age for the cohort.



APPENDIX C

In the calculation of θ*, the slope of the diagonal line, $\left(M - v_{\max d}\right)/\left(Sv_m - Sv_{\max d}\right)$, yields a negative value. Therefore, the absolute value sign is applied.

$$\theta \equiv \arctan\left|\frac{M - v_{\max\, d}}{Sv_M - Sv_{\max\, d}}\right|,$$

with M = the modal age at deaths on the survival curve; $v_{\max d}$ = the age corresponding to the point of maximum deceleration in attrition on the survival curve; Sv_M = the probability of survival at the modal age at deaths on the theoretical survival curve; and $Sv_{\max d}$ = the probability of survival at the age corresponding to the point of maximum deceleration in attrition on the theoretical survival curve.

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