

THREE SPECTROSCOPIC BINARIES IN THE BRIGHT STAR CATALOG SUPPLEMENT

C. D. Scarfe^{1,2}

Received November 28 2018; accepted February 28 2019

ABSTRACT

This paper presents spectroscopic orbits of three binaries, HD 121212, HD 148434 and HD 148912, with evolved primaries and periods of order a year, found in a survey of late-type stars listed in the Supplement to the Yale Bright Star Catalog. All the orbits were determined from observations made with the DAO 1.2-m telescope and coudé spectrograph. Observations were obtained using the radial velocity spectrometer until it was decommissioned in 2004, and since then using a CCD detector and cross-correlating the spectra with those of standard stars.

RESUMEN

Se presentan las órbitas espectroscópicas de tres binarias (HD 121212, HD 148434 y HD 148912) con primarias evolucionadas y períodos de cerca de un año, encontradas en el relevamiento de estrellas tardías listado en el suplemento al Yale Bright Star Catalog. Las órbitas se determinaron a partir de observaciones hechas en el telescopio de 1.2 m del DAO con el espectrógrafo coudé. Las observaciones se obtuvieron con el espectrómetro para velocidades radiales antes de su cancelación en 2004, y a partir de entonces con un detector CCD y mediante la correlación cruzada de los espectros con estrellas estándar.

Key Words: binaries:spectroscopic

1. INTRODUCTION

About thirty years ago the author began a program of observations of the radial velocities of late-type stars in the Supplement to the Yale Bright Star Catalog (Hoffleit et al. 1983), for which no radial velocity was listed in that publication. That program makes use of the coudé spectrograph of the Dominion Astrophysical Observatory's 1.22-m telescope, at a dispersion of 0.24 nm mm^{-1} . The three binaries whose orbits are presented here are among the first to be discovered in this program, in part due to their relatively short periods and consequent rapid changes in radial velocity.

2. BACKGROUND INFORMATION

2.1. HD 121212

HD 121212 (HIP 67803, CW CVn, $\alpha = 13^{\text{h}}52^{\text{m}}13^{\text{s}}$, $\delta = 33^{\circ}47'12''$ [2000]) is to be found in

the eastern part of Canes Venatici, following α CVn by almost exactly an hour at a closely similar declination. Its BV magnitude and colour were determined by *Hipparcos* and are listed in the Tycho-2 catalogue (Hog et al. 2000) as $V = 6.97$, $B - V = 1.49$, consistent with its MK type, K5 III.

The *Gaia* (2018) parallax of HD 121212 is $\pi = 2.31 \pm 0.05 \text{ mas}$, which yields $M_v = -1.21 \pm 0.28$, in good agreement with its classification as a late K giant. *Gaia* also measured its proper motion components, which are $-41.79 \text{ mas yr}^{-1}$ in right ascension and $-3.17 \text{ mas yr}^{-1}$ in declination. Together with the parallax these yield the velocity of 71.5 km s^{-1} perpendicular to the line of sight.

HD 121212 was found by *Hipparcos* (ESA 1997) to be a semiregular variable star of type SRD. However, its amplitude is $0^{\text{m}}.25$, much larger than that of most variables of that type (Adelman 2000). Over the years it did seem to the author to vary in brightness to a modest extent, but this seemed to be uncorrelated with the variation in radial velocity.

¹University of Victoria, BC, Canada.

²Guest worker, Dominion Astrophysical Observatory, Herzberg Institute of Astrophysics, National Research Council of Canada.

TABLE 1
STANDARD STARS USED FOR CROSS-CORRELATION

Standard Star	Spectral Type	$R.V.$ km s ⁻¹	Binaries		
			HD 121212	HD 148434	HD 148912
α Ari	K2 III	14.49 \pm 0.03		x	x
α Boo	K2 III	-5.30 \pm 0.03		x	x
α Cas	K0 II-III	-4.25 \pm 0.03		x	x
α Hya	K3 III	-4.40 \pm 0.06	x		x
α Tau	K5 III	54.26 \pm 0.04	x		x
β Gem	K0 III	3.28 \pm 0.02		x	x
β Oph	K2 III	-12.20 \pm 0.02		x	x
δ Oph	M1 III	-19.40 \pm 0.07	x		x
μ Psc	K4 III	34.45 \pm 0.05	x		x
16 Vir	K0 III	36.52 \pm 0.04		x	x
35 Peg	K0 III	54.36 \pm 0.03		x	x
HR 3145	K2 III	71.71 \pm 0.08		x	x

2.2. HD 148434

HD 148434 (HIP 80528, $\alpha = 16^h26^m22^s$, $\delta = 40^\circ48'34''$ [2000]) is located in the northern part of the constellation of Hercules, a little to the south of σ Her. Its BV magnitude and colour were found by *Hipparcos* (Hog et al. 2000) to be $V = 6.91$, $B - V = 1.11$, in agreement with its spectral type of K0 III.

The *Gaia* (2018) parallax of HD 148434 is $\pi = 7.31 \pm 0.03$ mas, which yields $M_v = 1.23 \pm 0.01$, supporting its classification as a giant. *Gaia* also measured the proper motion of HD 148434, obtaining $\mu_\alpha = 21.368 \pm 0.033$ mas yr⁻¹, $\mu_\delta = -53.462 \pm 0.035$ mas yr⁻¹. With the parallax these yield a tangential velocity of 36.2 km s⁻¹.

2.3. HD 148912

HD 148912 (HIP 80907, $\alpha = 16^h31^m21^s$, $\delta = 1^\circ18'32''$ [2000]) lies in the constellation of Ophiuchus, but close to its boundaries with both Hercules and Serpens, and near the bright star λ Oph. Its BV magnitude and colour are $V = 6.78$, $B - V = 1.39$ and its MK type is K2-3 III.

The *Gaia* (2018) parallax of HD 148912 is $\pi = 2.51 \pm 0.11$ mas, which yields $M_v = -1.22 \pm 0.10$, consistent with its status as a late-K giant. The proper motion components, also measured by *Gaia*, are $\mu_\alpha = -17.119 \pm 0.202$ mas yr⁻¹, $\mu_\delta = 22.117 \pm 0.138$ mas yr⁻¹. The tangential velocity determined from these and the parallax is 52.8 km s⁻¹.

3. OBSERVATIONS

Initially the radial velocities were determined at the telescope with the radial-velocity spectrometer (RVS) (Fletcher et al. 1982). Numerous observations

of IAU standard stars have been used to adjust the zero-point of the RVS system to the scale defined by Scarfe (2010). Since 2004, when the spectrometer was decommissioned, observations have been made through the same spectrograph optics, but using a CCD as detector, and reduced with a ‘pipeline’ program developed by D. Bohlender. Radial velocities were determined by averaging the results from cross-correlation with standard stars. The choice of standard stars for each binary is based on the precision of the cross-correlation, which in turn depends on the match in spectral type between the binary and the standards. The standards for each binary discussed here are listed in Table 1, where the velocities are on the scale and zero-point of Scarfe (2010), although the velocities differ very slightly from those in that paper, owing to the inclusion of slightly more data since its publication. The uncertainties are those of the mean value, and the binaries for which each standard was used are indicated.

For HD 121212, 95 observations were obtained with the RVS between JD 2447216 and 2452880, and 68 with the CCD between JD 2453117 and JD 2457497. The total span of 10281 days is just over 87 orbital cycles, so that the period is determined quite precisely, with a formal uncertainty of less than three minutes. The RVS velocities are presented in Table 2, and those obtained with the CCD in Table 3.

For HD 148434, 42 observations were obtained with the RVS between JDs 2447303 and 2452886, followed by 76 with the CCD between JDs 2453117 and 2457941. The span of 10638 days is just over 50 orbital cycles, and as a result the period of this binary is also determined fairly precisely, with a for-

TABLE 2
RVS RADIAL VELOCITIES OF HD 121212

Julian Date −2,400,000	Cycle No. & Phase	<i>R.V.</i> km s ^{−1}	<i>O</i> − <i>C</i> km s ^{−1}	Julian Date −2,400,000	Cycle No. & Phase	<i>R.V.</i> km s ^{−1}	<i>O</i> − <i>C</i> km s ^{−1}
47216.01	0.0553	−22.7	−0.6	50973.80	31.8653	5.1	−3.0 ^a
47259.88	.4267	−18.5	1.0	50976.74	.8903	5.0	0.0
47263.87	.4605	−13.7	0.7	51166.13	33.4935	−9.3	−0.1
47291.79	.6968	12.7	−1.8	51224.04	.9836	−10.9	−0.8
47303.77	.7982	14.0	−0.1	51279.88	34.4564	−15.4	−0.4
47324.72	.9755	−8.8	−0.1	51284.86	.4985	−8.8	−0.4
47346.77	1.1622	−33.9	0.5	51309.86	.7101	15.0	−0.0
47676.76	3.9556	−4.7	0.6	51317.80	.7773	13.4	−1.6
47694.74	4.1078	−30.0	−0.6	51325.79	.8450	10.2	−0.2
47725.76	.3704	−26.6	0.6	51333.74	.9123	1.4	−0.4
47940.02	6.1841	−36.4	−0.6	51344.74	35.0054	−15.0	−1.1
47962.96	.3783	−26.7	−0.5	51365.73	.1831	−34.9	0.8
47996.87	.6654	13.6	1.0	51370.73	.2254	−37.0	−0.3
48037.76	7.0115	−15.0	−0.2	51623.93	37.3688	−27.2	0.1
48059.76	.1978	−36.1	0.2	51634.88	.4614	−15.2	−1.0
48075.72	.3328	−30.6	0.7	51662.86	.6984	14.6	0.0
48080.72	.3752	−26.2	0.3	51689.79	.9263	−0.2	0.3
48388.80	9.9831	−10.8	−0.8	51697.72	.9934	−11.4	0.3
48418.73	10.2365	−36.5	0.2	51711.76	38.1123	−30.2	−0.4
48605.06	11.8138	13.6	0.5	51718.74	.1714	−35.2	−0.1
48681.97	12.4648	−14.4	−0.7	51730.77	.2732	−33.8	1.8
48730.81	.8782	7.5	0.9	51737.72	.3320	−31.1	0.3
48735.84	.9208	0.5	0.0	51785.67	.7379	15.3	−0.3
48780.75	13.3010	−34.3	−0.3	52009.89	40.6360	9.8	−0.2
48960.05	14.8188	13.0	0.3	52026.84	.7794	14.6	−0.4
48998.08	15.1407	−32.0	0.7	52033.79	.8383	11.3	0.3
49041.96	.5122	−6.1	0.2	52044.80	.9315	−0.2	1.1
49061.07	.6739	13.2	0.0	52052.76	.9988	−11.9	0.8
49084.90	.8756	6.6	−0.2	52073.74	41.1765	−34.7	0.6
49121.84	16.1883	−36.2	−0.3	52086.74	.2865	−34.5	0.5
49370.08	18.2897	−34.2	0.5	52096.73	.3711	−27.7	−0.6
49417.97	.6951	14.8	0.3	52150.67	.8277	11.0	−1.0
49442.93	.9064	3.8	1.2	52387.85	43.8355	11.2	−0.1
49459.84	19.0496	−22.0	−0.8	52417.79	44.0889	−26.5	0.4
49507.75	.4551	−15.7	−0.5	52438.74	.2662	−35.5	0.5
49557.71	.8780	7.2	0.7	52460.72	.4523	−15.2	0.4
49803.91	21.9621	−6.2	0.2	52465.73	.4947	−8.8	0.3
49849.80	22.3506	−28.8	0.6	52481.71	.6300	10.0	0.5
50125.04	24.6805	13.3	−0.4	52648.11	46.0385	−18.2	1.2
50160.95	.9845	−10.0	0.2	52702.98	.5031	−8.0	−0.2
50181.89	25.1618	−33.2	1.2	52761.84	47.0013	−14.1	−1.0
50232.76	.5924	4.4	−0.8	52768.83	.0604	−23.9	−1.1
50279.71	.9898	−13.3	−2.2 ^a	52774.78	.1108	−30.2	−0.5
50504.00	27.8884	7.6	2.4 ^a	52796.78	.2971	−36.1	−1.9
50577.79	28.5131	−5.9	0.3	52809.73	.4067	−22.8	−0.5
50917.93	31.3925	−26.0	−0.7	52817.72	.4744	−12.5	−0.3
50925.88	.4597	−14.4	0.1	52880.66	48.0071	−13.4	0.8
50931.83	.5100	−7.3	−0.7				

^aRejected observation.

mal uncertainty of a little more than an hour. The RVS data are presented in Table 4 and those from the CCD in Table 5.

For HD 148912, 68 observations were obtained with the RVS between JDs 2447263 and 2452886, followed by 82 with the CCD between JDs 2453117

TABLE 3
 CCD RADIAL VELOCITIES OF HD 121212

Julian Date −2,400,000	Cycle No. & Phase	<i>R.V.</i> km s ^{−1}	<i>O</i> − <i>C</i> km s ^{−1}	Julian Date −2,400,000	Cycle No. & Phase	<i>R.V.</i> km s ^{−1}	<i>O</i> − <i>C</i> km s ^{−1}
53117.84	50.0149	−15.93	−0.47	54531.97	61.9857	−10.26	0.17
53132.81	.1416	−33.12	−0.27	54561.91	62.2391	−36.57	0.07
53144.79	.2430	−36.43	0.16	54585.86	.4418	−17.31	−0.10
53165.72	.4202	−20.72	−0.29	54593.81	.5091	−6.64	0.14
53180.73	.5473	−0.96	0.06	54594.81	.5176	−5.07	0.40
53187.73	.6065	6.99	0.12	54602.77	.5850	4.53	0.33
53204.74	.7505	15.54	−0.05	54614.76	.6864	13.57	−0.40
53213.76	.8264	11.82	−0.28	54625.75	.7795	14.92	−0.03
53418.04	52.5561	0.40	0.15	54853.10	64.7040	14.72	−0.10
53425.01	.6151	8.24	0.38	54926.90	65.3287	−31.84	−0.14
53452.94	.8515	9.29	−0.40	54929.90	.3541	−29.60	−0.53
53468.96	.9871	−10.40	0.28	54966.81	.6666	13.01	0.32
53495.74	53.2138	−36.21	0.40	54987.74	.8438	10.80	0.30
53507.78	.3158	−33.37	−0.51	55013.73	66.0638	−23.37	−0.01
53515.78	.3835	−25.85	−0.35	55302.90	68.5116	−6.61	−0.22
53525.74	.4678	−13.37	−0.16	55310.87	.5791	3.88	0.46
53542.74	.6117	7.32	−0.15	55321.82	.6718	13.03	−0.03
53594.69	54.0515	−21.75	−0.28	55671.89	71.6351	10.38	0.41
53782.97	55.6453	11.07	0.14	56038.87	74.7416	15.72	0.11
53785.03	.6627	12.63	0.23	56391.90	77.7301	15.06	−0.46
53788.96	.6961	14.32	−0.15	56405.87	.8483	10.20	0.17
53823.94	.9921	−11.67	−0.13	56419.81	.9663	−6.53	0.57
53836.87	56.1015	−28.69	−0.13	56661.09	80.0089	−14.95	−0.52
53850.86	.2199	−36.56	0.12	56661.95	.0161	−16.39	−0.73
53860.80	.3041	−33.89	−0.11	56663.95	.0330	−18.56	−0.06
53871.81	.3973	−23.77	−0.11	56678.90	.1596	−34.28	0.00
53906.78	.6933	14.27	−0.07	56779.87	81.0143	−14.45	0.91
53916.75	.7777	14.52	−0.50	57089.96	83.6393	10.62	0.24
53927.74	.8707	7.90	0.43	57098.97	.7155	15.04	−0.17
54180.96	59.0143	−15.74	−0.38	57119.91	.8928	5.01	0.44
54238.78	.5037	−7.62	−0.01	57151.80	84.1627	−34.51	−0.01
54245.79	.5631	1.29	0.05	57167.79	.2981	−33.30	0.91
54253.75	.6305	9.34	−0.17	57465.01	86.8141	12.71	−0.35
54285.73	.9011	3.95	0.57	57496.84	87.0835	−26.30	−0.10

and 2457941. The span of 10678 days is over 22 orbital cycles, and as a result of this and the relatively high eccentricity, the period of this binary is also determined fairly precisely, with a formal uncertainty of a little less than an hour. The RVS data are presented in Table 6 and those from the CCD in Table 7.

4. ORBITS

For all three binaries, solutions were obtained from the RVS and CCD data separately, and weights based upon the standard deviations from those solutions were assigned for a solution from the combined data. The weight for the CCD data was always set to 1.0. In all cases the weight for the RVS

data was lower than for the CCD data, being 0.25, 0.18 and 0.10 respectively for HD 121212, HD 148434 and HD 148912. The orbital elements are presented in Table 8 for all three binaries, and their velocity curves are given in Figures 1, 2 and 3.

5. DISCUSSION

All three binaries discussed in this paper show no trace of faint companions in our data, and it must be concluded that the companions must be at least 2.5 magnitudes fainter than the primaries, especially if their spectral types are similar, since then they are likely to cause some slight distortion of the cross correlation function. Moreover, if the secondaries are main sequence stars a difference of only 2.5 magnitudes would indicate that the secondaries are of type

TABLE 4
RVS RADIAL VELOCITIES OF HD 148434

Julian Date −2,400,000	Cycle No. & Phase	<i>R.V.</i> km s ^{−1}	<i>O</i> − <i>C</i> km s ^{−1}	Julian Date −2,400,000	Cycle No. & Phase	<i>R.V.</i> km s ^{−1}	<i>O</i> − <i>C</i> km s ^{−1}
47303.87	0.5629	−4.4	−0.2	52184.63	.4274	−5.8	1.1
47324.83	0.6610	−2.6	−0.3	52387.93	24.3797	−8.1	−0.5
47328.81	0.6797	−2.0	−0.1	52417.89	.5201	−5.1	0.0
49508.84	10.8923	−1.2	0.1	52438.84	.6182	−2.8	0.3
50973.86	17.7553	−1.3	−0.3	52460.77	.7210	−1.0	0.3
51689.86	21.1096	−6.5	0.1	52465.80	.7445	−1.1	−0.1
51697.78	.1467	−6.5	0.8	52481.76	.8193	−1.0	−0.3
51718.80	.2451	−8.4	0.1	52488.72	.8519	−0.7	0.2
51730.77	.3012	−8.3	0.0	52516.69	.9830	−3.9	−0.7
51785.69	.5585	−5.0	−0.7	52537.66	25.0812	−4.7	1.2
51796.69	.6100	−3.4	−0.2	52544.64	.1139	−7.1	−0.4
51824.63	.7409	−0.9	0.2	52564.63	.2075	−8.0	0.2
51952.06	22.3379	−7.6	0.6	52579.60	.2776	−8.9	−0.4
52010.96	.6138	−2.7	0.4	52648.12	.5986	−2.6	0.9
52026.93	.6886	−1.6	0.2	52761.95	26.1319	−8.2	−1.2
52051.88	.8055	−0.8	−0.2	52774.85	.1923	−8.1	−0.1
52073.83	.9083	−1.7	−0.1	52796.84	.2953	−9.7	−1.4
52086.81	.9691	−2.6	0.2	52809.82	.3562	−6.0	1.9
52096.78	23.0158	−4.0	0.1	52817.80	.3935	−7.2	0.2
52150.66	.2682	−8.8	−0.3	52873.74	.6556	−2.5	−0.2
52163.68	.3292	−8.7	−0.5	52885.72	.7117	−0.9	0.5

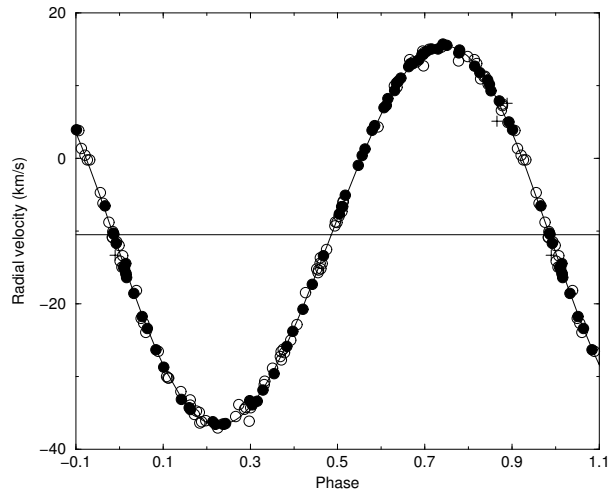


Fig. 1. The observed radial velocities of HD 121212, with the curve derived from the adopted elements drawn through them. The DAO CCD data are shown as filled circles and the spectrometer data as open circles. Rejected RVS velocities are shown as plus signs.

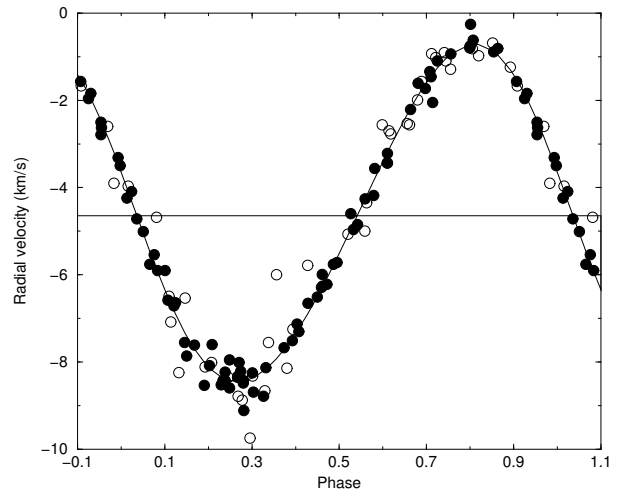


Fig. 2. The observed radial velocities of HD 148434, with the curve derived from the adopted elements drawn through them. The symbols have the same meaning as in Figure 1, except that there are no rejected data.

about A0 for HD 121212 and HD 148912, and about F5 for HD 148434. The former would have mass near $3 M_{\odot}$, and the latter, $1.3 M_{\odot}$. Either should be detectable in the system's colour indices, and indeed in the spectra of HD 121212 and 148912. Thus

we conclude that the secondaries are either main-sequence stars that are fainter than the primaries by more than 2.5 magnitudes, or slightly evolved subgiants, in which case their masses are only a little less than those of the primaries, given the evolutionary timescales of stars once they leave the main se-

TABLE 5
 CCD RADIAL VELOCITIES OF HD 148434

Julian Date −2,400,000	Cycle No. & Phase	<i>R.V.</i> km s ^{−1}	<i>O</i> − <i>C</i> km s ^{−1}	Julian Date −2,400,000	Cycle No. & Phase	<i>R.V.</i> km s ^{−1}	<i>O</i> − <i>C</i> km s ^{−1}
53117.97	27.7997	−0.76	−0.04	54695.71	.1908	−8.53	−0.52
53145.86	.9304	−1.84	0.12	54714.69	.2797	−8.42	0.01
53165.86	28.0240	−4.09	0.22	54724.68	.3265	−8.78	−0.58
53213.77	.2485	−7.95	0.47	54926.98	36.2742	−8.21	0.23
53258.71	.4590	−6.29	0.03	54966.94	.4614	−6.26	0.02
53425.09	29.2384	−8.23	0.15	54987.87	.5595	−4.26	−0.00
53515.86	.6637	−2.21	−0.03	54998.80	.6107	−3.44	−0.25
53525.82	.7103	−1.46	−0.02	55013.77	.6808	−1.61	0.28
53544.78	.7991	−0.80	−0.08	55050.72	.8539	−0.89	−0.01
53601.68	30.0657	−5.76	−0.30	55302.95	38.0355	−4.72	−0.09
53618.71	.1455	−7.55	−0.23	55321.94	.1245	−6.64	0.26
53619.69	.1501	−7.86	−0.46	55386.73	.4280	−6.65	0.23
53640.65	.2482	−8.59	−0.17	55393.78	.4610	−5.99	0.29
53647.63	.2810	−9.11	−0.68	55407.76	.5265	−4.60	0.36
53785.06	.9248	−1.96	−0.11	55425.71	.6106	−3.22	−0.03
53823.99	31.1071	−6.58	−0.07	55720.80	39.9929	−3.31	0.15
53836.96	.1679	−7.61	0.09	55766.76	40.2083	−7.60	0.59
53850.98	.2336	−8.42	−0.06	56106.75	41.8010	−0.26	0.46
53860.92	.2801	−8.48	−0.05	56148.74	.9977	−3.50	0.08
53871.90	.3316	−8.13	0.03	56170.71	42.1006	−5.90	0.46
53906.83	.4952	−5.72	−0.11	56405.93	43.2025	−8.08	0.05
53914.79	.5325	−4.96	−0.13	56419.92	.2681	−8.32	0.12
53916.83	.5420	−4.85	−0.22	56486.77	.5813	−3.56	0.24
53994.69	.9068	−1.57	−0.05	56513.73	.7075	−1.34	0.14
54004.67	.9536	−2.79	−0.33	56779.99	44.9549	−2.63	−0.14
54238.84	33.0506	−5.01	0.04	56853.76	45.3005	−8.25	0.11
54245.84	.0834	−5.90	0.03	56875.73	.4034	−7.13	0.16
54253.85	.1209	−6.71	0.11	57090.04	46.4073	−7.30	−0.08
54285.75	.2703	−8.01	0.43	57099.01	.4494	−6.51	−0.01
54292.75	.3031	−8.69	−0.34	57151.92	.6972	−1.73	−0.10
54307.72	.3732	−7.67	0.04	57232.74	47.0758	−5.54	0.19
54328.71	.4716	−6.22	−0.14	57478.99	48.2294	−8.52	−0.18
54351.67	.5791	−4.18	−0.34	57486.99	.2669	−8.35	0.09
54593.91	34.7139	−2.05	−0.65	57533.87	.4865	−5.76	0.02
54602.91	.7561	−0.94	0.00	57584.75	.7249	−1.10	0.16
54613.84	.8073	−0.62	0.10	57847.00	49.9534	−2.50	−0.04
54625.85	.8635	−0.81	0.15	57907.82	50.2384	−8.42	−0.04
54657.79	35.0132	−4.24	−0.23	57940.75	.3926	−7.51	−0.06

quence. One final possible alternative is that the secondaries are white dwarfs, which would again be very much fainter than the primaries, offering little hope of detection.

Despite these considerations, it might be possible to detect the companions interferometrically. The best chance of doing so, as suggested in the preceding paragraph, is in the case that they are evolved objects only slightly less massive than the primaries. We estimate below values of the angular size of the major semiaxes of the systems' orbits. As an example, we make the plausible assumption that the

primary star's mass is twice that of the sun for all three systems. We make use of the following two equations. The first is the equation of the mass function, in which M_1 is the primary's mass, M_2 is that of the secondary, and $q = M_2/M_1$.

$$\frac{f(M)}{M_1 \sin^3 i} = \frac{q^3}{(1+q)^2} \quad (1)$$

The second equation is for the projected angular size a of the major semiaxis, in terms of the parallax π , the mass ratio q , the inclination i , and the product

TABLE 6
RVS RADIAL VELOCITIES OF HD 148912

Julian Date −2,400,000	Cycle No. & Phase	<i>R.V.</i> km s ^{−1}	<i>O</i> − <i>C</i> km s ^{−1}	Julian Date −2,400,000	Cycle No. & Phase	<i>R.V.</i> km s ^{−1}	<i>O</i> − <i>C</i> km s ^{−1}
47263.97	0.6578	−68.2	0.7	51796.68	.0677	−70.7	0.0
48418.82	3.0553	−69.2	−0.3	51824.61	.1257	−75.1	0.4
50232.83	6.8211	−61.9	−0.0	51952.07	.3903	−75.4	−0.3
50246.83	.8502	−60.9	−0.9	52010.97	.5125	−72.6	0.1
50518.05	7.4132	−74.7	0.0	52026.94	.5457	−72.2	−0.2
50577.89	.5375	−72.7	−0.5	52051.89	.5975	−70.5	0.2
50609.85	.6038	−71.2	−0.7	52073.84	.6430	−68.9	0.5
50917.98	8.2435	−77.1	−0.2	52086.82	.6700	−68.9	−0.4
50925.96	.2601	−76.4	0.4	52096.78	.6907	−67.9	−0.0
50973.86	.3595	−76.2	−0.6	52150.65	.8025	−63.5	−0.6
51070.64	.5604	−72.0	−0.4	52163.69	.8296	−61.8	−0.4
51279.96	.9950	−56.5	−0.3	52184.64	.8731	−58.4	0.0
51289.93	9.0157	−60.8	−0.3	52387.94	11.2951	−78.1	−1.7
51309.92	.0572	−68.8	0.4	52417.91	.3573	−75.0	0.6
51317.93	.0738	−72.8	−1.2	52438.85	.4008	−74.8	0.2
51325.94	.0904	−74.4	−1.1	52460.77	.4463	−73.8	0.3
51333.85	.1068	−75.6	−1.1	52465.81	.4568	−74.0	−0.1
51344.80	.1296	−76.5	−0.7	52481.77	.4899	−73.2	0.1
51365.76	.1731	−76.5	0.2	52488.73	.5043	−72.2	0.8
51414.68	.2746	−77.4	−0.7	52516.68	.5624	−72.1	−0.5
51438.64	.3244	−76.8	−0.7	52527.67	.5852	−71.4	−0.4
51444.65	.3369	−76.9	−0.9	52544.63	.6204	−71.3	−1.3
51452.63	.3534	−76.1	−0.4	52564.62	.6619	−68.5	0.3
51624.00	.7092	−67.2	−0.0	52579.59	.6930	−67.7	−0.0
51632.01	.7258	−66.0	0.5	52648.13	.8353	−60.4	0.6
51662.98	.7901	−63.7	−0.1	52703.06	.9493	−52.8	0.6
51689.86	.8459	−60.4	−0.1	52761.94	12.0715	−71.5	−0.3
51697.82	.8624	−59.4	−0.3	52768.93	.0860	−73.2	−0.3
51710.80	.8894	−57.4	−0.2	52774.86	.0984	−73.5	0.5
51718.81	.9060	−55.9	0.1	52796.86	.1440	−77.6	−1.4
51730.78	.9309	−53.5	0.7	52808.80	.1688	−75.2	1.5
51737.75	.9453	−53.6	−0.2	52817.80	.1875	−77.2	−0.3
51785.70	10.0449	−66.5	0.4	52873.73	.3036	−75.5	0.9
51787.70	.0490	−67.5	0.2	52885.72	.3285	−75.6	0.4

$a_1 \sin i$, determined from the orbit.

$$a = \pi (a_1 \sin i) (1 + q^{-1}) \csc i. \quad (2)$$

As indicated above we assume that despite the luminosity differences between the stars, they are similar in mass and surface temperature, as was found for the components of HR 6046 by Scarfe et al. (2007), where the companion was detected with difficulty, despite being almost equal in mass to the primary. Here, let us assume that $q = 0.9$ for all three binaries. Then we solve equation 1 for the inclination of each orbit, and insert its cosecant into equation 2 along with the values of $a_1 \sin i$ from the orbit solutions, the parallaxes and the assumed values of q to find the projected major semiaxes.

An alternative, more direct, approach is to estimate the angular sizes of the systems' major semiaxes, using Kepler's Third Law and their parallaxes, by means of equation (3), in which a is the angular major semiaxis, π is the parallax, P is the period in years and M is the total mass of the system in solar units.

$$a = \pi P^{2/3} M^{1/3} \quad (3)$$

A note of caution - choice of too small a value for M , coupled with too low a value of the mass ratio q , may lead to values of $\sin i$ that exceed unity when the mass function is large, as it is for HD 121212. For example our choice of 3.8 solar masses for the total mass of that system does not permit the mass

TABLE 7
CCD RADIAL VELOCITIES OF HD 148912

Julian Date −2,400,000	Cycle No. & Phase	<i>R.V.</i> km s ^{−1}	<i>O</i> − <i>C</i> km s ^{−1}	Julian Date −2,400,000	Cycle No. & Phase	<i>R.V.</i> km s ^{−1}	<i>O</i> − <i>C</i> km s ^{−1}
53117.99	12.8107	−62.48	−0.02	54602.89	.8933	−56.99	−0.11
53144.92	.8666	−58.92	−0.05	54625.82	.9409	−53.66	−0.02
53165.84	.9100	−55.39	0.22	54657.76	16.0072	−58.77	−0.11
53180.82	.9411	−53.60	0.03	54695.70	.0860	−73.44	−0.55
53187.79	.9556	−53.23	0.00	54714.67	.1254	−75.37	0.17
53204.78	.9909	−55.92	−0.31	54724.66	.1461	−76.78	−0.53
53213.76	13.0095	−58.87	0.28	54927.00	.5662	−71.35	0.15
53251.68	.0882	−73.20	−0.10	54943.99	.6014	−70.47	0.11
53258.69	.1028	−73.84	0.44	54966.92	.6490	−68.98	0.21
53272.65	.1318	−75.80	−0.01	54987.83	.6925	−67.67	0.08
53283.61	.1545	−76.14	0.31	54998.80	.7152	−67.19	−0.28
53418.10	.4337	−74.52	−0.16	55013.76	.7463	−65.44	0.20
53453.05	.5063	−73.16	−0.26	55055.70	.8333	−61.03	0.08
53515.86	.6366	−69.80	−0.23	55302.99	17.3467	−75.93	−0.12
53525.85	.6574	−68.82	0.11	55321.96	.3861	−75.08	0.12
53542.79	.6926	−67.99	−0.25	55386.75	.5206	−72.13	0.45
53545.81	.6988	−67.77	−0.25	55393.74	.5351	−72.15	0.10
53594.72	.8004	−63.02	0.01	55407.73	.5641	−71.46	0.09
53601.69	.8148	−62.17	0.05	55720.84	18.2142	−76.78	0.21
53618.67	.8501	−60.15	−0.13	55766.73	.3094	−75.73	0.58
53619.68	.8522	−60.02	−0.14	56106.79	19.0154	−60.22	0.24
53640.62	.8957	−56.55	0.15	56148.72	.1024	−74.00	0.26
53647.61	.9102	−55.81	−0.21	56170.69	.1480	−76.05	0.25
53785.09	14.1956	−77.05	−0.10	56405.95	.6364	−69.71	−0.13
53824.02	.2764	−76.78	−0.11	56419.95	.6655	−68.51	0.16
53836.98	.3033	−76.56	−0.18	56486.75	.8042	−62.52	0.30
53850.96	.3323	−76.12	−0.11	56549.67	.9348	−53.78	0.17
53860.94	.3530	−75.74	−0.02	56799.87	20.4542	−73.90	0.07
53871.92	.3758	−75.37	−0.01	56853.75	.5661	−71.56	−0.06
53906.86	.4484	−74.13	−0.05	56875.71	.6117	−70.09	0.20
53916.81	.4690	−73.77	−0.09	57090.05	21.0566	−69.01	0.06
53994.65	.6306	−69.75	−0.00	57120.02	.1188	−75.17	0.06
54004.64	.6513	−69.13	−0.01	57160.92	.2038	−77.12	−0.14
54238.86	15.1376	−75.84	0.16	57232.73	.3528	−75.70	0.02
54245.86	.1521	−76.47	−0.07	57479.01	.8641	−59.13	−0.08
54263.87	.1895	−76.83	0.09	57487.01	.8807	−57.73	0.11
54285.77	.2350	−76.88	0.07	57533.89	.9780	−54.18	−0.11
54292.77	.2495	−77.05	−0.18	57584.76	22.0836	−72.72	−0.07
54307.75	.2806	−76.77	−0.14	57847.02	.6281	−69.59	0.23
54328.74	.3242	−76.09	0.03	57907.85	.7544	−65.57	−0.28
54351.66	.3718	−76.30	−0.87 ^a	57940.76	.8227	−61.73	0.03

^aRejected observation.

ratio to be as small as 0.6. One can avoid this pitfall by checking with equation 1.

Since our choice of primary mass and system mass ratio are the same for both the method using the mass function and that using Kepler's Third Law, it is not surprising that both methods yield the same results for the angular major semiaxes, 1.7 mas for HD 121212, 9.1 mas for HD 148434, and 5.4 mas for HD 148912. The angular separations at any or-

bit phase should be of this order of magnitude for our representative choice of masses, and any plausible masses and mass ratios are unlikely to yield very different results, as indicated by the weak dependence of equation 3 on the total mass. High inclinations like that of HD 121212 will make the separation significantly smaller at times, and large eccentricities like that of HD 148912 will give occasionally larger separations. The major axes estimated here

TABLE 8

TABLE 8 ORBITAL ELEMENTS

Object	HD 121212	HD 148434	HD 148912
P (days)	118.132 ± 0.002	213.464 ± 0.046	481.699 ± 0.040
T (J.D - 2,450,000)	3470.5 ± 1.4	4868.4 ± 4.8	3690.9 ± 3.1
K (km s^{-1})	26.16 ± 0.05	3.86 ± 0.04	11.88 ± 0.04
e	0.026 ± 0.002	0.070 ± 0.011	0.497 ± 0.002
ω (degrees)	94.2 ± 4.3	76.1 ± 8.1	48.2 ± 0.3
γ (km s^{-1})	-10.49 ± 0.04	-4.64 ± 0.03	-69.05 ± 0.02
S.E. (wt. 1) (km s^{-1})	0.341	0.253	0.197
$a_1 \sin i$ (Gm)	42.48 ± 0.09	11.31 ± 0.12	68.28 ± 0.25
$f(M)$ (M_\odot)	0.2194 ± 0.0013	0.00127 ± 0.00004	0.0548 ± 0.0006

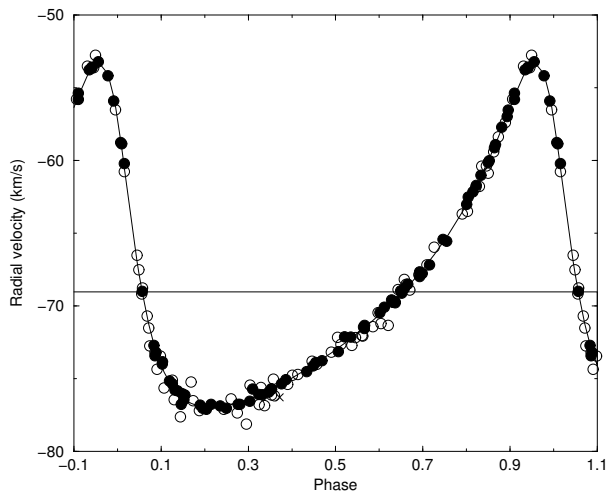


Fig. 3. The observed radial velocities of HR 148912, with the curve derived from the adopted elements drawn through them. The symbols have the same meaning as for Figure 1, except that a rejected CCD velocity is shown as a cross.

would make resolution of the pairs challenging for *Gaia* and very difficult, if not impossible, for earlier instruments. Nor do any of the inclinations estimated here offer much hope of detecting eclipses, despite the likely angular sizes of the giant primaries. Determination of the actual masses thus remains a challenge.

The author would like to acknowledge the National Research Council of Canada for the provision of observing time at the Dominion Astrophysical Observatory for a period now exceeding half of the century of that observatory's existence, and especially to thank members of the DAO staff, in particular D. Bohlender, D. Monin and the late L. Saddlemeyer for their greatly appreciated support and assistance. He would also like to thank a kindly referee for some helpful advice.

REFERENCES

- Adelman, S. J. 2000, IBVS, 4959, 1
 ESA 1997, The Hipparcos and Tycho Catalogs, SP 1200
 Fletcher, J. M., Harris, H. C., McClure, R. D., & Scarfe, C. D. 1982, PASP, 94, 1017
 Gaia Collaboration 2018, A&A, arXiv1804.09365
 Hoffleit, D., Saladyga, M. & Wlasuk, P. 1983, A Supplement to the Bright Star Catalogue (New Haven, CT: Yale Univ. Obs.)
 Hog, E., Fabricius, C., Makarov, V. V., et al. 2000, A&A, 355, L27
 Scarfe, C. D. 2010, Obs, 130, 214
 Scarfe, C. D., Griffin, R. F., & Griffin, R. E. M. 2007, MNRAS, 376, 1671