THREE-STAGE ENTRY GAME: THE STRATEGIC EFFECTS OF ADVERTISING

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Abstract: This paper analyzes the effects of investment in advertising in the three-stage entry game model with one incumbent and one potential entrant firm. It is shown that if a game theory is applied, under particular conditions, advertising can be used as a strategic weapon in the market entry game. Depending on the level of the advertising interaction factor, conditions for over-investment in advertising for strategic purposes are given. Furthermore, three specific cases are analyzed: strictly predatory advertising, informative advertising and the case when one firm's advertising cannot directly influence the other firm's profit. For each of them, depending on the costs of advertising and marginal costs, equilibrium is determined, and conditions under which it is possible to deter the entry are given. It is shown that if the value of the advertising interaction factor increases, power of using advertising as a weapon to deter entry into the market decreases. Thus, in the case of informative advertising, advertising cannot be used as a tool for deterring entry into the market, while in the case of predatory advertising, it can. Also, we have proved that in the case of strictly informative advertising an over-investment never occurs, while in the two other cases, there is always over-investment either to deter or to accommodate the entry.

Keywords: Three-stage entry game, advertising, strategic investment, over-investment, market equilibrium, entry deterrence, entry accommodation.

MSC: 91A05, 91A20, 91A80.

1. INTRODUCTION

In the last decades a growing attention has been paid to the role of advertising in the competition among firms. The impact that advertising is supposed to have on the

market structure, and particularly on the conditions of the entry, is one of the most controversial aspects in the economic literature. This is because there are two different categories of advertising which can be marked as informative and persuasive. The persuasive view holds that advertising primarily affects demand by changing tastes and creating brand loyalty. In other words, persuasive advertising creates differentiation among products [7], which at times may not be real [25]. The advertised product thus faces a less elastic demand. This elasticity effect suggests that advertising causes higher prices [1]. Furthermore, the persuasive view holds that advertising may deter entry.

Foundation of the informative view is laid by Ozga [20] and Stigler [23]. The informative view holds that advertising primarily affects demand by conveying information. The advertised product thus faces a more elastic demand. This elasticity effect suggests that advertising causes lower prices. Nelson [17] makes the distinction between experience goods and search goods. Nelson [18] further explains that a high level of advertising provides indirect information that the advertised goods are of a high quality. This signalling role of advertising is of a particular significance for experience goods. As Nelson [18, 19] and Demsetz [10] explain, a finding that profitability and advertising are positively associated may indicate only that firms of superior efficiency advertise more. Furthermore, the informative view holds that advertising is not used by established firms to deter entry; instead, advertising facilitates entry [24], since it is an important means through which entrants provide price and quality information to consumers. Although there are situations where advertising roles, awareness and persuasion, are used separately by marketers. Firms often use advertising as a mix element that informs and persuades simultaneously [3].

There are many important papers dealing with models of advertising competition and their effects on consumer behaviour and market performance. Dorfman and Steiner [12] offer an early model of optimal advertising as a function of market structure. According to the Dorfman-Steiner condition, a profit maximising firm will use advertising up to the point at which its marginal value product is equal to the price elasticity of the demand. Dixit and Norman [11] offer an influential welfare analysis of persuasive advertising. The complementary view also emerged during this research phase. Butters's classical model [4] marks the beginning of the theoretical literature on informative advertising. In Schmalensee [21] advertising informs consumers, and firms compete in quantities. He establishes that the incumbent under-invests so as to commit to be more aggressive in the post-entry game. Ishigaki [14] replaces Schmalensee's quantity competition framework by a price competition one, and finds that entry is at most blockaded but never effectively impeded. Cubbin and Domberger [9] have shown that in a static market the dominant firms are more likely to use advertising in order to respond to an entry. This implies that advertising is considered as one of the instruments employed by the incumbent firms if they wish to assume an antagonistic behavior, and to deter the entry. Furthermore, Bagwell and Ramey [2] analyze signalling games in which an incumbent firm can signal, with its advertising and its price to deter or accommodate entry of a potential entrant who is uncertain about the costs or demand conditions in the market. Moreover, they show that, in models of limit-pricing, a privately informed incumbent deters entry by over-investing in advertising so as to signal demand or cost conditions.

There have also been a large number of theoretical papers and many empirical studies trying to evaluate the impact of high advertising intensity on entry conditions.

Cubbin [8] has shown that advertising may, under very general assumptions, be a source of entry barriers. Ishigaki [15] develops models in which advertising not only informs consumer of brands, but also can influence consumer brand choices through repetition. By examining a multi-stage game in which two firms sequentially advertise before simultaneously setting price, Ishigaki shows that repetitive advertising can be an entry-deterrence weapon available to an incumbent in a subgame perfect equilibrium. Coccorese [6] shows that in a context of decreasing returns on advertising there are certain conditions of cost and demand under which a rational behaviour of established and potential firms may generate entry barriers. Banerjee and Bandyopadhyay [3] suggest that advertising is generally a nonviable competitive tool for smaller firms. They establish that, when the market shares of the firms are significantly different, then the unique equilibrium is attained, where only the large firm advertises. Chen et al. [5] analyze the effects of combative advertising, on the market power. They propose a model for combative advertising where advertising changes the distribution of consumer preferences in a tug-of-war. They show that, depending on the nature of consumer response, combative advertising can reduce price competition to benefit competing firms.

Nevertheless, it is undeniable that advertising activity exerts some influence on the behaviour of incumbents or potential firms. When a firm decides to advertise, its main aim is to change demand conditions, especially to reduce price competition through product differentiation. The effects of this single action spread over the whole market: the consumers, who buy the product of the advertiser, might not be the purchasers of other similar goods for a certain time, which leads to negative implications on the demand of the other firms. As a consequence, potential entrants could face an entry barrier, whose height is linked to some aspects such as brand loyalty and penetration costs.

In this paper, three specific cases of advertising, strictly predatory advertising, informative advertising and the case when advertising of one firm cannot directly influence the other firm's profit, are analyzed. Using a game-theoretic approach, we show that, under particular conditions of costs and demand, advertising can be used as a strategic weapon in entry game. Since advertising has long-term effects, an incumbent monopolist shall sometimes find it optimal to over-invest in promotional expenditures in order to deter or accommodate entry. We prove that the existence of an over-investment in advertising deeply depends on the type of advertising. Moreover, it is shown that in the case of strictly informative advertising an over-investment never occurs, while in the two other cases, there is always over-investment either to deter or accommodate entry.

Furthermore, it will be confirmed that in the case of predatory advertising, investment in advertising can be used for entry deterrence, but with increasing value of advertising interaction factor, the power of using advertising as a weapon to deter entry into the market decreases. Thus in the case of informative advertising, advertising cannot be used as a weapon for deterring entry into the market.

The paper is organized as follows. In Section 2 a model of a three-stage entry game is presented. Section 3 enlightens effects of a strategic over-investment and strategic under-investment in advertising for entry deterrence and accommodation purposes. The possible outcomes of the three-stage entry game, as well as the conditions of a strategic over-investment, are given in Section 4. Section 5 deals with some special cases of advertising and identifies the solutions of the game for strictly predatory and strictly informative advertising, as well as for the case when advertising by one firm has

no effect upon the profit of the other firm. Finally, Section 6 offers some concluding remarks.

2. THREE-STAGE ENTRY GAME MODEL

Since the hypothesis to be tested in this paper is that advertising can be used as a strategic weapon in an entry game, it is assumed that there is an established firm (firm 1) in the market (initially a monopoly), and a potential entrant (firm 2). The game is sequential. In the first stage the incumbent (firm 1) decides whether to invest in advertising or not. It can decide to invest either to maximise its own profit or to discourage the entry of the rival firm driving its profits to zero; but it can also decide to accommodate the entry, therefore choosing a level of advertising expenditure compatible with Cournot duopoly. In the second stage, the potential entrant (firm 2) observes the incumbent's choice of advertising level and then chooses whether or not to enter the market, and whether to advertise or not. In the third stage, either the incumbent is a monopolist or the incumbent and the entrant compete as duopolists. If the entry occurs, a Cournot-Nash equilibrium with quantity-setting will appear. The game tree is shown in Figure 1.
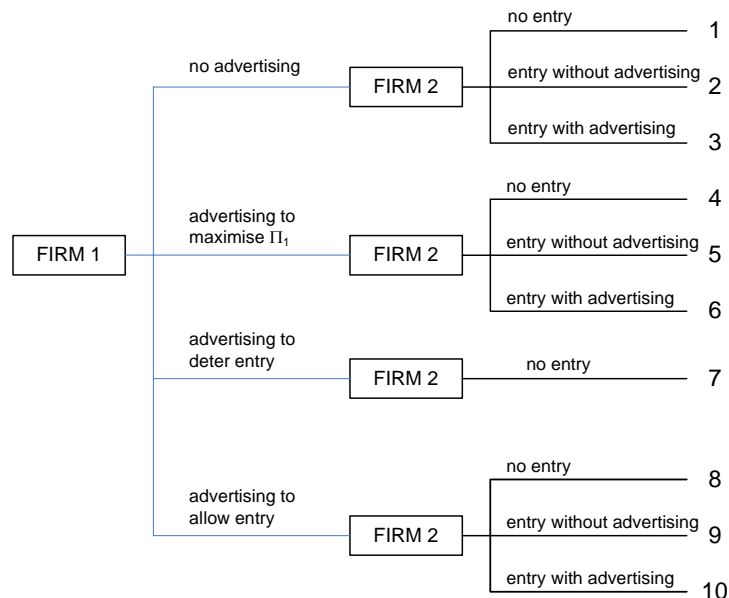

Figure 1: The game tree

This is a game with perfect information (the firm 2 observes the incumbent's level of advertising), and therefore it can be solved by backward induction. To find out the solutions of this game, we have to evaluate the profit of each of the two firms.

Let us suppose that a representative consumer maximises $U(q_1, q_2) - \sum_{i=1}^{2} p_i q_i$, where q_i is the quantity of goods i, and p_i is its price. U is assumed to be quadratic and strictly concave utility function [22]:

$$U(q_1, q_2) = a_1 q_1 + a_2 q_2 - \frac{1}{2}(b_1 q_1^2 + 2\gamma q_1 q_2 + b_2 q_2^2) \tag{1}$$

where a_i and b_i are positive, $i = 1, 2$. This utility function gives rise to a linear demand structure. Inverse demands are given by

$$p_i = a_i - b_i q_i - \gamma q_j, \quad i, j = 1, 2, \ i \neq j \tag{2}$$

where a_i is a reservation price, b_i is a marginal price and γ is a factor of substitutability. The goods are substitutes, independent, or complements according to whether $\gamma \gtrless 0$. When $a_1 = a_2 = a$ and $b_1 = b_2 = \gamma = b$, the goods are perfect substitutes. In this paper, this specialization will be used. Note that in this case, utility function is not strictly concave.

The main premise of this analysis is that advertising expenditures shift out the demand curve for the firm that affords them. This can be thought of as an increase in consumers' "willingness to pay", or as an increase in quantity demanded at a given price. This can be expressed by simple linear function:

$$a_i^A = a + A_i + \rho A_j, \quad i, j = 1, 2, \ i \neq j \tag{3}$$

where a_i^A is the reservation price after advertising. A_i represents the increase in the consumers' willingness to pay induced by its own advertising. It can also be regarded as the rise of price that firms can apply to every sold unit of goods. A_j represents changes in the consumers 'willingness to pay induced by competitor's advertising. $\rho \in [-1, 1]$ is the advertising interaction factor (advertising spillover), representing the relative response of one firm's advertising to another's. If ρ is positive, the advertising is cooperative, i.e. the advertising also assists the other firm; if ρ is negative, the advertising is predatory, i.e. the gain in the demand from advertising of one firm is the expense of another. If $\rho = 0$, then advertising by one firm has no effect upon the other firm. In other words, an additional unit of advertising expenditure by the firm i shifts outward its own demand curve, while the externality upon the rival firm depends on whether $\rho > 0$ (the firm j's curve shifts outwards), or $\rho < 0$ (the curve shifts inwards). Note that for $\rho = -1$, it is a case of perfect "business stealing" since the firm's advertising attracts only the consumers who would otherwise buy from the other firm [16].

As a result, the inverse demand functions can be written as:

$$p_i = a + A_i + \rho A_j - bq_i - bq_j, \quad i, j = 1, 2, \ i \neq j. \tag{4}$$

For simplicity, the cost of production is assumed to be equal to zero. The cost of advertising (c_A) is the same for the two firms and assumed to be quadratic, in order to

permit diminishing returns on advertising expenditures: as they increase, their effectiveness get lower and lower, because it gets more and more difficult to reach consumers who have not received the messages before. The cost of advertising can be expressed by:

$$c_A = \frac{v}{2} A_i^2, \; i = 1, 2 ,$$ (5)

where $v > 0$ is a constant. Given inverse demand and costs, the profit functions for the two firms are:

$$\Pi_i = (a + A_i + \rho A_j - bq_i - bq_j)q_i - \frac{v}{2} A_i^2, \quad i, j = 1, 2, \; i \neq j .$$ (6)

In the sequel, the profit functions will be used to reveal effects of strategic advertising on the entry, as well as to solve game equilibria, depending on advertising interaction factor and advertising costs.

3. STRATEGIC INVESTMENT IN ADVERTISING

In the entry game model, the incumbent's investment decision is said to be strategic if the firm 1 distorts its investment level away from the level which maximizes profits, in order to affect the behaviour of the firm 2. More precisely, investment is called strategic if the equilibrium value of the advertising differs from that which would have been chosen in a model where the firm 2 did not observe the firm 1's choice of advertising level. Two types of strategic investment can occur, strategic over-investment and strategic under-investment. In this section, the circumstances under which over-investment in advertising occurs will be given.

Let us first consider the situation where only the firm 1 (incumbent) invests in advertising. Then the profit functions for the firms can be expressed by:

$$\Pi_1(A_1, q_1(A_1), q_2(A_1)) \text{ and } \Pi_2(A_1, q_1(A_1), q_2(A_1))$$

As we mentioned above, in the first stage of the game, the firm 1 chooses a level of advertising A_1. Given A_1 quantities q_1 and q_2 are determined by Nash equilibrium: $\{q_1^*(A_1), q_2^*(A_1)\}$. We will look at the effects of A_1 on the Nash equilibrium, assuming that the Nash equilibrium is unique and stable.

If the firm 1 chooses A_1 such that: $\Pi_2(A_1, q_1^*(A_1), q_2^*(A_1)) < 0$, the entry is blocked; $\Pi_2(A_1, q_1^*(A_1), q_2^*(A_1)) = 0$, the entry is deterred; $\Pi_2(A_1, q_1^*(A_1), q_2^*(A_1)) > 0$, the entry is accommodated. Whether the incumbent wants to accommodate or to deter the entry depends on the comparison of the profit in the two cases.

Let us suppose that the incumbent wants to deter the entry of the firm 2. Then the incumbent chooses A_1 such that $\Pi_2(A_1, q_1^*(A_1), q_2^*(A_1)) = 0$. The question is: Which advertising level should the incumbent use to make firm 2's entry unprofitable? Let us take the total derivative of the firm 2's profits with respect to A_1:

$$\frac{d\Pi_2}{dA_1} = \frac{\partial \Pi_2}{\partial A_1} + \frac{\partial \Pi_2}{\partial q_1} \frac{\partial q_1^*}{\partial A_1} + \frac{\partial \Pi_2}{\partial q_2} \frac{\partial q_2^*}{\partial A_1} ,$$ (7)

where $\partial \Pi_2 / \partial q_2 = 0$. If $d\Pi_2 / dA_1 < 0$, then the investment in A_1 makes the firm 1 tough, and if $d\Pi_2 / dA_1 > 0$, then the investment in A_1 makes the firm 1 soft. According to Fudenberg and Tirole [13], if the investing makes the firm look tough, then to deter the entry the firm should over-invest, to look "top dog", very big and ready to fight. If the investing makes the firm look soft, then the firm should under-invest to deter entry, looking "lean and hungry".

Suppose that deterring the entry is too costly and that the incumbent decides to accommodate the firm 2's entry. In deterring the entry, A_1 is determined by post entry of the firm 2's profits. In accommodating the entry, the firm 1's behaviour in the first period (A_1) is dictated by the firm 1's profit $\Pi_1(A_1, q_1^*(A_1), q_2^*(A_1))$. The incentive to invest is given by the total derivative of this profit function with respect to A_1.

$$\frac{d\Pi_1}{dA_1} = \underbrace{\frac{\partial \Pi_1}{\partial A_1}}_{\substack{\text{direct effect} \\ \text{DE}}} + \underbrace{\frac{\partial \Pi_1}{\partial q_1}}_{=0} \frac{\partial q_1^*}{\partial A_1} + \underbrace{\frac{\partial \Pi_1}{\partial q_2} \frac{\partial q_2^*}{\partial A_1}}_{\substack{\text{strategic effect} \\ \text{SE}}} \tag{8}$$

The *direct effect* (DE) exists regardless of whether the firm 2 sees A_1 or not. Thus the firm 1 should over-invest if the *strategic effect* is positive (SE>0), and under-invest if the *strategic effect* is negative (SE<0).

Suppose that the entry is accommodated and the firm 2 enters and advertises. Now the profit functions can be expressed by:

$$\Pi_1(A_1, A_2, q_1^*(A_1, A_2), q_2^*(A_1, A_2)) \text{ and } \Pi_2(A_1, A_2, q_1^*(A_1, A_2), q_2^*(A_1, A_2)).$$

Differentiating with respect to A_i and A_j, and considering that $\dfrac{\partial \Pi_i}{\partial q_i} = 0$, we have:

$$d\Pi_i = \underbrace{\left[\frac{\partial \Pi_i}{\partial A_i} + \frac{\partial \Pi_i}{\partial q_j} \frac{\partial q_j^*}{\partial A_i} \right]}_{\substack{\text{effect of the advertising} \\ \text{of firm } i \text{ on its own profit}}} dA_i + \underbrace{\left[\frac{\partial \Pi_i}{\partial A_j} + \frac{\partial \Pi_i}{\partial q_j} \frac{\partial q_j^*}{\partial A_j} \right]}_{\substack{\text{the impact of the advertising} \\ \text{of the rival on the firm } i\text{'s profit}}} dA_j \tag{9}$$

The first bracket of $d\Pi_i$ contains the effect of the advertising of the firm *i* on its own profit; the second one shows the impact of the rival's advertising on the same profit. Both are formed by two parts: the direct effect and the strategic effect. If the second part is negative, the firm *j* will over-invest in advertising because this investment makes it tough. The firm *i* will over-invest for strategic purposes if the first part of $d\Pi_i$ has a positive strategic effect.

4. THE ENTRY GAME OUTCOMES

There are ten possible outcomes of the three-stage entry game, as shown in Figure 1. The advertising levels, as well as derivation of quantities and profits for the firm 1 (the incumbent) and the firm 2 (the entrant), associated with each pathway of the game tree, are given below.

Pathway 1. The firm 1 does not advertise, the firm 2 does not enter. This is a standard monopoly equilibrium where:

$$q_1 = \frac{a}{2b}, \quad q_2 = 0, \quad \Pi_1 = \frac{a^2}{4b}, \quad \Pi_2 = 0.$$

Pathway 2. The firm 1 does not advertise, the firm 2 enters but does not advertise. This is a standard Cournot outcome. In this case, the profit functions for the firm 1 and the firm 2 are:

$$\Pi_1 = (a - bq_1 - bq_2)q_1, \quad \Pi_2 = (a - bq_1 - bq_2)q_2 .$$

Thus, it follows that:

$$q_i = \frac{a}{3b}, \quad \Pi_i = \frac{a^2}{9b}, \quad i = 1, 2.$$

Pathway 3. The firm 1 does not advertise, the firm 2 enters and advertises. Now, the profit functions are:

$$\Pi_1 = (a + \rho A_2 - bq_1 - bq_2)q_1, \quad \Pi_2 = (a + A_2 - bq_1 - bq_2)q_2 - \frac{v}{2}A_2^2$$

and the outcomes are:

$$A_1 = 0, \quad q_1 = \frac{a(3bv - 2\rho^2 + 6\rho - 4)}{b(9bv - 2\rho^2 + 8\rho - 8)}, \quad \Pi_1 = \frac{a^2(3bv - 2\rho^2 + 6\rho - 4)^2}{b(9bv - 2\rho^2 + 8\rho - 8)^2},$$

$$A_2 = \frac{2a(2 - \rho)}{9bv - 2\rho^2 + 8\rho - 8}, \quad q_2 = \frac{3av}{9bv - 2\rho^2 + 8\rho - 8}, \quad \Pi_2 = \frac{a^2 v}{9bv - 2\rho^2 + 8\rho - 8}$$

Here the firm 2 invests in advertising in order to capture the market. It can be observed that there is an impact of the firm's 2 advertising (A_2) on the profits of both firms. But, the question is, under which circumstances over-investment will occur.
It is easy to check that

$$\frac{\partial \Pi_1}{\partial A_2} = \rho q_1, \quad \frac{\partial \Pi_1}{\partial q_2} = -bq_1, \quad \frac{\partial q_2^*}{\partial A_2} = \frac{2 - \rho}{3b} \quad \Rightarrow \quad \frac{d\Pi_1}{dA_2} = \frac{2q_1(2\rho - 1)}{3} \begin{cases} > 0 & \text{for } \rho > 1/2 \\ < 0 & \text{for } \rho < 1/2 \end{cases}$$

$$\frac{\partial \Pi_2}{\partial q_1} = -bq_2, \quad \frac{\partial q_1^*}{\partial A_2} = \frac{2\rho - 1}{3b} \quad \Rightarrow \quad SE = -q_2 \frac{2\rho - 1}{3} \begin{cases} > 0 & \text{for } \rho < 1/2 \\ < 0 & \text{for } \rho > 1/2 \end{cases},$$

If the $d\Pi_1 / dA_2$ is negative, the firm 2 will over-invest in advertising because this investment makes it tough. It is easy to check that $d\Pi_1 / dA_2$ is negative when advertising interaction factor (ρ) is less than ½. In the same interval, there are positive strategic effects. When $\rho > 1/2$, under-investment will occur.

Pathway 4. The firm 1 advertises to maximise its own profit, the firm 2 does not enter.

Given that $\Pi_1 = (a + A_1 - bq_1 - bq_2)q_1 - \frac{v}{2}A_1^2$, and $q_2 = 0$ the solutions are:

$$A_1 = \frac{a}{2bv-1}, \quad q_1 = \frac{av}{2bv-1}, \quad \Pi_1 = \frac{a^2 v}{2(2bv-1)}, \quad A_2 = 0, \quad q_2 = 0, \quad \Pi_2 = 0$$

Here firm 1 retains monopolistic position, and only direct effects of advertising will occur.

Pathway 5. The firm 1 advertises to maximise its own profit, the firm 2 enters but does not advertise. In this case the key assumption is that the incumbent firm doesn't know what kind of behaviour the potential entrant will adopt about the entry and advertising expenditures (the game is sequential). For this reason it seems reasonable that the firm 1 will invest in advertising in order to maximise its own profit, with no regard to the consequences of this action on the behaviour of the firm 2. Therefore, since $A_1 = a/(2bv-1)$ maximises Π_1 when the incumbent is the only firm in the market, it is:

$$\Pi_1 = (a + A_1 - bq_1 - bq_2)q_1 - \frac{v}{2}A_1^2 = (a + \frac{a}{2bv-1} - bq_1 - bq_2)q_1 - \frac{v}{2}\left(\frac{a}{2bv-1}\right)^2$$

$$\Pi_2 = (a + \rho\frac{a}{2bv-1} - bq_1 - bq_2)q_2 .$$

The solutions are:

$$A_1 = \frac{a}{2bv-1}, \quad q_1 = \frac{2abv + a(1-\rho)}{3b(2bv-1)}, \quad \Pi_1 = \frac{2(2abv + a(1-\rho))^2 - 9a^2 bv}{18b(2bv-1)^2},$$

$$A_2 = 0, \quad q_2 = \frac{2a(bv-1+\rho)}{3b(2bv-1)}, \quad \Pi_2 = \frac{4a^2(bv-1+\rho)^2}{9b(2bv-1)^2}$$

Pathway 6. The firm 1 advertises to maximise its profit, the firm 2 enters and advertises. As before, $A_1 = a/(2bv-1)$, which means that:

$$\Pi_1 = (a + \frac{a}{2bv-1} + \rho A_2 - bq_1 - bq_2)q_1 - \frac{v}{2}\left(\frac{a}{2bv-1}\right)^2$$

$$\Pi_2 = (a + A_2 + \rho\frac{a}{2bv-1} - bq_1 - bq_2)q_2 - \frac{v}{2}A_2^2$$

It follows that:

$$A_1 = \frac{a}{2bv-1}, \quad q_1 = \frac{a + A_2 + \rho\frac{a}{2bv-1} - \frac{3bvA_2}{2-\rho}}{b},$$

$$A_2 = \frac{2a(bv-1+\rho)}{(2bv-1)} \cdot \frac{2(2-\rho)}{(9bv + 8\rho - 2\rho^2 - 8)}, \quad q_2 = \frac{3vA_2}{2(2-\rho)}$$

Using the given quantities and advertising levels, the corresponding profits Π_1 and Π_2 can be calculated. Also, for this pathway we have

$$\frac{\partial \Pi_1}{\partial A_2} = \rho q_1, \quad \frac{\partial \Pi_1}{\partial q_2} = -bq_1, \quad \frac{\partial q_2^*}{\partial A_2} = \frac{2-\rho}{3b} \quad \Rightarrow \quad \frac{d\Pi_1}{dA_2} = \frac{2q_1(2\rho-1)}{3} \begin{cases} > 0 & \text{for } \rho > 1/2 \\ < 0 & \text{for } \rho < 1/2 \end{cases}$$

$$\cdot \; \frac{\partial \Pi_2}{\partial q_1} = -bq_2, \quad \frac{\partial q_1^*}{\partial A_2} = \frac{2\rho-1}{3b} \quad \Rightarrow \quad SE = -q_2 \frac{2\rho-1}{3} \begin{cases} > 0 & \text{for } \rho < 1/2 \\ < 0 & \text{for } \rho > 1/2 \end{cases},$$

It is easy to check that $d\Pi_1 / dA_2$ is negative for $\rho < 1/2$. In the same interval, there are positive strategic effects. Thus the firm 2 will over-invest in advertising for strategic purpose when advertising interaction factor is less than 1/2.

Pathway 7. The firm 1 advertises to lower Π_2 to zero in order to deter the entry of the firm 2. Here we have the following profit functions:

$$\Pi_1 = (a + A_1 - bq_1 - bq_2)q_1 - \frac{v}{2}A_1^2, \quad \Pi_2 = (a + \rho A_1 - bq_1 - bq_2)q_2$$

Imposing that the optimal value of A_1 has to make the entry of the firm 2 unprofitable, we obtain:

$$A_1 = \frac{a}{1-2\rho}, \quad q_1 = \frac{a(\rho-1)}{b(2\rho-1)}, \quad \Pi_1 = \frac{a^2(2-4\rho-2\rho^2-bv)}{2b(1-2\rho)^2}, \quad A_2 = 0, \; q_2 = 0, \; \Pi_2 = 0.$$

Based on the previous, it can be concluded that A_1 is positive for $\rho < 1/2$; q_1 is positive for $\rho < 1/2$ and $\rho > 1$. According to this, it can be concluded that the incumbent can advertise to lower Π_2 to zero in order to the deter entry of the rival when $\rho < 1/2$; otherwise cannot. Also, for this pathway we have

$$\frac{\partial \Pi_2}{\partial A_1} = \rho q_2, \quad \frac{\partial \Pi_2}{\partial q_1} = -bq_2, \quad \frac{\partial q_1^*}{\partial A_1} = \frac{(2-\rho)}{3b} \quad \Rightarrow \quad \frac{d\Pi_2}{dA_1} = \frac{2q_2(2\rho-1)}{3} \begin{cases} > 0 & \text{for } \rho > 1/2 \\ < 0 & \text{for } \rho < 1/2 \end{cases}$$

It can be observed that for $\rho < 1/2$ it follows $\dfrac{d\Pi_2}{dA_1} < 0$, and the investment in A_1 makes the firm 1 tough, which means that in order to deter the entry the firm 1 should over-invest.

Pathway 8. The firm 1 advertises to accommodate the entry and share the market with the rival; the firm 2 enters and advertises. The profit of the two firms can now be written as:

$$\Pi_i = (a + A_i + \rho A_j - bq_i - bq_j)q_i - \frac{v}{2}A_i^2, \quad i, j = 1, 2, \; i \neq j.$$

Maximization for q_i and A_i gives:

$$A_i = \frac{2a(2-\rho)}{9bv + 2\rho^2 - 2\rho - 4}, \quad q_i = \frac{3av}{9bv + 2\rho^2 - 2\rho - 4}, \quad i = 1, 2$$

$$\Pi_i = \frac{a^2 v(9bv - 2\rho^2 + 8\rho - 8)}{(9bv + 2\rho^2 - 2\rho - 4)^2}, \quad i = 1, 2$$

Moreover,

$$\frac{\partial \Pi_i}{\partial q_j} = -bq_i, \quad \frac{\partial q_j^*}{\partial A_i} = \frac{2\rho - 1}{3b}, \quad \frac{\partial q_j^*}{\partial A_j} = \frac{2 - \rho}{3b}, \quad \frac{\partial \Pi_i}{\partial A_i} = q_i - vA_i, \quad \frac{\partial \Pi_i}{\partial A_j} = \rho q_i.$$

Now, it follows:

$$d\Pi_i = \left[q_i - vA_i + (-bq_i)\frac{2\rho - 1}{3b} \right] dA_i + \left[\rho q_i + (-bq_i)\frac{2 - \rho}{3b} \right] dA_j.$$

On this pathway both firms will over-invest in advertising when $\rho < 1/2$. The firm j will over-invest for strategic purposes if $\rho q_i + (-bq_i)\frac{2 - \rho}{3b} < 0$. It is easy to check that this will happen for $\rho < 1/2$. On the other hand, the firm i will over-invest for strategic purposes if $q_i(1 - 2\rho)/3 > 0$, and this will happen for $\rho < 1/2$.

Pathway 9. The firm 1 advertises to accommodate the entry; the firm 2 enters but does not advertise. Similarly as for the pathway 5, the key assumption is that the incumbent firm does not know what kind of behaviour the potential entrant will adopt about entry and advertising expenditures. Therefore, since $A_1 = \frac{2a(2 - \rho)}{9bv + 2\rho^2 - 2\rho - 4}$ is the advertising level which accommodates the entry, it can be written:

$$\Pi_1 = (a + \frac{2a(2 - \rho)}{9bv + 2\rho^2 - 2\rho - 4} - bq_1 - bq_2)q_1 - \frac{v}{2}\left(\frac{2a(2 - \rho)}{9bv + 2\rho^2 - 2\rho - 4} \right)^2$$

$$\Pi_2 = (a + \rho\frac{2a(2 - \rho)}{9bv + 2\rho^2 - 2\rho - 4} - bq_1 - bq_2)q_2.$$

It follows that:

$$A_1 = \frac{2a(2 - \rho)}{9bv + 2\rho^2 - 2\rho - 4}, \quad q_1 = \frac{a + A_1(2 - \rho)}{3b}, \quad \Pi_1 = \frac{2(a + A_1(2 - \rho))^2 - 9A_1^2 bv}{18b},$$

$$A_2 = 0, \quad q_2 = \frac{a + A_1(2\rho - 1)}{3b}, \quad \Pi_2 = \frac{(a + A_1(2\rho - 1))^2}{9b}.$$

Pathway 10. The firm 1 advertises to accommodate the entry; the firm 2 does not enter. Given that $\Pi_1 = (a + A_1 - bq_1 - bq_2)q_1 - \frac{v}{2}A_1^2$ and $q_2 = 0$, the solution is:

$$A_1 = \frac{2a(2 - \rho)}{9bv + 2\rho^2 - 2\rho - 4}, \quad q_1 = \frac{a + A_1}{2b}, \quad \Pi_1 = \frac{(a + A_1)^2 - 2A_1^2 bv}{4b}, \quad A_2 = 0, \quad \Pi_2 = 0.$$

Based on the previous analysis the following can be summarized: in the pathways 1 and 2, neither company invest in advertising, while in other pathways at least one of them makes investment. In the pathways 4 and 10 one firm (firm 1) invests, while

the other does not enter the market, so that in these pathways only direct effects of investment occur. In the other pathways, there are strategic effects of advertising, but as for the pathways 3, 6, 7 and 8 if the parameter ρ is less then 1/2, there is an over-investment, regardless of the value of other parameters of the model. When this parameter is less than 1/2 there is a strategic under-investment.

When the parameter ρ is equal to 1/2, total derivative of firms i's and j's profits with respect to A_i $(i, j = 1, 2, \ i \neq j)$ are equal to 0, which means that in that case, the investment in advertising maximizes their own profit as well as profit of the rival.

According to the previous analysis, the following proposition has been proved.

Proposition 1. *When the advertising interaction factor is less than 1/2 ($\rho < 1/2$) and at least one of the firms on the duopoly market advertises, the optimal strategy is an over-investment for strategic purposes. When $\rho > 1/2$, the optimal strategy is an under-investment.*

5. THE MARKET EQUILIBRIUM SOLUTIONS

In the previous section it is shown that existence of over/under-investment in advertising deeply depends on advertising interaction factor (ρ). On the other hand, the equilibrium solution of the game depends not only on this factor, but on the value of marginal costs (b) and costs of advertising (v), too.

Let us first consider solutions to the game as a whole. For example, consider the pathways 4, 5 and 6. The firm 1 has made its decision in the first stage (advertise to maximise profits). The firm 2 knows the firm 1's decision and now has to make its own entry decision: it will surely choose the payoff which is the biggest for itself, i.e. it will compare its profitability under no entry, entry with advertising, and entry without advertising. The pathways 5 and 6 will be always preferred to the pathway 4, provided that corresponding profits and quantities take positive values. Furthermore, the profit for the pathway 6 could be compared with that for the pathway 5, and it is possible to determine the conditions under which the entry with advertising is preferred to the entry without advertising. In this way, given the firm 1's choice to maximise profits, the firm 2's preferred choice can be determined. A similar analysis could be carried out for the other groups of the pathways 1, 2, 3 and 8, 9, 10. Once the firm 2's choices have been determined, the choice of the firm 1 could be examined and so the conditions under which the firm 1 would prefer to advertise, to accommodate or deter entry, or not advertise at all, could be determined.

However, there occurs the problem of comparison of expressions with 3 variables, where one of them can take a negative value. Thus, solving analytically the game as a whole is very difficult, and here equilibrium analysis for three typical cases is done: (1) for the strictly predatory advertising ($\rho = -1$), (2) for the case of strictly informative advertising ($\rho = 1$), and (3) for the low spillover ($\rho = 0$).

5.1. The market equilibrium solutions for the strictly predatory advertising

Here it will be supposed that the advertising is strictly predatory i.e. firm's advertising attracts only consumers who would otherwise buy from the other firm ($\rho = -1$). Corresponding outcomes for every pathway from the game tree in Figure 1 are shown in Table 1.

Given that this is a game with complete information, and that the game can be solved by backward induction, the behaviour of the entrant must be considered first. We will first analyze the choice of the firm 2, given the firm 1's choice. It will enter the market only if its profit is positive.

Table 1. Outcomes for predatory advertising ($\rho = -1$)

Path	Profits	Quantities	Advertising level
1	$\Pi_1 = \dfrac{a^2}{4b}, \quad \Pi_2 = 0$	$q_1 = \dfrac{a}{2b}, \quad q_2 = 0$	
2	$\Pi_i = \dfrac{a^2}{9b}, \quad i = 1,2$	$q_i = \dfrac{a}{3b}, \quad i = 1,2$	
3	$\Pi_1 = \dfrac{a^2(bv-4)^2}{9b(bv-2)^2}$ $\Pi_2 = \dfrac{a^2v}{9(bv-2)}$	$q_1 = \dfrac{a(bv-4)}{3b(bv-2)}$ $q_2 = \dfrac{av}{3(bv-2)}$	$A_1 = 0, \ A_2 = \dfrac{2a}{3(bv-2)}$
4	$\Pi_1 = \dfrac{a^2v}{2(2bv-1)}, \quad \Pi_2 = 0$	$q_1 = \dfrac{av}{2bv-1}, \quad q_2 = 0$	$A_1 = \dfrac{a}{2bv-1}, \quad A_2 = 0$
5	$\Pi_1 = \dfrac{a^2(8b^2v^2+7bv+8)}{18b(2bv-1)^2}$ $\Pi_2 = \dfrac{4a^2(bv-2)^2}{9b(2bv-1)^2}$	$q_1 = \dfrac{2a(bv+1)}{3b(2bv-1)}$ $q_2 = \dfrac{2a(bv-2)}{3b(2bv-1)}$	$A_1 = \dfrac{a}{2bv-1}, \quad A_2 = 0$
6	$\Pi_1 = \dfrac{a^2(8b^2v^2-25bv+8)}{18b(2bv-1)^2}$ $\Pi_2 = \dfrac{4a^2v(bv-2)}{9(2bv-1)^2}$	$q_1 = \dfrac{2a(bv-1)}{3b(2bv-1)}$ $q_2 = \dfrac{2abv}{3b(2bv-1)}$	$A_1 = \dfrac{a}{2bv-1}$ $A_2 = \dfrac{4a}{3(2bv-1)}$
7	$\Pi_1 = \dfrac{a^2(8-bv)}{18b}, \quad \Pi_2 = 0$	$q_1 = \dfrac{2a}{3b}, \quad q_2 = 0$	$A_1 = \dfrac{a}{3}, \ A_2 = 0$
8	$\Pi_i = \dfrac{a^2(bv-2)}{9b^2v}, \quad i = 1,2$	$q_i = \dfrac{a}{3b}, \quad i = 1,2$	$A_i = \dfrac{2a}{3bv}, \quad i = 1,2$
9	$\Pi_1 = \dfrac{a^2(3b^2v^2+10bv+12)}{9b^3v^2}$ $\Pi_2 = \dfrac{a^2(bv-2)^2}{9b^3v^2}$	$q_1 = \dfrac{a(bv+2)}{3b^2v}$ $q_2 = \dfrac{a(bv-2)}{3b^2v}$	$A_1 = \dfrac{2a}{3bv}, \quad A_2 = 0$
10	$\Pi_1 = \dfrac{a^2(9b^2v^2+4bv+4)}{36b^3v^2}, \quad \Pi_2 = 0$	$q_1 = \dfrac{a(3bv+2)}{6b^2v}, \quad q_2 = 0$	$A_1 = \dfrac{2a}{3bv}, \ A_2 = 0$

Let us first suppose that incumbent chooses not to advertise. The options available to the entrant would be: no entry, enter but not advertise, enter and advertise. The corresponding profits are: $\Pi_2(1)=0$, $\Pi_2(2)=a^2/9b$, and $\Pi_2(3)=a^2v/(9(bv-2))$. $\Pi_2(3)$ and $q_2(3)$ have positive value for $bv>2$. Consequently, it is $\Pi_2(3)>\Pi_2(2)>\Pi_2(1)$, the firm 2 will choose to enter and advertise. For $bv<2$, the firm 2 could choose to enter but not advertise (pathway 2). However, if the firm 1 decides not to invest in advertising, the firm 2 can invest the amount analogous to the pathway 7, which would cause a drop in the firm 1's profit to zero (the firm 1 would be removed from the market), and the firm 2 would achieve higher profits than in the case of entering the market without investing in advertising. As a result, we can state the next proposition:

Proposition 2. *If the incumbent chooses not to advertise, optimal strategy to the rival is to enter and invest in advertising.*

Corollary. *Direct implication of the previous proposition is that for $\rho=-1$, the firm 1 always has to invest in advertising.*

Let us now suppose that the firm 1 advertises to maximise its own profit. The monopolistic choice of A_1 for the incumbent is $A_1=a/(2bv-1)$. Again, the options available to the entrant would be: no entry, enter but not advertise, enter and advertise. From Table 1, the corresponding profits are: $\Pi_2(4)=0$, $\Pi_2(5)=\dfrac{4a^2(bv-2)^2}{9b(2bv-1)^2}$ and

$\Pi_2(6)=\dfrac{4a^2v(bv-2)}{9(2bv-1)^2}$. The pathway 5 exhibits a positive value for q_2 when $bv<1/2$ and $bv>2$, and positive value for Π_2 when $bv>0$. The pathway 6 exhibits a positive value for q_2 when $bv>1/2$ and positive value for Π_2 when $bv>2$. Both pathways 5 and 6 exhibit positive values for q_2 and Π_2 when $bv>2$. In this interval, we can easily prove that $\Pi_2(6)>\Pi_2(5)$, thus the firm 2 will enter and advertise (Pathway 6). When $bv<1/2$, $\Pi_2(6)$ is negative while $q_2(5)$ and $\Pi_2(5)$ are positive, thus the entrant will choose to enter but not to invest in advertising (Pathway 5). For $1/2<bv<2$, $q_2(5)$ and $\Pi_2(6)$ are negative, so the firm 2 will choose not to enter (Pathway 4).

If the incumbent decides to set an A_1 to accommodate the entry of the firm 2, the options available to the entrant would be again: enter and advertise, enter but not advertise, no entry. The corresponding profits are $\Pi_2(8)$, $\Pi_2(9)$ and $\Pi_2(10)$ (see Table 1). $\Pi_2(8)$ is positive for $bv>2$. In the same interval, $\Pi_2(8)>\Pi_2(9)>\Pi_2(10)$, and thus the firm 2 will enter and advertise (Pathway 8). For $bv<2$, $\Pi_2(9)$ is positive, but $q_2(9)$ is negative, and thus the firm 2 will choose not to enter (Pathway 10).

The previous analysis proves the following:

Proposition 3. *If the incumbent chooses the amount of A_1 which maximises Π_1 in the monopolistic case, the rival will enter and invest in advertising when $bv>2$; enter but not advertise when $bv<1/2$, otherwise, it will not enter. If the incumbent chooses the*

amount of A_1 which accommodates the entry, the rival will enter and invest in advertising when $bv > 2$, otherwise, it will not enter.

Knowing what the firm 2 would choose for each of the possible choices of the incumbent, we will now determine the optimal incumbent's choice.

According to Proposition 2, the firm 1 must always invest in advertising to avoid to be put away by the firm 2. Furthermore, according to Proposition 3, when $bv < 1/2$ the firm 2 would choose either to enter but not advertise (Pathway 5) or not to enter (Pathway 10). Thus the firm 1 has to select among pathways 5, 7 or 10. In the same interval $q_1(5)$ is negative and thus the incumbent will compare $\Pi_1(7)$ and $\Pi_1(10)$. Regardless of the firm 1 decision, in this interval the entry of the firm 2 will be deterred. It is easy to prove that $\Pi_1(10) > \Pi_1(7)$, and the incumbent will set $A_1 = 2a/3bv$, thus making the entry of the rival unprofitable.

When $1/2 < bv < 2$, the incumbent will compare $\Pi_1(4)$, $\Pi_1(7)$ and $\Pi_1(10)$ because they are positive in this interval. In this interval the entrant would always choose not to enter (Pathways 4 or 10). It is easy to prove that $\Pi_1(4) > \Pi_1(7) > \Pi_1(10)$, so the incumbent will choose A_1 to maximise its monopolistic profit, the rival does not enter.

The propositions 2 and 3 have shown that for $bv > 2$ the optimal choice of the firm 2 is to enter and advertise (Pathways 3, 6 or 8). Accordingly, the firm 1 has to select one of the pathways 6, 7 or 8. It is easy to verify that $\Pi_1(7) > \Pi_1(6)$ for $(7 - \sqrt{41})/2 < bv < (7 + \sqrt{41})/2$ and $\Pi_1(7) > \Pi_1(8)$ for $bv < 3 + \sqrt{13}$. Therefore, the incumbent will invest in advertising to deter entry as long as $2 < bv < 3 + \sqrt{13}$, gaining profit $\Pi_1(7)$. When $bv > 3 + \sqrt{13}$, it is always $\Pi_1(8) > \Pi_1(7)$ and $\Pi_1(8) > \Pi_1(6)$, so a Cournot duopoly with advertising will emerge.

The previous analysis proves the following:

Proposition 4. *For $bv < 1/2$ the incumbent will deter the entry of the rival by setting $A_1 = 2a/3bv$. For $1/2 < bv < 2$ the incumbent chooses A_1 so as to maximise its monopolistic profit, the rival does not enter. For $2 < bv < 3 + \sqrt{13}$, the incumbent will deter the entry of the rival setting $A_1 = a/3$. For $bv > 3 + \sqrt{13}$, the incumbent will accommodate the entry of the rival, the rival will enter and invest in advertising.*

5.2. The market equilibrium solutions for the low spillover

Let us now suppose that advertising by one firm has no direct effect upon the other firm's profit ($\rho = 0$). Corresponding outcomes for every pathway from the game tree in Figure 1 are shown in Table 2. Analogously to the previous case, to solve the game, the behaviour of the entrant given the firm 1's choices must be considered first.

If the incumbent chooses not to advertise, the entrant will compare $\Pi_2(1) = 0$, $\Pi_2(2) = a^2/9b$, $\Pi_2(3) = a^2v/(9bv - 8)$. It is easy to check that for $bv > 8/9$, $\Pi_2(3) > \Pi_2(2) > \Pi_2(1)$ and thus the firm 2 will choose to enter and advertise (Pathway 3). For $bv < 8/9$, the best choice for the firm 2 is to invest amount analogous to the

pathway 7, which would cause a drop in incumbent's profit to zero (the incumbent would be removed from the market).

If the incumbent chooses the amount of A_1 which maximises Π_1 in the monopolistic case, the entrant will compare: $\Pi_2(4) = 0$, $\Pi_2(5)$ and $\Pi_2(6)$ (see Table 2). We can easily check that for $bv < 1/2$, $\Pi_2(6)$ is negative, so the entrant will choose to enter but not advertise (Pathway 5). For $bv > 1$, it is $\Pi_2(6) > \Pi_2(5) > \Pi_2(4)$. In the same interval, $q_2(6)$ and $\Pi_2(6)$ are positive, so potential entrant will choose to enter and advertise (Pathway 6). For $1/2 < bv < 1$, $q_2(6)$ and $q_2(5)$ are negative, so the firm 2 will not enter (Pathway 4).

Table 2. Outcomes for the low spillover ($\rho = 0$)

Path	Profits	Quantities	Advertising level
1	$\Pi_1 = \dfrac{a^2}{4b}$, $\Pi_2 = 0$	$q_1 = \dfrac{a}{2b}$, $q_2 = 0$	
2	$\Pi_i = \dfrac{a^2}{9b}$, $i = 1,2$	$q_i = \dfrac{a}{3b}$, $i = 1,2$	
3	$\Pi_1 = \dfrac{a^2(3bv-4)^2}{b(9bv-8)^2}$ $\Pi_2 = \dfrac{a^2v}{9bv-8}$	$q_1 = \dfrac{a(3bv-4)}{b(9bv-8)}$ $q_2 = \dfrac{3av}{9bv-8}$	$A_1 = 0$ $A_2 = \dfrac{4a}{9bv-8}$
4	$\Pi_1 = \dfrac{a^2v}{2(2bv-1)}$, $\Pi_2 = 0$	$q_1 = \dfrac{av}{2bv-1}$, $q_2 = 0$	$A_1 = \dfrac{a}{2bv-1}$, $A_2 = 0$
5	$\Pi_1 = \dfrac{2(2abv+a)^2 - 9a^2bv}{18b(2bv-1)^2}$ $\Pi_2 = \dfrac{4a^2(bv-1)^2}{9b(2bv-1)^2}$	$q_1 = \dfrac{2abv+a}{3b(2bv-1)}$ $q_2 = \dfrac{2a(bv-1)}{3b(2bv-1)}$	$A_1 = \dfrac{a}{2bv-1}$, $A_2 = 0$
6	$\Pi_1 = \dfrac{a^2v(36b^3v^3 - 141b^2v^2 - 155bv - 81)^2}{(2bv-1)^2(9bv-8)^2}$ $\Pi_2 = \dfrac{4a^2v(bv-1)^2}{(2bv-1)^2(9bv-8)}$	$q_1 = \dfrac{av(6bv-23)}{(2bv-1)(9bv-8)}$ $q_2 = \dfrac{6av(bv-1)}{(2bv-1)(9bv-8)}$	$A_1 = \dfrac{a}{2bv-1}$ $A_2 = \dfrac{8a(bv-1)}{(2bv-1)(9bv-8)}$
7	$\Pi_1 = \dfrac{a^2(2-bv)}{2b}$, $\Pi_2 = 0$	$q_1 = \dfrac{a}{b}$, $q_2 = 0$	$A_1 = a$, $A_2 = 0$
8	$\Pi_i = \dfrac{a^2v(9bv-8)}{(9bv-4)^2}$, $i = 1,2$	$q_i = \dfrac{3av}{9bv-4}$, $i = 1,2$	$A_i = \dfrac{4a}{9bv-4}$, $i = 1,2$
9	$\Pi_1 = \dfrac{a^2(81b^2v^2+16)}{9b(9bv-4)^2}$ $\Pi_2 = \dfrac{a^2(9bv-8)^2}{9b(9bv-4)^2}$	$q_1 = \dfrac{a(9bv+4)}{3b(9bv-4)}$ $q_2 = \dfrac{a(9bv-8)}{3b(9bv-4)}$	$A_1 = \dfrac{4a}{9bv-4}$, $A_2 = 0$
10	$\Pi_1 = \dfrac{81a^2b^2v^2}{4b(9bv-4)^2}$, $\Pi_2 = 0$	$q_1 = \dfrac{9av}{2(9bv-4)}$, $q_2 = 0$	$A_1 = \dfrac{4a}{9bv-4}$, $A_2 = 0$

If the incumbent decides to set an A_1 that accommodates the entry of the firm 2,

the entrant will compare: $\Pi_2(8) = \dfrac{a^2 v(9bv-8)}{(9bv-4)^2}$, $\Pi_2(9) = \dfrac{a^2(9bv-8)^2}{9b(9bv-4)^2}$ and $\Pi_2(10) = 0$.

When $bv < 4/9$, $q_2(8)$ is negative, so the entrant will choose to enter but not to invest in advertising (Pathway 9) gaining profit $\Pi_2(9)$. For $4/9 < bv < 8/9$, $\Pi_2(8)$ and $q_2(9)$ are negative, so potential entrant will choose not to enter (Pathway 10). $\Pi_2(8)$ and $q_2(8)$ are positive for $bv > 8/9$. In the same interval, $\Pi_2(8) > \Pi_2(9) > \Pi_2(10)$, and thus the firm 2 will enter and advertise (Pathway 8).

The previous analysis proves the following:

Proposition 5. *If the incumbent chooses not to advertise, the entrant will enter and invest in advertising. If the incumbent chooses the amount of A_1 which maximises Π_1 in the monopolistic case, the rival will enter and invest in advertising when $bv > 1$; enter but not advertise when $bv < 1/2$, otherwise, it will not enter. If the incumbent chooses the amount of A_1 which accommodates entry, the rival will enter and invest in advertising when $bv > 8/9$, enter but not advertise when $bv < 4/9$, otherwise, it will not enter.*

Let us now determine the optimal choice for the incumbent. According to the Proposition 5, the firm 1 must always invest in advertising to avoid to be put away by the firm 2. Furthermore, when $bv < 4/9$, the firm 2 would choose either to enter end advertise (Pathway 3) or to enter but not advertise (Pathways 5 or 9). Thus the firm 1 will compare $\Pi_1(5), \Pi_1(7)$ and $\Pi_1(10)$). In the same interval $q_1(5)$ and $q_1(9)$ are negative and thus the incumbent will set $A_1(7) = a$ to deter the entry of the firm 2.

When $4/9 < bv < 1/2$, the firm 2 would choose to enter end advertise (Pathway 3), enter but not advertise (Pathway 5) or not to enter (Pathway 10). In the same interval $q_1(5)$ is negative and thus the firm 1 will compare $\Pi_1(7)$ and $\Pi_1(10)$. When $1/2 < bv < 8/9$ the firm 2 would choose either to enter end advertise (Pathway 3), or not to enter (Pathways 4 or 10). Thus the firm 1 has to select among the pathways 4, 7 or 10. Regardless of what the firm 1 decides, for $4/9 < bv < 8/9$, the entry of the firm 2 will be deterred. It is easy to check that $\Pi_1(10) > \Pi_1(7)$ and $\Pi_1(10) > \Pi_1(4)$, so the firm 1 will set $A_1 = 4a/(9bv-4)$ to deter the entry of the firm 2.

According to the Proposition 5, when $8/9 < bv < 1$, the firm 2 would choose either to enter and invest in advertising (Pathways 3 or 8) or not to enter (Pathway 4). Therefore the incumbent will compare $\Pi_1(4)$, $\Pi_1(7)$ and $\Pi_1(8)$. It is easy to prove that $\Pi_1(4)$ is greater than $\Pi_1(7)$ and $\Pi_1(8)$, so the incumbent will choose $A_1 = a/(2bv-1)$ to maximise its monopolistic profit, and the rival does not enter.

The proposition 5 has shown that for $bv > 1$ the optimal choice of the firm 2 is to enter and advertise (Pathways 3, 6 or 8). Accordingly, the firm 1 has to select one of the pathways 6, 7 or 8. For $1 < bv < 2$, $q_1(6)$ is negative and thus the incumbent will

compare $\Pi_1(7)$ and $\Pi_1(8)$. It is easy to verify that $\Pi_1(7) > \Pi_1(8)$ for $1 < bv < 1.80161$ and $\Pi_1(8) > \Pi_1(7)$ for $1.80161 < bv < 2$.

For $bv > 2$, $\Pi_1(7)$ is negative, so the incumbent will compare $\Pi_1(6)$ and $\Pi_1(8)$. It is easy to check that in this interval $\Pi_1(8) > \Pi_1(6)$ (note that $q_1(6) < 0$ for $bv < 23/6$) so the incumbent will choose to accommodate the entry of the firm 2.

The previous analysis proves the following:

Proposition 6. *For $bv < 4/9$ the incumbent will deter the entry of the rival setting $A_1 = a$. For $4/9 < bv < 8/9$ the incumbent will choose $A_1 = 4a/(9bv - 4)$ to accommodate the entry of the rival, the rival does not enter. For $8/9 < bv < 1$ the incumbent chooses $A_1 = a/(2bv - 1)$ so as to maximise its monopolistic profit, the rival does not enter. For $1 < bv < 1.80161$ the incumbent will deter the entry of the rival setting $A_1 = a$. For $bv > 1.80161$, the incumbent will accommodate the entry of the rival, the rival will enter and invest in advertising.*

5.3. The market equilibrium solutions for the strictly informative advertising

Let us now suppose that advertising is strictly informative, i.e. advertising also assists the other firm ($\rho = 1$). Corresponding outcomes for every pathway from the game tree in Figure 1 are shown in Table 3. Again, to solve the game, we will first consider the behaviour of the entrant given the firm 1's choices.

Table 3. Outcomes for strictly informative advertising ($\rho = 1$)

Path	Profits	Quantities	Advertising level
1	$\Pi_1 = \dfrac{a^2}{4b}$, $\Pi_2 = 0$	$q_1 = \dfrac{a}{2b}$, $q_2 = 0$	
2	$\Pi_i = \dfrac{a^2}{9b}$, $i = 1,2$	$q_i = \dfrac{a}{3b}$, $i = 1,2$	
3	$\Pi_1 = \dfrac{9a^2b^2v^2}{b(9bv - 2)^2}$, $\Pi_2 = \dfrac{a^2v}{9bv - 2}$	$q_1 = \dfrac{3abv}{b(9bv - 2)}$, $q_2 = \dfrac{3av}{9bv - 2}$	$A_1 = 0$, $A_2 = \dfrac{2a}{9bv - 2}$
4	$\Pi_1 = \dfrac{a^2v}{2(2bv - 1)}$, $\Pi_2 = 0$	$q_1 = \dfrac{av}{2bv - 1}$, $q_2 = 0$	$A_1 = \dfrac{a}{2bv - 1}$, $A_2 = 0$
5	$\Pi_1 = \dfrac{8a^2b^2v^2 - 9a^2bv}{18b(2bv - 1)^2} = \dfrac{a^2v(8bv - 9)}{18(2bv - 1)^2}$ $\Pi_2 = \dfrac{4a^2bv^2}{9(2bv - 1)^2}$	$q_1 = \dfrac{2av}{3(2bv - 1)}$ $q_2 = \dfrac{2av}{3(2bv - 1)}$	$A_1 = \dfrac{a}{2bv - 1}$, $A_2 = 0$
6	$\Pi_1 = \dfrac{a^2v(72b^3v^3 - 81b^2v^2 + 36bv - 4)}{2(2bv - 1)^2(9bv - 2)^2}$ $\Pi_2 = \dfrac{8a^2b^2v^3(4bv - 1)}{(2bv - 1)^2(9bv - 2)^2}$	$q_1 = \dfrac{6abv^2}{(2bv - 1)(9bv - 2)}$ $q_2 = \dfrac{6abv^2}{(2bv - 1)(9bv - 2)}$	$A_1 = \dfrac{a}{2bv - 1}$ $A_2 = \dfrac{4abv}{(2bv - 1)(9bv - 2)}$

7	$\Pi_1 = -\dfrac{a^2(4+bv)}{2b}$, $\Pi_2 = 0$	$q_1 = 0$, $q_2 = 0$	$A_1 = -a$, $A_2 = 0$
8	$\Pi_i = \dfrac{a^2 v(9bv-2)}{(9bv-4)^2}$, $i = 1,2$	$q_i = \dfrac{3av}{9bv-4}$, $i = 1,2$	$A_i = \dfrac{2a}{9bv-4}$, $i = 1,2$
9	$\Pi_1 = \dfrac{a^2(81b^2v^2 - 54bv + 4)}{9b(9bv-4)^2}$ $\Pi_2 = \dfrac{a^2(9bv-2)^2}{9b(9bv-4)^2}$	$q_1 = \dfrac{a(9bv-2)}{3b(9bv-4)}$ $q_2 = \dfrac{a(9bv-2)}{3b(9bv-4)}$	$A_1 = \dfrac{2a}{9bv-4}$, $A_2 = 0$
10	$\Pi_1 = \dfrac{a^2(81b^2v^2 - 44bv + 4)}{4b(9bv-4)^2}$, $\Pi_2 = 0$	$q_1 = \dfrac{a(9bv-2)}{2b(9bv-4)}$, $q_2 = 0$	$A_1 = \dfrac{2a}{9bv-4}$, $A_2 = 0$

If the incumbent chooses not to advertise, the entrant will compare $\Pi_2(1) = 0$, $\Pi_2(2) = a^2/9b$, and $\Pi_2(3) = a^2 v/(9bv-2)$. $\Pi_2(3)$ and $q_2(3)$ have positive values for $bv > 2/9$. In the same interval, $\Pi_2(3) > \Pi_2(2) > \Pi_2(1)$, and thus the firm 2 will choose to enter and advertise (Pathway 3). For $bv < 2/9$, the firm 2 will chose to enter but not advertise (Pathway 2).

If the incumbent chooses the amount of A_1 which maximises Π_1 in the monopolistic case, the entrant would compare: $\Pi_2(4) = 0, \Pi_2(5)$ and $\Pi_2(6)$ (see Table 3). We can easily check that for $bv < 1/2$, $q_2(5)$ and $\Pi_2(6)$ are negative, so the entrant will choose not to enter (Pathway 4). For $1/2 < bv < 1 + \sqrt{5}/3$, $\Pi_2(6) > \Pi_2(5) > \Pi_2(4)$. In the same interval, $q_2(6)$ and $\Pi_2(6)$ are positive, so the potential entrant will choose to enter and advertise (Pathway 6). For $bv > 1 + \sqrt{5}/3$, $\Pi_2(5) > \Pi_2(6)$, and thus the firm 2 will enter but not advertise (Pathway 5).

If the incumbent decides to set an A_1 that accommodates the entry of the firm 2, the entrant will compare: $\Pi_2(8) = \dfrac{a^2 v(9bv-2)}{(9bv-4)^2}$, $\Pi_2(9) = \dfrac{a^2(9bv-2)^2}{9b(9bv-4)^2}$ and $\Pi_2(10) = 0$. When $bv < 2/9$, $q_2(8)$ is negative, so the entrant will choose to enter but not invest in advertising (Pathway 9) gaining profit $\Pi_2(9)$. For $2/9 < bv < 4/9$, $q_2(8)$ and $q_2(9)$ are negative, so potential entrant will choose not to enter (Pathway 10). $\Pi_2(8)$ is positive for $bv > 4/9$. In the same interval, $\Pi_2(8) > \Pi_2(9) > \Pi_2(10)$, and thus the firm 2 will enter and advertise (Pathway 8).

The previous analysis proves the following:

Proposition 7. *If the incumbent chooses not to advertise, the entrant will enter and invest in advertising when $bv > 2/9$, otherwise it will enter but not advertise. If the incumbent chooses the amount of A_1 which maximises Π_1 in the monopolistic case, the rival will enter and invest in advertising when $1/2 < bv < 1 + \sqrt{5}/3$; enter but not advertise when*

$bv > 1 + \sqrt{5}/3$, *otherwise, it will not enter. If the incumbent chooses the amount of* A_1 *which accommodates the entry, the rival will enter and invest in advertising when* $bv > 4/9$, *enter but not advertise when* $bv < 2/9$, *otherwise, it will not enter.*

Let us now determine the optimal incumbent's choice, knowing what the firm 2 would choose for each of the possible incumbent's chooses.

According to the Proposition 7, when $bv < 2/9$ the firm 2 would choose either to enter but not advertise (Pathways 2 or 9) or not to enter (Pathway 4). Thus the firm 1 has to select among the pathways 2, 4 or 9. In the same interval $\Pi_1(4)$ is negative and thus the incumbent will compare $\Pi_1(2)$ and $\Pi_1(9)$. It is easy to check that $\Pi_1(2) > \Pi_1(9)$, so the incumbent will not invest in advertising.

When $2/9 < bv < 4/9$, the incumbent will compare $\Pi_1(2)$, $\Pi_1(4)$ and $\Pi_1(10)$. In the same interval $\Pi_1(4)$ is negative. It is easy to check that $\Pi_1(2) > \Pi_1(10)$, so the incumbent will not invest in advertising.

According to the Proposition 7, when $4/9 < bv < 1/2$ the firm 2 would choose either to enter and invest in advertising (Pathways 3 or 8) or not to enter (Pathway 4). In the same interval $\Pi_1(4)$ is negative and thus the incumbent will compare $\Pi_1(3)$ and $\Pi_1(8)$. It is easy to check that $\Pi_1(8) > \Pi_1(3)$, and the incumbent will set $A_1 = 2a/3bv$ to accommodate the entry of the firm 2.

The Proposition 7 has shown that for $1/2 < bv < 1 + \sqrt{5}/3$ the optimal choice for the firm 2 is to enter and advertise. Accordingly, the firm 1 has to select one of the pathways 3, 6, or 8. It is easy to verify that $\Pi_1(8) > \Pi_1(6) > \Pi_1(3)$ and the incumbent will set $A_1 = 2a/3bv$ to accommodate the entry of the firm 2.

For $bv > 1 + \sqrt{5}/3$ the optimal choice for the firm 2 is either to enter and advertise (Pathways 3 or 8) or to enter but not advertise (Pathway 5). It is easy to verify that it is $\Pi_1(8) > \Pi_1(3) > \Pi_1(5)$ and the incumbent will set $A_1 = 2a/3bv$ to accommodate the entry of the firm 2.

The previous analysis proves the following:

Proposition 8. *For* $bv < 4/9$ *the incumbent will not invest in advertising, the rival will enter but not advertise. For* $bv > 4/9$ *the incumbent will hoose* $A_1 = 2a/3bv$ *to accommodate the entry of the rival, the rival will enter and invest in advertising.*

5.4. Discussion

Based on the previous analysis of the three special cases, we can sum up the market equilibria as follows: In the case of strictly predatory advertising ($\rho = -1$), the entry is deterred for $bv < 3 + \sqrt{13}$ and accommodated for $bv > 3 + \sqrt{13}$; for the low advertising spillover ($\rho = 0$) the entry is deterred for $bv < 1.80161$ and accommodated for $bv > 1.80161$; and for the strictly informative advertising ($\rho = 1$) the entry is always accommodated regardless of what the value of bv is.

It should be noted that the entry is more likely to occur as b and/or v increase. It is clear that when v is higher advertising is more costly, which lowers profits and thus discourages its use for entry deterrence purposes. When b increases, the possibility of the entry is higher because the demand curve of each firm is steeper and steeper, so a given amount A of advertising investment from the incumbent has less and less effect on the quantity sold by the rival.

However, it can be noted that the threshold value of bv to which advertising can be used for the entry deterrence decreases with increasing value of the parameter ρ. This threshold is the highest in the case of predatory advertising (approximately 6.61), continues to decline as spillover decreases, while in the case of informative advertising (explicitly for $\rho > 1/2$) it is impossible to deter entry.

Now let us consider whether or not there is an over-investment for each of the above-mentioned cases, as well as the conditions under which it occurs.

In the case of strictly predatory advertising we have proved that for $bv < 2$ the firm 1 will over-invest in advertising to avoid to be put away by the firm 2. For $2 < bv < 3 + \sqrt{13}$ there is an over-investment in advertising in order to prevent the entry, because $A_1(7) > A_1(8) > A_1(6)$; actually, if $bv > 2$ the firm 2 definitely enters and advertises, so that now the pathway 7 must be compared with the pathways 6 and 8. If $bv > 3 + \sqrt{13}$, since there is no possibility to deter the rival's entry and its investment in advertising, the existing firm has to afford promotional expenses in any case, even if the best choice for both of them would be to refrain from advertising ($\Pi_i(8) < \Pi_i(2)$, $i = 1, 2$): once more there is an over-investment in advertising from the firm 1, but now it has to accommodate entry. In this interval both firms earn lower profits than they would under Cournot duopoly. Hence, strategic implications force them to afford promotional expenditures, and the resulting Nash equilibrium is not the Pareto-efficient solution: the strategic interaction between rational players who behave non-cooperatively does not yield the optimal outcome for neither of them.

In the case of the low advertising spillover, we proved that for $bv < 8/9$ the firm 1 will over-invests in advertising to avoid to be put away by the firm 2. For $8/9 < bv < 1$ there is over-investment in advertising in order to prevent entry, because $A_1(4) > A_1(7)$ and $A_1(4) > A_1(8)$; actually, if $8/9 < bv < 1$ the firm 2 would either enter and advertise or not enter, so that now the pathway 4 must be compared with the pathways 3, 7 and 8. We have verified that for $1 < bv < 1.80161$ there is over-investment in advertising in order to prevent entry, because $A_1(7) > A_1(6) > A_1(8)$. If $bv > 1.80161$, since there is no possibility to deter the rival's entry and its investment in advertising, the existing firm has to afford promotional expenses in any case, even if the best choice for both of them would be to refrain from advertising ($\Pi_i(8) < \Pi_i(2)$, $i = 1, 2$): once more there is an over-investment in advertising from the firm 1, but now it has to accommodate the entry.

The observation, that for predatory advertising as well as for the low advertising spillover (exactly for $\rho < 1/2$) there is always overinvestment in strategic purposes is in the line with the Propositions 1.

Finally, we have proved that in the case of strictly informative advertising an over-investment never occurs, no matter what the value of bv is. Precisely, according to the Proposition 6 for $bv < 4/9$ neither the firm 1 nor the firm 2 invests in advertising. For $bv > 4/9$, both firms invest in advertising. It is easy to check that for $bv > 8/27$, $\Pi_i(8) > \Pi_i(2)$, $i = 1, 2$, and thus there is under-investment in order to accommodate the entry. These findings are in line with the Proposition 1.

6. CONCLUSIONS

In this paper we consider the effects of investment in advertising in the three-stage entry game model with one incumbent and one potential entrant. The decisions to be made at the three stages are: the incumbent decides to advertise or not, the new firm (potential entrant) decides whether to enter and whether to advertise. Then if there is the entry, in the third stage, a Cournot-Nash equilibrium in quantities appears.

In our model, it is assumed that an additional unit of advertising expenditure by the firm i shifts outward its own demand curve while the externality upon the rival firm depends on whether advertising interaction factor is positive (the firm j's curve shifts outwards) or negative (the curve shifts inwards)

The objective of the paper is to show that, under certain conditions, advertising can be used as a strategic weapon in the entry game. Our model has shown that in a context of decreasing returns on advertising there are certain conditions under which rational established and potential firms may find it optimal to over-invest as well as to under-invest in advertising for strategic purposes. Specifically, according to this model, the pressure of the potential entry and the effect that advertising is supposed to have on the demand imply that, when the advertising interaction factor is less than 1/2, the incumbent firm will always over-invest in promotional activities, while the entry may be possible depending on the slope of demand function and the cost of advertising. When advertising interaction factor is greater than 1/2, the optimal choice is under-investment.

Furthermore, three special cases, strictly predatory advertising, informative advertising and the case when advertising of one of the firms cannot directly influence the other firm's profit, were analyzed. For each of them, depending on the costs of advertising and marginal costs, equilibrium is determined, and conditions under which it is possible to deter the entry are given.

It is proved that in the case of predatory advertising, investment in advertising can be used for entry deterrence, but with increasing value of advertising interaction factor, power of using advertising as a weapon to deter the entry into the market decreases. So, in the case of informative advertising, advertising cannot be used as a weapon for deterring the entry into the market.

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