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Throughput and Delay Analysis of Half-Duplex IEEE 802.11 Mesh Networks

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***Abstract*— Emerging technologies for mesh networks can provide users with last-mile service to an access point by forwarding data through wireless relays instead of through expensive wireline infrastructure. While an extensive amount of literature on the subject has been amassed in the last decade, existing papers model network traffic flow solely as a function of routing topology, neglecting contention at the Media Access Control layer; as a result, the inbound flow to a relay station is independent of the transmission success rate from forwarding stations. This leads to overestimation of traffic flow, especially at network operation approaching full capacity, and in turn makes for inaccuracies in predicting throughput and delay. In our model, the inbound flow depends on the transmission success rate as well. Other novel contributions are the incorporation of a half-duplex contention model we developed in previous work, which captures both uplink and downlink traffic, and a generic framework to represent any mesh routing topology (minimum-hop, minimum-airtime, etc.)**

***Index Terms*— Multi-hop; contention**

I. Introduction

The increasing demand for residential broadband access has drawn attention to emerging technologies such as the IEEE 802.11s [1], which is a draft amendment for mesh networks currently in its final stage of approval. Mesh networks can provide user stations with last-mile service to an access point by forwarding data through wireless relays instead of through expensive wireline infrastructure. A practical application of notable importance is the Smart Grid [2].

While an extensive amount of literature has been amassed on IEEE 802.11 mesh networks in the last decade, here we can cite only a sample of representative works due to space constraints. One class of papers computes bounds on throughput and delay based on simple geometrical models for contention access between stations rather than on the binary exponential backoff schemes of the Media Access Control (MAC) protocol [3], [4]. In a second class, performance metrics for a generic link are first generated based on the MAC protocol and then extrapolated for traffic in multi-hop networks [5], [6]. In yet another class of papers, the opposite occurs: the forwarded traffic flows to relay stations are computed based on a given routing topology; then the throughput and delay are calculated using a link contention model applied to local traffic: in Croce [7], the total end-to-end network flow is segmented into a fraction at each relay assuming a Poisson frame arrival rate; Yan [8] uses an approximation for the traffic flow "diffusing" throughout the network; Huang [9] partitions the deployment area of the stations into concentric rings whose uplink traffic to the center increases in proportion to the amount forwarded from those in rings farther out; along the same lines, as in our paper, Hu [10] models the uplink flow increasing with smaller hop distance.

The aforementioned papers model network traffic flow solely as a function of the mesh routing topology, neglecting contention; as a result, the inbound flow to a relay station is independent of the transmission success rate from forwarding stations. This leads to overestimation of traffic flow, especially at network operation approaching full capacity, and in turn makes for inaccuracies in predicting throughput and delay. In our model, the inbound traffic also accounts for the transmission success rate from forwarding stations. To our knowledge, only one other paper models this by embedding the routing topology in a Markov-state analysis [11]; however, the paper only computes bounds on throughput assuming homogeneous traffic at the stations, so the effect of tiered forwarding loads is not captured. All these papers either consider only unidirectional traffic flow, assume a full-duplex structure, or do not treat the topic at all. Specifically, our contributions are:

- an end-to-end contention model which accounts for the transmission success rate of forwarded traffic;
- the incorporation of a half-duplex IEEE 802.11 MAC layer model that we developed in [12] in support of the work presented here to capture both uplink and downlink network flows;
- a generic framework to represent any mesh routing topology (minimum-hop, minimum-airtime, etc.) in the contention model.

The paper reads as follows: in Sections II and III, we first consider a single-hop network in which all the stations lie within the coverage range of an access point. Section II describes the physical layer analysis to determine the coverage range while Section III provides an overview of our half-duplex MAC layer model. Section IV contains the main contribution of this paper in which the network is generalized to include stations that lie outside of the access point's range and so must route to it through other relay stations. Section V presents analytical results for the throughput and delay of an example mesh network, followed by conclusions.

II. Physical layer coverage analysis

The coverage range r_{cov} is defined as the maximum distance at which an access point can communicate with another network station. In this section, we determine its value for a given outage probability and a set of physical (PHY) layer operating parameters. The outage probability is the probability that the received signal-to-noise-ratio (SNR) is below the required SNR to operate the link. The required SNR depends on the information data rate and serves as an input to the analysis.

The received SNR is modeled as a combination of a deterministic component, based on transmitter-receiver range r, and a random component due to shadowing and small-scale fading. The deterministic component can be evaluated using a link budget approach. In terms of the commonly used E_b/N_0, the ratio of the received energy per bit to the power spectral density of the noise, it is expressed in decibels as

$$\overline{\left(\frac{E_b}{N_0}\right)_{\mathrm{rx,dB}}(r)} = EIRP_{\mathrm{dBm}} + G_{\mathrm{r,dBi}} - PL_{\mathrm{dB}}(r) - L_{\mathrm{s,dB}} - N_{0,\mathrm{dBm/Hz}} - R_{\mathrm{b,dB-b/s}}, \quad (1)$$

where $EIRP_{\mathrm{dBm}}$ is the effective isotropic radiated power from the transmitter (dBm), $G_{\mathrm{r,dBi}}$ is the receiver antenna gain (dBi), $PL_{\mathrm{dB}}(r)$ is the path loss at distance r (dB), $L_{\mathrm{s,dB}}$ is the cumulative system loss due to cabling or other implementation losses (dB), $N_{0,\mathrm{dBm/Hz}}$ is the power spectral density of the noise (dBm/Hz), and $R_{\mathrm{b,dB-b/s}}$ is the physical layer information data rate (dB-b/s). Using a simple path loss model, the term for path loss in (1) is calculated as

$$PL_{\mathrm{dB}}(r) = PL_0 + 10n\log_{10}(r), \quad (2)$$

where n is the path loss exponent and PL_0 is the reference path loss at 1 m.

The outage probability $P_{\mathrm{out}}(r)$ at range r is defined as the probability that the received SNR $(E_b/N_0)_{\mathrm{rx,dB}}(r) = \overline{\left(\frac{E_b}{N_0}\right)_{\mathrm{rx,dB}}(r)} + X$ is less than the required SNR $(E_b/N_0)_{\mathrm{cov,dB}}$:

$$P_{\mathrm{out}}(r) = P\left[\left(\frac{E_b}{N_0}\right)_{\mathrm{rx,dB}}(r) < \left(\frac{E_b}{N_0}\right)_{\mathrm{cov,dB}}\right], \quad (3)$$

where $X = X_{s,\mathrm{dB}} + X_{f,\mathrm{dB}}$ is the combined random attenuation due to shadowing and fading. Based on the assumption of lognormal shadowing, the shadowing attenuation $X_{s,\mathrm{dB}}$ is modeled as a zero-mean Gaussian random variable with standard deviation σ; and, based on the assumption of Nakagami-m fading, the fading attenuation X_f is modeled as a unit-mean Gamma-distributed random variable and $X_{f,\mathrm{dB}} = 10\log(X_f/10)$. We assume that the shadowing and fading are mutually independent and independent of those on other links.

Given a uniform distribution of the stations around the access point, the average outage probability over the coverage area bounded by r_{cov} is obtained by weighting (3) by the probability density function of the station-to-AP distance as

$$\overline{P_{\mathrm{out}}} = \int_0^{r_{\mathrm{cov}}} P_{\mathrm{out}}(r)\, f_r(r)\, dr, \quad (4)$$

where $f_r(r) = 2r/r_{\mathrm{cov}}^2$. Then r_{cov} can be found numerically through the equation above to meet a given $\overline{P_{\mathrm{out}}}$.

III. The half-duplex IEEE 802.11 MAC model

The MAC-layer performance metrics of interest for an individual class of nodes X in the network are the reliability, R^X, the average application data throughput, S^X, and the mean link delay, D^X. In this section, we consider a single-hop network composed from N stations lying within the coverage area of a single access point. The stations attempt to communicate with the AP using the IEEE 802.11 protocol at the MAC layer. Numerous models of this protocol have been discussed in the literature, starting with Bianchi's seminal paper in 2000 [13]. Most models of this class attempt to capture the behavior of the protocol's backoff and retransmission mechanism.

The IEEE 802.11 protocol uses Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA), which relies on a backoff counter. When a frame becomes available for transmission, the protocol will choose a backoff value in the window between zero and the initial maximum backoff value W_0. The protocol decrements this counter each slot interval, but it will hold the countdown while other stations are transmitting. The frame is sent when the counter reaches zero. Unsuccessful (i.e. unacknowledged) transmissions result in the station's doubling its maximum backoff counter value, using the new maximum to choose a random backoff, and beginning a new countdown. The maximum backoff during the ith transmission attempt, $0 \le i \le \alpha$, is

$$W_i = \begin{cases} 2^i W_0, & i \le M \\ 2^m W_0, & i > M \end{cases} \quad (5)$$

where $i = 0$ denotes the initial attempt, α is the limit on retransmissions, and M is the maximum number of times the window is doubled.

In [12], we developed an extension to the Bianchi model that incorporates novel elements such as allowing for both uplink and downlink traffic between the stations and the access point. This accounts for the fact that any node can be in either transmit or receive mode, but cannot perform both functions simultaneously. Thus, an AP that is transmitting to a station will not receive any frames that are being sent on the uplink, i.e., from a station to the AP; this will result in unacknowledged frames that the stations will treat as collisions. The outputs of this analysis are the frame transmission failure probability for a single transmission attempt $P^X_{\mathrm{fail_N}}$ when there are N stations, and the mean MAC layer service time

$$\frac{1}{\mu_{\mathrm{MAC,X}}} = \sum_{i=0}^{\alpha} (1 - P^X_{\mathrm{fail_N}})(P^X_{\mathrm{fail_N}})^i \left[i\mu_C + \mu^X_{W_i} + \mu_S\right] + P^{\alpha+1}_{\mathrm{fail_N}}\left[(\alpha+1)\mu_C + \mu^X_{W_\alpha}\right], \quad (6)$$

where X is an identifier that takes the value ST if the node in question is a station or AP if it is the access point. The mean time to successfully send a frame and the mean time taken up by a lost frame are μ_S and μ_C, respectively, and $\mu^X_{W_i}$ is the mean time a node of type X spends in backoff stages 0 through i.

Using the approach in [14], we use the mean MAC service time to compute the set of state probabilities $\{p^X_n\}^K_{n=0}$, where $K-1$ is the capacity of the node's transmission buffer, and n is the number of frames in the node, including the frame in service and all the frames in the buffer. We use an M/M/1/K queueing model, thus the state probabilities are

$$p^X_n = \left(\frac{\lambda^X}{\mu^X_{\mathrm{MAC}}}\right)^n \Big/ \sum_{j=0}^{K}\left(\frac{\lambda^X}{\mu^X_{\mathrm{MAC}}}\right)^j, \quad (7)$$

where λ^X is the frame arrival rate at the X class of nodes.

All the performance metrics follow directly from the state probabilities and so are functions of N. The node's blocking probability is

$$P^X_B = P(\mathrm{bufferfull}) = \mathrm{p}^X_K. \quad (8)$$

We use P^X_B and $P^X_{\mathrm{fail_N}}$ to get the reliability

$$R^X = (1 - P^X_B)(1 - (P^X_{\mathrm{fail_N}})^{\alpha+1}). \quad (9)$$

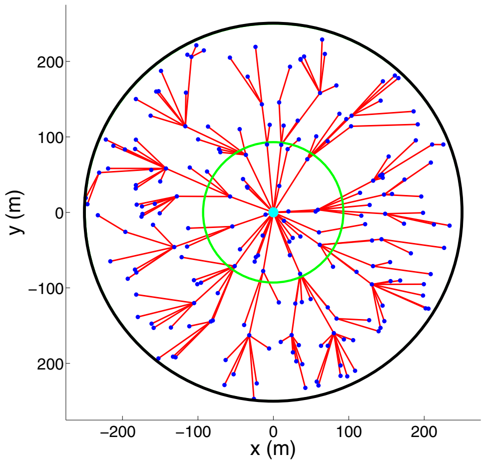


Fig. 1. The geographical network: a single trial of an example minimum-hop mesh network with N = 200 stations uniformly distributed within the deployment area bound by r_{dep} = 250 m (black circle). The routes from the stations (blue) to the access point (cyan) are shown in red for r_{cov} = 93 m (green circle). The corresponding set of hop probabilities is $P^{\text{hop}}_{\{1,2,3,4\}} = \{0.14, 0.33, 0.49, 0.04\}$ (H = 4) and the expected number of stations within the range of the AP is 28.

This is the probability that a frame that the MAC layer receives from the higher layers is not dropped because of a full buffer and that it is successfully transmitted either during the initial transmission attempt or during one of the maximum of α allowed retransmissions. Using the reliability, we get the average application throughput in bit/s, which is

$$S^X = \lambda^X L^X R^X, \tag{10}$$

where L^X is the number of application bits in a frame, so that $\lambda^X L^X$ is the offered application load in bit/s. Note that it is possible to achieve saturation (i.e. S^X remains constant as λ^X increases) if R^X decreases with increasing λ^X so that their product is a constant. Finally, the mean MAC-layer delay follows from Little's Law [15] and is

$$D^X = E\{\text{packetdelay}\} = l^X / \left[\lambda^X(1 - \mathrm{P}^X_{\mathrm{B}})\right], \tag{11}$$

where $l^X = \sum_{n=0}^{K} n\, p^X_n$ is the average number of frames in the node, including the one in service.

Using this mechanism for generating S^X, R^X, and D^X for a single hop, we will show how to build up expressions for throughput, reliability, and delay over multiple hops in a mesh network. This analysis uses the fact that less-than-perfect reliability results in the attrition of flows over a multi-hop link, so that the total flow is not merely the sum of the contributions of each relay.

IV. Mesh Network

In this section, we generalize the network to include stations lying within the circular area bounded by the deployment range $r_{\text{dep}} > r_{\text{cov}}$. Source stations outside of the AP's coverage range route to it through other relay stations in the network in a mesh topology. In our framework, we decompose the path from the source to the AP into a chain of hops and at each hop the relay contends with the other stations in its coverage area for transmission access. The contention at each hop is represented by our half-duplex MAC model.

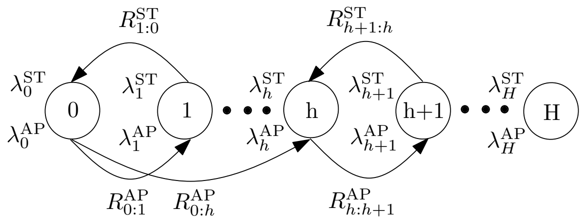


Fig. 2. The hop chain: the uplink and downlink traffic flows over the links between the hops.

A. Mesh routing topology

The mesh routing topology between the stations and the access point is determined by the distribution of the stations and the pairwise link metric for route selection. In this paper, we consider minimum-hop routing; specifically, if a station lies within the coverage range of another station, the stations are *neighbors* and the link metric is 1, otherwise the link metric is ∞. Then the minimum-hop routes can be computed through Dijkstra's shortest path algorithm [16]. If, as an alternative, the airtime link metric [17] is used, Dijkstra's algorithm yields the minimum-airtime routes. Other common link metrics are bit error and (negative) throughput.

In our framework, the routing topology is characterized by a set of *hop probabilities*. The hop probability P^{hop}_h is the probability that any network station routes to the access point through h hops. In the minimum-hop mesh network for uniformly distributed stations, there is no closed-form for the hop probabilities for $h > 2$ [18]. So we resort to Monte Carlo trials. The advantage of this method is that the hop probabilities can be calculated for any distribution of the stations and any set of parameters, including non-circular coverage areas when using directional antennas. In a single trial, the locations of the N stations are generated randomly and the minimum-hop routes to the AP are computed for each station. The hop probability P^{hop}_h is calculated by recording the number of stations routing to the AP through h hops and then dividing by N. The hop probability is refined by averaging the results over all the trials. The trials return P^{hop}_h, $h = 1 \ldots H$, where H is the maximum number of probable hops, i.e. for some threshold value $P^{\text{hop}}_h \approx 0$, $h > H$. Also, the probability that a station is disconnected from the AP is

$$P^{\text{hop}}_{\emptyset} = 1 - \sum_{h=1}^{H} P^{\text{hop}}_h. \tag{12}$$

A single trial of an example minimum-hop network for a given set of parameters is shown in Fig. 1. The PHY layer parameters in Table I were used to compute r_{cov}.

B. Mesh performance metrics

We can analyze the average performance of the mesh at each hop by collapsing the two-dimensional geographical network into a one-dimensional *hop chain* via the hop probabilities, as explained in the sequel. Fig. 2 depicts the uplink and downlink traffic flows over the links between the hops. The access point lies at $h = 0$.

1) Uplink traffic: The uplink *effective* arrival rate λ^{ST}_h of a relay station at hop h is defined as its individual station arrival rate λ^{ST} plus the uplink rate forwarded from the stations at $h+1$.

It represents the aggregate rate due to the traffic towards the AP. In recursive form, it can be written as

$$\lambda_h^{\mathrm{ST}} = \lambda^{\mathrm{ST}} + \lambda_{h+1}^{\mathrm{ST}} \left(\frac{P_{h+1}^{\mathrm{hop}}}{P_h^{\mathrm{hop}}} \right) R_{h+1:h}^{\mathrm{ST}}. \tag{13}$$

The second term on the right side of the equation is the forwarded rate $\lambda_{h+1}^{\mathrm{ST}}$ from a single station multiplied by the expected number of forwarding stations per relay expressed as a ratio of hop probabilities. The term is conditioned by the *link reliability* $R_{h+1:h}^{\mathrm{ST}}$, which is the probability that a forwarded message was received by the relay. Given $\lambda_H^{\mathrm{ST}} = \lambda^{\mathrm{ST}}$, λ_h^{ST} is computed through (13) in cascade from $h = H-1 \ldots 1$. Since the AP does not generate nor forward uplink traffic, $\lambda_0^{\mathrm{ST}} = 0$.

Consecutive relays on the route to the AP are subject to common contention from stations lying within the intersection of both their coverage areas; however, since we assume independent shadowing and fading between links, the contention affects the relays independently. Thus the link reliabilities on the route are also independent. Conditions favoring this assumption are a low or bursty packet arrival rate; also, a low station density makes for a less populated intersection area. Then the uplink reliability $R_{h:i}^{\mathrm{ST}}$ from a station at h to an uplink station at i can be expressed as a product of the link reliabilities between the two:

$$R_{h:i}^{\mathrm{ST}} = \prod_{j=i}^{h-1} R_{j+1:j}^{\mathrm{ST}}. \tag{14}$$

It follows from (10) that the uplink throughput is the station arrival rate multiplied by the station frame size L^{ST} and the uplink reliability:

$$S_{h:i}^{\mathrm{ST}} = \lambda^{\mathrm{ST}} L^{\mathrm{ST}} R_{h:i}^{\mathrm{ST}}. \tag{15}$$

The uplink delay $D_{h:i}^{\mathrm{ST}}$ from a station at h to an uplink station at i can be expressed recursively by decomposing it into the delay from h to $i+1$ plus the delay from $i+1$ to i:

$$D_{h:i}^{\mathrm{ST}} = D_{h:i+1}^{\mathrm{ST}} + D_{i+1:i}^{\mathrm{ST}} R_{h:i+1}^{\mathrm{ST}} (1 - P_{B,i+1:i}^{\mathrm{ST}}). \tag{16}$$

The delay $D_{i+1:i}^{\mathrm{ST}}$ in the second term is conditioned by $R_{h:i+1}^{\mathrm{ST}}$, the probability that $i+1$ received the frame from h. It is also conditioned by the probability that the station at $i+1$ did not drop the message due to a full buffer, where $P_{B,i+1:i}^{\mathrm{ST}}$ is the blocking probability. If either of those two events occur, the second term is null and there is no incremental delay to the next hop incurred by a frame originating at h.

2) Downlink traffic: The downlink effective arrival rate

$$\lambda_h^{\mathrm{AP}} = \frac{\lambda^{\mathrm{AP}}}{N} \left(\frac{\sum_{i=h+1}^{H} P_i^{\mathrm{hop}}}{P_h^{\mathrm{hop}}} \right) R_{0:h}^{\mathrm{AP}} \tag{17}$$

has a similar format to (13), except that on the downlink the relays do not generate their own frames; rather, they simply forward those sent from the AP, where λ^{AP}/N is the AP arrival rate per unit sink station. The ratio of hop probabilities accounts for the total number of downlink stations ($i = h+1 \ldots H$) to which the relay must individually forward frames. The rate is conditioned by the probability $R_{0:h}^{\mathrm{AP}}$ that the frame was received by the relay from the AP. The aggregate AP arrival rate over all the sink stations is $\lambda_0^{\mathrm{AP}} = \lambda^{\mathrm{AP}}$ and since stations at hop H do not generate nor forward downlink traffic, $\lambda_H^{\mathrm{AP}} = 0$.

Similar to (14), the downlink reliability from a station at h to a downlink station at i can be expressed as a product of the link reliabilities between the two

$$R_{h:i}^{\mathrm{AP}} = \prod_{j=h}^{i-1} R_{j:j+1}^{\mathrm{AP}}, \tag{18}$$

and the downlink throughput follows as

$$S_{h:i}^{\mathrm{AP}} = \lambda^{\mathrm{AP}} L^{\mathrm{AP}} R_{h:i}^{\mathrm{AP}}. \tag{19}$$

The downlink delay can be computed recursively as

$$D_{h:i}^{\mathrm{AP}} = D_{h:i-1}^{\mathrm{AP}} + D_{i-1:i}^{\mathrm{AP}} R_{h:i-1}^{\mathrm{AP}} (1 - P_{B,i-1:i}^{\mathrm{AP}}). \tag{20}$$

Finally, the network average $\bar{\zeta}$ of any performance metric ζ_h over all hops is provided from the hop probabilities as

$$\bar{\zeta} = \zeta_{\emptyset} P_{\emptyset}^{\mathrm{hop}} + \sum_{h=1}^{H} \zeta_h P_h^{\mathrm{hop}}. \tag{21}$$

In this paper, we assume that the reliability, throughput, and delay of a disconnected station is zero, so the associated performance metric $\zeta_{\emptyset}$ is also zero.

Note that our framework is general enough to support different routing schemes. For example, as opposed to unicasting on the downlink, if the access point broadcasts instead, this changes the ratio of hop probabilities in (17) to 1. Or, the framework can support multiple access points by indexing the hop probabilities accordingly and adding respective terms to the equations for the uplink and downlink effective arrival rates.

C. Hop contention

In the single-hop network, our MAC model for contention between the AP and the N stations performs a mapping of a set of input parameters to a set of output metrics[1], which we denote as $C\{(N, \lambda^{\mathrm{ST}}); \lambda^{\mathrm{AP}}\} \rightarrow \{(R^{\mathrm{ST}}, R^{\mathrm{AP}}); (D^{\mathrm{ST}}, D^{\mathrm{AP}}); (P_B^{\mathrm{ST}}, P_B^{\mathrm{AP}})\}$. In the mesh network, under the previously stated assumption of independent link reliabilities, the same MAC model can be applied separately at each hop between the relay at h and the other stations within its coverage range; it follows that the input parameters and the output metrics are then indexed according to the hop index h.

The first of the three input parameters (substituting for λ^{AP}) is the effective arrival rate of the relay at h given as

$$\lambda_h = \lambda_h^{\mathrm{ST}} + \lambda_h^{\mathrm{AP}}. \tag{22}$$

Observe that, as opposed to the AP, the relay must account for the combined traffic from both the uplink and the downlink. The second input parameter (substituting for N) is the average number of stations N_h within the relay's range for which it will back off if it detects a transmission. For large deployment areas, i.e. $r_{\mathrm{dep}} \gg r_{\mathrm{cov}}$, N_h is constant at lower hops; however, it drops off at higher hops due to edge effects when the coverage area extends beyond the deployment area. The third input parameter (substituting for λ^{ST}) is the effective arrival rate of the N_h stations. Recall that in the single-hop network, the N stations all lie at $h = 1$ and so have the same effective rate λ^{ST}; however, in the minimum-hop mesh network, the N_h stations can lie at any of three hops $i = h-1 \ldots h+1$ according to [18]. By denoting $P_{i|h}^{\mathrm{hop}}$ as the

[1]Here we list only the input parameters and output metrics which pertain directly to the mesh network.

conditional probability that a relay at h has a station at i within its range, rather we use the *average* effective rate over the three hops:

$$\tilde{\lambda}_h = \sum_{i=h-1}^{h+1} \lambda_i P^{\text{hop}}_{i|h}. \tag{23}$$

The number N_h and the set of conditional hop probabilities can be computed during the Monte Carlo trials. Then the three input parameters completely characterize the contention at h with mapping $C_h\{(N_h, \tilde{\lambda}_h); \lambda_h\} \rightarrow \{(R^{\text{ST}}_{h+1:h}, R^{\text{AP}}_{h:h+1}); (D^{\text{ST}}_{h+1:h}, D^{\text{AP}}_{h:h+1}); (P^{\text{ST}}_{B,h+1:h}, P^{\text{AP}}_{B,h:h+1})\}$.

It is important to note that the input parameter $\lambda_h = \lambda^{\text{ST}}_h + \lambda^{\text{AP}}_h$ to the contention at h depends on the output metrics from all contentions. By the same token, the link reliability output metrics ($R^{\text{ST}}_{h+1:h}$,$R^{\text{AP}}_{h:h+1}$) affect the input parameters to all contentions. This can be seen through the recursive equation (13), where the uplink term λ^{ST}_h is actually a function of the output metric $R^{\text{ST}}_{h+1:h}$ from the same contention at h and not just of those cascaded from higher hops; also, the downlink term λ^{AP}_h in (17) is a function of the link reliability output metrics from all lower hops via $R^{\text{AP}}_{0:h}$. This means that the input parameters and the output metrics at all hops are interdependent and so must be computed collectively. To this end, we minimize the convex[2] objective function

$$\sum_{h=0}^{H-1} |R^{\text{ST,k+1}}_{h+1:h} - R^{\text{ST,k}}_{h+1:h}| + |R^{\text{AP,k+1}}_{h:h+1} - R^{\text{AP,k}}_{h:h+1}| \tag{24}$$

in the $2H$-dimensional space through a numerical method, where k denotes the iteration index. The first step is to initialize all the link reliabilities arbitrarily to 1. Then at iteration k of the method, $(R^{\text{ST,k}}_{h+1:h}, R^{\text{AP,k}}_{h:h+1})$ are used to compute $(\lambda^{\text{ST,k}}_h, \lambda^{\text{AP,k}}_h)$ through (13) and (17). The input parameters $\tilde{\lambda}^k_h$ and λ^k_h to the contention at h can then be computed through (22) and (23), mapping to new link reliabilities $(R^{\text{ST,k+1}}_{h+1:h}, R^{\text{AP,k+1}}_{h:h+1})$. At iteration $k+1$, if the new link reliabilities lower the objective function, they are accepted; otherwise, the bisection method [19] is used to find new link reliabilities which do so. The iterations continue until the objective function is minimized to within a desired tolerance. The final values of the uplink and downlink effective rates are provided from the final link reliabilities.

Recall that we used a modified MAC model that extended the single class $X \in \{\text{ST}\}$ of transmitting entities to two classes as $X \in \{\text{ST}; \text{AP}\}$ so as to include a transmitting access point as well. For a more precise analysis of our mesh network, our half-duplex MAC model can be trivially modified in the same manner to distinctively include all classes of stations that lie within the range of a relay at h as $X \in \{i|^{h+1}_{i=h-1}; h\}$. Then the contention at h between the relay and the stations maps as $C_h\{(N_{i|h}, \lambda_i)\,|^{h+1}_{i=h-1}; \lambda_h\} \rightarrow \{(R^{\text{ST}}_{i:h}, R^{\text{AP}}_{h:i}), (D^{\text{ST}}_{i:h}, D^{\text{AP}}_{h:i}), (P^{\text{ST}}_{B,i:h}, P^{\text{AP}}_{B,h:i})\}\,|^{h+1}_{i=h-1}$, where $N_{i|h}$ is the expected number of stations at i given h and can be rounded to $N_{i|h} = [P^{\text{hop}}_{i|h} \cdot N_h]$. As a result, the returned values $(R^{\text{ST}}_{h+1:h}, R^{\text{AP}}_{h:h+1})$ in (13) and (17) are then specific to the stations at $h+1$ rather than average values over all the stations in the range, as when using the average effective rate $\tilde{\lambda}_h$.

Finally, one should keep in mind the three non-overlapping channels of 802.11 in a practical mesh implementation. This feature would be particularly beneficial in enhancing spectral efficiency. By staggering the channels between hops, any sequence of three hops operates on different channels, effectively reducing the average number of contending stations within the coverage range of a relay.

[2]Due to space constraints, the proof is not provided here.

TABLE I

SELECT STATION AND AP PARAMETERS FOR THE EXAMPLE NETWORK

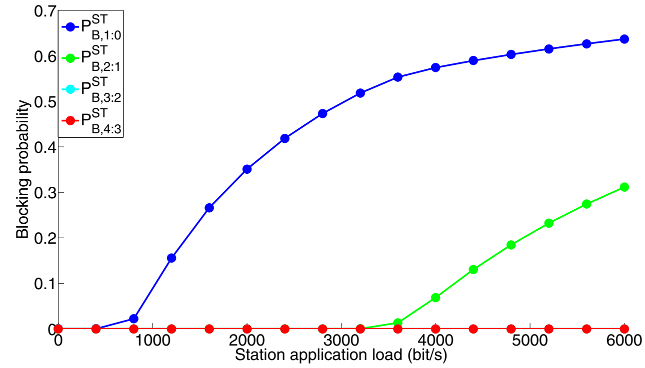
PHY		MAC	
$EIRP$	20 dBm	W_0	32
G_r	5 dB	K	51
$(E_b/N_0)_{cov}$	10.4 dB	M	5
n	3	α	7
σ	8	$L^{\text{ST}}, L^{\text{AP}}$	10 Kbit/frame
m	1	$L_{overhead}$	384 bit/frame
f_c	2.4 GHz	802.11 mode	{'b', 1Mbit/s}
BW	11 MHz	MAC scheme	RTS/CTS

V. NUMERICAL RESULTS

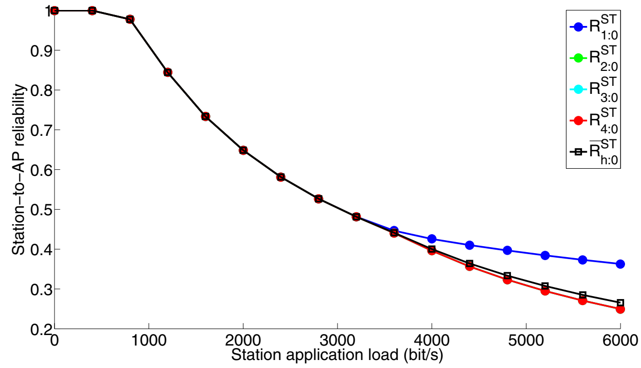
This section presents the mesh performance metrics generated for an example minimum-hop network. A single trial of the network is illustrated in Fig. 1. Table I provides select PHY and MAC parameters for the stations and the AP set equally. Fig. 3 displays the uplink performance metrics versus the station application load $\lambda^{\text{ST}} L^{\text{ST}}$. The values on the abscissa corresponds to the station arrival rate λ^{ST} varying from 0 to 0.6 frame/s while the AP arrival rate was fixed at λ^{AP} = 20 frame/s (λ^{AP}/N = 0.1 frame/s). Fig. 3(a) shows the uplink blocking probabilities of the stations labeled according to hop index. Before 400 bit/s, all the stations route to the AP with no congestion; however, after this first breakpoint, the stations at h = 1 begin to drop frames, as indicated by $P^{\text{ST}}_{B,1:0}$'s increase from zero. These stations break down first because they have the greatest effective arrival rate due to forwarded traffic to and from all stations at higher hops. At about 3200 bit/s, the stations at h = 2 also begin to break down whereas for this load range the stations at h = 3 and 4 maintain zero blocking probabilities. Note that the dropped frames at h = 2 alleviate the load on the stations at h = 1, which continue to drop more frames with additional station application load, but at a reduced rate of increase.

The same two breakpoints can be observed in Fig. 3(b) illustrating the station-to-AP reliabilities. Before 400 bit/s, they are all equal to one; thereafter, the breakdown at h = 1 results in a drop-off of $R^{\text{ST}}_{1:0}$. Even though the other link reliabilities $R^{\text{ST}}_{h+1:h}$, $h = 1 \ldots 3$ remain at one, indicating no congestion at higher hops, the respective station-to-AP reliabilities $R^{\text{ST}}_{h:0}$, which are products of $R^{\text{ST}}_{1:0}$, follow suit until the next breakpoint. At 3200 bit/s, $R^{\text{ST}}_{2:1}$ then also drops off from one, pulling down $R^{\text{ST}}_{2:0} = R^{\text{ST}}_{2:1} R^{\text{ST}}_{1:0}$ yet faster. Again, since the reliabilities $R^{\text{ST}}_{3:0}$ and $R^{\text{ST}}_{4:0}$ are products of $R^{\text{ST}}_{2:0}$, they follow suit. In addition, note that $R^{\text{ST}}_{1:0}$ continues to decrease, but at a lower rate since the forwarded load from h = 2 is alleviated. Also plotted here is the average reliability over the hops $\bar{R}^{\text{ST}}_{h:0}$. Fig. 3(c) shows the station-to-AP throughput given through (19), which exhibits the same behaviors as in Fig. 3(b).

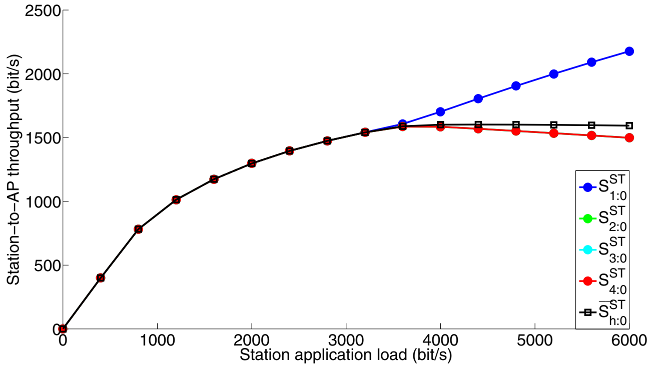
Finally, Fig. 3(d) displays the station-to-AP delays. Before the first breakpoint, the values increase with load, with $D^{\text{ST}}_{h:0} < D^{\text{ST}}_{h+1:0}$, as expected (see zoom); thereafter, contention builds up at the stations at h = 1, increasing their MAC service time; by 1200 bit/s, they start dropping their own frames, including those forwarded from higher hops. When a frame is dropped, it carries no additional delay onto lower hops; this explains why the delays



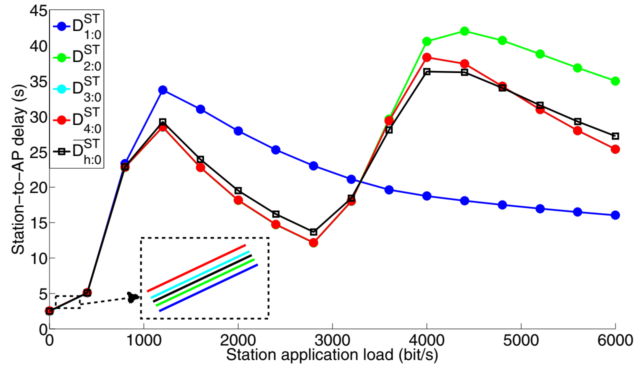
(a) Blocking probability



(b) Station-to-AP reliability



(c) Station-to-AP throughput



(d) Station-to-AP delay

Fig. 3. Uplink performance metrics versus station application load

then start to decrease with additional load and why the stations at higher hops then have shorter delays. At the second breakpoint, contention builds up at the stations at $h = 2$ as well, making for a sharp rise at $h = 2 \ldots 4$ due to increased service time, followed by a descent due to dropped frames; this alleviates the load on the stations at $h = 1$ whose delay continues to decrease, but at a lower rate.

The downlink performance metrics exhibit the same trends as those for the uplink.

VI. Conclusions

In this paper, we have described an analytical framework to compute end-to-end performance metrics such as throughput and delay for bi-directional traffic between N transmitting stations and a transmitting access point routing through a minimum-hop mesh network. Numerical results were generated for an example network to highlight the nuances of the framework. Along the way, we described extensions to include other routing topologies, broadcast, multiple access points, and multiple channels.

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