

# Throughput Improvement of STS-Based MC DS-CDMA System with Variable Spreading Factor

V. Vakil\* and H. Aghaeinia\*

**Abstract:** The throughput enhancement of Space-Time Spreading (STS)-based Multicarrier Direct Sequence Code Division Multiple Access (MC DS-CDMA) system is investigated in this paper. Variable Spreading Factor (VSF) is utilized to improve the data throughput of the system. In this contribution, an analytical approach is proposed to compute a new expression for the Bit Error Rate (BER) performance of the STS-based MC DS-CDMA system against pre-dispreading Signal-to-Noise Ratio (SNR) for different values of spreading factor (SF). The other contribution of the paper is deriving a new closed form expression for computing the throughput enhancement and the BER performance of the VSF STS-based MC DS-CDMA system over Rayleigh fading channel. It is demonstrated that using VSF method in STS-based MC DS-CDMA system improves the throughput of the system by keeping the BER performance at the target level.

**Keywords:** Frequency diversity, MC DS-CDMA, Space-Time Spreading, Spreading Factor, Throughput, Variable.

## 1 Introduction

Recent studies in wireless communication attempt to increase the capacity, data rate and performance of the systems. Diversity is a technique that provides several replicas of the transmitted signal in independent fading channels. In this paper, space diversity and frequency diversity are considered for CDMA system. The Space-Time Spreading method [1] is used as the transmit diversity technique for MC DS-CDMA system. In MC DS-CDMA system, multicarrier modulation is combined with time domain spreading, where DS spreading is performed to spread subcarriers' signals [2-6]. In [2] the BER performance of the multiantenna MC DS-CDMA system is investigated, when communicating over correlated time-selective Rayleigh fading channels. [3] introduces a multicarrier modulation scheme for wideband CDMA systems that performs the spreading in the time and frequency domain simultaneously. In [4-6] MC DS-CDMA using STS-assisted transmit diversity is investigated in the context of broadband communications over frequency-selective Rayleigh-fading channels. Multicarrier

modulation makes it possible to achieve frequency diversity through performing S-depth interleaving.

In addition, variable rate transmission techniques have been proposed to improve the performance of wireless systems in supporting several services. Adaptive rate transmission is a technique in which the information rate varies according to the quality of the channel [7-12]. In [7] the multi processing-gain, multi-channel, multi-modulation and multi-chip rate schemes are studied and compared. The error performances of multicode and variable spreading gain schemes are analyzed and compared in W-CDMA in [8]. In [9], a generic performance analysis for the multicode and variable spreading factor multirate schemes in MC-CDMA is presented. In [10, 11] adaptive rate transmissions are investigated in the context of CDMA system using variable spreading factors (VSF). In these works, the transmission rate is adapted in response to the multiuser interference (MUI) level. In [12] the throughput maximization problem for CDMA uplink is formulated as an optimization problem in terms of the spreading gain and transmit power of the users, and is solved using a nonlinear programming approach.

Using diversity techniques results in increasing the capacity of CDMA system and applying adaptive rate transmission to this system enhances the data throughput of each user. Using both techniques, improves the performance of CDMA system in

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\* The Authors are with the Department of Electrical Engineering, Amirkabir University of Technology, Tehran, Iran.

E-mails: [vakil@aut.ac.ir](mailto:vakil@aut.ac.ir), [aghaeini@aut.ac.ir](mailto:aghaeini@aut.ac.ir).

supporting high data services [13-21]. In [13] a closed form expression of the average BER is derived for a Rake receiver based adaptive-modulation scheme over frequency selective slow Rayleigh fading channels. In [14], the average bit error rate and the average throughput for adaptive modulation/transmit diversity combined schemes are derived for the CDMA system over frequency-selective Rayleigh fading channels. The average throughput of W-CDMA systems with Multiple-input-multiple-output and adaptive modulation and coding is evaluated in [15] by simulation and theoretical analysis.

In our previous works, STS as the space-time diversity technique is used for CDMA system with adaptive rate transmission techniques, in different fading channels [16-21]. The throughput improvement of Adaptive Modulated STS-based CDMA system is investigated in [16-18]. Variable spreading factor is used to improve the throughput of the same system in [19, 20]. In [21], the combination of these adaptive rate methods is proposed for throughput enhancing of the STS-based CDMA system.

Future mobile radio systems require multirate transmission capability to efficiently support various services with wide range of data rates. Our contribution in this paper is using Variable Spreading Factor, as an efficient adaptive rate technique in combination with space-time diversity and frequency diversity to increase the throughput and performance of the system. Furthermore, from the BER curves of the STS-based MC DS-CDMA system versus pre-dispreading SNR some critical thresholds are obtained which are used in rate adaptation procedure. Hence, an expression is proposed for calculating the BER performance of the system for different values of spreading factor. The other contribution of the paper is deriving a new closed form expression for computing the throughput enhancement and the BER performance of VSF STS-based MC DS-CDMA system.

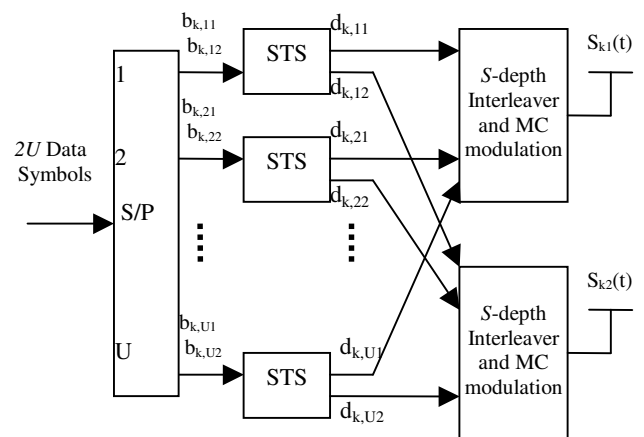
The rest of paper is organized as follows. Section 2 describes the applied STS scheme for data transmission over frequency selective Rayleigh fading channel. In section 3, Variable Spreading Factor method is described. The BER performance of STS-based MC DS-CDMA system as well as the average pre-dispreading SNR of the system is calculated in section 4. Then in section 5, an analytical approach is proposed to compute the throughput and BER performance of VSF system. Section 6 contains numerical results and discussions. Finally, section 7 concludes the paper.

## 2 Data Transmission in STS-based MC DS-CDMA System

The system considered in this paper is an orthogonal MC DS-CDMA scheme [2, 4-6] using U.S number of subcarriers, two transmitter antennas and one receiver antenna. The transmitter schematic due to  $k^{\text{th}}$  user is shown in Fig. 1. As shown in this figure, a block of  $2U$  data symbols is serial-to-parallel converted to  $U$  parallel sub-blocks. The symbols in each sub-block are space-time spread using the STS method of [1]. Each STS block generates 2 parallel signals to be mapped to the 2 transmitter antennas. The  $U$  STS output signals which will be transmitted using the same antenna are then interleaved by an  $S$ -depth interleaver, so that each STS signal is transmitted on  $S$  subcarriers. The interleaver guarantees that the same STS signal is transmitted by the specific  $S$  subcarriers having the maximum possible frequency spacing, so that they experience independent fading and hence achieve maximum frequency diversity. Finally as shown in the figure, the Inverse Fast Fourier Transform (IFFT) is invoked to carry out multicarrier modulation, and each IFFT block's output signal is transmitted through one of the transmitter antennas. The  $k^{\text{th}}$  user's transmitted signal is:

$$\underline{S}_k(t) = \sqrt{\frac{P}{2US}} \sum_{u=1}^U \sum_{s=1}^S \begin{bmatrix} b_{k,u1} c_{k,1} + b_{k,u2}^* c_{k,2} \\ b_{k,u2} c_{k,1} - b_{k,u1}^* c_{k,2} \end{bmatrix} \times \exp(j2\pi x_{(s-1)U+u} t) e^{j\theta_k} \quad (1)$$

where  $b_{k,u1}$  and  $b_{k,u2}$  are the data symbols in the  $u^{\text{th}}$  sub-block due to  $k^{\text{th}}$  user, and  $c_{k,1}$ ,  $c_{k,2}$  are the  $k^{\text{th}}$  user's  $2N \times 1$  orthogonal unit-norm spreading codes. One possible choice of the code-vectors  $\underline{c}_{k,1}$  and  $\underline{c}_{k,2}$  is:



**Fig. 1.** The transmitter schematic of the STS-based MC DS-CDMA

$$\begin{aligned} \underline{c}_{k,1} &= [\underline{c}_{k,0}^T \quad \underline{c}_{k,0}^T]^T \\ \underline{c}_{k,2} &= [\underline{c}_{k,0}^T \quad -\underline{c}_{k,0}^T]^T \end{aligned} \quad (2)$$

where  $\underline{c}_{k,0}$  is an  $N \times 1$  spreading code, specified to  $k^{\text{th}}$  user.  $P$  is the transmitted power and  $\theta_k$  is the carrier phase containing modulation phase.

The channel is assumed to be slowly varying frequency-selective fading, which can be modeled as a tapped delay line as represented in [22]. The channel model between the  $i^{\text{th}}$  transmitter antenna and the  $k^{\text{th}}$  user's receiver antenna is given in Fig. 2. The MC DS-CDMA system is designed such that the U.S subcarrier signals each experiences flat fading and the subcarrier signals conveying the same symbol will experience independent fading [4-6].

The subchannel response due to subcarrier specified with frequency  $f_{(s-1)U+u}$  is:

$$\begin{aligned} h_{(us)i,k}(t) &= h_{(us)i,k} \exp(j\xi_{(us)i,k}) \\ u &= 1, 2, \dots, U \quad s = 1, 2, \dots, S \end{aligned} \quad (3)$$

where  $h_{(us)i,k}$  and  $\xi_{(us)i,k}$  are the amplitude and phase of each sub-channel, respectively. The coefficients  $\{h_{(us)i,k}\}$  are independent i.i.d. random variables obeying the Rayleigh distribution with probability density function as equation (4).

$$f_{\alpha}(\alpha) = \frac{\alpha}{\alpha_0^2} e^{-\alpha^2/(2\alpha_0^2)} \quad (4)$$

The phase variables  $\{\xi_{(us)i,k}\}$  are i.i.d. random variables with uniform distribution in  $[0, 2\pi)$ .

The receiver schematic of the  $k^{\text{th}}$  user is shown in Fig. 3. If the total number of  $K$  user signals is transmitted synchronously over Rayleigh fading channels, the received signal at the given mobile station can be expressed as:

$$\begin{aligned} R(t) &= \sqrt{\frac{P}{2US}} \sum_{k=1}^K \sum_{u=1}^U \sum_{s=1}^S [(b_{k,u1} c_{k,1} + b_{k,u2}^* c_{k,2}) \\ &\times h_{(us)1,k} \exp(j\psi_{(us)1,k}) + (b_{k,u2} c_{k,1} - b_{k,u1}^* c_{k,2}) \\ &\times h_{(us)2,k} \exp(j\psi_{(us)2,k})] \exp(j2\pi f_{(s-1)U+u} t) \\ &+ N(t) \end{aligned} \quad (5)$$

in which  $\psi_{(us)i,k} = \xi_{(us)i,k} + \theta_{(us)i,k}$  is uniformly distributed in  $[0, 2\pi)$  [23].

Let the first user be the user-of-interest, the subscript of the reference user is omitted for notational convenience. We also change the subscripts  $u, s$  for  $u=1, 2, \dots, U$  and  $s=1, 2, \dots, S$  to  $u'=1, 2, \dots, US$ . At the receiver side, in order to detect the transmitted data symbol, the FFT based multicarrier demodulation and Space-Time dispreading should be performed. Equation (6) expresses this operation as:

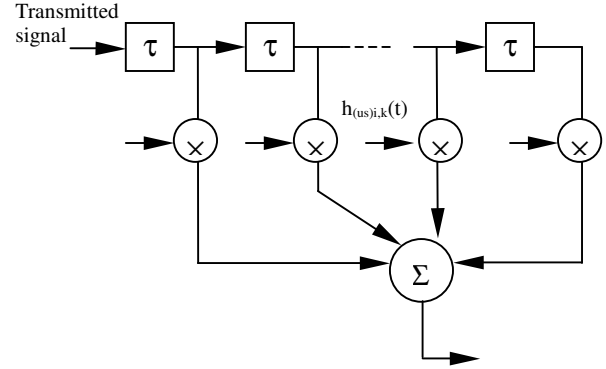


Fig. 2. The tapped delay-line model of frequency-selective channel

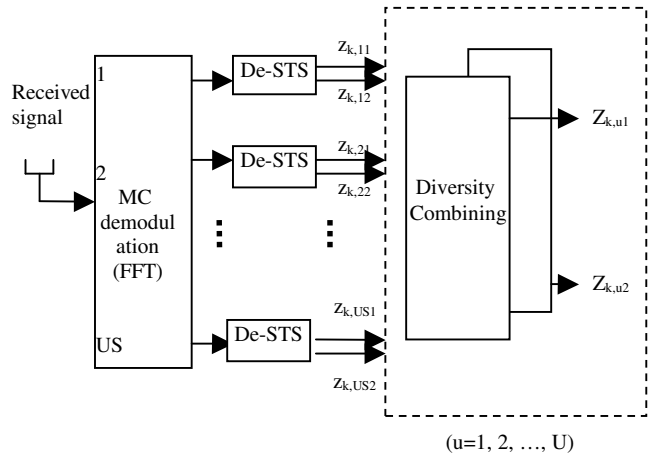


Fig. 3. The receiver schematic of the STS-based MC DS-CDMA

$$d_{u'1} = \int_0^{2UT_s} R(t) c_i(t) \exp(-j2\pi f_{u'}) dt \quad (6)$$

where  $T_s$  is the symbol duration of transmitted symbols. When substituting (5) into above equation, it can be shown that:

$$\begin{aligned} d_{u'1} &= \sqrt{\frac{2UP}{S}} T_s [b_{u'1} h_{u'1} \exp(j\psi_{u'1}) \\ &+ b_{u'2} h_{u'2} \exp(j\psi_{u'2})] + J_{u'1} \\ d_{u'2} &= \sqrt{\frac{2UP}{S}} T_s [b_{u'2}^* h_{u'1} \exp(j\psi_{u'1}) \\ &- b_{u'1}^* h_{u'2} \exp(j\psi_{u'2})] + J_{u'2} \end{aligned} \quad (7)$$

where  $\mathbf{J}_{u'i}$  contains all type of interference signals. By assuming orthogonal multicarrier signals, orthogonal STS codes, synchronous transmission of all users and slow flat fading of each subcarrier, no other interference signal is considered except Gaussian noise. Therefore, regarding the approach proposed in [1], if we define:

$$\underline{\mathbf{d}}_{u'} = \begin{bmatrix} \mathbf{d}_{u'1} \\ \mathbf{d}_{u'2}^* \end{bmatrix} \text{ and } \mathbf{H}_{u'} = \begin{bmatrix} \mathbf{h}_{u'1} e^{j\psi_{u'1}} & \mathbf{h}_{u'2} e^{j\psi_{u'2}} \\ -\mathbf{h}_{u'2}^* e^{-j\psi_{u'2}} & \mathbf{h}_{u'1}^* e^{-j\psi_{u'1}} \end{bmatrix}, \text{ the}$$

dispread signal of each subcarrier can be expressed as:

$$\underline{\mathbf{d}}_{u'} = \sqrt{\frac{2UP}{S}} \mathbf{T}_S \mathbf{H}_{u'} \underline{\mathbf{b}} + \underline{\mathbf{N}}_{u'} \quad (8)$$

where  $\underline{\mathbf{N}}_{u'} = [\mathbf{N}_{u'1} \quad \mathbf{N}_{u'2}^*]^T$  is the noise vector and:

$$\mathbf{N}_{u,i}(t) = \int_0^{2UT_s} \mathbf{N}(t) \mathbf{c}_{1,i}(t) \exp(-j2\pi f_u t) dt \quad (9)$$

It is a complex Gaussian distributed variable having zero mean and a variance of  $4UN_0T_s$ .

Regarding the method proposed in [1] for detecting complex data symbols, by multiplying equation (8) by  $\mathbf{H}_{u'}^H$  the decision variable  $\underline{\mathbf{z}}_{u'}$  is calculated as equation (10), in which  $\mathbf{H}_{u'}^H$  is the complex conjugate transpose of  $\mathbf{H}_{u'}$ .

$$\underline{\mathbf{z}}_{u'} = \begin{bmatrix} \mathbf{z}_{u'1} \\ \mathbf{z}_{u'2} \end{bmatrix} = \sqrt{\frac{2UP}{S}} \mathbf{T}_S \mathbf{H}_{u'}^H \mathbf{H}_{u'} \underline{\mathbf{b}} + \mathbf{H}_{u'}^H \underline{\mathbf{N}}_{u'} \quad (10)$$

Letting  $\underline{\mathbf{h}}_{u'i}$  denote the  $i^{\text{th}}$  column of matrix  $\mathbf{H}_{u'}$ ,  $\mathbf{z}_{u'i}$  can be formulated as:

$$\mathbf{z}_{u'i} = \sqrt{\frac{2UP}{S}} \mathbf{T}_S \mathbf{b}_i \sum_{i=1}^2 |\mathbf{h}_{u'i}|^2 + \underline{\mathbf{h}}_{u'i}^H \underline{\mathbf{N}}_{u'} \quad (11)$$

After combining the replicas of the same signal transmitted on  $S$  subcarriers, the decision variable for detecting data sequence at the receiver is:

$$\mathbf{Z}_{ui} = \sum_{s=1}^S \mathbf{z}_{((s-1)U+u)i} \quad ; \quad u = 1, 2, \dots, U \quad (12)$$

### 3 Variable Spreading Factor

Variable Spreading Factor is a technique used for increasing the average throughput of the system. Regarding the quality of channel, it uses several spreading codes with different spreading factors [6, 10-12, 19, 20]. In VSF system, the chip rate of the CDMA users is kept constant throughout the transmission, while bit rate is varied by using spreading codes with different Spreading Factors (SF). When the channel quality is high, a low SF is used in order to increase the throughput and conversely, when the channel is hostile, a high SF is employed for maintaining a target BER performance.

In all adaptive rate transmission techniques, a metric corresponding to near-instantaneous channel quality is required. When the instantaneous channel gain is known, the transmitter can adapt the rate and/or power relative to the instantaneous channel state. The quality metric is compared with the pre-determined thresholds. The transmitter utilizes the result of this comparison to choose the appropriate spreading code for the next symbol.

In this paper, we use the instantaneous output SNR ( $\gamma$ ) as the adaptation metric. When BER curves versus output SNR are plotted for different SFs, it is observed that the curves are completely overlapped. Hence, the required thresholds cannot be distinguished. This phenomenon is mentioned in [6] with the explanation that SNR varies proportionally to SF. Therefore, similar to [6, 12, 19, 20] we use the BER curves versus pre-dispreading SNR to determine the switching thresholds. Pre-dispreading SNR ( $\gamma_p$ ) is the SNR before despreading the received sequence. Despreading leads to noise power reduction by the factor of SF, so  $\gamma_p$  is related to  $\gamma$  via  $\gamma = \text{SF} \times \gamma_p$ .

### 4 BER Analysis of STS-based MC DS-CDMA System

The first important step in improving the throughput of the system is to determine the thresholds by which the selected metric would be compared. Therefore, the BER performance of the STS-based MC DS-CDMA system against pre-dispreading SNR should be initially computed for different values of SF. In order to compute the BER performance of a system in fading channels, it should be considered that fading impacts are reflected in the variations of the instantaneous SNR. The average probability of error in terms of pre-dispreading SNR is then calculated as:

$$P_b = \int_0^{\infty} P(\gamma_p) f'_{\gamma_p}(\gamma_p) d\gamma_p \quad (13),$$

in which  $P(\gamma_p)$  is the probability of error of the performed modulation in AWGN channel.  $\gamma_p$  is the pre-dispreading SNR per bit and  $f'_{\gamma_p}(\gamma_p)$  is the probability density function of  $\gamma_p$ .

Therefore, to analyze the STS-based MC DS-CDMA system  $f'_{\gamma_p}(\gamma_p)$  should be computed. In the same method as proposed in [6] and based on Gaussian estimation stated in [24], the instantaneous output SNR is calculated as:

$$\gamma = \frac{E[\mathbf{Z}_{ui}]^2}{\text{var}[\mathbf{Z}_{ui}]} \quad (14)$$

for a given set of complex channel transfer factor estimates. The instantaneous pre-dispreading SNR then could be calculated as

$\gamma_p = \frac{1}{SF} \gamma$ . Also,  $Z_{ui}$  has a mean value given by:

$$E[Z_{ui}]^2 = \frac{2U}{S} E_s T_s \left( \sum_{s=1}^S \sum_{i=1}^2 |h_{((s-1)U+u)i}|^2 \right)^2 \quad (15)$$

where  $E_s = PT_s$  is the symbol energy. The variance of  $Z_{ui}$  can be written as:

$$\begin{aligned} \text{var}[Z_{ui}] &= E \left[ \left( \sum_{s=1}^S \underline{h}_{((s-1)U+u)i}^H \underline{N}_{((s-1)U+u)i} \right)^2 \right] \\ &\times \left( \sum_{s=1}^S \underline{h}_{((s-1)U+u)i}^H \underline{N}_{((s-1)U+u)i} \right)^* \end{aligned} \quad (16)$$

Based on the assumption that the interference imposed by the AWGN constitutes independent random variables, the above equation can be expressed as:

$$\begin{aligned} \text{var}[Z_{ui}] &= \sum_{s=1}^S \sum_{i=1}^2 |h_{((s-1)U+u)i}|^2 \text{var}[N_{((s-1)U+u)i}] \\ &= 4UN_0 T_s \sum_{s=1}^S \sum_{i=1}^2 |h_{((s-1)U+u)i}|^2 \end{aligned} \quad (17)$$

The ratio  $\frac{E[Z_{ui}]^2}{\text{var}[Z_{ui}]}$  for QPSK data symbol is the output SNR per symbol, which can now be expressed as:

$$\gamma_{\text{perSym}} = \frac{E_s}{2SN_0} \sum_{s=1}^S \sum_{i=1}^2 |h_{((s-1)U+u)i}|^2 \quad (18)$$

The output SNR per bit is given as  $\gamma = \frac{\gamma_{\text{perSym}}}{\log_2(M)}$  after

replacing  $\frac{E_s}{N_0}$  by  $\log_2(M) \frac{E_b}{N_0}$  in equation (18). M is 4

for QPSK modulation. Defining  $\bar{\gamma}_{si} = \frac{\Omega E_b}{2S N_0}$  and

$\gamma_{si} = \bar{\gamma}_{si} \frac{h_{((s-1)U+u)i}^2}{\Omega}$  where  $\Omega = E[h_{((s-1)U+u)i}^2]$ , the

instantaneous SNR per bit is expressed as  $\gamma = \sum_{s=1}^S \sum_{i=1}^2 \gamma_{si}$ .

Hence, the instantaneous pre-dispreading SNR per bit can be computed as  $\gamma_p = \sum_{s=1}^S \sum_{i=1}^2 \gamma_{psi}$  in which

$\gamma_{psi} = \bar{\gamma}_{psi} \frac{h_{((s-1)U+u)i}^2}{\Omega}$  and  $\bar{\gamma}_{psi} = \frac{1}{SF} \bar{\gamma}_{si}$ . Therefore, the average output SNR can be calculated as:

$$\bar{\gamma} = \sum_{s=1}^S \sum_{i=1}^2 \bar{\gamma}_{si} = 2S \bar{\gamma}_{si} \quad (19)$$

and the average pre-dispreading SNR can be expressed as  $2S \bar{\gamma}_{psi}$ .

The BER performance of the system is calculated as  $P_b = \int_0^\infty P(\gamma_p) f'_{\gamma_p}(\gamma_p) d\gamma_p$  where  $\gamma_p = \sum_{s=1}^S \sum_{i=1}^2 \gamma_{psi}$ . The BER

performance of QPSK modulations in AWGN channel is expressed in the form of Q-functions as  $Q(\sqrt{2(SF)\gamma_p})$ . If the alternative representation of

$Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp(-\frac{x^2}{2\sin^2\theta}) d\theta$  which simplifies the analysis is considered, it can be shown that:

$$\begin{aligned} P_b &= \int_0^\infty Q\left(\sqrt{2SF \sum_{s=1}^S \sum_{i=1}^2 \gamma_{psi}}\right) f'_{\gamma_p}(\gamma_p) d\gamma_p \\ &= \int_0^\infty \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{2SF \sum_{s=1}^S \sum_{i=1}^2 \gamma_{psi}}{2\sin^2\theta}\right) d\theta f'_{\gamma_p}(\gamma_p) d\gamma_p \\ &= \frac{1}{\pi} \int_0^\infty \int_0^{\frac{\pi}{2}} \prod_{s=1}^S \prod_{i=1}^2 \exp\left(-SF \frac{\gamma_{psi}}{\sin^2\theta}\right) d\theta f'_{\gamma_p}(\gamma_p) d\gamma_p \end{aligned} \quad (20)$$

Based on the fact of independency, if the following equation is considered:

$$\begin{aligned} f'_{\gamma_p}(\gamma_p) d\gamma_p &= f'_{\gamma_{p11} \dots \gamma_{pS2}}(\gamma_{p11}, \dots, \gamma_{pS2}) d\gamma_{p11} \dots d\gamma_{pS2} \\ &= f'_{\gamma_{p11}}(\gamma_{p11}) d\gamma_{p11} \dots f'_{\gamma_{pS2}}(\gamma_{pS2}) d\gamma_{pS2} \end{aligned} \quad (21)$$

If  $f'_{\gamma_p}(\gamma_p)$  in expression (20) is replaced with above equation, it can now be written as equation (22).

$$\begin{aligned} P_b &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{s=1}^S \prod_{i=1}^2 \exp\left(-SF \frac{\gamma_{psi}}{\sin^2\theta}\right) d\theta \\ &\times \int_0^\infty \dots \int_0^\infty f'_{\gamma_{p11}}(\gamma_{p11}) d\gamma_{p11} \dots f'_{\gamma_{pS2}}(\gamma_{pS2}) d\gamma_{pS2} \\ &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{s=1}^S \prod_{i=1}^2 \int_0^\infty \exp\left(-SF \frac{\gamma_{psi}}{\sin^2\theta}\right) f'_{\gamma_{psi}}(\gamma_{psi}) d\gamma_{psi} d\theta \end{aligned} \quad (22)$$

In order to compute this expression, the Moment Generating Function (MGF) method is used. The MGF is associated with the fading PDF and is defined by

$M_\gamma(p) = \int_0^\infty e^{p\gamma} f_\gamma(\gamma) d\gamma$  [25, 26]. Since the MGF of  $\gamma$

is the Laplace transform of  $f_\gamma(\gamma)$  and Laplace transform is available in closed form, this representation allows evaluation of the above equation in the simplest possible way.

The BER expression of (22) can be represented as equation (23), by defining  $M_{\gamma_{psi}}\left(-\frac{SF}{\sin^2\theta}\right)$  as

$$\begin{aligned} \int_0^\infty \exp\left(-SF \frac{\gamma_{psi}}{\sin^2\theta}\right) f'_{\gamma_{psi}}(\gamma_{psi}) d\gamma_{psi} \\ P_b = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{s=1}^S \prod_{i=1}^2 M_{\gamma_{psi}}\left(-\frac{SF}{\sin^2\theta}\right) d\theta \end{aligned} \quad (23)$$

As all sub-channels are considered to be Rayleigh distributed, using [25] the PDF of their corresponding output SNRs can be expressed as

$$f_{\gamma_{si}}(\gamma_{si}) = \frac{1}{\bar{\gamma}_{si}} \exp\left(-\frac{\gamma_{si}}{\bar{\gamma}_{si}}\right) \quad \text{with the MGF of}$$

$$M_{\gamma_{si}}(s) = \frac{1}{1 - s\bar{\gamma}_{si}}. \quad \text{Since } f'_{\gamma_{psi}}(\gamma_{psi}) \text{ is the PDF of each}$$

sub-channel's pre-dispreading SNR, it could be computed using the probability density function of the corresponding output SINR;  $f_{\gamma_{si}}(\gamma_{si})$ . It is performed

due to the linear relation of  $\gamma_{psi} = \frac{1}{SF} \gamma_{si}$ . This leads to

$$\text{calculate the desired PDF as } f'_{\gamma_{psi}}(\gamma_{psi}) = \frac{1}{\bar{\gamma}_{psi}} \exp\left(-\frac{\gamma_{psi}}{\bar{\gamma}_{psi}}\right).$$

Therefore, it can be shown that

$$M_{\gamma_{psi}}\left(-\frac{SF}{\sin^2\theta}\right) = \left(1 + \frac{SF}{\sin^2\theta} \bar{\gamma}_{psi}\right)^{-1}. \quad \text{If this expression is}$$

replaced in equation (23), it now can be expressed as expression (24) which proposes a new closed form for computing the BER performance of STS-based MC DS-CDMA system in frequency selective Rayleigh fading channel.

$$P_b = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left( \frac{\sin^2\theta}{(SF)\bar{\gamma}_{psi} + \sin^2\theta} \right)^{2S} d\theta \quad (24)$$

The above equation demonstrates that the BER performance of the system depends on the number transmitted antennas, which is equal to 2, and the total number of subcarriers used to extract frequency diversity. This expression can be used to evaluate the BER of the system against pre-dispreading SNR for different values of spreading factor. The resultant curves are essential to determine the thresholds by which the selected metric would be compared in VSF procedure.

## 5 Throughput Analysis of VSF STS-based MC DS-CDMA system

In this section, the average throughput of VSF STS-based MC DS-CDMA system is computed over Rayleigh fading channel.

Using what proposed in [6] and used in [19, 20], it could be stated that in a triple-mode VSF scheme at a specific value of  $\gamma$  when SF is switched between 256, 128 and 64,  $\gamma_p$  is the lowest when SF=256 is employed.

Hence, in VSF CDMA scheme  $\gamma_p$  is maintained at the value associated with the highest spreading factor of SF=256. The transmitted power is multiplied by a factor of  $1/f$  for the two other cases; in which  $f$  is 2 for SF=128 and 4 for SF=64. Therefore, in VSF system the SNR associated with the highest SF is used in SF adaptation procedure, which is performed as Table 1. In this table,  $N_s$  is the number of transmitted symbols in each

transmission course,  $\{l_{ip}\}$  are the predetermined thresholds achieved by using the BER curves and  $l_i = 256 \times l_{ip}$ .

**Table 1.** SF adaptation procedure

Selection rule	SF	$f$	$N_s$
$l_1 \leq \gamma < l_2$	256	1	1
$l_2 \leq \gamma < l_3$	128	2	2
$l_3 \leq \gamma < l_4$	64	4	4

Considering the SF adaptation procedure as described in table 1, it is apparent that the number of transmitted bits is dependent on the probability of selecting the appropriate SF. Therefore, the average throughput of the system as the average number of transmitted bits in each transmission course, when QPSK modulation is used, can be computed as follows.

$$B(\bar{\gamma}, \{l_1, l_2, l_3, l_4\}) = 2F(l_1, l_2) + 4F(l_2, l_3) + 8F(l_3, l_4) \quad (25)$$

$$F(\alpha, \beta) = \int_{\alpha}^{\beta} f_{\gamma}(\gamma) d\gamma$$

In order to compute the above equation  $f_{\gamma}(\gamma)$  should be calculated as a closed form expression and the equation (21) and the MGF method is not functional anymore. The SNR expression due to our STS MC DS-CDMA system is  $\gamma = \sum_{s=1}^S \sum_{i=1}^2 \gamma_{si}$ . Since

$\gamma_{si} = \bar{\gamma}_{si} \frac{h_{((s-1)U+u),i}^2}{\Omega}$  and  $h_{((s-1)U+u),i}$  is Rayleigh distributed,  $h_{((s-1)U+u),i}^2$  is a Chi-square distributed random variable [22]. Therefore  $\gamma_{si}$  has a Chi-square probability function with the characteristic function of

$$\varphi_{\gamma}(j\omega) = \frac{1}{(1 - j\omega\bar{\gamma}_{si})} \quad \text{where } \bar{\gamma}_{si} \text{ is identical for all}$$

values of  $s$  and  $i$ . On the other hand, the summation of  $2S$  statistically independent Chi-square distributed random variables has a Chi-square probability function with  $4S$  degrees of freedom with characteristic function of

$$\varphi_{\gamma}(j\omega) = \frac{1}{(1 - j\omega\bar{\gamma}_{si})^{2S}} \quad [22]. \quad \text{Therefore, the probability}$$

density function of  $f_{\gamma}(\gamma)$  due to our STS MC DS-CDMA system could be computed as expression (26).

$$f_{\gamma}(\gamma) = \frac{1}{(2S-1)! \bar{\gamma}_{si}^{2S}} \gamma^{(2S-1)} e^{-\gamma/\bar{\gamma}_{si}} \quad (26)$$

In equation (25)  $\int f_{\gamma}(\gamma) d\gamma$  can now be computed as:

$$\int f_{\gamma}(\gamma) d\gamma = \frac{-\exp(-\gamma/\bar{\gamma}_{si})}{(2S-1)! \bar{\gamma}_{si}^{(2S-1)}} \left[ \gamma^{(2S-1)} - \frac{(2S-1)\gamma^{(2S-1)}}{\frac{-1}{\bar{\gamma}_{si}}} \right. \\ \left. + \dots + \frac{(-1)^{(2S-1)}(2S-1)!}{\left(\frac{-1}{\bar{\gamma}_{si}}\right)^{(2S-1)}} \right] \quad (27)$$

If the above equation is replaced in expression (25), the average throughput of the system can be calculated.

To compute the BER of the VSF system, it should be considered that it depends on the number of transmitting bits and the probability of selecting the appropriate SF. Therefore, the BER performance of the system can be expressed in a closed form analogous to what has been performed in [17, 18] for adaptive modulation method:

$$P_e(\bar{\gamma}) = \frac{1}{B(\bar{\gamma}, \{I_i\})} \left( 2 \int_{I_{1p}}^{I_{2p}} P_{\text{QPSK}}(\gamma_p) f'_{\gamma_p}(\gamma_p) d\gamma_p \right. \\ \left. + 4 \int_{I_{2p}}^{I_{3p}} P_{\text{QPSK}}(\gamma_p) f'_{\gamma_p}(\gamma_p) d\gamma_p + 8 \int_{I_{3p}}^{I_{4p}} P_{\text{QPSK}}(\gamma_p) f'_{\gamma_p}(\gamma_p) d\gamma_p \right) \quad (28)$$

To propose the above expression as a closed form equation, let define:

$$P_{r,\text{QPSK}}(\bar{\gamma}_p; \{\alpha, \beta\}) = \int_{\alpha}^{\beta} P_{\text{QPSK}}(\gamma_p) f'_{\gamma_p}(\gamma_p) d\gamma_p \quad (29)$$

$f'_{\gamma_p}(\gamma_p)$  can be calculated as equation (30) if the relation

$\gamma_p = \frac{1}{\text{SF}} \gamma$  and equation (26) is considered.

$$f'_{\gamma_p}(\gamma_p) = \frac{1}{(2S-1)! \bar{\gamma}_{psi}^{2S}} \gamma_p^{(2S-1)} e^{-\gamma_p / \bar{\gamma}_{psi}} \quad (30)$$

To compute the BER of the system,  $P_{r,\text{QPSK}}(\bar{\gamma}_p; \{\alpha, \beta\})$  is calculated as below:

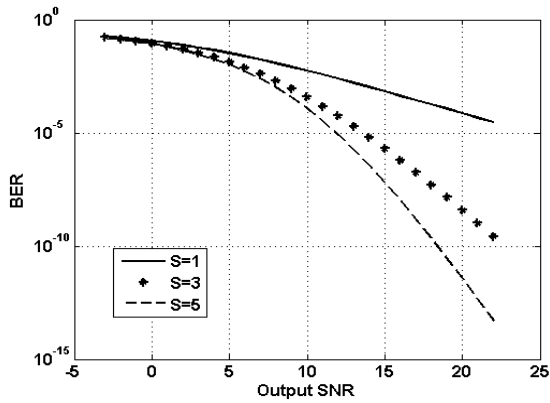
$$P_{r,\text{QPSK}}(\bar{\gamma}_p; \{\alpha, \beta\}) = \int_{\alpha}^{\beta} Q(\sqrt{2(\text{SF})\gamma_p}) \\ \times \frac{1}{(2S-1)! \bar{\gamma}_{psi}^{2S}} \gamma_p^{(2S-1)} e^{-\gamma_p / \bar{\gamma}_{psi}} d\gamma_p = \\ \frac{1}{(2S-1)! \bar{\gamma}_{psi}^{2S}} \int_{\alpha}^{\beta} Q(\sqrt{2(\text{SF})\gamma_p}) \gamma_p^{(2S-1)} e^{-\gamma_p / \bar{\gamma}_{psi}} d\gamma_p \quad (31)$$

After replacing this equation in (25), the BER performance of the VSF MC DS-CDMA system can be calculated.

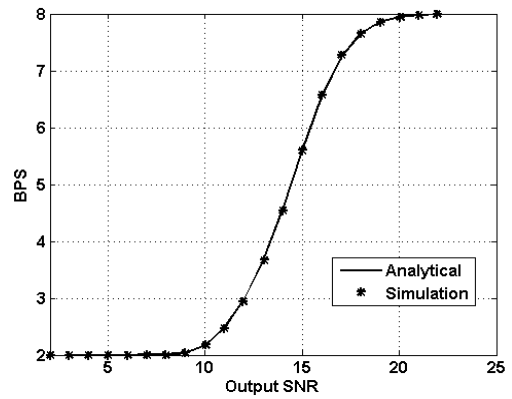
## 6 Numerical Results and Discussion

In this section, some analytical results due to numerical computing the proposed expressions are provided. Fig. 4 shows the BER performance of the STS-based MC DS-CDMA system due to equation (24) for SF=64. QPSK modulation is assumed in frequency-selective Rayleigh fading channel with  $\alpha_0 = 0.5$ . In this figure, the influence of frequency diversity order through performing interleaving with different depths is considered. As it is apparent in the figure, the performance of the system is improved for higher frequency diversity orders. Regarding these results, the interleaver's depth is considered 3, for investigating the VSF system.

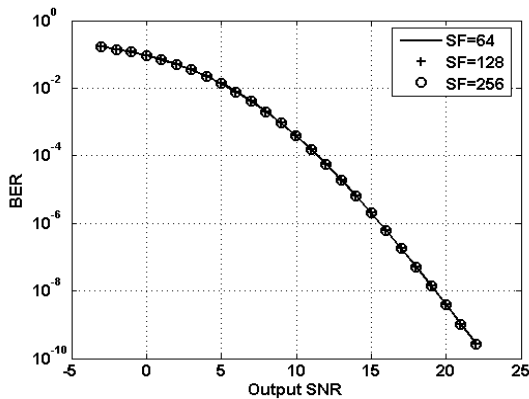
Fig. 5 shows the BER curves of the STS-based MC DS-CDMA system versus output SNR for different SFs. As declared in section III, it is observed that the curves are completely overlapped. Therefore, we use the BER curves versus pre-dispreading SNR to determine the switching thresholds. In Fig. 6, the analytical results due to calculation of expression (24) versus the average pre-dispreading SNR are presented. These curves are used to determine the threshold levels for VSF procedure. The thresholds are selected so that the BER of the system remains below 0.1%. The resultant thresholds are  $\{I_{ip}\} = \{-27.09, -12.08, -9.072, \infty\}$ . Using these values, the VSF procedure is performed as described in Table 1. The average throughput of the VSF STS-based MC DS-CDMA system is given in figure 7 through simulation and numerical computation of equation (25). BPS represents the average number of the transmitted bits per symbol and is a measure of throughput [6]. This figure shows that using VSF method in STS-based MC DS-CDMA system improves the throughput of the system. The matching of the simulation and analytical results is apparent in this figure. It proves that the analytical expression proposed in this paper can be used to evaluate the throughput of the VSF STS-based MC DS-CDMA system in frequency-selective Rayleigh channel. The BER performance of the system due to numerical computation of equation (28) is shown in Fig. 8 in comparison with the non-VSF system with SF=64. The BER curves in this figure are nearly the same, since the BER performance of the system versus output SNR -as shown in Fig. 5- is not affected by the SF value.



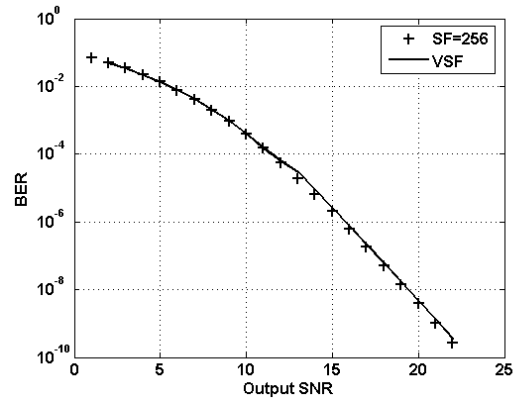
**Fig. 4.** BER performance of STS-based MC DS-CDMA system in Rayleigh channel for SF=64 and  $\alpha_0 = 0.5$  (considering the influence of frequency diversity order).



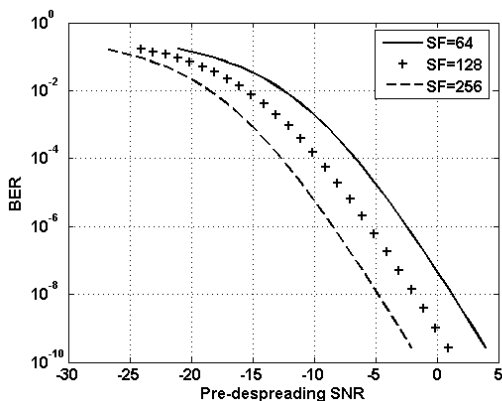
**Fig. 7.** The average throughput (BPS) of the VSF STS-based MC DS-CDMA system in Rayleigh fading channel for S=3 and  $\alpha_0 = 0.5$ .



**Fig. 5.** BER performance of STS-based MC DS-CDMA system in Rayleigh fading channel for S=3 and  $\alpha_0 = 0.5$  (comparing different values of SF).



**Fig. 8.** The BER performance of the VSF STS-based MC DS-CDMA system in Rayleigh fading channel for S=3 and  $\alpha_0 = 0.5$ .



**Fig. 6.** BER performance of STS-based MC DS-CDMA system, against pre-dispreading SNR, in Rayleigh fading channel for S=3 and  $\alpha_0 = 0.5$  (comparing different values of SF).

## 7 Conclusions

In this paper, the application of Variable Spreading Factor is investigated for STS-based MC DS-CDMA system to improve the data throughput of the system over frequency selective Rayleigh fading channel. In this contribution, an analytical approach is proposed to compute a new expression for the BER performance of the system with respect to pre-dispreading SNR for different values of spreading factor. The resultant BER curves are necessary for determining the required switching thresholds for VSF part. Using the thresholds obtained in this way, Variable Spreading Factor method was performed on STS-based MC DS-CDMA system to enhance the average throughput of the system. An analytical approach was then proposed to compute the throughput enhancement and the BER of the VSF system. The novel expressions presented in this paper simplify the evaluation of the proposed system with no need to more calculations and simulations. Changing the



parameters in these expressions leads to easier analysis of the VSF STS-based MC DS-CDMA system, in a flexible manner with less complexity.

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**Vahideh Vakil** graduated in communication engineering from Ferdowsi University of Mashad, Iran, in 2000. In 2003, she received her M.Sc. from Ferdowsi University of Mashad, Iran. She finished her Ph.D. in Communication System engineering in Amirkabir University of Technology, in 2008. From 2007 until present, she is a faculty member in Bahar institute of higher education of Mashad, where she is an assistant professor. Her research interests include work in CDMA systems, diversity techniques, adaptive rate transmission, and OFDM systems.



**Hassan Aghaeinia** received his B.Sc. degree from Amirkabir University of Technology (Tehran Polytechnic) in electronic engineering, in 1987. In 1989 he finished his M.Sc. in Amirkabir University of Technology. Then, in 1992 he received the M.Sc. from Valenciennes University (UVHC), Valenciennes, France. Then he continued his studies towards Ph.D. in UVHC in electronic engineering. He finished his Ph.D. in 1996. From 1996 till present, he is a faculty member in Amirkabir University of Technology, where he is an assistant professor in Communication Engineering Group. His research includes work in digital communications & spread spectrum systems, advanced communication systems, and digital image processing & image communication.