Throughput Maximization of Ad-hoc Wireless Networks Using Adaptive Cooperative Diversity and Truncated ARQ

Lin Dai, *Member, IEEE,* and Khaled B. Letaief, *Fellow, IEEE*

***Abstract*—We propose a cross-layer design which combines truncated ARQ at the link layer and cooperative diversity at the physical layer. In this scheme, both the source node and the relay nodes utilize an orthogonal space-time block code for packet retransmission. In contrast to previous cooperative diversity protocols, here cooperative diversity is invoked only if the destination node receives an erroneous packet from the source node. In addition, the relay nodes are not fixed and are selected according to the channel conditions using CRC. It will be shown that this combination of adaptive cooperative diversity and truncated ARQ can greatly improve the system throughput compared to the conventional truncated ARQ scheme and fixed cooperative diversity protocols. We further maximize the throughput by optimizing the packet length and modulation level and will show that substantial gains can be achieved by this joint optimization. Since both the packet length and modulation level are usually discrete in practice, a computationally efficient algorithm is further proposed to obtain the discrete optimal packet length and modulation level.**

***Index Terms*—Cooperative diversity, truncated ARQ, cross-layer design, adaptive resource allocation, MIMO, ad-hoc networks.**

I. Introduction

THE use of multiple antennas at both the transmitter and receiver can bring significant capacity gains [1]. Unfortunately, this could be impractical in an ad-hoc wireless network, due to the size of the node or the mobile unit. In order to overcome this limitation, a new form of spatial diversity, whereby diversity gains are achieved via the cooperation of nodes, has been proposed. The main idea behind this approach, which is called cooperative diversity, is to use orthogonal relay transmission to achieve diversity gain. In particular, each node has one or several partners. The node and its partner(s) are responsible for transmitting not only their own information, but also the information of their partner(s). Therefore, a virtual antenna array is obtained through the use of the relays' antennas without complicated signal design or adding more antennas at the nodes. Sendonaris *et al* proposed the idea of cooperative diversity and applied it into CDMA cellular systems. In [2-3], they presented an information-theoretic model, where two nodes cooperate by transmitting each bit over two successive bit intervals. Their results showed that node cooperation increases the sum-rate over non-cooperative transmission for ergodic fading links, given that the channel state information is available at the transmitter. Laneman and Wornell further extended the above work [4-5]. They thoroughly investigated the performance gain of cooperative diversity in ergodic and non-ergodic scenarios and presented several cooperative protocols, including amplify-and-forward, decode-and-forward, selection relaying and space-time-coded cooperation. Besides, Hunter *et al* introduced coding (RCPC codes [6-7] or turbo codes [8]) into the cooperation. This coded cooperative diversity has been shown to be able to achieve significant performance gains over the amplify-and-forward and decode-and-forward protocols [7]. Following a different approach, Stefanov and Erkip designed channel codes that can fully exploit the diversity gains of user cooperation [9]. Other important work includes cooperative regions analysis for coded cooperative protocol [10], diversity-multiplexing tradeoff analysis on cooperative protocols [11], space-time code design criteria for amplify-and-forward relay channels [12], capacity and symbol error rate analysis [13-14] and cross-layer optimization for energy-constrained cooperative networks [15].

In most of the present cooperative protocols, no restrictions are imposed on the selection of relays. Therefore, when the channel between the source node and the relay node ($s-r$ channel) is poor, cooperative diversity may result in even worse performance than the non-cooperative case due to severe error propagation. In [4], a selection relaying protocol with two nodes cooperation was proposed, where the relay forwards the source node's information only if the $s-r$ channel fading coefficient is above a given threshold. In other words, the node is selected to be a relay only when its corresponding $s-r$ channel is good enough. Obviously, such selective protocol can achieve better performance than the fixed ones [4]. However, it is usually not trivial to select a suitable threshold since it depends on the actual value of the channel fading coefficients. A higher threshold will reduce the possible performance gain while a lower one will allow more error propagation which also degrades the performance.

Automatic Repeat Request (ARQ) protocol at the link

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L. Dai is with the Department of Electrical Engineering, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon Tong, Hong Kong, China (e-mail: lindai@cityu.edu.hk).

K. B. Letaief is with the Department of Electrical and Computer Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, China (e-mail: eekhaled@ee.ust.hk).

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layer is an effective means to overcome the channel fading, where Cyclic Redundancy Check (CRC) is usually used for error check and retransmissions are requested if the packet is received erroneously [16-17]. In practice, the maximum number of retransmissions is usually limited so as to minimize the delay and buffer size and such variant ARQ is called truncated ARQ protocol [16]. Despite the improved reliability, the truncated ARQ scheme requires more transmission time. It will be shown that the throughput of the truncated ARQ scheme is exactly the same as that of the direct transmission. Actually as long as the channel statistics remains unchanged during the direct transmission and retransmissions, no throughput gains can be achieved via ARQ.

In this paper, we propose a cross-layer design which combines truncated ARQ at the data link layer and cooperative diversity at the physical layer. We will show that through this combination, adaptive cooperative diversity gain can be achieved without any specific threshold and error propagation is therefore avoided. Besides, the channel quality is significantly improved in the retransmissions by using relays so that substantial throughput gains can be obtained. In particular, in this new scheme, Q idle nodes around the source node are defined as *relay candidates*. These nodes also receive the packet transmitted from the source node to the destination node and check the CRC results. Only the ones who detect the correct CRC are selected to be relays and involved in the possible retransmission, with both the source node and the relays utilizing a suitable orthogonal space-time block code (STBC) to retransmit this packet. It can be seen that this new scheme is adaptive to the $s-r$ channels by virtue of the CRC bits instead of some specific threshold and so no error propagation will be incurred by relaying. Besides, high efficiency can be achieved since node cooperation is adopted only when the destination node fails to detect the nodes correctly. As a result, this scheme, which is referred to as *Selective Cooperative diversity with ARQ* (SCA), can be expected to bring significant performance gain over the previous ARQ-only or fixed cooperative diversity schemes. Another scheme which combines truncated ARQ and fixed node cooperation is also considered in this paper. It will be shown that this scheme, which is referred to as *Fixed Cooperative diversity with ARQ* (FCA), requires lower complexity than SCA, while it may cause some performance loss due to error propagation.

Throughput is defined as the data rate successfully received and regarded as a key measure of QoS for wireless systems [18]. In this paper, we focus on the throughput at the data link layer where end-to-end delivery of the packet should be guaranteed. The loss due to retransmissions of the packet is also included here. The throughput expressions of SCA and FCA are derived and compared to that of the pure truncated ARQ scheme. It will be shown that when the $s-r$ channels are perfect, both SCA and FCA can achieve substantial gains over the truncated ARQ scheme thanks to cooperative diversity. However, with poor $s-r$ channels, the performance of FCA will deteriorate rapidly and even become worse than the truncated ARQ scheme due to error propagation. On the other hand, SCA will always achieve the highest throughput among all the schemes because of its adaptability to the $s-r$ channels.

Throughput is usually affected by many design parameters, including symbol rate, modulation level, packet length, the maximum number of retransmissions, and power level. In this paper, we further maximize the throughput by optimizing the packet length L and the modulation level b. It is shown that substantial gains can be obtained with the joint optimization of L and b. Besides, it is found that at low SNR we could only optimize the packet length to get an optimal throughput, whereas at high SNR, optimizing the modulation level can more effectively maximize the throughput.

In practice, the packet length L and the modulation level b should be discrete and under some constraints according to the specific transmission schemes. Exhaustive search over the discrete optimal L^* and b^* will result in prohibitive complexity. In this paper, we further present a computationally efficient algorithm, *low-complexity discrete optimization algorithm* (L-DOA), by which the discrete optimal values of L^* and b^* can be found with a rather low complexity. The comparison between the optimal throughput with continuous optimal L^* and b^* and that of discrete optimal L^* and b^* shows that in most cases, the resulting two throughput curves coincide very well, which implies that the discretization of L^* and b^* results in very slight throughput loss.

This paper is organized as follows. The channel model and the details of the proposed SCA and FCA are provided in Section II. Section III presents the throughput comparison of SCA, FCA and the pure truncated ARQ scheme. The throughput is further maximized by optimizing the packet length L and the modulation level b in Section IV. Section V presents the optimization process over discrete parameters. Finally, Section VI summarizes and concludes this paper.

II. Cooperative Diversity with Truncated ARQ

In this section, we propose a new cross-layer design which combines truncated ARQ at the link layer and cooperative diversity at the physical layer.

A. Scheme Description

We consider an ad-hoc network with K nodes and assume that each node is equipped with only one antenna. Q idle nodes are assumed to be available as the possible relays for the source node during the packet transmission. Throughout this paper, these Q nodes are referred to as *relay candidates*.[1] Particularly, the source node transmits a data packet with a C-bit CRC attached. The destination node detects CRC and then sends an acknowledgement that is either positive (ACK) or negative (NACK) back to the source node. At the same time, all the Q relay candidates check the CRC and the ones who get positive results are selected to be relays. If the packet is correctly detected by the destination node (with ACK feedback), the source node continues to transmit a new data packet and the above process is repeated. Otherwise, retransmission will start. Both the source node and the relays will jointly retransmit the packet by utilizing a suitable orthogonal STBC. The retransmission continues

[1]The assumption that Q idle nodes are available as relay candidates can be satisfied with a proper multiuser scheduling strategy. However, a detailed study of the issues involved with the specific scheduling strategies is out of the scope of this paper.

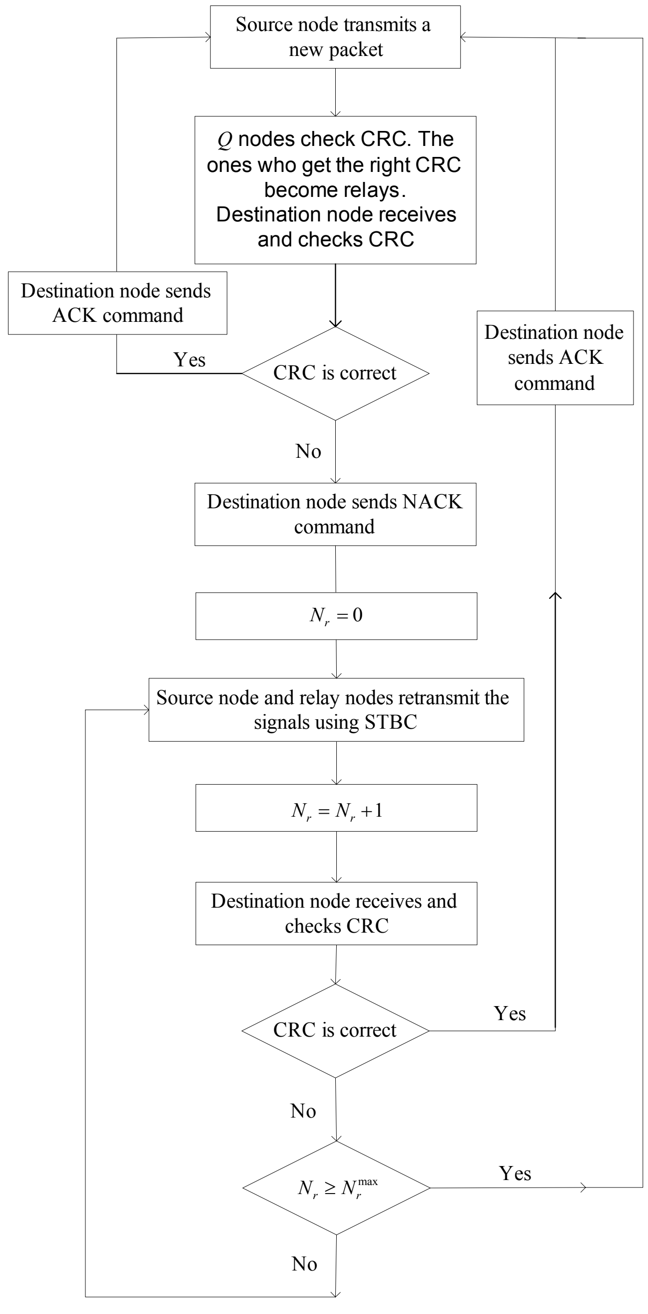

Fig. 1. Flow chart of SCA.

until the packet is successfully delivered, or the number of retransmissions exceeds $N_r^{\max}$ which is a preset parameter indicating the maximum number of retransmissions allowed per packet. The detailed flow chart of this new scheme is shown in Fig. 1.

It can be seen that this new scheme can adapt to the $s-r$ channels thanks to the use of CRC bits. Only the relay candidates who correctly detect the packet are selected to be relays. Adaptive cooperative diversity gain is actually achieved and error propagation can be avoided. Besides, node cooperation is adopted only when the destination node fails to detect the packet correctly. Higher efficiency can therefore be achieved compared to the previous cooperative diversity protocols. As a result, it can be expected that this proposed SCA scheme can bring significant throughput gains over those ARQ-only or fixed cooperative diversity schemes. For the sake of comparison, FCA, is also proposed in this paper, where truncated ARQ is combined with fixed node cooperation. In contrast to SCA, in FCA the relays are pre-assigned and always fixed during the whole transmission. In the retransmission, the relays send their estimates instead of the original signals. Therefore, FCA requires lower complexity than SCA, while it may cause some performance loss due to error propagation, which will be shown in Section III.

In SCA, the number of relays v may vary in each packet transmission. In this paper, the STBC scheme is dependent on the value of v, i.e., we use $(v+1)$-symbol STBC in the retransmission. It should be distinguished from the space-time block coded protocol proposed in [5], where a $(Q+1)$-symbol STBC is adopted and for each cooperative transmission, $(v+1)$ columns are selected from the code matrix. It can be checked that this space-time block coded protocol can achieve the same diversity gain as ours and the complexity is rather low since each relay candidate (also the source node) is allocated a fixed orthogonal coding pattern regardless of v. However, it may lead to low efficiency. For instance, assume that $Q=2$ and 1 relay is selected ($v=1$). With the space-time block coded protocol in [5], the rate R is only $3/4$.[2] Instead, the Alamouti's scheme can be adopted in our case so as to achieve the full rate $R=1$. It should be also noticed that in our scheme, the relays should send a message to notify both the source node and the destination node before each retransmission. The source node then assigns the STBC columns to the relays. This node communication will result in some additional delay and overhead compared to the conventional ARQ or previous cooperative diversity protocols. However, considering that the transmission is performed in the unit of packet (over one hundred symbols per packet, for example), this overhead can be neglected since it only requires several bits.

B. Channel Model

The communication between a source and a destination node is assumed to be over a flat Rayleigh fading channel and facilitated by v relays which are selected from Q relay candidates, as shown in Fig. 2. In addition, perfect channel knowledge is assumed to be available at the receiver side only, through the use of training sequences.

At time slots $t_0+1, ..., t_0+\varsigma$, the source node sends a packet $x^s_{t_0+1}, \ldots, x^s_{t_0+\varsigma}$ with transmission power P_t per symbol, where $x^s_{t_0+i}$ is an M-QAM modulated symbol and $\varsigma = L/b$ is the number of symbols per packet with a total packet length of L bits and a modulation level of $b = log_2 M$ bits. The signal received by the destination node at time slot t_0+i, $i=1, ..., \varsigma$, is then given by

$$y^d_{t_0+i} = h_0 x^s_{t_0+i} + z_0 \tag{1}$$

where the channel gain h_0 is assumed to be a complex Gaussian random variable with zero-mean and variance σ_0^2. Here σ_0^2 accounts for the effect of large-scale path loss

[2]Assume that the [3,4,3] STBC code given in [23] (pp. 2485, Eqn. (99)) is adopted. For a k-symbol-T-slot STBC, the rate R is equal to k/T.

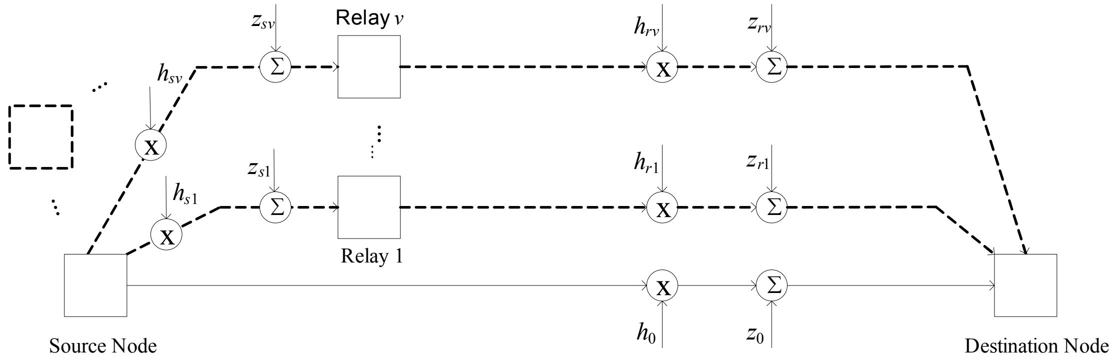

Fig. 2. System model. Q idle nodes are assumed to be available as the relay candidates.

and shadowing [5-6]. Also, z_0 represents the additive white Gaussian noise with zero-mean and variance N_0. At the j-th relay candidate, $j = 1, \ldots, Q$, the received signal is given by

$$y^{r_j}_{t_0+i} = h_{sj} x^s_{t_0+i} + z_{sj} \tag{2}$$

where the noise z_{sj} and the fading coefficient h_{sj} are complex Gaussian random variables with zero-mean and variance N_0 and $\sigma^2_{s_j}$, respectively, $j = 1, ..., Q$. In the following, the transmission of the packet $x^s_{t_0+1}, ..., x^s_{t_0+\varsigma}$ is referred to as *direct transmission*.

If the destination node fails to detect the packet correctly, retransmission will start at time slot $t_L + 1$. Let N_r be the number of retransmissions, $1 \le N_r \le N_r^{\max}$. The received signal at the destination node at time slot $t_L + i$ is then given by

$$y^d_{t_L+i} = h_0 x^s_{t_L+i} + z_0 + \sum_{j=1}^{v} h_{rj} x^{r_j}_{t_L+i} + z_{rj} \tag{3}$$

for $i = 1, ..., N_r\varsigma/R$, where R is the rate of STBC. $x^s_{t_L+i}$ and $x^{r_j}_{t_L+i}$ are the space-time block coded symbols transmitted by the source node and the j-th relay node with the transmission power $P_t/(v+1)$, respectively. The additive noise z_{rj} and the fading coefficient h_{rj} are complex Gaussian random variables with zero-mean and variance N_0 and $\sigma^2_{r_j}$, respectively, $j = 1, ..., v$. In this paper, it is assumed that the large scale fading of each $r-d$ channel (the channel between the relay and the destination node) is the same as that of the $s-d$ channel (the channel between the source node and the destination node), i.e., $\sigma^2_{r_1} = \cdots = \sigma^2_{r_v} = \sigma^2_0$.[3]

Throughout the paper, the following symbols and notations will be used.

γ_{sd}: Average SNR per symbol of the direct transmission
γ_{sr}: Average SNR per symbol of the retransmission
γ^i_{ss-r}: Average SNR per symbol of the i-th $s-r$ channel
p_{pd}: Average packet error rate (PER) of the direct transmission
p_{pr}: Average PER of the retransmission
p^i_{ps-r}: Average PER of the i-th $s-r$ channel
p_{sd}: Average symbol error rate (SER) of the direct transmission
p_{sr}: Average SER of the retransmission
p^i_{ss-r}: Average SER of the i-th $s-r$ channel

III. Throughput Analysis

In this section, the throughput of both SCA and FCA is analyzed. For the sake of comparison, the throughput expression of the pure truncated ARQ scheme is also provided.[4]

A. Throughput of the Truncated ARQ Scheme

Assume that the total length of a data packet is L, where a C-bit CRC is attached and a square M-QAM is adopted with $b = \log_2 M$ bit/symbol. The symbol rate R_s is assumed to be constant and thus omitted in the following. For a point-to-point single transmission, the throughput is then given by [18]

$$T = \frac{L-C}{L} b(1-p_{sd})^{L/b} \tag{4}$$

where p_{sd} is the SER of the direct transmission.

With truncated ARQ, retransmission will start if the packet is detected erroneously and will continue until the packet is successfully delivered or the number of retransmissions N_r exceeds $N_r^{\max}$. In the retransmission, the data rate will be reduced since the packet is repeated. Therefore, the throughput of the truncated ARQ scheme can be obtained as

$$T_{ARQ} = \frac{L-C}{L} b \cdot P_A/\bar{N}_A \tag{5}$$

where

$$P_A = 1 - p_{pd}(p_{pr})^{N_r^{\max}} \tag{6}$$

[3]The difference in the large-scale fading among the $r-d$ channels and the $s-d$ channel can be easily compensated for by power control, i.e., different transmission power levels can be allocated to each relay or the source node according to its corresponding $\sigma^2_{r_j}$ or σ^2_0 . For simplicity, we assume that the large scale fading effect is the same and so equal power allocation is adopted in this paper.

[4]In this section, we adopt a rather abstract system model. For example, we only consider uncoded M-QAM and the additional overhead in ARQ signaling (such as ACK and NACK) is neglected. Nevertheless, the mathematical framework is general and the throughput analysis can be easily extended to the specific systems.

is the packet successful rate. $\bar{N}_A$ is the average number of transmissions per packet, which is given by

$$\bar{N}_A = 1 \cdot (1 - p_{pd}) + \sum_{i=2}^{N_r^{\max}} i \cdot p_{pd}(p_{pr})^{i-2}(1 - p_{pr}) + (N_r^{\max} + 1) \cdot \left(p_{pd}(p_{pr})^{N_r^{\max}-1}(1 - p_{pr}) + p_{pd}(p_{pr})^{N_r^{\max}} \right) \quad (7)$$

In (7), p_{pd} and p_{pr} are given by

$$p_{pd} = 1 - (1 - p_{sd})^{L/b} \text{and} p_{pr} = 1 - (1 - p_{sr})^{L/b}. \quad (8)$$

The channel statistics does not change in the retransmissions. Therefore, the average SER of the retransmission p_{sr} should be equal to p_{sd}. As a result, we have $p_{pr} = p_{pd}$ and (5) can then be simplified as

$$T_{ARQ} = \frac{L - C}{L} b \cdot (1 - p_{pd}). \quad (9)$$

From (9) and (4), it can be seen that the truncated ARQ scheme has exactly the same throughput as the direct transmission. Despite the improved reliability for each packet, the truncated ARQ scheme requires more transmission time. As long as the channel statistics keeps constant in the retransmissions, no throughput gain can be achieved. Nevertheless, we will show that by combining the truncated ARQ scheme and cooperative diversity, the SER of the retransmission will be improved greatly so that significant throughput gains can be obtained.

In this paper, it is assumed that the $s - d$ channel is a flat Rayleigh fading channel. As such, the closed-form expression for the average SER of M-QAM is given by [19] (Eqn. (49), pp. 1331)

$$p_{sd} = 2\left(1 - \frac{1}{\sqrt{2^b}}\right)\left(1 - \sqrt{\frac{g\gamma_{sd}}{1 + g\gamma_{sd}}}\right) + \left(1 - \frac{1}{\sqrt{2^b}}\right)^2 \cdot \left[\frac{4}{\pi}\sqrt{\frac{g\gamma_{sd}}{1 + g\gamma_{sd}}} \arctan\left(\sqrt{\frac{1 + g\gamma_{sd}}{g\gamma_{sd}}}\right) - 1\right], \quad (10)$$

where $g = \frac{3}{2(2^b - 1)}$. γ_{sd} is the average SNR per symbol of the direct transmission, which is given by $\gamma_{sd} = \sigma_0^2 P_t / N_0$. Therefore, by substituting (10) into (8) and (9), the throughput of the pure truncated ARQ scheme, T_{ARQ}, can be computed.

B. Throughput of FCA

In FCA, v pre-assigned relays and the source node are used in the retransmission and both utilize a $(v + 1)$-symbol orthogonal STBC to send the packet together. Therefore, the throughput of FCA can be obtained as

$$T_{FCA} = \frac{L - C}{L} b \cdot P_F / \bar{N}_F \quad (11)$$

where P_F is the packet successful rate which can be computed by (6). $\bar{N}_F$ is the average number of transmissions per packet, which is given by

$$\bar{N}_F = 1 \cdot (1 - p_{pd}) + p_{pd} \cdot \left[\begin{array}{c} \sum_{i=2}^{N_r^{\max}} (1 + (i-1)/R) \cdot (p_{pr})^{i-2}(1 - p_{pr}) \\ + (1 + N_r^{\max}/R) \cdot (p_{pr})^{N_r^{\max}-1} \end{array} \right] \quad (12)$$

where R is the rate of STBC (for a k-symbol-T-slot STBC, $R = \frac{k}{T}$). In this case, the average SER of the retransmission $p_{sr} \neq p_{sd}$ since multiple antenna transmission is adopted in the retransmission. In FCA, the retransmission can be regarded as $(v + 1)$-transmit-1-receive STBC with M-QAM symbols over Rayleigh fading channels. From [20], we know that the average SER of m-transmit-n-receive STBC with M-QAM in Rayleigh fading channels is given by

$$p_s = \frac{2q\phi_r(g)}{\sqrt{\pi}} \frac{\Gamma(mn + 1/2)}{\Gamma(mn + 1)} \cdot {}_2F_1\left\{mn; \frac{1}{2}; mn + 1; \frac{1}{1 + g\gamma_s}\right\} - \frac{2q^2}{\pi} \frac{\phi_r(2g)}{2mn + 1} F_1\left\{1, mn, 1; mn + \frac{3}{2}; \frac{1 + g\gamma_s}{1 + 2g\gamma_s}, \frac{1}{2}\right\}, \quad (13)$$

where γ_s is the average SNR per symbol, $q = 1 - 1/\sqrt{2^b}$, $\phi_r(s) \triangleq (1 + s\gamma_s)^{-mn}$, and ${}_2F_1(a, b; c; x)$ is the Gauss hypergeometric function defined as

$${}_2F_1(a, b; c; x) \triangleq \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n n!} x^n. \quad (14)$$

In the above equation, $(a)_n = \Gamma(a + n)/\Gamma(a)$ with $\Gamma(\cdot)$ denoting the Gamma function. Likewise, $F_1(a, b, b'; c; x, y)$ is the Appell hypergeometric function defined as

$$F_1(a, b, b'; c; x, y) \triangleq \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} \frac{(a)_{n+k} (b)_n \left(b'\right)_k}{(c)_{n+k}\, n!k!} x^n y^k. \quad (15)$$

Therefore, p_{sr} can be obtained by substituting $m = v + 1$, $n = 1$, and $\gamma_s = \gamma_{sr}$ into (13).

The computation of γ_{sr} should include the effect of error propagation since in FCA the relay nodes retransmit their estimates instead of the original signals. As a result, γ_{sr} will not be given by $\frac{\gamma_{sd}}{(v+1)R}$. Instead, the expression of γ_{sr} of FCA is given in Theorem 1.

Theorem 1: The average SNR per symbol of the retransmission in FCA, γ_{sr}, is given by

$$\gamma_{sr} = \frac{\gamma_{sd}}{a\gamma_{sd} + (v + 1)R} \quad (16)$$

where

$$a = \frac{24v}{2^b - 1}(1 - 2^{-b/2}) \cdot \left[\begin{array}{c} \left(1 - \sqrt{\frac{g\gamma_{ss-r}}{1+g\gamma_{ss-r}}}\right) + (1 - 2^{-b/2}) \cdot \\ \left(\frac{2}{\pi}\sqrt{\frac{g\gamma_{ss-r}}{1+g\gamma_{ss-r}}} \arctan\left(\sqrt{\frac{1+g\gamma_{ss-r}}{g\gamma_{ss-r}}}\right) - 1\right) \end{array} \right], \quad (17)$$

assuming that the average SNR per symbol of the i-th $s - r$ channel $\gamma_{ss-r}^i = \gamma_{ss-r} = \sigma_s^2 P_t / N_0$ for $i = 1, ..., v$.

Proof: See the Appendix.

By combining Theorem 1 and (11-12), the throughput of FCA, T_{FCA}, can be obtained.

C. Throughput of SCA

In SCA, the number of relay nodes v is not fixed. Only the candidates who detect the correct CRC results are involved in the retransmission. Therefore, the throughput expression of SCA should be written as

$$T_{SCA} = \frac{L-C}{L} b \cdot P_S / \bar{N}_S \tag{18}$$

where P_S and $\bar{N}_S$ are the packet successful rate and the average number of transmissions per packet, respectively. They are given by

$$P_S = \sum_{j=0}^{Q} \left(1 - p_{pd}(p_{pr}(j))^{N_r^{\max}}\right) \cdot P(v=j) \tag{19}$$

and

$$\bar{N}_S = \sum_{j=0}^{Q} N(j) \cdot P(v=j) \tag{20}$$

with $N(j)$ given by

$$N(j) = 1 - p_{pd} + p_{pd} \cdot \left[\begin{array}{c} \sum_{i=2}^{N_r^{\max}} (1+(i-1)/R(j)) \cdot p_{pr}(j)^{i-2}(1-p_{pr}(j)) \\ +(1+N_r^{\max}/R(j)) \cdot p_{pr}(j)^{N_r^{\max}-1} \end{array} \right] \tag{21}$$

for $j = 0, \ldots, Q$.

In (21), $R(j)$ is the rate of a $(j+1)$-symbol STBC (let $R(0) = 1$) and $p_{pr}(j)$ is the PER of the j-relay retransmission.[5] In SCA, no error propagation is introduced by relays. Therefore, with a j-relay retransmission ($j > 0$), $p_{sr}(j)$ can be obtained via (13) by substituting $m = j+1$, $n = 1$, and $\gamma_s = \gamma_{sr}(j) = \frac{\gamma_{sd}}{(j+1)R(j)}$. Otherwise, $p_{sr}(j) = p_{sd}$ (for $j = 0$).

$P(v = j)$ is the probability that j relay candidates detect the correct CRC results and is given by

$$P(v=0) = p_{ps-r}^1 \cdot p_{ps-r}^2 \cdot \cdots \cdot p_{ps-r}^Q \tag{22}$$

$$\begin{aligned} P(v=1) = {} & (1-p_{ps-r}^1) \cdot p_{ps-r}^2 \cdot \cdots \\ & \cdot p_{ps-r}^Q + p_{ps-r}^1 \cdot (1-p_{ps-r}^2) \cdot \cdots \\ & \cdot p_{ps-r}^Q + \cdots + p_{ps-r}^1 \cdot p_{ps-r}^2 \cdot \cdots \\ & \cdot (1-p_{ps-r}^Q) \end{aligned} \tag{23}$$

$$\vdots$$

where p_{ps-r}^i is the PER of the i-th $s-r$ channel, $i = 1, ..., Q$, and we have $p_{ps-r}^i = 1-(1-p_{ss-r}^i)^{L/b}$. p_{ss-r}^i is the SER of the i-th $s-r$ channel and can be computed by

$$p_{ss-r}^i = 2\left(1-\frac{1}{\sqrt{2^b}}\right)\left(1-\sqrt{\frac{g\gamma_{ss-r}^i}{1+g\gamma_{ss-r}^i}}\right) + \left(1-\frac{1}{\sqrt{2^b}}\right)^2 \cdot \left[\frac{4}{\pi}\sqrt{\frac{g\gamma_{ss-r}^i}{1+g\gamma_{ss-r}^i}} \arctan\left(\sqrt{\frac{1+g\gamma_{ss-r}^i}{g\gamma_{ss-r}^i}}\right) - 1\right] \tag{24}$$

where $g = \frac{3}{2(2^b-1)}$ and γ_{ss-r}^i is the average SNR per symbol of the i-th $s-r$ channel which is given by $\gamma_{ss-r}^i = \sigma_{s_i}^2 P_t/N_0$, $i = 1, \ldots, Q$.

[5]Note that $p_{pr}(j)$ is now indexed by j since in SCA the PER of the retransmission is dependent on the number of nodes involved in the retransmission.

When $\sigma_{s_i}^2 = \sigma_s^2, i = 1, ..., Q$, a general expression of $P(v = j)$ can be obtained as

$$P(v=j) = C_Q^j \left(1-p_{ps-r}\right)^j \left(p_{ps-r}\right)^{Q-j} \tag{25}$$

where $p_{ps-r} = p_{ps-r}^i$, for all $i = 1, ..., Q$, and $j = 0, ..., Q$.

Finally, by combining (18-25), the throughput of SCA, T_{SCA}, can be obtained.

D. Throughput Comparison

Assume that $Q = 2$ idle nodes are available as the relay candidates. $C = 16$ bit CRC is assumed to be adopted with a cyclic generator polynomial of $g_{CRC16}(D) = D^{16} + D^{12} + D^5 + 1$. Assume that the packet length L is 120 bits and QPSK is adopted (i.e., $b = 2$ bits). The maximum number of retransmissions $N_r^{\max}$ is assumed to be 3. For simplicity, $\sigma_{s_i}^2$ is assumed to be equal to σ_s^2, for $i = 1, \ldots, Q$. Therefore, we have $\gamma_{ss-r} = \gamma_{ss-r}^i$, $i = 1, \ldots, Q$.

Fig. 3 presents the numerical and simulation results on the throughput of the pure truncated ARQ, FCA with 1 relay, FCA with 2 relays and SCA when the average SNR per symbol of the $s-r$ channels γ_{ss-r} is 20dB. The x-axis "SNR" is referred to the average SNR per symbol of the direct transmission γ_{sd}. A good match can be observed between the simulation and numerical curves, despite slight discrepancy in the case of FCA. From Fig. 3 it can be also seen that both FCA and SCA perform much better than the truncated ARQ scheme. Here γ_{ss-r} is assumed to be 20dB, which indicates rather good $s-r$ channels. In this case, cooperative diversity gain can be fully exploited to improve the SER of the retransmission and so substantial throughput gains can be expected to be achieved by SCA and FCA over the truncated ARQ scheme. As Fig. 3 shows, SCA always achieves the highest throughput, which is due to its adaptability to the $s-r$ channels. It should be noticed that at high SNR, FCA with 1 relay gets better performance than the 2 relay case. This is because with 3 nodes cooperation, the STBC is not full rate.[6] When the SNR of the $s-d$ channel (also the $r-d$ channels) is high enough (which implies a good diversity gain), rate loss will significantly influence the throughput. Therefore, although in the low SNR regime FCA with 2 relays can achieve a better throughput, this throughput will become less than that of the 1 relay case when SNR is high enough.

Fig. 4 shows the case when γ_{ss-r} decreases to 15dB. Here the $s-r$ channels are not good enough and therefore the performance of FCA deteriorates rapidly due to the effect of error propagation. When SNR is high which indicates a good $s-d$ channel, FCA even gets a worse throughput than the truncated ARQ scheme. In contrast, SCA still achieves the highest throughput among all the schemes and a significant gain can be observed.

We further consider the case of $\gamma_{ss-r} = 10$ dB, which indicates even worse $s-r$ channels. In this case, FCA cannot work since cooperative diversity gain is overwhelmed by the effect of severe error propagation. Its SER of retransmission is always worse than that of the truncated ARQ scheme. Therefore, as Fig. 5 shows, the throughput of FCA is much

[6]In this paper, we take the [3,4,3] STBC code given in [23] (pp. 2485, Eqn. (99)). Therefore, the rate R is 3/4.

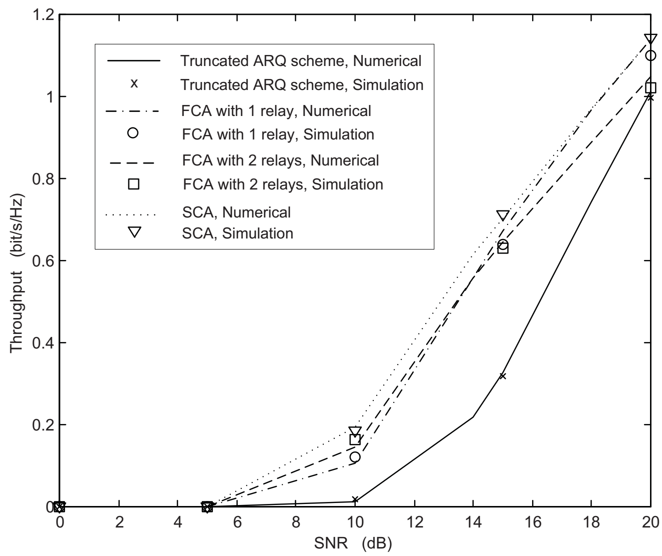

Fig. 3. Throughput of the truncated ARQ scheme, FCA with 1 relay, FCA with 2 relays and SCA when γ_{ss-r}=20dB with $L = 120$ and $b = 2$.

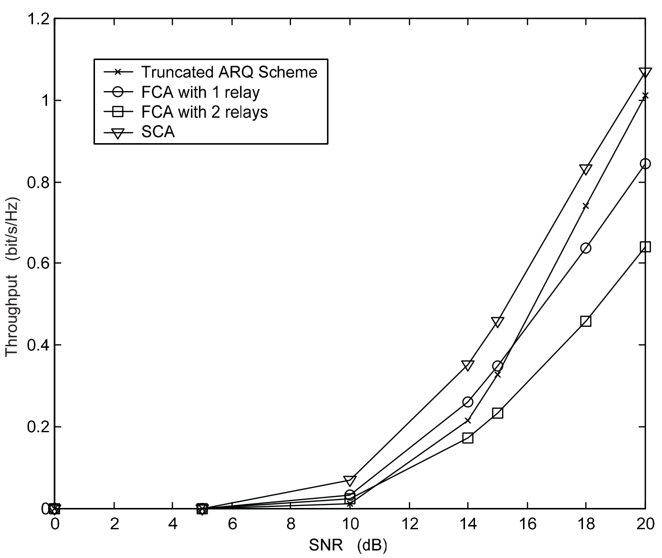

Fig. 4. Throughput of the truncated ARQ scheme, FCA with 1 relay, FCA with 2 relays and SCA when γ_{ss-r}=15dB with $L = 120$ and $b = 2$.

lower than that of the truncated ARQ scheme. SCA again has the best performance. However, due to the bad quality of $s-r$ channels, SCA seldom uses relays and therefore it has nearly the same throughput as the pure truncated ARQ scheme.

From the above discussion, we can conclude that SCA can always achieve a significant throughput gain irrespective of whether the $s-r$ channels are good or not. In contrast, the performance of FCA highly depends on the quality of the $s-r$ channels. With poor $s-r$ channels, the performance will deteriorate rapidly due to error propagation and may be even worse than the pure truncated ARQ scheme. This can be more clearly seen in Fig. 6, where the average SNR per symbol of the $s-d$ channel γ_{sd} is fixed to be 15dB. Here the x-axis is given by $\rho = \gamma_{ss-r}/\gamma_{sd}$. It can be seen that FCA can achieve a higher throughput than the truncated ARQ scheme only when ρ is larger than 1.5. This can give us some insights on the selection of cooperation region when

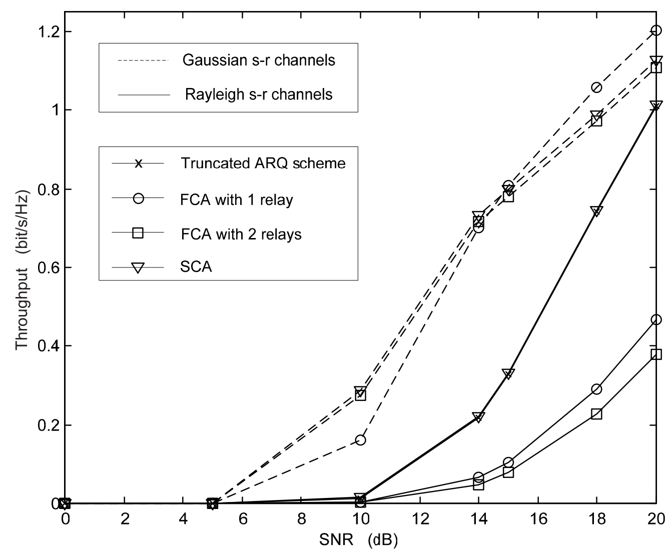

Fig. 5. Throughput of the truncated ARQ scheme, FCA with 1 relay, FCA with 2 relays and SCA with Gaussian $s-r$ channels and Rayleigh $s-r$ channels when $\gamma_{ss-r} = 10$ dB with $L = 120$ and $b = 2$.

fixed cooperative diversity is adopted. SCA again achieves the highest throughput and substantial gains can be observed for all the values of ρ.

From Fig. 6, it can be also seen that a higher ρ indicates a better performance gain of SCA or FCA over the truncated ARQ. In other words, more cooperative diversity gain can be achieved with a larger ratio of σ_s^2 to σ_0^2. Neglecting the effect of shadowing, this implies that the relays should be located close to the source node. Furthermore, it is found that the performance gain will be even more significant if fewer scatters exist between the source node and the relays. As shown in Fig. 5, with the assumption of Gaussian $s-r$ channels, substantial gains can be achieved by both SCA and FCA although γ_{ss-r} is only 10dB. In contrast, in Rayleigh fading $s-r$ channels, no performance gain can be obtained by SCA and FCA with the same γ_{ss-r}. Therefore, we conclude that Q relay candidates should be selected from those located around the source node so as to achieve better performance.

It should be noticed that in Gaussian $s-r$ channels, FCA with 1 relay performs the best among all the schemes at high SNR. A closer observation shows that in this case SCA is very likely to choose $Q = 2$ relays since the $s-r$ channels are with good quality. Therefore, it has very slight performance gain over FCA with 2 relays and both of them suffer from the rate loss of 3-symbols STBC.

IV. Throughput Optimization using Adaptive Techniques

The throughput expressions of the truncated ARQ scheme, FCA and SCA have been given by (9), (11) and (18), respectively. These expressions clearly depend on two important parameters: the packet length L and the modulation level b. As we know, a small packet length indicates that most packets arrive without errors but at the cost of a large packet overhead. A large L implies higher efficiency while the packet is more susceptible to errors which cause more retransmissions. As for the modulation level b, a packet with a low modulation level

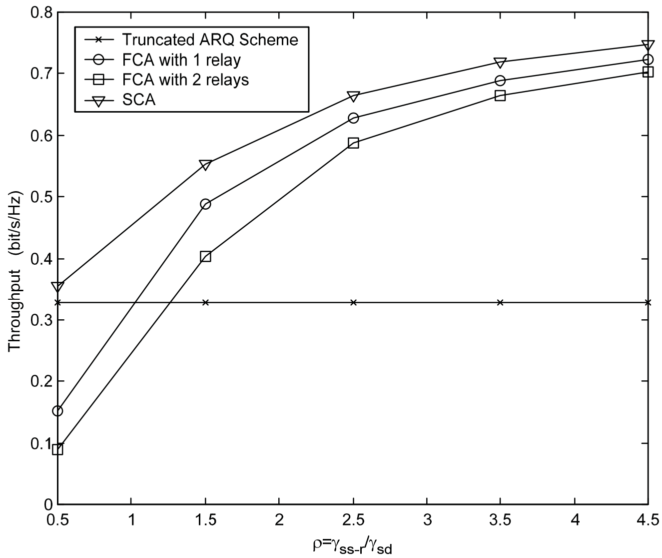

Fig. 6. Throughput of the truncated ARQ scheme, FCA with 1 relay, FCA with 2 relays and SCA when γ_{sd}=15dB with $L = 120$ and $b = 2$.

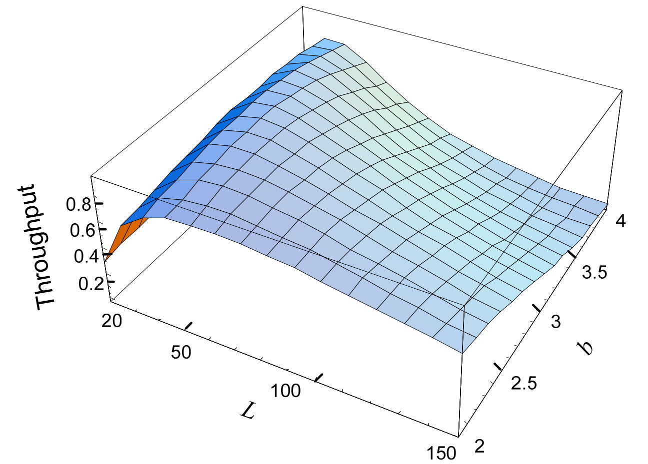

Fig. 7. Throughput plane of SCA versus L and b with γ_{sd}=15dB and γ_{ss-r}=20dB

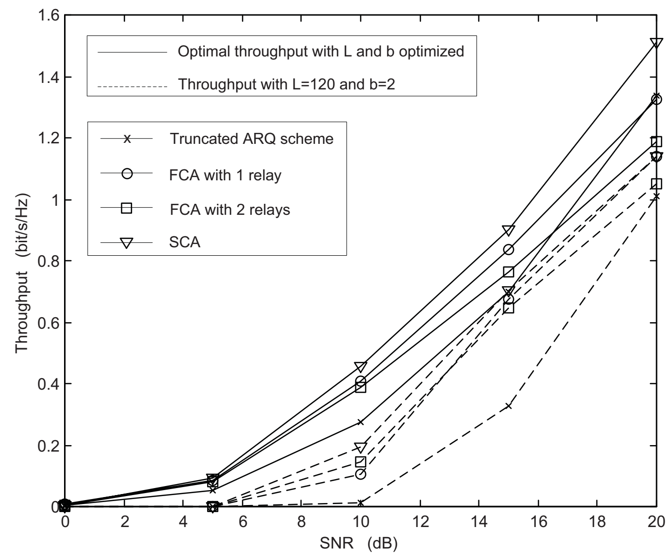

Fig. 8. Optimal throughput vs. throughput with fixed L and b of the truncated ARQ scheme, FCA with 1 relay, FCA with 2 relays and SCA when γ_{ss-r}=20dB

is more robust but may result in inefficient use of the channel. On the other hand, a packet with a high modulation level is more liable to error but carries more information per symbol. Therefore, an appropriate L or b is desired so as to improve the throughput.

In this section, we will further maximize the throughput by optimizing L and b. Since the throughput expressions are neither concave nor convex, an analytical solution for the optimal packet length L^* and modulation level b^* is hard to be obtained. Therefore, we resort to simulations to observe how throughput varies with L and b first. In this section, we always assume that both L and b take continuous values.

Fig. 7 shows the selected sample when SCA is adopted with γ_{ss-r}=20dB and γ_{sd}=15dB. It can be seen that the throughput plane is rather smooth, which indicates a good match between the local maximum and the global maximum. Actually, we have conducted extensive simulations of these three schemes, i.e., the truncated ARQ scheme, FCA and SCA, and found that the throughput plane is always smooth. Therefore, based on the above observation, we resort to the Method of Hooke and Jeeves [22] to obtain the optimal L^* and b^* as well as the maximum throughput. Fig. 8 shows the optimized throughput curves of the truncated ARQ scheme, FCA with 1 relay, FCA with 2 relays and SCA when γ_{ss-r} is 20dB. It can be seen that with the joint optimization of L and b, the throughput can be improved greatly whichever scheme is adopted. SCA again achieves the best throughput and substantial gains can be observed over all the other schemes. The throughput of FCA with 1 relay and 2 relays are also significantly improved by the joint optimization. However, in contrast to the case with a fixed L of 120 and a fixed b of 2, FCA with 2 relays obtains a lower optimal throughput than that of the truncated ARQ scheme at high SNR. A closer observation shows that in this case the optimal packet length L^* of these schemes is around 50, which is much lower than 120. At high SNR, the communication link provided by the $s-d$ channel is reliable enough for such a short packet. Therefore, although better diversity gain can be achieved by node cooperation, the rate loss plays a more important role here. On the other hand, this also indicates that $\rho = \gamma_{ss-r}/\gamma_{sd}$ should be large so as to assure that performance gain can be achieved by FCA. SCA always obtains the best performance regardless of the quality of $s-r$ channels. The results with a lower γ_{ss-r} are similar and so we omit them here.

We take the example of the truncated ARQ scheme to further show the throughput gains brought by the joint optimization of L and b. As Fig. 9 shows, the optimal throughput curve with both L and b optimized coincides with the suboptimal one with only L optimized (b is fixed to be 2) in the low SNR regime. At high SNR, the suboptimal throughput curve with only b optimized approaches that of the optimal one. This implies that at low SNR we could only optimize L to get an optimal throughput, whereas at high SNR, optimizing b can more effectively optimize the throughput. Besides, we compare the optimal packet length L^* with b fixed and that

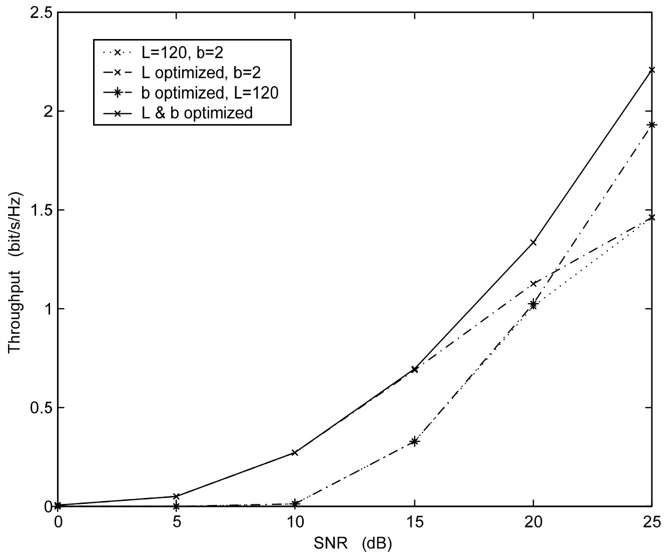

Fig. 9. Throughput optimization of the truncated ARQ scheme

TABLE I
OPTIMAL L^* AND b^* SEARCHED BY L-DOA FOR THE TRUNCATED ARQ, FCA AND SCA ($\gamma_{ss-r} = 20$ DB)

SNR (γ_{sd})		0dB	5dB	10dB	15dB	20dB
Truncated ARQ Scheme	L^*	20	22	30	42	40
($u = i$)	b^*	2	2	2	2	4
FCA with $v = 1$ relay	L^*	20	24	36	48	60
($u = 2i$)	b^*	2	2	2	2	2
FCA with $v = 2$ relay	L^*	24	24	36	54	60
($u = 3i$)	b^*	2	2	2	2	2
SCA with $Q = 2$ relay	L^*	24	24	36	48	48
($u = 6i$)	b^*	2	2	2	2	4

with b optimized. It is found that with b optimized, the optimal packet length L^* can be greatly decreased in the high SNR regime since in this case, we can use a higher modulation instead of increasing the packet length. On the other hand, the comparison of the optimal modulation level b^* with L fixed and that with L optimized shows that with L optimized, even higher modulation can be adopted so as to improve the throughput. We do not present the figures due to limited space.

V. Optimization over Discrete Parameters

In the above analysis, L and b are always assumed to be continuous. However, since square M-QAM is adopted, b should actually be even, i.e. $b = 2k$, $k = 1, 2, 3, \cdots$. Besides, L should be larger than C and be an integer multiple of b, i.e. $L > C$ and $L = ub$, where u is an integer. For SCA and FCA, there are even more constraints on L due to the adoption of STBC. For example, for FCA with 1 relay, u should be an integer multiple of 2 since a 2-symbol STBC is adopted in the retransmission. For SCA with $Q = 2$, u should further be an integer multiple of 6 since both 2-symbol and 3-symbol STBC may be adopted.

Exhaustive search over the discrete optimal L^* and b^* will result in prohibitive complexity. In this section, we present a computationally efficient algorithm to find the discrete optimal L^* and b^*. Particularly, in the new algorithm, we start searching the optimal packet length L^* with a fixed b=2 and for each incremental b, we compute the maximum throughput by optimizing L. The whole process will terminate when the throughput begins to fall. This is based on the observation that the local throughput and the global throughput always perfectly match when b varies and L is fixed or L varies and b is fixed. This new algorithm shall be referred to as *low-complexity discrete optimization algorithm* (L-DOA) and is described below.

Algorithm 1 Low-complexity discrete optimization algorithm (L-DOA))

Let $L = ub$ where u takes on different values for different schemes. $u = i$, $i = 1, 2, 3, \ldots$, for the truncated ARQ scheme, $u = 2i$ for FCA with 1 relay, $u = 3i$ for FCA with 2 relays, and $u = 6i$ for SCA with $Q = 2$. $T[l, b]$ represents the throughput with a packet length l and a modulation level b.
Initialization: $k = 1$, $T^* = 0$;
loop
    $b_k = 2k$. Find the optimal packet length l^* under the given b_k.
    Let $a = \lfloor l^*/(ub_k) \rfloor$.
    Let $L_1 = a \cdot ub_k$ and $L_2 = (a+1) \cdot ub_k$.
    Compare $T[L_1, b_k]$ and $T[L_2, b_k]$.
    if $T[L_1, b_k] \geq T[L_2, b_k]$ **then**
        $L_k = L_1$, $T_k = T[L_1, b_k]$;
    else
        $L_k = L_2$, $T_k = T[L_2, b_k]$;
    end if

    if $T_k > T^*$ **then**
        $T^* = T_k$, $L^* = L_k$, $b^* = b_k$;
    else
        stop;
    end if
    $k = k + 1$
end loop

It can be seen that with L-DOA, the joint optimization of L and b is decoupled so that the complexity can be reduced dramatically. Table 1 lists the discrete optimal values of L^* and b^* for the truncated ARQ scheme, FCA with 1 relay, FCA with 2 relays and SCA with $Q = 2$ under a γ_{ss-r} of 20dB. The corresponding throughput curves are plotted in Fig. 10. For comparison, the optimal throughput curves with continuous optimal L^* and b^* are also shown. It can be seen that a good match between the throughput curves with continuous optimal L^* and b^* and that with discrete optimal L^* and b^* can be achieved whichever scheme is adopted, indicating that the discretization of L^* and b^* results in very slight throughput loss. It also indicates the superior performance of L-DOA. A closer observation shows that a slight throughput loss will be incurred in the cases of SCA with γ_{sd}=15dB and FCA and the truncated ARQ scheme with γ_{sd}=20dB. Actually, this is because the optimal b^* is around 3 in these cases. By restricting b to be even, a throughput loss will be incurred. Nevertheless, the two corresponding throughput curves perform a perfect match in most cases. The corresponding results with the other values of γ_{ss-r} are similar and so we omit them here due to limited space.

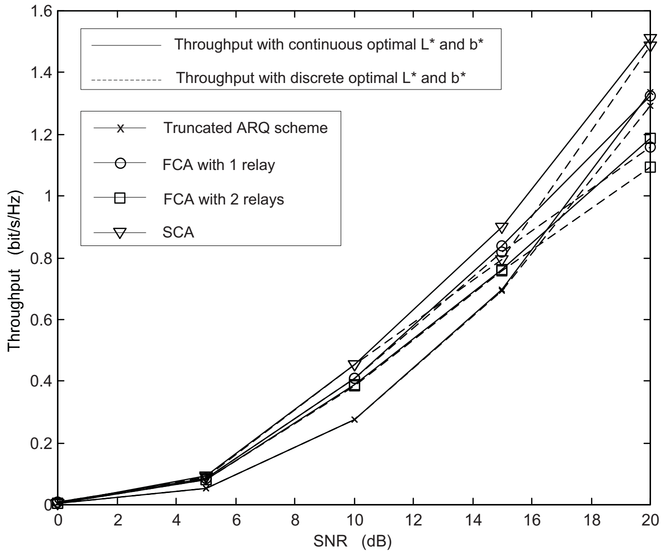

Fig. 10. Throughput with continuous optimal L^* and b^* and with discrete optimal L^* and b^* of the truncated ARQ scheme, FCA with 1 relay, FCA with 2 relays and SCA when γ_{ss-r}=20dB.

VI. Conclusions

In this paper, we proposed a cross-layer design combining truncated ARQ at the link layer and cooperative diversity at the physical layer, which has been shown to be able to greatly improve the system throughput. The throughput expressions of the proposed SCA scheme, FCA scheme and the pure truncated ARQ scheme are derived and verified by simulation results. The comparison of these three schemes showed that the proposed SCA scheme can always achieve the highest throughput and effectively avoid error propagation. The throughput is further maximized by optimizing the packet length L and the modulation level b and it was found that substantial gains can be achieved by this joint optimization. Besides, since both L and b are usually discrete and under some constraints in practice, we proposed a low-complexity discrete optimization algorithm (L-DOA) to search the discrete optimal L^* and b^*. It was then shown that the discretization of L^* and b^* results in very slight throughput loss.

It must be noted that the above performance analysis did not take into consideration some important effects such as the Doppler spread and frequency selectivity. Hence, extending this work to consider these effects is of potential interest. Another interesting extension of this work is to analyze the performance in multiuser scenarios with specific scheduling strategies.

Appendix A
Proof of Theorem 1

Let X denote an m-antenna-T-time-slot-k-symbol STBC. It can be then represented as

$$\mathbf{X} = [\mathbf{A_1 s} + \mathbf{B_1 s}^*, \mathbf{A_2 s} + \mathbf{B_2 s}^*, ..., \mathbf{A_T s} + \mathbf{B_T s}^*] \quad (26)$$

where $\mathbf{s}$ is a $k \times 1$ complex variable vector and $\mathbf{A_i}, \mathbf{B_i}$ are constant coefficient matrices in $\mathcal{R}^{m \times k}$. Assume n antennas at the receiver side. Then, the $n \times T$ receive signal matrix $\mathbf{Y}$ can be written as

$$\mathbf{Y} = \sqrt{\frac{SNR}{m}} \mathbf{HX} + \mathbf{N} \quad (27)$$

where $SNR = \sigma_0^2 P_t / N_0$, $\mathbf{H}$ and $\mathbf{N}$ are the $n \times m$ channel gain matrix and $n \times T$ noise matrix, respectively. All h_{ij}, $i = 1, \ldots, n$, $j = 1, \ldots, m$ and n_{ij}, $i = 1, \ldots, n$, $j = 1, \ldots, T$ are i.i.d. complex-valued Gaussian random variables with zero-mean and unit variance. After a series of linear transformation, we have

$$\mathbf{r} = \sqrt{\frac{SNR}{m}} \mathcal{H} \cdot \mathbf{s} + \mathbf{z} \quad (28)$$

where

$$\mathcal{H} = \begin{bmatrix} \mathbf{HA_1} + \mathbf{H}^* \mathbf{B_1}^* \\ \vdots \\ \mathbf{HA_T} + \mathbf{H}^* \mathbf{B_T}^* \end{bmatrix} \quad (29)$$

From the property of $\{\mathbf{A}_j\}_{j=1,\ldots,T}$ and $\{\mathbf{B}_j\}_{j=1,\ldots,T}$, we have

$$\mathcal{H}^+ \cdot \mathcal{H} = trace(\mathbf{H}^+ \mathbf{H}) \cdot \mathbf{I_k} \quad (30)$$

Therefore,

$$\tilde{\mathbf{r}} = \sqrt{\frac{SNR}{m}} \alpha \cdot \mathbf{s} + \tilde{\mathbf{z}} \quad (31)$$

where $\tilde{z}_i \sim \mathcal{N}_c(0, \alpha)$, $i = 1, ..., k$, and $\alpha = trace(\mathbf{H}^+ \mathbf{H})$. Obviously, is a central chi-square distributed variable with $2mn$ degrees of freedom. From (31), it can be seen that orthogonal space-time block decoding transforms a MIMO channel into multiple equivalent SISO channels. Therefore, $\mathbf{s}$ can be easily detected using a one-dimension MLD.

In FCA, v relay nodes and the source node use a $v + 1$ symbol STBC to send the signals together. Therefore, here we have $m = k = v + 1$ and $n = 1$. Note that in FCA, the relay nodes are pre-assigned and CRC will not be checked to assure reliable information forwarding. As a result, in the re-transmission only the source node transmits the original signal vector $\mathbf{s}$. At each relay, the estimate $\hat{\mathbf{s}}$ instead of the original signal vector $\mathbf{s}$ is transmitted, where $\hat{\mathbf{s}} = [\hat{s}_0, \hat{s}_1, ..., \hat{s}_v]'$. After decoding, the equivalent SISO channel model becomes

$$\tilde{\mathbf{r}} = \mu\alpha \cdot \mathbf{s} + \widehat{\mathbf{z}} \quad (32)$$

where $\mu = \sqrt{\gamma_{sd}/(v+1)}$ and α is a central chi-square distributed variable with $2(v + 1)$ degrees of freedom. We have

$$\widehat{\mathbf{z}} = \mu \mathcal{H}^+ \mathcal{H}_r \cdot \begin{bmatrix} \hat{s}_0 - s_0 \\ \hat{s}_1 - s_1 \\ \vdots \\ \hat{s}_v - s_v \end{bmatrix} + \tilde{\mathbf{z}} \quad (33)$$

and

$$\mathcal{H}_r = \begin{bmatrix} \mathbf{HA_{1r}} + \mathbf{H}^* \mathbf{B_{1r}}^* \\ \vdots \\ \mathbf{HA_{Tr}} + \mathbf{H}^* \mathbf{B_{Tr}}^* \end{bmatrix} \quad (34)$$

where $\mathbf{A_{ir}}$ ($\mathbf{B_{ir}}$) is obtained by replacing the first row of $\mathbf{A_i}$ ($\mathbf{B_i}$) with zeros, $i = 1, ..., T$. Here the new equivalent noise $\widehat{z}$ includes both the additive white noise $\tilde{\mathbf{z}}$ and the effect of error estimates. It can be further obtained that

$$\begin{aligned} var(\widehat{\mathbf{z}}) &= var(\hat{s}_i - s_i)\mathbf{I_{v+1}} \cdot \mathcal{H}^+ \mathcal{H}_r \mathcal{H}_r^+ \mathcal{H} \\ &+ \alpha \mathbf{I_{v+1}} \le \alpha \mathbf{I_{v+1}} \left(var(\hat{s}_i - s_i)\psi + 1 \right) \end{aligned} \quad (35)$$

where ψ is a central chi-square distributed variable with $2v$ degrees of freedom.

Let $D\left(s_i - \hat{s}_i\right)$ denote the distance between s_i and $\hat{s}_i$, then we have

$$var\left(\hat{s}_i - s_i\right) = D\left(s_i - \hat{s}_i\right) \cdot P\left(s_i \to \hat{s}_i\right). \tag{36}$$

Here we assume that errors always happen between two neighbors. Therefore, the minimum distance $D_{\min}\left(s_i - \hat{s}_i\right)$ is used instead of $D\left(s_i - \hat{s}_i\right)$. Let E_s denote the average energy of the constellation. From [21], it follows that for M-QAM symbols

$$D_{\min}\left(s_i - \hat{s}_i\right) = \frac{6E_s}{2^b - 1}. \tag{37}$$

Besides, we have assumed that the $s-r$ channel is flat Rayleigh fading. According to [21],

$$\begin{aligned} P\left(s_i \to \hat{s}_i\right) = &\, 2\left(1 - \frac{1}{\sqrt{2^b}}\right)\left(1 - \sqrt{\frac{g\gamma_{ss-r}}{1 + g\gamma_{ss-r}}}\right) \\ &+ \left(1 - \frac{1}{\sqrt{2^b}}\right)^2 \\ &\cdot \left[\frac{4}{\pi}\sqrt{\frac{g\gamma_{ss-r}}{1 + g\gamma_{ss-r}}}\arctan\left(\sqrt{\frac{1 + g\gamma_{ss-r}}{g\gamma_{ss-r}}}\right) - 1\right], \end{aligned} \tag{38}$$

where γ_{ss-r} is the average SNR per symbol of the $s-r$ channel and is given by $\gamma_{ss-r} = \sigma_s^2 P_t / N_0$.

Therefore, by combining (32-38), the instantaneous SNR per symbol of the retransmission, $\tilde{\gamma}_{sr}$, should be given by

$$\tilde{\gamma}_{sr} = \frac{\alpha\gamma_{sd}}{\frac{6}{2^b - 1}w\gamma_{sd} + (v+1)R} \tag{39}$$

where

$$w = \psi\left\{\begin{array}{c} 2\left(1 - \frac{1}{\sqrt{2^b}}\right)\left(1 - \sqrt{\frac{g\gamma_{ss-r}}{1+g\gamma_{ss-r}}}\right) + \left(1 - \frac{1}{\sqrt{2^b}}\right)^2 \\ \cdot\left[\frac{4}{\pi}\sqrt{\frac{g\gamma_{ss-r}}{1+g\gamma_{ss-r}}}\arctan\left(\sqrt{\frac{1+g\gamma_{ss-r}}{g\gamma_{ss-r}}}\right) - 1\right] \end{array}\right\}.$$

It can be easily checked that with a large v, $\sqrt{var(w)}/E(w) \to 0$, implying that the fluctuation around the mean of w can be neglected. Therefore, w is replaced by $E(w)$ and $\tilde{\gamma}_{sr}$ can be further written as $\tilde{\gamma}_{sr} = \frac{\alpha\gamma_{sd}}{a\gamma_{sd} + (v+1)R}$, where a is given by (17). Finally, from [20] the average SNR per symbol of the retransmission γ_{sr} can be obtained, as shown in (16).

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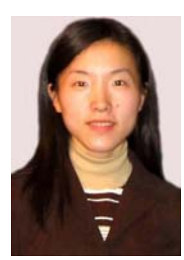

Lin Dai (S'00-M'03) received the B.S. in electronic engineering from Huazhong University of Science and Technology, Wuhan, China, in 1998 and the M.S. and Ph. D. degrees in electrical and electronic engineering from Tsinghua University, Beijing, China, in 2000 and 2002, respectively. She was a postdoctoral fellow at the Hong Kong University of Science and Technology and University of Delaware. Since 2007, she has been with City University of Hong Kong, where she is an assistant professor. Her research interests include wireless communications, information and communication theory. She received the Best Paper Award at IEEE WCNC 2007.

Khaled B. Letaief (S'85-M'86-SM'97-F'03) received the BS degree *with distinction* in Electrical Engineering from Purdue University at West Lafayette, Indiana, USA, in December 1984. He received the MS and Ph.D. Degrees in Electrical Engineering from Purdue University, in August 1986, and May 1990, respectively. From January 1985 and as a Graduate Instructor in the School of Electrical Engineering at Purdue University, he has taught courses in communications and electronics.

From 1990 to 1993, he was a faculty member at the University of Melbourne, Australia. Since 1993, he has been with the Hong Kong University of Science and Technology where he is currently Chair Professor and Head of the Electronic and Computer Engineering Department. He is also the Director of the Hong Kong Telecom Institute of Information Technology. His current research interests include wireless and mobile networks, Broadband wireless access, OFDM, Cooperative networks, Cognitive radio, MIMO, and Beyond 3G systems. In these areas, he has published over 300 journal and conference papers and given invited keynote talks as well as courses all over the world.

Dr. Letaief served as consultants for different organizations and is the founding Editor-in-Chief of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He has served on the editorial board of other prestigious journals including the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS - WIRELESS SERIES (as Editor-in-Chief). He has been involved in organizing a number of major international conferences and events. These include serving as the Co-Technical Program Chair of the 2004 IEEE International Conference on Communications, Circuits and Systems, ICCCS'04; General Chair of the 2007 IEEE Wireless Communications and Networking Conference, WCNC'07; as well as the Technical Program Co-Chair of the 2008 IEEE International Conference on Communication, ICC'08.

In addition to his active research and professional activities, Professor Letaief has been a dedicated teacher committed to excellence in teaching and scholarship. He received the *Mangoon Teaching Award* from Purdue University in 1990; the Teaching Excellence Appreciation Award by the School of Engineering at HKUST (4 times); and the Michael G. Gale Medal for Distinguished Teaching (*Highest university-wide teaching award* and only one recipient/year is honored for his/her contributions).

He is a Fellow of IEEE, an elected member of the IEEE Communications Society Board of Governors, and an IEEE Distinguished lecturer of the IEEE Communications Society. He also served as the Chair of the IEEE Communications Society Technical Committee on Personal Communications and is currently serving as the Chair of the Steering Committee of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He is the recipient of the 2007 *IEEE Communications Society Publications Exemplary Award.*