Mon. Not. R. astr. Soc. (1988) **231**, 823–831

Tidal enhancement by a binary companion of stellar winds from cool giants

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Accepted 1987 October 14. Received 1987 October 1; in original form 1987 July 13

Summary. Certain RS CVn binaries show 'mass inversion' (i.e. the more evolved star being the less massive) before Roche lobe overflow (RLOF) has taken place. We find that the mass loss rates of ordinary giants are too small by a large factor to produce the observed masses of Z Her, the best example of this phenomenon. We postulate that the originally less massive secondary has tidally enhanced the rate of mass loss by stellar wind from the primary and investigate a simple model of this enhancement over a wide range of initial mass ratios and periods. We find that the period/mass ratio space divides into three regions: (i) no Roche lobe overflow occurs before a white dwarf is produced leading to systems like FF Aqr and AY Cet; (ii) RLOF occurs after the mass ratio q has been reduced from $q_0>1$ to $q<0.7$ after which slow (nuclear time-scale) mass transfer takes place allowing Algols to be formed from late Case B systems without drastic mass transfer; (iii) RLOF occurs while $q>0.7$ resulting in rapid (hydrodynamic time-scale) mass transfer probably leading to common envelope evolution and possibly to coalescence.

1 Introduction

Several binaries of the RS CVn type are eclipsing double-lined systems with rather well determined masses (Popper & Ulrich 1977; Popper 1980). Most of these systems consist of a red subgiant (~K0IV) and an F or G main-sequence star; the masses are nearly equal with the more evolved star usually being slightly more massive. This greater mass is in accordance with expectation, since even the larger more evolved star is usually smaller than its Roche lobe by a factor of 2 or more. But in table 6 of Popper's magisterial review, three systems out of 20 have a primary (by which we mean, unconventionally, the originally more massive star, i.e. the one which is more evolved) which is less massive than the secondary, by ~3–10 per cent. Among these three is Z Her:

(K0IV + F5IV, $1.10+1.22\,M_\odot$, $2.6+1.6\,R_\odot$, $4+4\,L_\odot$; 3.99 day).

We concentrate on this system because it is the only one of the three in which Popper's tabulated data are not qualified by a colon indicating uncertainty.

Even the secondary in Z Her is quite evolved: the zero age main-sequence radius for a $1.22\,M_\odot$ star is $\sim 1.0\,R_\odot$, for a fairly normal population I composition of $X=0.7$, $Z=0.02$. A glance at an isochrone for 3.5 Gyr (Morgan & Eggleton 1978, fig. 3) suggests that with a colour of $B-V=0.47$ (Popper 1980) it is at or very near the 'hook' produced by central hydrogen exhaustion. In fact both components of Z Her fit fairly comfortably on this isochrone with masses of 1.29 and $1.23\,M_\odot$. This suggests that mass loss is concentrated strongly towards the Hayashi track, so that the secondary has lost very little mass while the primary has lost ~15 per cent of its initial mass. Since it requires roughly 1 Gyr for the primary to have evolved to its present colour from the point where it once had the secondary's present colour, the mass loss rate averaged over this interval must have been $\sim 2\times 10^{-10}\,M_\odot\,\mathrm{yr}^{-1}$. We emphasize that this appears to us to be a rather well-determined number; it would be difficult to vary either the amount of mass that must have been lost, or the time in which it must have been lost, by any substantial amount without doing considerable violence to otherwise acceptable stellar modelling.

Reimers (1975) measured mass loss rates from circumstellar lines in the spectra of red giants. He found that an empirical fit to the mass loss rates is

$$\dot{M} = -4\times 10^{-13}\,\frac{RL}{M}\,(M_\odot\,\mathrm{yr}^{-1}), \tag{1}$$

with R, L and M in solar units. For the primary of Z Her, with parameters as listed in the first paragraph, we obtain $\sim 4\times 10^{-12}\,M_\odot\,\mathrm{yr}^{-1}$. Since this $|\dot{M}|$ should increase with evolution, the average over the previous 1 Gyr must have been still less. Thus equation (1) gives too small an $|\dot{M}|$ by at least a factor of 50. But equation (1), despite the fact that its functional form was produced from dimensional arguments (it assumes that a fixed fraction of the star's luminosity goes into lifting mass out of that star's potential well) and that the empirical coefficient is uncertain by at least a factor of 2 (Reimers 1975), cannot be very far wrong for single giants since it accounts well for the fact that white dwarf masses are typically $0.6-0.7\,M_\odot$, while their precursor masses must be typically 1–2 $M_\odot$.

An obvious explanation of the discrepancy is that mass loss is enhanced by the presence of a binary companion. This would presumably be through the agency of tidal friction and dynamo activity. Whereas a single giant would tend to spin down as it evolves (partly because its moment of inertia increases and partly because its wind, corotating to some Alfvén radius, brakes the star magnetically), a giant in a binary can be expected to spin up when its radius becomes large enough, relative to its Roche lobe radius, for tidal friction to become important. In Section 2 we use a very simple and heuristic model of enhancement, which we calibrate by reference to Z Her.

If mass loss by red giants can be enhanced by two orders of magnitude through the presence of a binary companion, then the consequences for the evolution of moderately close binaries, in particular those expected to undergo late Case B evolution (requiring $P\sim 2$–20 day, depending on the primary mass in the range 1–3 $M_\odot$) are profound. The primary may quite easily lose its entire red giant envelope before it even expands to its Roche lobe radius, and may thus avoid Roche lobe overflow altogether. We explore this possibility in Section 3, and in Section 4 we present our conclusions.

2 A simple model for enhanced mass loss

In this paper we attempt to avoid the complexity of the physical processes involved in enhanced mass loss by assuming simply that the rate of mass loss from a single star will be augmented by a factor which depends on some power of the ratio R/R_L, where R is the stellar radius and R_L is the radius of the Roche lobe around it. According to Zahn (1975) and Campbell & Papaloizou

(1983), the torque in tidal friction depends on $(R/R_{\rm L})^6$, and so we make a guess that the tidal enhancement of $\dot{M}$ can be modelled by Reimers' formula with an extra term:

$$\dot{M} = -4\times 10^{-13}\,\frac{RL}{M}\left\{1+B\times\min\left[\left(\frac{R}{R_{\rm L}}\right)^6,\ \frac{1}{2^6}\right]\right\}, \tag{2}$$

where R, L and M are in solar units and the time is in yr. We include saturation at $R=\frac{1}{2}R_{\rm L}$ since we expect the giant to be in complete corotation by then. The parameters of Z Her already quoted give $R/R_{\rm L}\sim 0.5$. Hence if $|\dot{M}|$ is to be increased by a factor in excess of 50 we require $B \geqslant 3000$.

To obtain a more definite value for B, we use the stellar evolution code of Eggleton (1971), somewhat updated, to follow the evolution of two stars of initial masses 1.30 and 1.22 $M_\odot$, both subject to equation (2). We did not attempt anything as sophisticated as a least-squares fit, because our first attempt gave very reasonable agreement. With $B=10^4$, after 3.41 Gyr the parameters of the system were

$$(1.14+1.22\,M_\odot,\ 2.7+1.8\,R_\odot,\ 4.1+4.1\,L_\odot).$$

These are to be compared with the observed parameters quoted in Section 1. That B is larger by a factor ~ 3 than our previous lower limit is a consequence of the fact that the current $|\dot{M}|$ must be somewhat larger than its average over the last ~ 1 Gyr.

We now investigate the effect on late Case B evolution of enhanced stellar wind as indicated by equation (2) with $B\sim 10^4$. Rather than compute any more models in detail, we use a number of rules of thumb for red giant evolution based on detailed stellar modelling. We deal only with the situation where the primary is already on the Hayashi track when significant mass loss (let alone mass transfer) begins.

For population I Hayashi-track stars, we find that radius as a function of luminosity is reasonably well approximated by

$$R=(10^{-0.555}\,L^{0.4}+10^{-0.0833}\,L^{0.667})\,M^{-0.27} \tag{3}$$

in solar units; while for red giants and supergiants with degenerate cores luminosity and core mass as functions of time are reasonably well approximated by

$$10^{11}\,\dot{M}_{\rm c}=L=\frac{10^{6.06}\,M_{\rm c}^{7.62}}{1+10^{1.16}\,M_{\rm c}^{5.62}+10^{1.26}\,M_{\rm c}^{6.62}}, \tag{4}$$

where $M_{\rm c}$ and L are in solar units and the time is in years. Equations (2)–(4) almost entirely determine the evolution of the primary. We need only a further pair of equations to govern the Roche lobe radius $R_{\rm L}$ and the orbital separation d:

$$\frac{R_{\rm L}}{d}=\frac{0.49\,q^{2/3}}{0.6\,q^{2/3}+\ln(1+q^{1/3})} \tag{5}$$

and

$$(M+M')\,d=\text{const},\qquad (M+M')^2P=\text{const}. \tag{6}$$

Equation (5) is from Eggleton (1983), and equation (6), involving the primary mass M and the secondary mass M' $(M/M'=q)$ indicates how the separation and period vary as one (or both) components lose mass by stellar wind.

Equation (6) assumes that the mass lost by a component removes from the system the same specific angular momentum as resides in the component's orbital motion. Thus it requires that (i) the star is small compared with the separation, and (ii) the wind is not magnetically linked to the primary out to distances comparable to the separation. The latter is probably the least satisfactory, but we defer discussion of this point until later. Equations (2)–(6) can be integrated rather easily by a stepwise procedure. However, one further input, the appropriate starting condition, is needed. We begin with $t=0$ at the base of the giant branch, where we approximate the core mass M_c by that value that gives a red giant luminosity equal to that of the zero age main-sequence star given by

$$L=\frac{10^{3.8}M^{2.5}+10^{4.3}M^{5.5}}{10^{4.4}+10^{3.6}M^{2.5}+M^{4.5}}. \tag{7}$$

The evolution approximations considered in this paper do not account for subgiant evolution before the shell burning phase is established. This is important in the case of Z Her but becomes less important for more massive stars and larger periods. Nor do they account for later stages in the evolution when the envelope has become very thin and equations (2) and (3) no longer apply.

Note also that Zahn (1975) finds the tidal synchronization rate to be proportional to $(1/q^2)(R/d)^6$ and in obtaining formula (2) we have used the approximation $R_L/d \propto q^{1/3}$ which is relatively accurate for $q \leqslant 1$. We feel that using $(R/R_L)^6$ for the full range of q is more suitable in this application but once again emphasize that the formula is only an educated guess.

3 Results

According to the simple stellar evolution considered here we would expect a lone red giant to lose mass from its envelope, in accordance with Reimers' formula, whilst its core grows as a nuclear burning shell moves outwards until its entire envelope has been lost and the core remains as a white dwarf. By considering single giants in this way reasonable masses are obtained for white dwarf remnants: for example, single stars of initial masses 1, 2 and 4 $M_\odot$ leave remnants of 0.419, 0.574 and 0.862 $M_\odot$ respectively. If the star is a component of a binary system this process will be interrupted if the giant fills its Roche lobe and overflows. This overflow can be of two types:

(i) If the mass ratio is still large enough ($q>0.7$) then mass exchange will cause the giant to overfill its Roche lobe even more resulting in catastrophic overflow on a hydrodynamic time-scale.

(ii) If, however, enough mass has been lost already and $q<0.7$ the star will continue to just fill its Roche lobe, since the lobe expands more than the star as the star loses mass but the star expands owing to nuclear burning, and overflow will proceed on the much slower nuclear time-scale.

We have investigated this by setting $B=0$ in the mass loss equation (equation 2) and evolving until either the star fills its Roche lobe, $R=R_L$, or it has lost its entire envelope, $M=M_c$. We illustrate the results over a range of initial periods and mass ratios for an initial primary mass of 2 $M_\odot$ by plotting a graph of initial period against initial mass ratio (Fig. 1) and distinguishing three regions:

(i) a white dwarf is formed before RLOF;
(ii) slow (nuclear time-scale) RLOF occurs;
(iii) rapid (hydrodynamic time-scale) RLOF occurs.

As expected, red giants in very close binaries always fill their Roche lobes very quickly and hence with $q>0.7$ and in wide binaries they never fill them before losing their envelopes. In addition stars of high mass ratio form white dwarfs after their cores have already exceeded 0.7 times the mass of the secondary: for a 2 $M_\odot$ star we have $M_{c,\,tms} \approx 0.2\,M_\odot$ and thus the core mass is already greater than the secondary mass when $q>2\times 0.7/0.2=7$. Consequently we find a wedge shaped region between the other two in which slow overflow occurs.

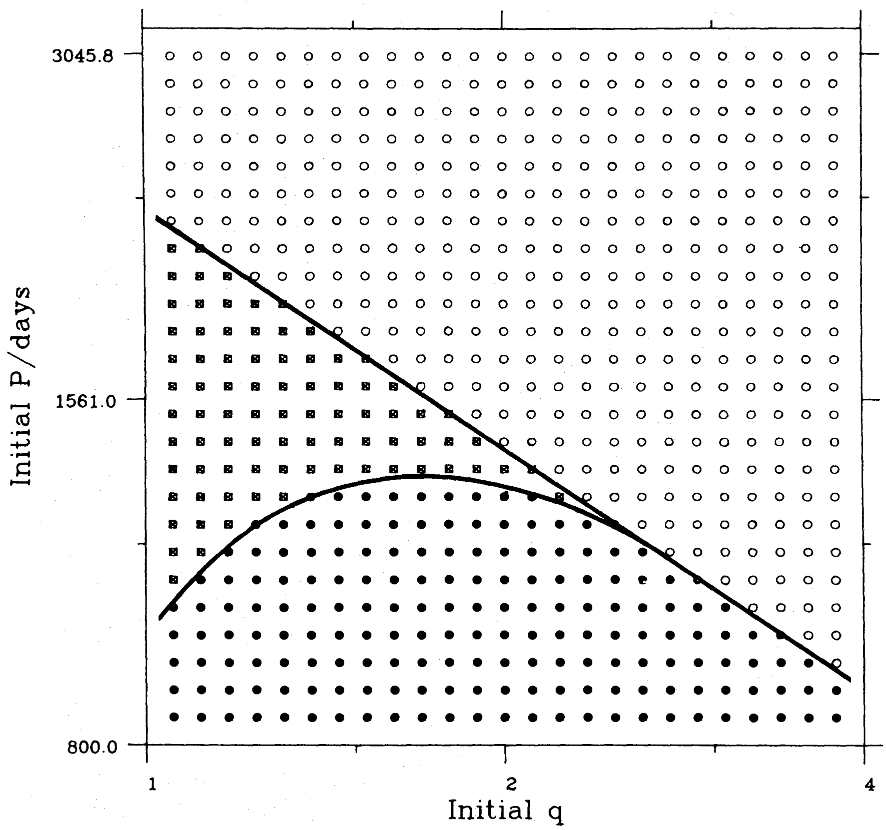

Figure 1. A plot showing the outcome of close binary evolution without enhanced mass loss (i.e. $B=0$ in equation 2) for an initial primary mass of $2\,M_\odot$ for various initial periods (P) and initial mass ratios ($q>1$). Open circles indicate that the envelope is lost before Roche lobe overflow (RLOF) begins; crossed squares indicate that RLOF begins while $q>0.7$; and filled circles indicate that RLOF begins while $q>0.7$.

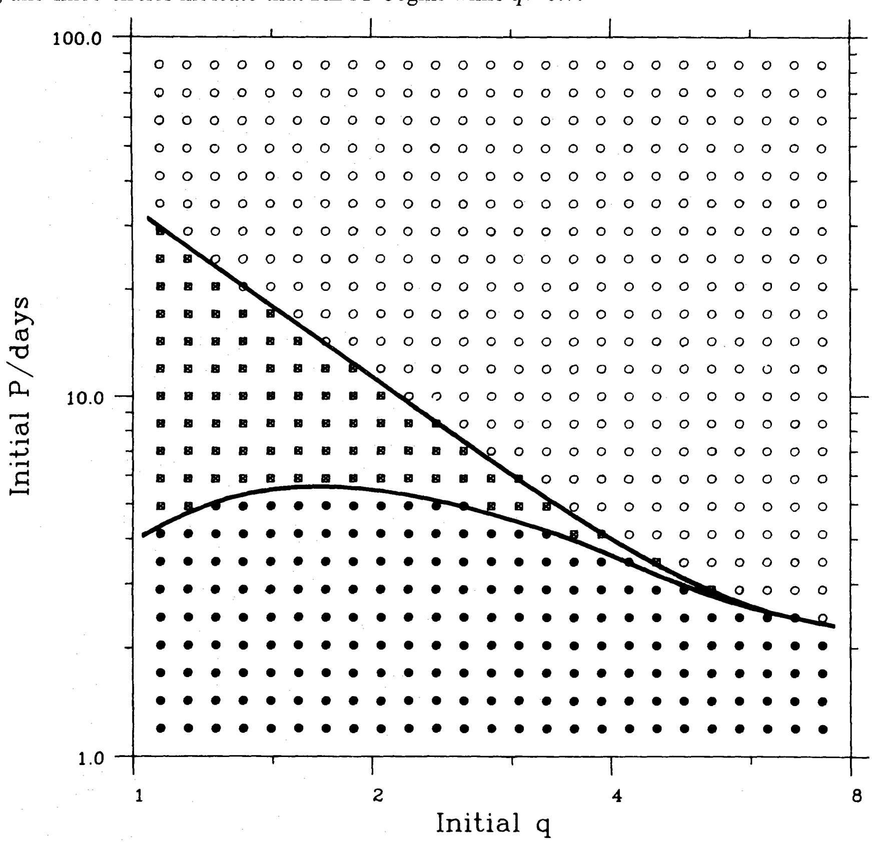

Figure 2. Similar to Fig. 1, but with binary-enhanced mass loss ($B=10\,000$). The interesting boundaries occur at much shorter periods.

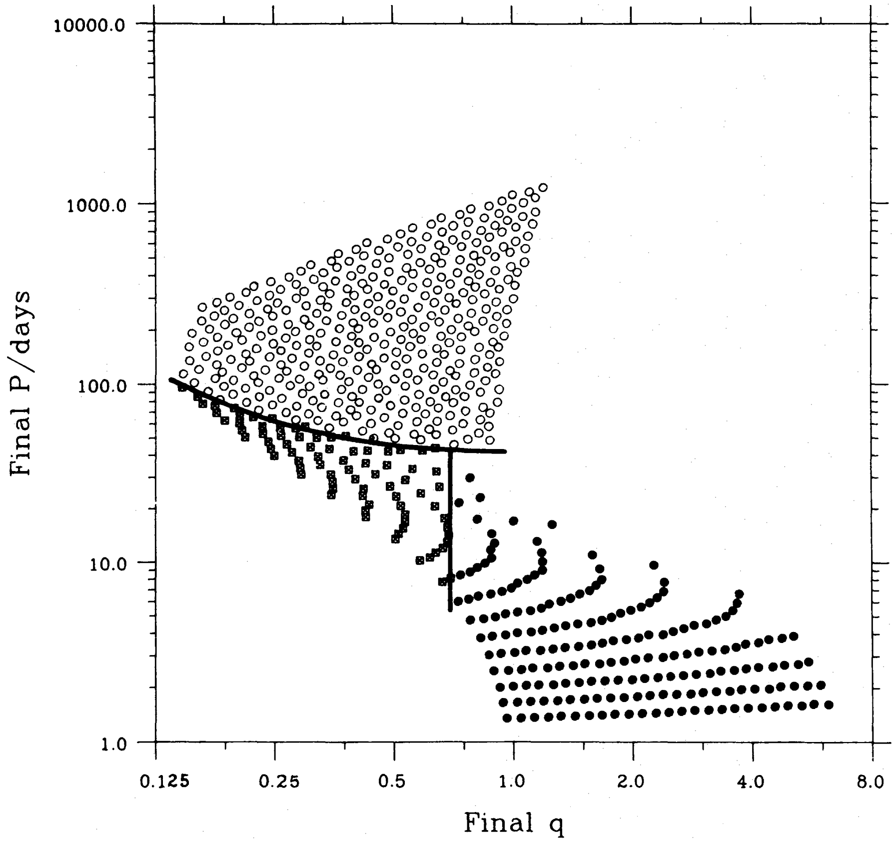

Figure 3. As Fig. 2 but plotting *final* period against *final* mass ratio.

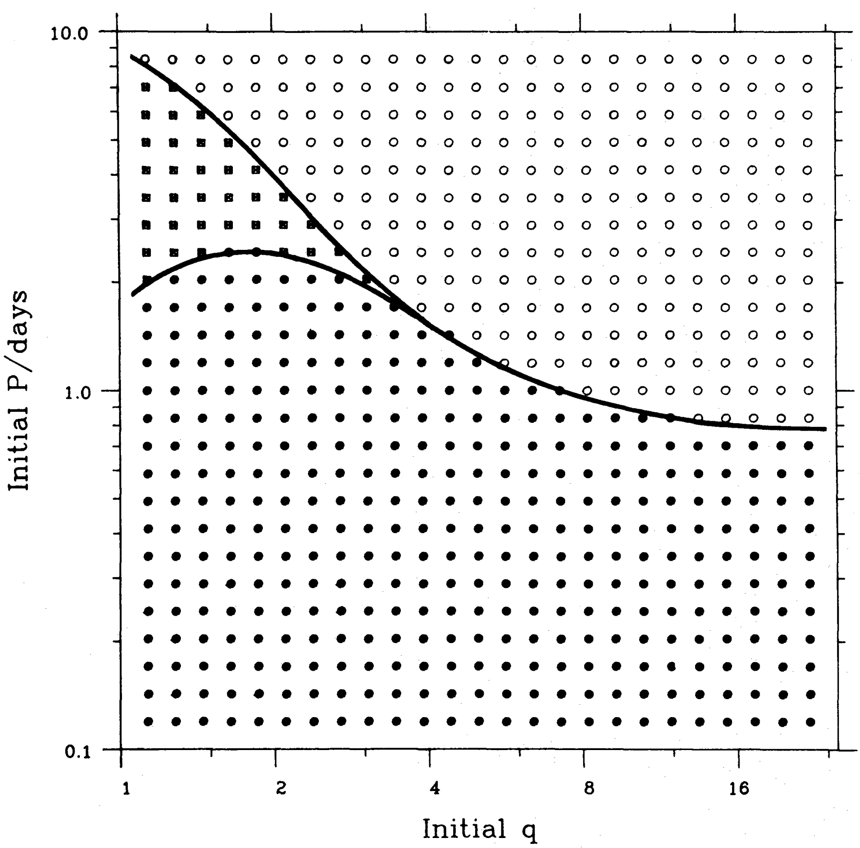

Figure 4. As Fig. 2 but for a $1.3\,M_\odot$ primary.

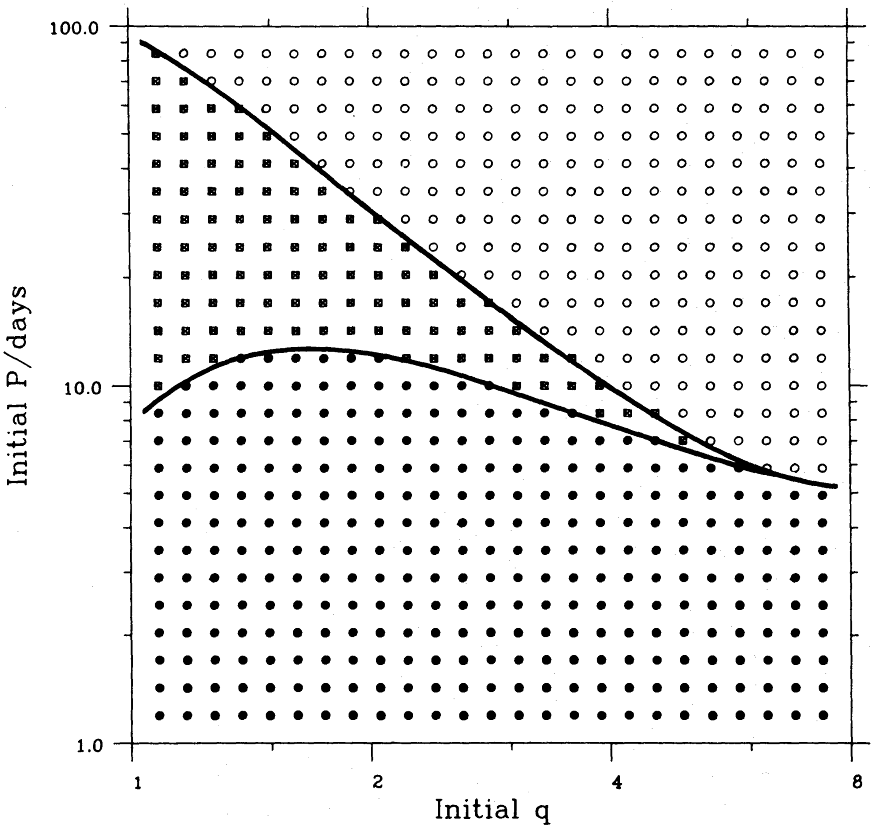

Figure 5. As Fig. 2 but for a $3\,M_\odot$ primary.

Now if enhanced mass loss is included at a rate consistent with our detailed model of Z Her we find this wedge shaped region moved down in period by about two orders of magnitude (Fig. 2). We now find that a star of two solar masses must be in a binary of period less than $\sim$40 day if it is to fill its Roche lobe at all and must have a period less than $\sim$6 day to overflow catastrophically. We also include a graph of final period against final mass ratio (Fig. 3) where final refers to the point at which we stop the evolution – that is:

(i) when the entire envelope is lost for stars that do not fill their Roche lobes, and
(ii) when the star fills its Roche lobe for those that do.

We find that there is a minimum final period of $\sim$50 day if the star is never to have filled its Roche lobe. Conversely we find that stars in binaries with periods that have already exceeded $\sim$400 day will never fill their Roche lobes. Similarly if the period is greater than $\sim$120 day the star will never overflow catastrophically.

We also present plots for a $1.3\,M_\odot$ (Fig. 4) and a $3\,M_\odot$ (Fig. 5) initial primary mass to illustrate the effect on the regions of varying this mass. We find that the wedge shaped region moves to higher period as the primary mass is increased. This is expected since a more massive star will evolve more rapidly and consequently have less time to lose mass before filling its Roche lobe.

4 Discussion

By comparing Figs 1 and 2 we deduce that enhancement of mass loss can drastically affect the evolution of a binary system. We note that this enhancement can be used to explain existing problems such as the formation of Algols and some types of precataclysmic variables as well as the observed mass inversion in RS CVn type stars.

The simple solution to the Algol paradox, that the initially more massive star in a close binary evolves to become a giant that fills its Roche lobe and consequently transfers enough matter to the initially less massive secondary to become the much less massive star, suffers from the problem that a red giant that is more massive than its companion should, on filling its Roche lobe, lose mass catastrophically, owing both to the expansion of its own envelope and also to the contraction of its Roche lobe. Such a process is very likely to lead to a common envelope scenario and probably to coalescence. If, however, by a process such as we are postulating the giant can lose enough mass to reduce the mass ratio to $q<0.7$ before filling its Roche lobe the transfer will proceed on the much slower nuclear time-scale. This will allow both stars to respond gradually and therefore will not radically alter the nature of the system in any way other than the transfer of mass. Systems that would evolve in this way would lie in the wedge shaped regions on our graphs. We note that without enhanced mass loss the periods of these systems would in general be >1000 day whereas with enhanced mass loss they are reduced to between 1 and 10 day for the $1.3\,M_\odot$ primary, 5 and 100 day for the $2\,M_\odot$ and 10 and 300 day for the $3\,M_\odot$. The periods of typical Algols are found to lie in these ranges (Popper 1980).

The application to cataclysmic variables is in the formation of precataclysmic systems with both a low mass white dwarf and a low mass red dwarf. Common-envelope evolution (Paczyński 1976) can probably account for some cataclysmic variables and some pre-cataclysmic binaries such as the close binary nuclei of certain planetary nebulae. However, it would have difficulty in accounting for any system which had a low-mass ($\leqslant 0.7\,M_\odot$) white dwarf or a low-mass ($\leqslant 0.2\,M_\odot$) red dwarf companion. 'Star-planet' evolution (Livio & Soker 1984) might explain systems with low mass red dwarf companions but still with a rather massive white dwarf. However, the interesting pre-cataclysmic binary AA Dor ($\equiv$LB3459) is an eclipsing sdO + low-mass (black?) dwarf, with estimated masses $0.2\,M_\odot + 0.04\,M_\odot$ and period 0.26 day (Conti, Dearborn & Massey 1981). It is hard to see how this could be formed by either the common-envelope or the star-planet scenario since both appear to require the primary to be a supergiant that would consequently have a core mass $\geqslant 0.7\,M_\odot$. DS1 (Drilling 1985) is another but less extreme case. Here the subdwarf's mass is again uncomfortably low, at 0.4–0.7 $M_\odot$, for either of these evolutionary paths. Now if we consider a system that has an initial primary mass of $1.3\,M_\odot$ and a mass ratio of $q=25$ so that the secondary has a mass of $\sim 0.05\,M_\odot$ and evolve the primary with our enhanced mass loss we find that we can produce a white dwarf with mass $\sim 0.25\,M_\odot$ and final period ~ 10 day without any mass transfer. We might expect a process of magnetic braking (Verbunt 1984) to reduce this period to that of AA Dor and eventually bring the system into a cataclysmic state within the age of the universe provided that the magnetic fields involved are large enough.

In conclusion, we again stress that the simple calculations discussed in this paper show that a process of mass loss enhancement owing to a binary companion is important in the evolution of multiple systems. To obtain more reliable quantitative results the physics of the wind generation, the importance of magnetic fields and the effect of tides must be considered in more detail. In particular we expect the matter lost from the star to corotate to some Alfvén radius (Mestel & Spruit 1987) and consequently carry off more angular momentum than we have assumed. This will drive the stars closer together, decreasing the size of the Roche lobes and consequently causing overflow to occur earlier, moving our region boundaries to higher initial periods. This process is currently under investigation.

Acknowledgments

CAT is very grateful for the financial support of the Science and Engineering Research Council while carrying out this work.

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