

# Tide-induced groundwater fluctuation in a coastal leaky confined aquifer system extending under the sea

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**Abstract.** This paper presents the analytical solution of groundwater response to tidal fluctuation in a coastal multilayer aquifer system consisting of a leaky confined aquifer, a semipermeable layer, and an unconfined aquifer. The semipermeable layer, or the roof of the leaky confined aquifer, extends under the sea. Comparisons are made with solutions derived previously, which considered only a single aquifer, zero extension of the roof, or infinite roof length, and demonstrate that the previous solutions are special cases of the solution presented here. A hypothetical example is used to discuss the impact of the dimensionless roof length, dimensionless leakage, and tidal efficiency on the groundwater level fluctuations in the inland part of the confined aquifer. The fluctuation decreases significantly with the roof length when the roof length is small but is insensitive to the change of roof length when the roof length is greater than a threshold. The impacts of leakage from the offshore and inland portions of the confining unit are different. Leakage from the offshore portion tends to increase the fluctuation of the groundwater level, while leakage from the inland portion tends to decrease the fluctuation. The fluctuation increases as the tidal efficiency increases. This fact is significant only when the roof length is great and leakage is small.

## 1. Introduction

With social and economic development in coastal areas, various coastal hydrogeological, engineering, and environmental problems arise. Among them are, for example, seawater intrusion, stability of coastal engineering structures, beach dewatering for construction purposes, and deterioration of the marine environment [Farrell, 1994; Svitil, 1996; Delwyn *et al.*, 1998; Akpofure *et al.*, 1984; Carr and Van der Kamp, 1969]. Solving these problems very often requires studies on the coastal hydrogeological conditions, which include aquifer parameter estimation and interaction between groundwater and seawater.

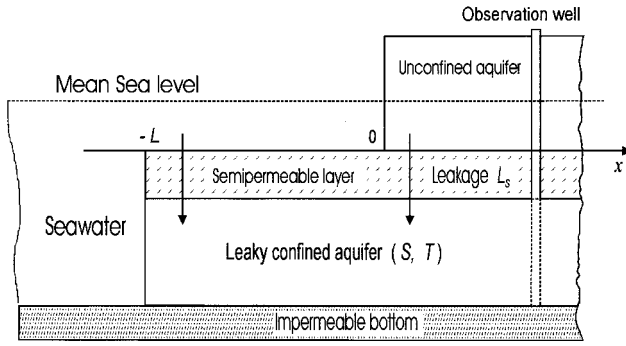
The study on the dynamic relation between seawater and coastal groundwater has become an active research area since the 1950s [e.g., Jacob, 1950; Ferris, 1951; Carr and Van der Kamp, 1969; Pandit *et al.*, 1991; Sun, 1997]. Most of the researchers assumed that there was only one aquifer. Jiao and Tang [1999] considered a multilayer system consisting of an unconfined aquifer, a leaky confined aquifer, and a semiconfining unit between them. Their study showed that the leakage significantly damps the fluctuations of groundwater level in the confined aquifer. Therefore, in the case where there is considerable leakage from the unconfined aquifer the lack of tidal fluctuations cannot be considered to be indicative of poor hydraulic connection between the confined aquifer and the seawater. These previous studies were all based on the assumption that the boundary of the aquifer coincides with the coastal line determined by the mean sea level. In reality, however, this assumption is not always valid because the roof of the aquifer,

i.e., the overlying confining layer, may extend for a certain distance under the sea owing to complicated coastal environments of sedimentation. It is of great importance to estimate the roof length since it represents a key boundary condition in the coastal groundwater flow model for seawater intrusion studies or coastal groundwater resources evaluation. Under the extreme assumption that the roof length is infinite, Van der Kamp [1972] derived a solution to describe the groundwater fluctuation in the aquifer. Li and Chen [1991a] considered the situation where the roof length is finite. Both studies assumed that there is no leakage from the confining layer. Li and Chen [1991b] took into account the leakage from the seawater through the offshore part of the confining unit. So far, there is no analytical solution which can consider the leakage of both seawater through the offshore part of the confining layer and the groundwater from the unconfined aquifer through the inland part of the confining layer. In this paper, an exact analytical solution is derived to describe groundwater level fluctuation in a leaky confined aquifer with a roof extending under the sea. The solution is a generalization of the solutions by Li and Chen [1991b] and Jiao and Tang [1999]. It has advantages over the solution by Li and Chen [1991b] in the sense that it includes the leakage from the entire roof and over that by Jiao and Tang [1999] in the sense that it considers the roof extending under the sea. An attempt is made to understand the impact of roof length, leakage, and tidal efficiency on groundwater level fluctuations.

## 2. Conceptual Model and Analytical Solution

Consider a coastal aquifer system consisting of an unconfined aquifer, a leaky confined aquifer, and a semipermeable layer between them (Figure 1). The unconfined aquifer terminates at the coast while the semipermeable layer extends over a certain distance under the sea. All the layers extend landward infinitely. The bottom of the leaky confined aquifer is imper-

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**Figure 1.** An idealized leaky confined aquifer system extending under the sea.

meable. The study here uses the assumption introduced by *Jiao and Tang* [1999] that the shallow unconfined aquifer has a large specific yield which can effectively damp the tidal effect so that the tidal fluctuation in the unconfined aquifer is negligible compared to that in the confined aquifer. This assumption is supported by numerous field studies by previous researchers [e.g., *White and Roberts*, 1994; *Millham and Howes*, 1995; *Chen and Jiao*, 1999], and detailed discussion is given by *Jiao and Tang* [1999]. On the basis of this assumption the water table of the unconfined aquifer is assumed to be constant and equal to the mean sea level, which is chosen as the datum for the hydraulic head in this paper.

Let the  $x$  axis be positive landward and perpendicular to the coastal line with their intersection being the origin of the axis (Figure 1). All the layers are horizontal, homogeneous, and of constant thickness. Assume that the flow in the confined aquifer is horizontal and that there is vertical leakage through the semiconfining unit with negligible storage. These assumptions were introduced by *Hantush and Jacob* [1955] in their benchmark paper about leaky aquifer systems and used by *Jiao and Tang* [1999] in coastal aquifer systems. It is further assumed that the density difference between the groundwater and the seawater can be neglected owing to its slight impact on groundwater level fluctuation [*Li and Chen*, 1991a]. According to these assumptions and the theories of leaky, elastic aquifers proposed by *Hantush and Jacob* [1955] and *Jacob* [1950] the mathematical models of *Jiao and Tang* [1999] and *Li and Chen* [1991a] can be combined directly to obtain the following new model to describe the groundwater fluctuations in the conceptual model shown in Figure 1:

Offshore aquifer

$$S \frac{\partial h}{\partial t} = T \frac{\partial^2 h}{\partial x^2} + ST_e \frac{dh_s}{dt} + L_s(h_s - h), \quad (1)$$

$$-\infty < t < +\infty, \quad -L < x < 0,$$

Inland aquifer

$$S \frac{\partial h}{\partial t} = T \frac{\partial^2 h}{\partial x^2} - L_s h, \quad -\infty < t < +\infty, \quad x > 0, \quad (2)$$

Boundary and continuity conditions

$$h(-L, t) = h_s(t) = A \cos(\omega t), \quad (3)$$

$$\lim_{x \downarrow 0} h(x, t) = \lim_{x \uparrow 0} h(x, t), \quad (4)$$

$$\lim_{x \downarrow 0} \frac{\partial h}{\partial x} = \lim_{x \uparrow 0} \frac{\partial h}{\partial x}, \quad (5)$$

$$h(+\infty, t) = 0, \quad (6)$$

where  $h(x, t)$ ,  $S$ , and  $T$  are hydraulic head [ $L$ ], storativity (dimensionless), and transmissivity [ $L^2 T^{-1}$ ] of the aquifer, respectively;  $L_s$  is specific leakage [ $T^{-1}$ ] of the semiconfining layer defined as  $K_1/b_1$  with  $b_1$  and  $K_1$  denoting the thickness [ $L$ ] and vertical hydraulic conductivity [ $L T^{-1}$ ] of the semipermeable layer, respectively;  $T_e$  is tidal efficiency (dimensionless) [*Jacob*, 1950];  $h_s(t)$  is hydraulic head of the sea tide [ $L$ ];  $A$  is amplitude [ $L$ ] of the tidal change;  $\omega$  is angular velocity (or frequency) [ $T^{-1}$ ] of tide and equals  $2\pi/t_0$  with  $t_0$  being the tidal period [ $T$ ], the time between high and low tides [*Todd*, 1980].

Equation (1) indicates that the groundwater level fluctuation in the portion of the leaky confined aquifer extending under the sea is caused by (1) its elastic compression and expansion due to the tidal loading rate of the seawater above the offshore part of the aquifer ( $ST_e(dh_s/dt)$ ), (2) the leakage from the overlying seawater ( $L_s(h_s - h)$ ), and (3) the sea tidal fluctuation at the boundary  $x = -L$  as expressed by (3). Equation (2) shows that the groundwater level fluctuation in the inland part of the leaky confined aquifer depends on the leakage ( $-L_s h$ ) from the unconfined aquifer. Both parts of the confined aquifer are interrelated through the continuity conditions of hydraulic head and groundwater flux set by (4) and (5), respectively. Equation (6) states that the tide has no effect far inland as  $x$  approaches infinity.

For convenience of discussion, two new parameters, the confined aquifer's tidal propagation parameter  $a[L^{-1}]$  and the dimensionless leakage  $u$ , are introduced:

$$a = \sqrt{\omega S / 2T} = \sqrt{\pi S / T t_0}, \quad (7a)$$

$$u = L_s / \omega S. \quad (7b)$$

Then the solution to the boundary value problem in (1)–(6) can be written as follows (see Appendix 1 for the derivation):

$$h(x, t) = A e^{-pax} \left[ \left( R_1 + \frac{\lambda}{2} \right) \cos(\omega t - qax) + \left( I_1 - \frac{\mu}{2} \right) \sin(\omega t - qax) \right] \\ = AC_e e^{-pax} \cos(\omega t - qax - \varphi), \quad x \geq 0, \quad (7c)$$

$$h(x, t) = A e^{-pax} [R_1 \cos(\omega t - qax) + I_1 \sin(\omega t - qax)] \\ + \lambda A \cos(\omega t) - \mu A \sin(\omega t) - \frac{1}{2} A e^{pax} [\lambda \cos(\omega t + qax) \\ + \mu \sin(\omega t + qax)], \quad -L < x < 0, \quad (7d)$$

where

$$0 < C_e = \sqrt{(R_1 + \lambda/2)^2 + (I_1 - \mu/2)^2} \leq 1 \quad (7e)$$

is dimensionless and defined as the comprehensive tidal efficiency of the leaky confined aquifer system. It is obvious that  $C_e > 0$ , but it is not so obvious that  $C_e \leq 1$ , the proof of which can be found in Appendix 2. In (7e),  $\lambda$  and  $\mu$  are two dimensionless constants defined as

$$\lambda = \frac{u^2 + T_e}{u^2 + 1}, \quad \mu = -\frac{(1 - T_e)u}{u^2 + 1}, \quad (7f)$$

$R_1$  and  $I_1$  are also dimensionless and given by

$$R_1 = e^{-paL}[(1 - \lambda) \cos(qaL) - \mu \sin(qaL)] + \frac{1}{2}e^{-2paL}[\lambda \cos(2qaL) + \mu \sin(2qaL)], \quad (7g)$$

$$I_1 = e^{-paL}[(1 - \lambda) \sin(qaL) + \mu \cos(qaL)] + \frac{1}{2}e^{-2paL}[\lambda \sin(2qaL) - \mu \cos(2qaL)]; \quad (7h)$$

$p$  and  $q$  are determined by the dimensionless leakage  $u$  and equal

$$p = \sqrt{1 + u^2} + u, \quad q = \sqrt{1 + u^2} - u; \quad (7i)$$

$\varphi$  is the fixed phase shift (dimensionless) defined by

$$\varphi = \arctan \frac{2I_1 - \mu}{2R_1 + \lambda}. \quad (7j)$$

Here the word “fixed” means that  $\varphi$  is a constant independent of  $x$  and  $t$ .

From (7a)–(7j) it can be seen that four independent parameters, the confined aquifer’s tidal propagation parameter  $a$ , dimensionless leakage  $u$ , roof length  $L$ , and tidal efficiency  $T_e$ , are involved in the model. Equation (7c) shows that the groundwater fluctuation at a fixed inland location  $x_0$  is also sinusoidal. It can be determined by the groundwater fluctuation amplitude  $A_{x0}[L]$  and the time lag  $t_{\text{lag}}[T]$  of groundwater response to tidal fluctuation, that is,

$$h(x_0, t) = A_{x0} \cos[\omega(t - t_{\text{lag}})]. \quad (8)$$

Comparison of (8) and (7c) yields

$$A_{x0} = AC_e(u, aL, T_e) \exp[-p(u)ax_0], \quad (9a)$$

$$t_{\text{lag}} = \frac{1}{\omega} [q(u)ax_0 + \varphi(u, aL, T_e)]. \quad (9b)$$

The parameters in the parentheses after  $C_e$ ,  $p$ ,  $q$ , and  $\varphi$  highlight their dependency relationship. Theoretically, by solving (9a) and (9b), any two unknowns among the four parameters  $a$ ,  $u$ ,  $L$ , and  $T_e$  can be estimated if the other two are given.

### 3. Discussion of the Solution

Equations (7c) and (7d) are the solutions for the groundwater heads in inland and offshore parts of the aquifer, respectively. Since most field studies on coastal aquifers focus on the inland part of the aquifer and there may be no observation of groundwater heads available in the offshore area, only (7c) is discussed in this paper.

#### 3.1. Comparison With Existing Solutions

From (7f), (7i), and (7b) one has

$$\begin{aligned} \lambda|_{L=0} &= T_e, & \mu|_{L=0} &= 0, \\ p|_{L=0} &= p|_{u=0} = q|_{L=0} = q|_{u=0} = 1. \end{aligned} \quad (10)$$

Using (7g) and (7h) leads to

$$R_1|_{L=0} = 1 - \frac{1}{2}\lambda, \quad I_1|_{L=0} = \frac{1}{2}\mu; \quad (11)$$

$$\lim_{L \rightarrow +\infty} R_1 = \lim_{L \rightarrow +\infty} I_1 = 0. \quad (12)$$

On the basis of (7c) and (10)–(12), the following discussion can be made.

If  $L_s = 0$ ,  $L = 0$ , then in view of (10), (11), and (7c), one finds that

$$h(x, t) = Ae^{(-ax)} \cos(\omega t - ax), \quad x \geq 0, \quad (13)$$

which is essentially the same as the equation introduced by *Jacob* [1950] with  $a$  defined as (7a), considering that in the original equation of *Jacob* [1950], only the cosine functions both in solution (13) and boundary condition (3) are replaced by sine functions. In this case, the groundwater fluctuation is determined by the single parameter  $a$  related to the confined aquifer. This is why  $a$  is called the aquifer’s tidal propagation parameter.

If  $L_s = 0$ ,  $L = +\infty$ , then in view of (10), (12), and (7c), it follows that

$$h(x, t) = \frac{1}{2}T_e Ae^{-ax} \cos(\omega t - ax), \quad x \geq 0, \quad (14)$$

which is the result of the work by *Van der Kamp* [1972].

If  $L = 0$ , then in view of (11) and (7c), one has

$$h(x, t) = Ae^{-pax} \cos(\omega t - qax), \quad x \geq 0, \quad (15)$$

which equals the result derived by *Jiao and Tang* [1999].

When  $L_s = 0$ , by means of (10) and (7c), it follows that

$$h(x, t) = AC_0 e^{-ax} \cos(\omega t - ax - \varphi_0), \quad x \geq 0, \quad (16a)$$

where

$$\begin{aligned} C_0 &= \{[e^{-aL}(1 - T_e) \cos(aL) + \frac{1}{2}T_e(1 + e^{-2aL} \cos(2aL))]^2 \\ &\quad + [e^{-aL}(1 - T_e) \sin(aL) + \frac{1}{2}T_e e^{-2aL} \sin(2aL)]^2\}^{1/2}, \end{aligned} \quad (16b)$$

$$\varphi_0 = \arctan$$

$$\frac{2(1 - T_e) \sin(aL) + T_e e^{-aL} \sin(2aL)}{2(1 - T_e) \cos(aL) + T_e [e^{aL} + e^{-aL} \cos(2aL)]}. \quad (16c)$$

The above solution (equation (16a)) should be the same as that by *Li and Chen* [1991a] because the assumptions used here are exactly the same as those in their paper. However, it is different from their solution (see equations (21b) and (21e) of *Li and Chen*). In order to find out the discrepancy both the derivations presented here and those of *Li and Chen* [1991a] are examined carefully. It appears that their result is questionable because their solution defined by their equations (21a), (21c), and (21d) does not satisfy the boundary condition (4b) [see *Li and Chen*, 1991a, pp. 99, 101]. Further examination shows that the mistakes occurred [*Li and Chen*, 1991a, pp. 100, 101] when the constants  $C_1$ ,  $C_2$ , and  $C_3$  were determined. A major problem is that  $C_1$ ,  $C_2$ , and  $C_3$  should be complex numbers but were mistaken as real numbers in their paper. Similar mistakes were also made by *Li and Chen* [1991b].

#### 3.2. Approximation for Small Roof Length

If the dimensionless parameter  $paL$  ( $\geq qaL$ , as can be seen from (7i)) is small enough for its terms of high order greater than or equal to two to be neglected, that is, if

$$e^{-paL} \approx 1 - paL, \quad e^{-2paL} \approx 1 - 2paL, \quad (17a)$$

$$\cos(qaL) \approx \cos(2qaL) \approx 1, \quad (17b)$$

$$\sin(qaL) \approx qaL, \quad \sin(2qaL) \approx 2qaL,$$

then the complex expressions of  $C_e$  and  $\varphi$  defined by (7e), (7j), (7g), and (7h) can be simplified as follows:

$$C_e \approx \sqrt{(1 - paL)^2 + (qaL)^2} \approx e^{-paL}, \quad (18a)$$

$$\varphi = \arctan \frac{qaL}{1 - paL} \approx \frac{qaL}{1 - paL} \approx qaL. \quad (18b)$$

Substituting (18a) and (18b) back into (7c) yields

$$h(x, t) = A \exp[-pa(x + L)] \cos[\omega t - qa(x + L)], \quad (19)$$

$$x \geq 0.$$

Comparing (19) with (15), where the roof length is zero, it is found that when the roof length  $L$  is small enough, the groundwater fluctuation in the inland aquifer behaves as if the coastal line were extended seaward to the end of the roof, where  $x = -L$ . Numerically, if  $paL \leq 0.1$ , then  $qaL \leq paL \leq 0.1$ ; hence the approximate equalities of (17a), (17b), and (18a) will hold with relative errors of  $\sim 2\%$ , and the approximate equalities of (18b) will hold with absolute error of  $\sim 1\%$ . This accuracy is believed to be acceptable for the practical use of (19). Hence a threshold value of  $L$  can be chosen to be

$$L_L = 0.1/ap(u), \quad (20)$$

so that if  $L \leq L_L$ , the end of the roof can be regarded as the coastal line. As can be seen from (18a) and (18b), when  $L \leq L_L$ , both  $C_e$  and  $\varphi$  are independent of the tidal efficiency  $T_e$ , and the number of independent parameters of the aquifer system is reduced to only three, i.e.,  $a$ ,  $L$ , and  $u$ . This situation can be easily explained by the mechanism responsible for the groundwater level fluctuations in this aquifer system: The fluctuation caused by the sea tidal loading term  $T_e S(dh_s/dt)$  in (1) can be neglected if the roof length  $L$  is very small.

### 3.3. Approximation for Great Roof Length

If  $L = +\infty$ , then in view of (7c), (7e), (12), (7f), and (7j), it follows that

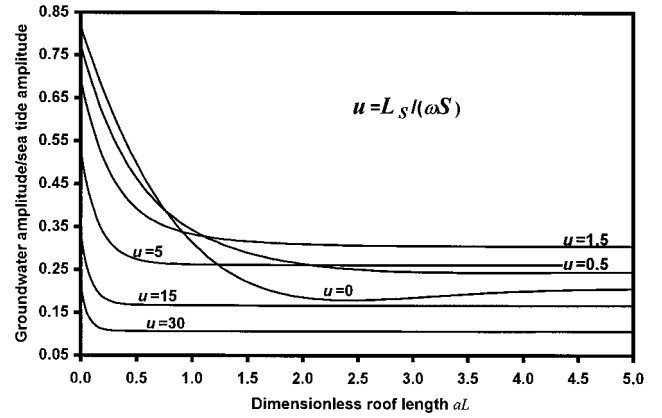
$$h(x, t) = C_\infty A e^{-pax} \cos(\omega t - qax - \varphi_\infty), \quad x \geq 0, \quad (21a)$$

where

$$C_\infty = \lim_{L \rightarrow +\infty} C_e = \frac{1}{2} \sqrt{\lambda^2 + \mu^2} = \frac{1}{2} \sqrt{\frac{u^2 + T_e^2}{u^2 + 1}}, \quad (21b)$$

$$\varphi_\infty = \lim_{L \rightarrow +\infty} \varphi = \arctan \frac{(1 - T_e)u}{u^2 + T_e}. \quad (21c)$$

Equations (21a), (21b), and (21c) are helpful in analyzing the influence of the roof length on the inland groundwater fluctuation. For a real aquifer system, there exists a finite threshold value  $L_U$  of the roof length  $L$ . When  $L \geq L_U$ , both the comprehensive tidal efficiency  $C_e$  and the phase shift  $\varphi$  will be close to  $C_\infty$  and  $\varphi_\infty$ , respectively, so that the groundwater fluctuation from (21a) is approximately the same as that from (7c). Under this circumstance the tidal propagation in the inland aquifer will behave as if the roof length  $L$  were infinite. Obviously, the threshold value  $L_U$  depends upon the detection limit of the water level measurement device, the parameters  $a$ ,  $u$ , and  $T_e$  of an actual aquifer system. An attempt has been



**Figure 2.** Change of ratio of groundwater level to sea tide amplitude at inland location  $ax_0 = 0.2$  with dimensionless roof length for different leakage when  $T_e = 0.5$ .

made to derive an equation similar to (20) to approximate the threshold value  $L_U$ . However, the equation is found to be very complicated and is not included here because of space limitation.

### 3.4. Discussion of the Analytical Solution by Hypothetical Example

A hypothetical example is used to demonstrate how the inland groundwater fluctuations are influenced by the roof length, leakage, and tidal efficiency. Two important parameters, tidal propagation parameter  $a$  and dimensionless leakage  $u$  (see (7a) and (7b)), are introduced in this paper. On the basis of the nine sets of aquifer parameters obtained from various pumping tests in real leaky confined aquifer systems [Jiao and Tang, 2001], the typical ranges of  $a$  and  $u$  are estimated to be  $10^{-3}$  to  $10^{-1} \text{ m}^{-1}$  and 0 to 10, respectively. For this hypothetical study a larger range of  $u$  (0–30) is used. The sea tide is assumed to be diurnal with an angular frequency  $\omega = 0.253 \text{ h}^{-1}$ , and the dimensionless distance ( $ax_0$ ) of the piezometer at the inland area is 0.2 from the coastline. The relative fluctuation and amplitude of the groundwater level, which are defined as  $h(x_0, t)/A$  and  $A_{x_0}/A$ , respectively, will be discussed in sections 3.4.1 and 3.4.2.

**3.4.1. Effect of roof length and leakage.** As shown in Figure 2, for any dimensionless leakage the amplitude at  $ax_0 = 0.2$  is reduced significantly as the dimensionless roof length  $aL$  increases when it is small. This is because the propagation of the sea tide at the boundary  $x = -L$  into the aquifer will be damped considerably as the roof length increases. When  $aL$  is greater than a certain value, the amplitude approaches a constant and is no longer sensitive to the roof length. As discussed in section 3.3, there is a threshold value of roof length. When the roof length is greater than that threshold value, the water level fluctuation will behave as if the roof length were infinite.

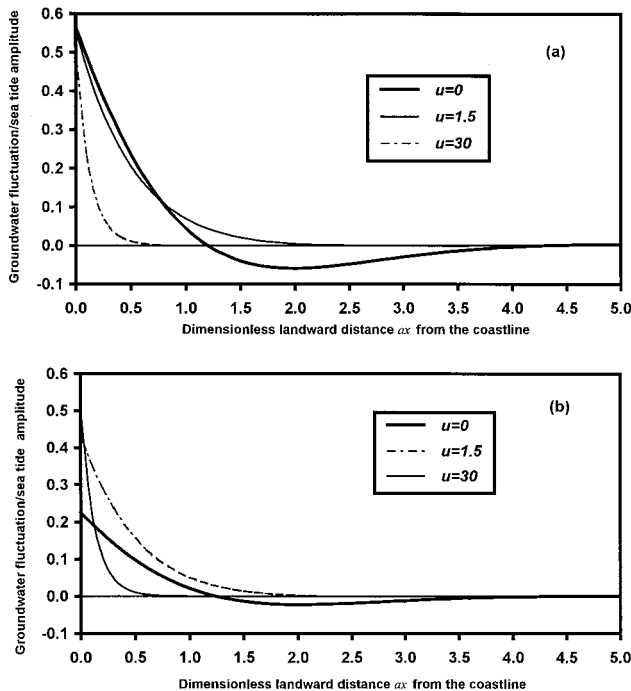
Figure 2 also shows that the dimensionless leakage  $u$  is an important damping factor to the amplitude of the fluctuation in the observation well when  $u$  is great. For small  $u$  the impact of leakage on the amplitude is complicated. The curve for  $u = 0$  indicates that the amplitude first decreases quickly and then increases slightly as dimensionless roof length increases. In the absence of leakage the water level fluctuation is controlled mainly by two factors: the boundary condition at  $x = -L$  and the compression due to the change of the loading of seawater above the roof. A longer roof length will create greater com-



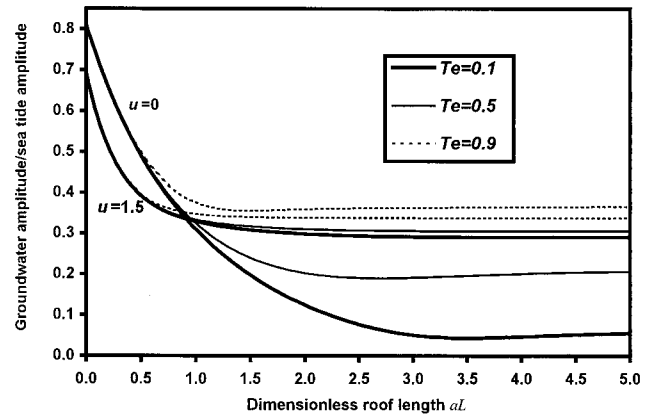
pression and then larger groundwater fluctuation. This also indicates that the compression effect is significant only when the leakage is very small and the roof length sufficiently great.

The amplitude shown in Figure 2 increases as  $u$  is increased from 0 to 1.5, but it decreases as  $u$  is increased further. This is because the effects of the leakage from the inland and offshore portions of the semipermeable layer are different. The leakage from the inland portion tends to damp the water level fluctuation in the confined aquifer because the water table in the unconfined aquifer is constant, while the leakage from the offshore part will enhance the fluctuation by transferring the sea wave into the confined aquifer. When the dimensionless leakage is small and the roof length is great, the enhancing effect due to the offshore leakage is dominant. Therefore the amplitude of the fluctuation in the confined aquifer shown in Figure 2 increases with leakage. However, when the leakage is great, the fluctuation in the confined aquifer will be overwhelmingly controlled by the inland leakage, and consequently, larger leakage will lead to smaller fluctuations.

Figure 3 shows clearly the impact of roof length and leakage on the fluctuations in the inland aquifer. As shown in Figure 3a, when  $aL = 0.5$ , the fluctuation is progressively smaller as dimensionless leakage  $u$  increases. For  $u = 0$  the tidal fluctuation is evident at the dimensionless distance of  $ax = 3$ . However, when  $u$  is increased to 30, fluctuations may be undetectable for the dimensionless distance  $>0.5$ . Figure 3a shows that when  $aL$  is small, the impact of leakage on fluctuations is the same as that given by *Jiao and Tang* [1999], who did not consider the roof length. Figure 3b shows the fluctuations inland when the dimensionless roof length  $aL$  is increased to 1.8. In the case of long roof length but no leakage ( $u = 0$ ) the confined aquifer virtually has no hydraulic con-



**Figure 3.** Change of ratio of inland groundwater fluctuation to sea tide amplitude with dimensionless landward distance from coastline at  $t = 0$  for different leakage when tidal efficiency  $T_e = 0.5$ , and when dimensionless roof length  $aL =$  (a) 0.5 and (b) 1.8.



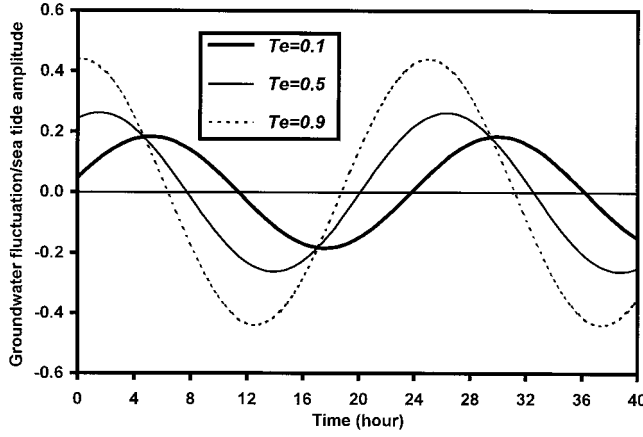
**Figure 4.** Change of ratio of groundwater level to sea tide amplitudes with dimensionless roof length  $aL$  at inland location  $ax_0 = 0.2$  for different tidal efficiency  $T_e$  and dimensionless leakage  $u$ .

nection with the sea. The tidal fluctuation is transferred to the aquifer only through the compression due to the loading rate of the seawater above the roof. The fluctuation is small but propagates far inland. When  $u$  is slightly increased to  $u = 1.5$ , the amplitude is increased significantly because the tidal fluctuation of the sea can be transferred to the aquifer through leakage over the roof under the sea. However, the distance over which the inland aquifer is disturbed by the sea tide is considerably reduced owing to the damping effect of the leakage in the inland area. When  $u$  is further increased to 30, the damping effect of the inland leakage is dominant, and the influential distance of the sea tide into the land is even shorter.

**3.4.2. Effect of tidal efficiency.** The tidal efficiency reflects the fluctuation of groundwater level caused by compression of both the aquifer skeleton and groundwater due to the loading rate of seawater above the roof of the confining layer (see equation (1)). Figure 4 shows how the amplitude in the piezometer at  $ax_0 = 0.2$  changes with the dimensionless roof length and dimensionless leakage for different tidal efficiency. There are two sets of curves in Figure 4, with  $u = 0$  and 1.5, respectively. For each set, three curves with  $T_e = 0.1, 0.5, 0.9$  are shown. The impact of the tidal efficiency on fluctuation is significant only when the leakage is small and the roof length is great. When  $aL$  is  $<0.8$ , the amplitude is insensitive to the change of the tidal efficiency. When  $aL$  is great and  $u = 0$ , the relative amplitude is sensitive to the change of  $T_e$ . When  $u$  is increased to 1.5, the relative amplitude is much less sensitive to the change of  $T_e$ . Figure 5 shows the change of groundwater level fluctuations with time at  $ax_0 = 0.2$  for different  $T_e$  when  $u = 0.1$  and  $aL = 1.8$ . As the tidal efficiency increases, the fluctuations increase and the time lag of the groundwater level response to the sea tide decreases.

## 4. Summary

Exact analytical solutions are derived to investigate the influences of tidal efficiency, roof length, and leakage of the semipermeable layer on tide-induced groundwater fluctuations in a coastal leaky confined aquifer system. The solutions are compared with those developed by *Jacob* [1950], *Van der Kamp* [1972], *Li and Chen* [1991a], and *Jiao and Tang* [1999]. The comparison shows that all the previous solutions can be regarded as special cases of the solution presented in this paper.



**Figure 5.** Change of ratio of groundwater fluctuation to sea tide amplitude with time at the inland location  $ax_0 = 0.2$  for different tidal efficiency  $T_e$  when  $aL = 1.8$  and  $u = 0.1$ .

In addition, the study here also points out possible mistakes in the solutions derived by *Li and Chen* [1991a, 1991b]. Approximations of the analytical solutions are derived when the roof length is very large or small.

Detailed discussion using a hypothetical example is carried out to understand the influence of various parameters on the behavior of the groundwater level fluctuations in the inland part of the confined aquifer. Roof length and leakage are two dominant factors controlling the groundwater fluctuations, while the tidal efficiency is significant only when the roof is long and the leakage is weak. Among the three parameters, the leakage has the most complicated impact on the groundwater level fluctuation. The leakage from the confining layer extending under the sea increases the fluctuation of water level in the confined aquifer, while that from the inland part decreases the fluctuation since the water table in the overlying unconfined aquifer remains virtually constant owing to large specific yield. How the leakage will eventually impact the fluctuation behavior in the confined aquifer depends on the superposition of the different effects of the leakage from the offshore and inland portions of the confining layer.

The amplitude and time lag of the fluctuation of groundwater level are described by two equations with four independent aquifer parameters: the aquifer's tidal propagation parameter, the dimensionless leakage, the roof length, and the tidal efficiency. Any two of them can be estimated if the other two are known.

### Appendix 1: Derivation of the Solution to (1)–(6)

Let  $H(x, t)$  be a complex function of the real variables  $x$  and  $t$  that satisfies (1)–(6) after  $h_s(t) = A \cos(\omega t)$  in (1)–(3) is replaced by  $A \exp(i\omega t)$ . Because  $h(x, t)$  is the solution to (1)–(6), it follows that

$$h(x, t) = \text{Re} [H(x, t)], \quad (\text{A1})$$

where  $\text{Re}$  denotes the real part of the complex expression, and  $i = \sqrt{-1}$ .

Now suppose

$$H(x, t) = AX(x) \exp(i\omega t), \quad (\text{A2})$$

where  $X(x)$  is an unknown function of  $x$ . Substituting (A2) into the six equations which  $H(x, t)$  satisfies, then dividing the results by  $A \exp(i\omega t)$ , yields

$$X''(x) - \frac{(i\omega S + L_s)}{T} X(x) = -\frac{i\omega T_e S + L_s}{T}, \quad (\text{A3})$$

$$-L < x < 0,$$

$$X''(x) - \frac{(i\omega S + L_s)}{T} X(x) = 0, \quad x > 0, \quad (\text{A4})$$

$$X(-L) = 1, \quad (\text{A5})$$

$$\lim_{x \downarrow 0} X(x) = \lim_{x \uparrow 0} X(x), \quad (\text{A6})$$

$$\lim_{x \downarrow 0} X'(x) = \lim_{x \uparrow 0} X'(x), \quad (\text{A7})$$

$$X(+\infty) = 0. \quad (\text{A8})$$

The general solutions to (A3) and (A4) are

$$X(x) = C_1 \exp[-a(p + iq)x] + C_2 \exp[a(p + iq)x] + \lambda + i\mu, \quad -L < x < 0, \quad (\text{A9a})$$

$$X(x) = C_3 \exp[-a(p + iq)x] + C_4 \exp[a(p + iq)x], \quad x > 0, \quad (\text{A9b})$$

where  $a$ ,  $p$  and  $q$ ,  $\lambda$  and  $\mu$  are constants defined by (7a), (7i), and (7f), respectively, and  $C_1$ ,  $C_2$ ,  $C_3$ , and  $C_4$  are four unknown complex constants. By means of (A5)–(A8), after some routine calculation, one obtains

$$C_4 = 0, \quad C_1 = R_1 - iI_1, \quad C_2 = -\frac{1}{2}(\lambda + i\mu), \quad (\text{A10})$$

$$C_3 = (R_1 + \frac{1}{2}\lambda) - i(I_1 - \frac{1}{2}\mu),$$

where  $R_1$  and  $I_1$  are constants given by (7g) and (7h), respectively. Now first substituting (A10) back into (A9a) and (A9b), then using the result to determine  $H(x, t)$  by means of (A2), and finally calculating the real part  $h(x, t)$  of  $H(x, t)$  in view of (A1), the solution  $h(x, t)$  of (1)–(6) is obtained and given by (7c) and (7d).

### Appendix 2: Proof of the Inequality $C_e \leq 1$

From the definition of  $C_3$  in (A10) and (7e) it is seen that  $C_e = |C_3|$ . In view of the definitions of  $C_1$ ,  $C_2$ , and  $C_3$  in (A10), it follows that

$$C_3 = C_1 - C_2 = (1 - \lambda - i\mu) \exp[-(p + iq)L] + \frac{1}{2}(\lambda + i\mu) \exp[-2(p + iq)L] + \frac{1}{2}(\lambda + i\mu) = \exp[-(p + iq)L] + \frac{1}{2}(\lambda + i\mu)\{1 - \exp[-(p + iq)L]\}^2; \quad (\text{A11})$$

hence, by using the triangular inequality, one has

$$|C_3| \leq e^{-pL} + \frac{1}{2}|\lambda + i\mu|\{1 - \exp[-(p + iq)L]\}^2. \quad (\text{A12})$$

Using (21b), one finds that

$$|\lambda + i\mu| \leq \sqrt{\lambda^2 + \mu^2} = \sqrt{\frac{u^2 + T_c^2}{u^2 + 1}} \leq 1. \quad (\text{A13})$$

Combining (A12) and (A13) yields

$$|C_3| \leq e^{-pL} + \frac{1}{2} |1 - \exp[-(p + iq)L]|^2 = e^{-pL} + \frac{1}{2} [1 - 2e^{-pL} \cos(qL) + e^{-2pL}] = f(\ell), \quad (\text{A14})$$

where

$$f(\ell) = e^{-\ell} + \frac{1}{2} \left[ 1 - 2e^{-\ell} \cos\left(\frac{q}{p}\ell\right) + e^{-2\ell} \right], \quad \ell = pL. \quad (\text{A15})$$

Differentiating (A15) with respect to  $\ell$  yields

$$\begin{aligned} f'(\ell) &= -e^{-\ell} + e^{-\ell} \cos\left(\frac{q}{p}\ell\right) + \frac{q}{p} e^{-\ell} \sin\left(\frac{q}{p}\ell\right) - e^{-2\ell} \\ &= e^{-\ell} \left[ -1 - e^{-\ell} + \sqrt{1 + \left(\frac{q}{p}\right)^2} \cos\left(\frac{q}{p}\ell - \frac{\pi}{4}\right) \right] \\ &\leq e^{-\ell} \left[ -1 - e^{-\ell} + \sqrt{2} \cos\left(\frac{q}{p}\ell - \frac{\pi}{4}\right) \right], \end{aligned} \quad (\text{A16})$$

from which one obtains

$$f'(\ell) \leq e^{-\ell} (-1 - e^{\ln 2} + \sqrt{2}) < 0, \quad \ell \leq \ln 2.$$

On the other hand, one has  $f(0) = 1$ . Hence  $f(\ell) < 1$  holds for  $0 < \ell \leq \ln 2$ .

When  $\pi/3 \geq \ell > \ln 2$ , because  $\cos(q/p)\ell \geq \cos \ell \geq \cos(\pi/3) = \frac{1}{2}$ ,  $e^{-\ell} \leq \frac{1}{2}$ , from (A15), it follows that

$$\begin{aligned} f(\ell) &= \frac{1}{2} + \frac{1}{2} e^{-\ell} \left[ 2 - 2 \cos\left(\frac{q}{p}\ell\right) + e^{-\ell} \right] \leq \frac{1}{2} + \frac{1}{2} e^{-\ell} (2 - 2 \\ &\quad \times \frac{1}{2} + \frac{1}{2}) \leq \frac{1}{2} + \frac{3}{4} e^{-\ell} < 1. \end{aligned}$$

When  $\pi/2 \geq \ell > \pi/3$ , substituting  $e^{-\pi/3} < 0.36$  and  $\cos(q/p)\ell \geq \cos \ell \geq 0$  into (A15) leads to

$$\begin{aligned} f(\ell) &= e^{-\ell} + \frac{1}{2} \left[ 1 - 2e^{-\ell} \cos\left(\frac{q}{p}\ell\right) + e^{-2\ell} \right] \\ &\leq 0.36 + \frac{1}{2} (1 + 0.36^2) = 0.9248 < 1. \end{aligned}$$

Last, if  $\ell > \pi/2$  because  $e^{-\pi/2} < 0.208$  and  $\cos(q/p)\ell \geq -1$ , one finds that

$$\begin{aligned} f(\ell) &= e^{-\ell} + \frac{1}{2} \left[ 1 - 2e^{-\ell} \cos\left(\frac{q}{p}\ell\right) + e^{-2\ell} \right] \leq 0.208 \\ &\quad + \frac{1}{2} (1 + 2 \times 0.208 + 0.208^2) = 0.937632 < 1. \end{aligned}$$

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