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Tides on Europa, and the thickness of Europa's icy shell

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[1] It has been shown previously that measurements of tides on Jupiter's moon Europa can be used to determine whether there is a liquid ocean beneath this moon's icy outer shell. In this paper we examine the further possibility of constraining the thickness of the icy shell in the case where a liquid ocean exists, by combining measurements of tidal gravity obtained from tracking an orbiting spacecraft with measurements of vertical tidal surface displacements obtained from a precise onboard altimeter. By simulating a 1-month Europa mapping mission we demonstrate that this combination of tidal measurements would provide a much better estimate of ice thickness than could be obtained using either tracking or altimeter measurements alone. The thickness value inferred from the combined data would also require an estimate of the shear modulus of Europa's icy shell. This introduces an additional uncertainty in the thickness estimate that is approximately proportional to the uncertainty in the inverse of the shear modulus.

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1. Introduction

[2] It has long been known that the surface of Jupiter's moon Europa is mainly composed of water ice. Various observations from the Galileo spacecraft, most notably from magnetometry [*Kivelson et al.*, 2000], support the hypothesis that this fractured, icy surface overlies a liquid water ocean. Details on geometry and physical properties of the shell and ocean, however, remain uncertain. Models of the internal structure of Europa based on gravitational field estimates inferred from perturbations of Galileo's orbit suggest that the total thickness of the combined ice+ocean layer is probably between 80 and 170 km [*Anderson et al.*, 1998]. The thickness of the icy shell has been estimated on a regional basis from a diverse collection of surface features [*Hoppa et al.*, 1999b; *Turtle and Pierazzo*, 2001; *Schenk*, 2002; *Nimmo et al.*, 2003]. Thermal models [*Ojakangas and Stevenson*, 1989; *Hussmann et al.*, 2002; *Tobie et al.*, 2003; *Spohn and Schubert*, 2003] permit global estimates, but such estimates are based on assumptions about internal structure that have considerable uncertainty. Regional thickness estimates range from kilometers to tens of kilometers. However, Europa's shell thickness has not been estimated reliably on a global basis. Such estimation will require a dedicated Europa orbiter mission.

[3] The presence of a liquid ocean would significantly increase tidal amplitudes on Europa [see, e.g., *Yoder and Sjogren*, 1996; *Edwards et al.*, 1997; *Chyba et al.*, 1998; *Moore and Schubert*, 2000; *Wu et al.*, 2001]. As a result, measuring tidal amplitudes is one of the primary motivations for a Europa orbiter mission. The goal of this paper is to examine whether tidal measurements could provide information on the thickness of the icy shell, using the current state of knowledge that a liquid ocean almost certainly exists. *Moore and Schubert* [2000] noted that tidal amplitudes should vary linearly with ice thickness, where the coefficient of that linear term depends on the shear modulus of the ice. We concur with that conclusion. However, we find that uncertainties in the material properties of the ocean and underlying rocky mantle would make it difficult to identify that linear term using observations of tidal gravity or tidal displacements alone. Here we combine theoretical analysis with a simulation of a nominal 1-month-long orbital mapping mission to investigate what could be learned about Europa's icy shell using modern techniques in geophysics and space geodesy. We find that by combining measurements of tidal gravity obtained from tracking an orbiting spacecraft, with measurements of vertical tidal surface displacements obtained from an onboard altimeter, it should be possible to extract that linear term to a high degree of accuracy, and so to constrain the product of the thickness and shear modulus of the ice. Direct measurement of Europa's tidal displacements even in the case of a fully solid body is within current capability, as demonstrated for Mars with an orbital laser altimeter [*Zuber et al.*, 1992; *Smith et al.*, 2001] and the application of crossover analysis

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[*Neumann et al.*, 2001] in the precision orbit determination process.

2. Tides and Love Numbers

[4] By far the largest tides on Europa are those caused by the gravitational attraction of Jupiter. Europa's rotation is synchronous with its orbital motion, both having periods of T = 3.55 (Earth) days. Nonsynchronous rotation could exist; but, if so, its period would be at least 10^4 years [*Hoppa et al.*, 1999a], which means any nonsynchronous drift would be negligible during a satellite lifetime. For synchronous rotation, tidal variability requires an eccentric orbit and has period T. To lowest order in eccentricity, ϵ, and assuming Europa's orbital obliquity vanishes, the tidal potential is [see *Kaula*, 1964]

$$V_T(r,\theta,\lambda,t) = A\left(\frac{r}{R}\right)^2\left[\left(1-3\cos^2\theta\right)\cos(nt) + \sin^2\theta\,[3\cos(nt)\cos(2\lambda)+4\sin(nt)\sin(2\lambda)]\right], \quad (1)$$

where $A = 3GM\epsilon R^2/4a^3$. Here r, θ, λ are the radius, colatitude, and eastward longitude; the time t is assumed to be zero at perijove; M is Jupiter's mass; and R, a, and n are Europa's radius, semimajor axis, and mean motion ($n = \sqrt{GM/a^3} = 2\pi/T$). Note that V_T varies periodically with time at a period of $2\pi/n$ = 3.55 *days*.

[5] Let U (θ, λ, t) and Φ (θ, λ, t) be, respectively, the vertical tidal surface displacement and the perturbation in the gravitational potential at the surface caused by the tidally perturbed density field of Europa. Then, since V_T is the sum of spherical harmonics of degree 2 ($(1 - 3\cos^2\theta)$ is a harmonic of degree 2, order 0; $\sin^2\theta\cos(2\lambda)$ and $\sin^2\theta\sin(2\lambda)$ are harmonics of degree 2, order 1),

$$U = h\,V_T(R,\theta,\lambda,t)/g \qquad \Phi = k\,V_T(R,\theta,\lambda,t), \quad (2)$$

where g is the gravitational acceleration at the European surface and h and k are Love numbers of degree 2 [*Love*, 1927; *Munk and MacDonald*, 1960; *Lambeck*, 1990]. h and k depend on the internal structure of Europa, and would be much larger if Europa had a liquid ocean than if it did not [see, e.g., *Yoder and Sjogren*, 1996; *Edwards et al.*, 1997; *Chyba et al.*, 1998; *Moore and Schubert*, 2000; *Wu et al.*, 2001]. Altimeter observations of tidal surface displacements could provide h, and gravity field solutions for the tidal potential could provide k, and so both measurement types could provide information about the presence of a liquid ocean.

[6] In this paper we suppose Europa has a liquid ocean underlying its icy outer shell. Our goal is to understand how observations of h and k could be used to provide information on the icy shell in general, and on the thickness of that shell in particular. To this end, we compute h and k for a wide variety of Europan structural models and parameter values. We use two methods, one analytical and one numerical, to compute the Love numbers.

3. Analytical Method

[7] We use an analytical method, described in this section, to find the Love numbers for a simplified structural model of Europa. Our goal is to develop an initial understanding of how the Love numbers depend on the parameters of the icy shell. The results described below suggest that although observations of h or k alone are unlikely to be useful for constraining ice thickness, there is a linear combination of h and k that could provide a meaningful estimate.

[8] Assume that Europa consists of a uniform, incompressible icy shell overlying a uniform, incompressible liquid ocean; and that the ocean overlies a rigid mantle. Inertial forces are ignored, which is equivalent to assuming the tidal period is infinite. Coriolis and centrifugal forces are also ignored, which is equivalent to ignoring the dynamical effects of Europa's rotation. Solutions are obtained by adding together linearly independent analytical solutions of the differential equations of motion (these solutions are tabulated by *Yuen et al.* [1982]), so that the combined solution satisfies all external and internal boundary conditions. This combined solution is found using Mathematica, to obtain the explicit dependence on Europa's structural parameters.

[9] Define R = 1565 km and $\bar{\rho}$ = 2989 kg/m^3 [*Anderson et al.*, 1998] as the radius and average density of Europa, ρ_o and ρ_i as the densities of the liquid ocean and the icy shell, d as the thickness of the icy shell, R_m as the radius of the rocky mantle (so that $R - R_m$ is the thickness of the combined liquid ocean and icy shell), and μ_i as the shear modulus of the icy shell. Although the exact analytical expressions for h and k are complicated, they simplify in the limit of $d/R \ll 1$ (i.e., a thin icy shell) to

$$h = h_0 + h_1\frac{d}{R}, \; k = k_0 + k_1\frac{d}{R}, \quad (3)$$

where

$$h_0 = \frac{5\bar{\rho}}{5\bar{\rho}-3\rho_o}, \; k_0 = \frac{3\rho_o}{5\bar{\rho}-3\rho_o}, \quad (4)$$

$$h_1 = \frac{75\bar{\rho}[R^2G\pi(16\bar{\rho}-3\rho_o)(\rho_i-\rho_o)-6\mu_i]}{11R^2G\pi(5\bar{\rho}-3\rho_o)^2\rho_o}, \quad (5)$$

$$k_1 = \frac{15[11R^2G\pi(8\bar{\rho}-3\rho_o)(\rho_i-\rho_o)-18\mu_i]}{11R^2G\pi(5\bar{\rho}-3\rho_o)^2}. \quad (6)$$

[10] Note that if we define $\Delta = 1 + k - h$, then

$$\Delta = \left[\frac{15R^2G\pi(16\bar{\rho}-11\rho_o)(\rho_o-\rho_i)+90\mu_i}{11R^2G\pi(5\bar{\rho}-3\rho_o)\rho_o}\right]\frac{d}{R}. \quad (7)$$

The quantity $-\Delta \times V_T/g$ is the height of Europa's outer surface above the surface of constant potential. Thus the result that $\Delta = 0$ if there is no icy shell (i.e., if $d = 0$), implies that in the limit of infinite period and no rotation, which is what we are assuming in this model, the tides of an uncovered ocean are equilibrium: i.e., at all times during the tidal cycle the ocean surface is a surface of constant gravitational potential.

[11] The fact that Δ vanishes when $d = 0$, whereas h and k do not, implies that observations of Δ are likely to be more

Table 1. Reference Model Parameter Values[a]

Parameter	Value
ρ_o	1000 kg/m^3
ρ_i	920 kg/m^3
ρ_c	5150 kg/m^3
μ_i	2.0×10^9 Pa
κ_i	1.0×10^{10} Pa
κ_o	2.0×10^9 Pa
μ_m	6.0×10^{10} Pa
κ_m	1.2×10^{11} Pa
κ_c	5.0×10^{11} Pa
R	1565 km
R_m	1465 km
R_c	700 km

[a]Subscripts *i*, *o*, *m*, and *c*, refer to the icy shell, liquid ocean, rocky mantle, and liquid core, respectively.

useful than observations of h or k alone for constraining the ice thickness. The h_1 (d/R) and k_1 (d/R) terms in (3) are almost certain to be much smaller than h_0 and k_0, respectively. Relatively small uncertainties in h_0 and k_0 due, for example, to uncertainties in ρ_o, could map into large uncertainties in d, even in the absence of observational errors in h and k.

[12] For example, Table 1 lists numerical values of the parameters appearing in (3)–(7), and of other parameters that we will introduce in section 4, that we use to define a "reference model". The values of ρ_o and ρ_i are those for pure water. The correct value of μ_i is uncertain. From laboratory experiments involving periodic loading of unfractured, saline ice, *Cole and Durell* [1995] concluded that as the forcing period increases, the shear modulus for ice at −30°C approaches an asymptotic value of about 2×10^9 Pa. We have adopted this number as our reference value. However, Cole and Durell's experiments do not consider loading at periods greater than 2.5–3 hours, which is far shorter than the 3.55 day tidal period on Europa; and their results for ice at higher temperatures seem to imply values of μ_i that may be smaller by a factor of 2 or more. Furthermore, there is evidence from surface faulting on Europa, that the shear modulus of the outer surface of the ice could be up to a factor of 5 smaller than Cole and Durell's estimate, due conceivably to surface fracturing [*Nimmo and Schenk*, 2006]. Using observations of tidal flexure of terrestrial ice shelves, *Vaughan* [1995] obtained values for Young's modulus consistent with a still smaller value of μ_i: $\mu_i \approx 0.3 \pm 0.1 \times 10^9$ Pa. *Schmeltz et al.* [2002] conducted a similar tidal flexure study, and found values of μ_i of between 0.3 and 1.3×10^9 Pa, with different values for different shelves, and even for the same shelves but for different time spans. *Schmeltz et al.* [2002] concluded that the large strains associated with tidal flexure of ice shelves is likely to induce significant visco-plastic effects which would affect these estimates of μ_i based on flexure observations. Because the tidal strains in the Europan icy shell are 1–2 orders of magnitude smaller than the tidal strains within terrestrial ice shelves, we instead adopt *Cole and Durell*'s [1995] value of μ_i for our reference value.

[13] Using the Table 1 values in (3)–(7), we obtain

$$h = 1.25 - 3.9\,\frac{d}{R}\,,\; k = 0.25 - 0.85\,\frac{d}{R},\; \Delta = 3.0\,\frac{d}{R}. \quad (8)$$

Suppose, though, that the ocean+ice layer was composed instead of the eutectic system H_2O-$MgSO_4$-Na_2SO_4 brine, one of the possibilities proposed by *Kargel et al.* [2002]. Then ρ_o = 1208 kg/m^3 and ρ_i = 1144 kg/m^3, so that

$$h = 1.32 - 3.5\,\frac{d}{R}\,,\; k = 0.32 - 0.90\,\frac{d}{R},\; \Delta = 2.6\,\frac{d}{R}. \quad (9)$$

[14] The difference between, say, k_0 in these two cases is not large (0.32 versus 0.25), but this difference could have a substantial impact on the value of ice thickness inferred from observations of k. For example, suppose the ocean+ice layer were composed of pure H_2O, but that the Love number observations were interpreted under the erroneous assumption that this layer was composed of Kargel et al.'s brine. Further, suppose the icy shell thickness, d, was close to zero. Then observations would yield h = 1.25, k = 0.25, and Δ = 0. If those observations were erroneously interpreted using (9), the values inferred for d would be $d = (1.32-1.25)/3.5 \times R \approx 30$ km using h, and $d \approx 120$ km using k. However, the value of d inferred from observations of Δ would, correctly, = 0 km. In the more general case of nonzero ice thickness, these errors in the assumed ocean and ice densities would cause d to be overestimated by 3.0/2.6 = 15% using Δ, while causing much larger overestimates of 30 km +11% using h, and 120 km − 6% using k. The 30 km and 120 km terms come from errors in h_0 and k_0 respectively, and they cause the total errors in d inferred from h and k to be far larger than the errors inferred from Δ.

[15] The errors in d caused by this mismodeling of h_0 and k_0, scale inversely with the value of μ_i. For example, if we use $\mu_i = 1 \times 10^9$ Pa, instead of the reference value 2×10^9 Pa, then our results for h, k, and Δ in the case of a H_2O-MS-NS ocean+ice layer, become

$$h = 1.32 - 1.9\,\frac{d}{R}\,,\; k = 0.32 - 0.53\,\frac{d}{R}\,,\; \Delta = 1.4\,\frac{d}{R} \quad (10)$$

so that the coefficients of the d/R terms are about half those in (9). The d/R coefficients in the case of a pure H_2O layer are similarly half those shown in (8). For this reduced value of μ_i, errors in the assumed ocean and ice densities would cause d to be overestimated by 60 km +16% using h and 210 km − 3% using k.

[16] Of the parameters required in (3)–(7), by far the most uncertain are d and μ_i. For the reference values listed in Table 1, $[90\,\mu_i]/[15R^2G\pi\,(16\bar{\rho} - 11\rho_o)(\rho_o - \rho_i)] \approx 8$, so that the numerator in (7) is dominated by the term proportional to μ_i. In this case, to an accuracy of 10–15%,

$$\Delta \approx \left[\frac{90\mu_i}{11R^2G\pi(5\bar{\rho} - 3\rho_o)\rho_o}\right]\frac{d}{R}, \quad (11)$$

so that observations of Δ would mostly provide a constraint on the product $\mu_i d$. If the product $\mu_i d$ is too large, large enough that $\mu_i d/R \approx 2R^2G\rho_o\bar{\rho}$ ($\approx 10^9$ Pa using the reference model values) or larger, then terms second order in d/R, neglected in (7), are of equal importance to the first-order terms and so should be included in the expression for Δ. This means, for example, that if $d \approx 30$ km, then (7) is a good approximation if $\mu_i \ll 5 \times 10^{10}$ Pa. Since 5×10^{10} Pa is about 250 times the reference value of μ_i, we conclude that (7) is adequate for values of d of this order or less.

[17] Small values of μ_i also lead to problems. For example, if μ_i is as small as implied by the tidal flexure studies described above, then (11) would not be a valid approximation for (7). In the still more extreme case that μ_i is on the order of $dRG\,\bar{\rho}\,(\rho_o - \rho_i)$ ($\approx 10^6$ Pa for a 30 km icy shell; or about 2000 times smaller than our reference value) or less, then higher-order terms in d/R should, again, be included in the approximation (7).

4. Numerical Method

[18] Although the analytical results described above provide insight into the Love numbers' dependence on key parameters of the ice shell, they are based on an incomplete model of Europa's structure. We extend those results by using a numerical method to compute Love numbers for a more general structural model: one that includes a liquid core and nonrigid rocky mantle underlying the ocean+ice layer, and that is compressible throughout. Our numerical results, described in this section, show that the overall conclusions inferred from the analytic method remain valid in this more general case.

[19] Our numerical method is based on the standard algorithms used by geophysicists to compute Love numbers for a stratified, compressible, and self-gravitating Earth. Our numerical code, in fact, is a modified version of the code employed by *Dahlen* [1976] to compute terrestrial Love numbers. We assign uniform material properties (i.e., the density ρ, shear modulus μ, and bulk modulus κ) to each region: the icy shell, liquid ocean, rocky mantle, and liquid core. The model allows us to include material properties that vary smoothly with radius, and it would perhaps be worthwhile to use an equation of state to do that when interpreting real data. However, for this pilot study, we elect instead to employ uniform material properties within each layer.

[20] Our reference numerical values for these parameters are listed in Table 1. Descriptions of ρ_i, ρ_o, and μ_i are given in section 3. For the liquid core we use *Anderson et al.*'s [1998] values for radius (R_c) and density (ρ_c) that assume an Fe-FeS mixture; and we use a value of κ_c that roughly coincides with the value of κ at the top of the Earth's liquid core [*Dziewonski and Anderson*, 1981]. For the icy shell and liquid ocean we use values of the bulk moduli, κ_i and κ_o, that are consistent with laboratory measurements as summarized by *Fletcher* [1970] for water ice and *Dorsey* [1940] for liquid water. The density in the rocky mantle is adjusted so that the mean density of Europa = 2989 kg/m^3 [*Anderson et al.*, 1998]. So, for example, if the icy shell thickness is 10 km and the other radii and densities are as given in Table 1, then the mantle density = 3214 kg/m^3. This value varies accordingly when we modify the thickness of the icy shell, the radius of the mantle, and the density and radius (and, for that matter, the existence) of the liquid core. To estimate the shear and bulk moduli for the mantle, we use Birch's law [see, e.g., *Poirier*, 2000] to estimate the seismic *P*-wave velocity v_p using an assumed Europan mantle density of ρ_m = 3214 kg/m^3; and we assume a ratio of v_p to v_s (shear wave velocity) of 1.78 [*Anderson and Bass*, 1984] to then obtain v_s. Using these estimates we obtain the values of μ_m and κ_m listed in Table 1.

[21] The solid black lines in Figures 1a–1c show our results for h, k, and Δ, as functions of ice thickness d, computed using the reference values listed in Table 1. The other lines included in Figures 1a–1c show results computed using modifications of the Table 1 values: values of ρ_o and ρ_i consistent with the eutectic H_2O-$MgSO_4$-Na_2SO_4 brine proposed by *Kargel et al.* [2002]; results for both an increase and a decrease in the shear modulus, μ_m, of the rocky mantle; and results for a model without a fluid core. We also generated results for plausible differences in κ_o, κ_m, κ_c, and R_m, though none of these caused significant changes in any of h, k, or Δ, and so the results are not shown. Results caused by changes in the ice parameters, μ_i, κ_i, and ρ_i, are discussed below. Results are shown only for $d \leq 30$ km, so that it's easier to see the effects of model differences.

[22] Note, as anticipated in section 3, that h, k, and Δ vary approximately linearly with d, and that Δ vanishes when $d = 0$ while h and k do not. Figures 1a and 1b show that h_0 and k_0 (the values of h and k when $d = 0$; see (3)) are now sensitive not only to the ocean and ice densities, but also to μ_m and to whether Europa has a fluid core or not. These sensitivities would introduce significant errors into any estimate of d made using observations of h or k alone.

[23] For example, suppose observations give $h = 1.25$ and $k = 0.26$, so that $\Delta = 0.01$. The horizontal dotted lines in Figures 1a–1c correspond to those values. Those lines in Figures 1a and 1b illustrate the wide range of possible values of d that could be inferred from observations of h or k alone, depending on what was assumed about other parameters. The presence or absence of a fluid core could make a difference in the inferred value of d of about 3 km when using h and 12 km when using k; and varying μ_m between 0.4×10^{12} Pa and 1.3×10^{12} Pa, a somewhat arbitrarily chosen but not implausible range of values, could make a difference of about 7 km when using h and 31 km when using k. The difference between the results computed assuming a pure H_2O ice+ocean layer and a eutectic H_2O-MS-NS layer is much the same as found using the analytical method in section 3: about 36 km using h and well over 100 km using k. As described in section 3, all these differences scale approximately inversely with μ_i; so that, for example, if $\mu_i = 1 \times 10^9$ Pa, instead of the reference value 2×10^9 Pa, then these differences in d approximately double.

[24] Figure 1c shows that the corresponding impact on values of Δ would be far less: the difference between the two models for the chemistry of the ice+ocean layer would cause an error in d of 15–20%, about the same as found using the analytical method; while differences caused by uncertainties in μ_m or in whether there is a fluid core or not, would be negligible.

[25] The slope of Δ as a function of d/R is on the order of 2.7 for small values of d. The difference between this value and the slope of 3.0 obtained using the analytical model (8) is almost entirely due to compressibility of the icy shell, which is included in the numerical model but not in the analytical model. We find that the effects of compressibility can be approximated reasonably well by adding a term to Δ that is inversely proportional to κ_i. Altogether, we approximate $\Delta \approx \Delta_a$, where

$$\Delta_a = \left[\frac{15R^2G\pi(16\bar{\rho} - 11\rho_o)(\rho_o - \rho_i) + 90\mu_i}{11R^2G\pi(5\bar{\rho} - 3\rho_o)\rho_o} - 1.4\frac{\mu_i}{\kappa_i}\right]\frac{d}{R}. \tag{12}$$

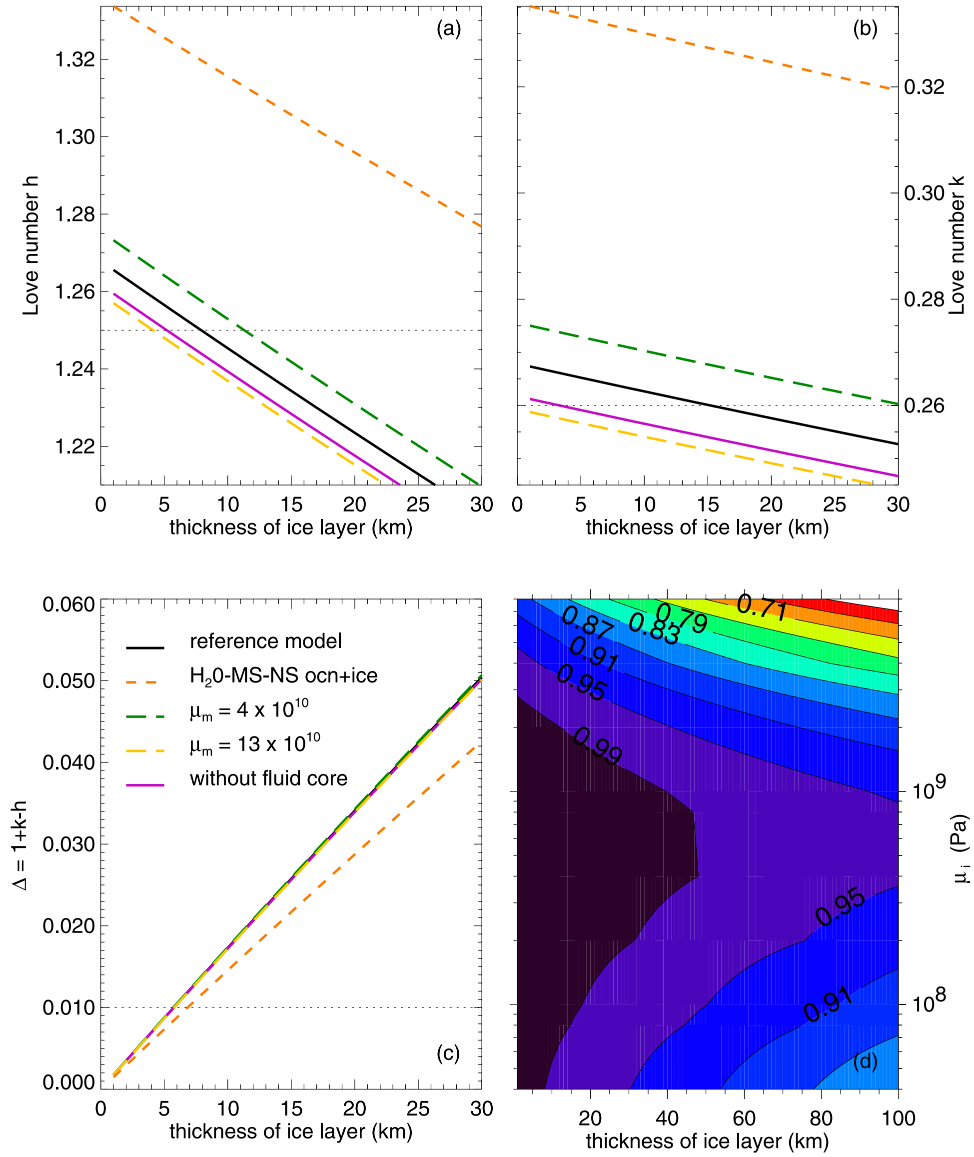

Figure 1. (a, b) Results for the Love numbers h and k as functions of ice thickness, d, computed using the numerical method described in the text. (c) Linear combination $\Delta = 1 + k - h$. Results are shown for the reference model described in Table 1, and for modifications of that reference model as described in the text. The results in all three cases vary approximately linearly with ice thickness. The horizontal, dotted lines in Figures 1a–1c, correspond to arbitrarily chosen values of h (=1.25) and k (=0.26), and are included to illustrate the advantage of using observations of Δ to constrain ice thickness, rather than observations of h or k alone. (d) A contour plot of Δ/Δ_a as a function of d and μ_i, where Δ_a is given by (12). Values from the reference model (Table 1) have been used for all other variables. The results in this panel are an indication of how good an approximation Δ_a is for Δ.

Figure 1d is a contour plot of Δ/Δ_a as a function of d and μ_i, computed using the reference model values (Table 1) for all other variables. The results show the approximation (12) begins to break down for large values of d, both when μ_i is small and, especially, when μ_i is large. This is caused by the neglect of second- and higher- order terms in d/R, and is discussed in section 3.

[26] For values of μ_i between the reference model value (2×10^9 Pa) and a factor of 10 or more smaller than the reference value, and for any plausible values of the other model parameters, Δ is approximated to within 10% (and even better for values of d of ≈30 km or less) by (12). For the reference value of μ_i, the 90 μ_i term dominates the bracket in (12), so that (11) becomes a good approximation for Δ. Because this approximation (11) breaks down for values of μ_i that are either much smaller or much larger than the reference value, it's an oversimplification to conclude that observations of Δ would constrain μ_i d. However, because by far the most uncertain parameter in (12), other than d, is μ_i, it is fair to conclude that the dominant error in any estimate of d caused by inadequacies in model parameters, would be caused by errors in μ_i. Also, at least for

values of μ_i close to the reference value, the corresponding uncertainty in d would be approximately proportional to the uncertainty in $1/\mu_i$.

5. Ocean Tides: Are They Equilibrium?

[27] Our conclusion that Δ is proportional to d is based on the results from our Europan tide models that predict that when there is no icy shell the tidal response of the ocean is equilibrium. For an equilibrium ocean, if the tidal potential at the surface is $V_T(\theta, \lambda, t)$ (1), then the tidal variation in sea surface height is

$$\bar{\eta}(\theta, \lambda, t) = (1 + k)\, V_T(\theta, \lambda, t)/g. \tag{13}$$

[28] Our tide models assume an infinite forcing period and ignore the Coriolis force. Those are not good assumptions when computing diurnal ocean tides on the Earth. To estimate the impact of these simplifying assumptions on Europa, we construct an independent model of the ocean tides using the Laplace tidal equations (LTE) on a sphere [e.g., *Lamb*, 1945], for an ocean without continental boundaries and of uniform depth. These equations describe tides in an incompressible ocean with no overlying solid shell, in the shallow water limit: i.e., when the ocean thickness, $H \ll R$. The LTE's include the effects of the Coriolis force and of a finite tidal period, and are the same equations routinely used to model ocean tides on Earth. For Europa, $H/R \approx 100/1565 = 6\%$. This is not nearly as small a ratio as for the Earth, where the corresponding ratio is about 0.06%. However, our expectation is that it is small enough to make the LTE's useful for resolving the issue of whether Europan ocean tides, with no icy shell, should be close to equilibrium.

[29] We force the LTE's with the applied potential $(1 + k)\, V_T$. We follow sections 213–221 of *Lamb* [1945], and expand the solution to first order in the parameter $\beta = 4\Omega^2 R^2/gH$, where Ω is Europa's rotation rate (= n, Europa's mean motion). For an ocean of 100 km depth, $\beta \approx 0.03$, so that $\beta \ll 1$. We find that, to first order in β, the tidal perturbation in sea surface height, η (θ, λ, t), is

$$\eta \approx \bar{\eta} + \frac{\beta}{12}\frac{A(1+k)}{g}\left[\cos(nt)\left(\cos^4\theta - \frac{\cos^2\theta}{2} - \frac{1}{30}\right) - \frac{3}{2}\left(\cos(nt)\,\cos(2\lambda) + \frac{4}{3}\sin(nt)\,\sin(2\lambda)\right)\sin^2\theta\cos^2\theta\right], \tag{14}$$

where $\bar{\eta}$ is the equilibrium tide in (13). (For diurnal tides on the Earth, $\beta \approx 20$, so the first-order expansion (14) is not a useful approximation, and η is not approximately equal to $\bar{\eta}$.)

[30] The fact that the ocean tide is not exactly an equilibrium tide has implications for the inferred value of Δ. Δ would be determined by setting $-\Delta V_T/g$ equal to the height of Europa's surface above the equipotential surface. In the case where the thickness of the icy shell on Europa, d, is negligible, this differential height is $\eta - \bar{\eta}$. Since $\eta - \bar{\eta}$ does not vanish, the inferred value of Δ would not be zero. Specifically, Δ in this case would be determined from

$$-\Delta V_T/g = \eta - \bar{\eta}\ . \tag{15}$$

Because the left- and right-hand sides of this equation depend on colatitude (θ) in different ways, there is no latitude-independent solution for Δ. Instead, a solution would be obtained by expanding each side of this equation in spherical harmonics, Y_l^m, and equating common coefficients. The left-hand side includes only Y_2^0 and Y_2^2 terms (the $1 - 3\cos^2\theta$ and $\sin^2\theta$ terms in (1), respectively). Equating Y_2^0 terms in (15), and using (14) for η, gives $\Delta = 5\beta/168 \approx 10^{-3}$; and equating Y_2^2 terms gives $\Delta = 3\beta/128 \approx 8 \times 10^{-4}$. Using the reference model values, each of these solutions for Δ would be interpreted as implying a value for d of about 600 meters, instead of the correct value: $d = 0$ km. This inferred value of d would vary approximately linearly with μ_i (see (12)), and inversely with the ocean depth H through the definition of β. Although this argument assumes that the correct value of d is 0, the tentative implication is that no matter what the value of d, the effects of ignoring Coriolis and inertial forces in the liquid ocean lead to an error in d of less than 1 km when the ocean thickness is on the order of 100 km or more. The error could be larger than this for a thinner ocean; if the icy shell occupied most of the total ice+ocean layer, for example. Furthermore, this ocean model does not include the effects of the overlying icy shell back on the ocean: effects that are likely to be more important if the shell thickness is large.

6. Other Complications

[31] The tidal models described in sections 3 and 4 ignore certain complicating effects in the solid portions of Europa (the icy shell and the rocky core), as well. Some of those effects are clearly unimportant. For example, rotational and inertial forces are omitted in those solid regions, just as they are in the ocean. Those forces affect solid body tides on Earth at only the 0.3% level [*Wahr*, 1981], and their impact on Europan tides is apt to be even smaller, given Europa's slower rotation rate and longer tidal period. Our models also assume the density and elastic parameters are constant throughout the rocky core, which is certainly an oversimplification. For the Earth, for example, these parameters increase by about 30% going from the top to the base of the lower mantle (a region where the effects of phase changes are not likely to be important). However, for Europa, the general insensitivity of Δ to any rocky core parameters (see section 4) suggests that this, too, is not a limiting assumption.

[32] Vertical stratification in the icy shell could be more important, particularly if it involves the ice shear modulus, μ_i. Suppose, for example, that the ice consists of two layers of identical densities and bulk moduli, but where the lower layer has a shear modulus of $\mu_{iL} = 2.0 \times 10^9$ Pa, after *Cole and Durrell* [1995], and the upper layer has a shear modulus of $\mu_{iU} = 0.4 \times 10^9$ Pa, which is near the lower limit for the outer surface suggested by *Nimmo and Schenk* [2006]. Suppose the depth to the upper-layer/lower-layer boundary is D. We use the numerical model described in section 4 to compute Δ in this case, using the reference model parameters shown in Table 1, and assuming a total ice thickness of $d = 25$ km. Results are shown in Figure 2, as a function of D, where $D = 0$ km and $D = 25$ km correspond to uniform μ_i values of 2.0×10^9 Pa and 0.4×10^9 Pa, respectively, throughout the entire icy shell.

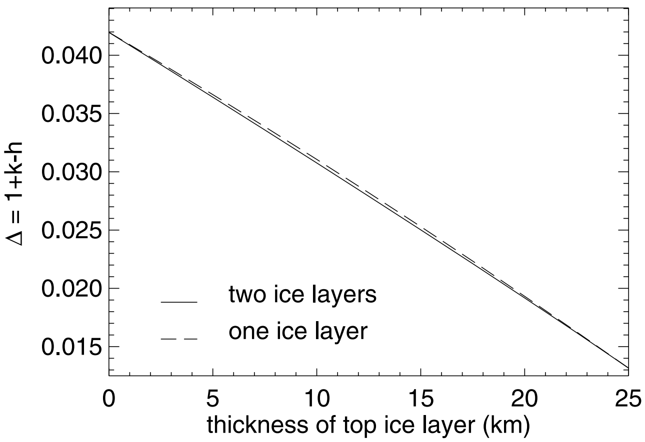

Figure 2. The solid line shows results for Δ computed for an icy shell (25 km thickness) that consists of two layers with different shear moduli. The *x* axis is the depth to the lower-layer/upper-layer interface. The dashed line shows similar results, but where the shear modulus is uniform throughout the icy shell, with a value equal to the icy shell average in the two layer case.

[33] The results in sections 3 and 4 show that for a single layer Δ is approximately proportional to $\mu_i d$. We find we obtain this same result in the two layer case, if we replace μ_i with $\overline{\mu_i} = (\mu_{iU}D + \mu_{iL}\,(d - D))/d$ = the average value of μ_i throughout the icy shell. This is illustrated in Figure 2, where the dashed line shows the values of Δ computed for the same Europa parameters used to compute the solid line, but where μ_i is replaced by the constant $\overline{\mu_i}$ throughout the entire icy shell. The good agreement between the solid and dashed lines suggests that (12) is a good approximation for Δ even in the case of a radially stratified μ_i within the icy shell, as long as we interpret μ_i in (12) as the average value of μ_i throughout the shell.

[34] One final complication ignored in our models, is anelasticity in the icy shell. Anelasticity is also ignored in the rocky core, but the general insensitivity of Δ to core structure implies that this is not likely to introduce significant errors.

[35] In the icy shell, where an anelastic Q as small as 10 has been considered [*Ojakangas and Stevenson*, 1989], the effects could be more important. In that case, (12) would still be a good approximation for Δ, but μ_i would now be complex, with real and imaginary parts that are affected by anelasticity. κ_i would remain unaffected, assuming the anelastic mechanisms involve dissipation of shear energy rather than bulk energy. The imaginary part of μ_i would cause Δ to have an imaginary part, which would lead to a phase lag between the tidal forcing and tidal response. Thus, measurements of the phase lag in Δ could provide information on Q in the icy shell.

7. Measuring Europa's Tides

[36] Altimetry and spacecraft tracking can be analyzed jointly to directly measure tidal displacements on Europa [cf. *Wu et al.*, 2001]. Here we describe and simulate an analogous experiment that could in principle be implemented in a 1-month orbital mapping mission at Europa. The experiment is based on a mission scenario consistent with that of a previously proposed Europa Orbiter mission. The results are meant to be simply demonstrative. In this scenario, Europa's tidal parameters are determined through coupled analysis of laser altimetry and microwave tracking data. The approach provides the strongest solutions for the orbit of the spacecraft, the shape of Europa, the gravity field of Europa, and the Love numbers k and h.

[37] The numerical simulation described below suggests the tidal parameters can be recovered to levels recommended by the Europa Science Definition Team [*Chyba et al.*, 1998]: ±0.0005 for k, and better than ±0.01 for h. This accuracy goal for k implies a spacecraft positional accuracy of order 10 m along track and better than 1 m radially; the accuracy goal for h implies a positional accuracy of 1 m or better in the radial direction. The present-day capability in orbit determination of planetary spacecraft though microwave tracking alone is approaching the 1-m level in the radial direction [*Lemoine et al.*, 1999] after improvement of all the forces acting on the spacecraft, particularly the gravity field. Achieving such accuracy typically takes weeks in the mapping mission dedicated to gravity field improvement. Not only will this amount of time be unavailable at Europa, but a high-quality gravity field must be obtained during the same time available for obtaining the tidal parameters. Of concern are errors in the gravity field that cause errors in the spacecraft position that are not removable by additional tracking [*Tapley and Rosborough*, 1985]. These errors have been observed for Earth-orbiting spacecraft and detected in orbits of the Mars Global Surveyor (MGS) spacecraft orbiting Mars. For MGS, high-precision gravity models developed at both JPL and GSFC [*Smith et al.*, 1999] show internal accuracies for the radial position of MGS at the submeter level [*Lemoine et al.*, 2001; *Yuan et al.*, 2001; *Konopliv et al.*, 2006]. However, comparison of these orbits with the MGS laser altimeter [*Zuber et al.*, 1992; *Smith et al.*, 2001] at over 2 million altimeter crossover locations show radial errors at the 3- to 5-m level. Inclusion of laser altimetry in the gravity model solution improves the gravity field and the orbits, and can largely remove these errors [*Rowlands et al.*, 1999].

[38] Altimeter crossover analysis is a powerful method for accurate determination of the spacecraft orbit and time-varying shape of a planet (or moon). As shown schematically in Figure 3, crossovers are locations on the surface of a planet where the orbit ground track crosses over a previous track. At these locations a measure of the radius of the planet is obtainable from both orbits. A difference in the estimated radius at the crossover location could be the result of (1) an error in one or both orbits, (2) an instrumental error, (3) a change in the radius of the planet, such as one due to tides, between the times of the altimeter observations, or (4) a combination of all these.

[39] Of particular importance is that the crossover observation is independent of the static radius of the planet but remains sensitive to any time-varying component of the shape due, for example, to tides. Since the crossover observations are also sensitive to orbital errors, they can be used as an observation of the spacecraft radial position in the orbit determination process [*Shum et al.*, 1998]. Thus, crossovers can be combined with the Doppler tracking of

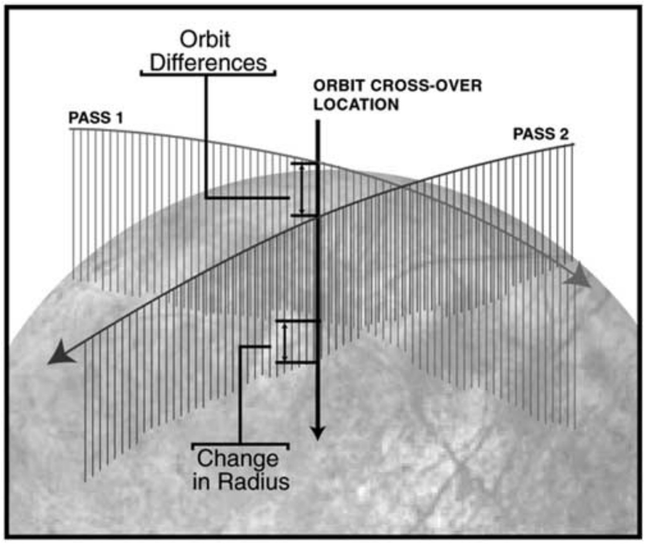

Figure 3. Schematic illustration of the altimeter crossover approach. By measuring, at points where two orbits cross, the range from the spacecraft to the surface from altimetry and the radius of the spacecraft from the planetary center of mass from Doppler tracking, the time-varying component of topography can be unambiguously determined.

the spacecraft both to improve the orbit and to estimate tidal changes in the shape of the planet.

[40] We have conducted a full simulation of the recovery of h and k for a 30-day, high-inclination, 200-km altitude Europa mission. We construct simulated tracking and altimeter data from the following a priori models. We use the Earth's Moon (without the center-of-mass/center-of-figure offset) as the model for Europa gravity and topography (the Moon may not be like Europa, but it provides a reasonable proxy for Europa's size and shape), a Jupiter-induced Europa tidal model that assumes $k = 0.2$ and $h = 1.0$, and the IAU model for the precession, nutation, and rotation of Europa (no physical librations). We assume that range-rate measurements (Doppler tracking) of the Europa spacecraft by the 3-station DSN (Deep Space Network) are available for the full 30-day mission, including the occultation of Europa behind Jupiter and of the spacecraft behind Europa. To be conservative, we did not include DSN measurements of range, which provide an additional constraint on spacecraft position. In the simulation, no allowance is given for the time spent pointing at Earth, but the data used in the simulation are less than would be expected in a nominal laser altimeter mapping mission. The DSN data are given a 0.1 mm/s accuracy for a 60-second integration. Small daily accelerations are included to represent spacecraft momentum dumps. The altimeter observations are simulated at 1-Hz, 1-m accuracy, and are acquired for the full 30-day mission. Once the simulated data are constructed, we discard all the a priori models, including GM, the gravity, topography, and tidal models, and we analyze the (simulated) tracking data in 4-day periods to provide initial estimates for GM and the spacecraft orbit. Iterating this process many times leads to a preliminary gravity model solution, as well as to orbits for the spacecraft for the 30-day mission. The (simulated) altimeter data were "hung" on the orbits and a preliminary topography model was obtained for Europa. The Love numbers k and h were estimated in subsequent iterations of this solution.

Table 2. Recovered Values for the Tides Without Altimetry[a]

Soln	Param	Recovered Value	Formal Sigma	Error
15 days	k	0.2135	0.0002	6.3%
15 days	h	0.9534	0.001	4.7%

[a]The values inputs to the simulation were $k = 0.2$ and $h = 1.0$.

Table 3. Correlations Between Gravity and k[a]

	$C_{2,0}$	$C_{2,1}$	$C_{2,2}$	$S_{2,1}$	$S_{2,2}$
k	0.71	−0.04	0.998	−0.01	0.95
h	0.99	−0.05	0.999	−0.04	0.95

[a]High correlations indicate that parameters are inseparable in inversion.

[41] The simulation revealed that solving directly for the tidal Love numbers, k and h, without using the altimetric crossovers to improve the orbit, does not yield the level 1 accuracy objectives. The principal gravity and topographic distortion of Europa is due to Jupiter's tidal forcing. Because Europa is in a synchronous orbit, most of the tidal distortion is a permanent deformation of the body. Table 2 demonstrates that separating the tidal shape from the non-tidal shape is difficult because of high correlation between estimates of the tidal parameters and the second-degree coefficients in the gravity and topographic expansions (Table 3). Such correlation is common when attempting to recover multiple parameters from a single spacecraft in orbit. Adding the altimeter crossovers to the Doppler data in the orbit determination process provides improved orbital accuracy, not only in the radial direction but particularly in the cross-track and along-track directions. The simulated solution behavior was entirely consistent with observational experience on MGS [*Rowlands et al.*, 1999]. Attempts for a simultaneous recovery of the spacecraft orbit and tidal parameters using altimetric crossovers were fully successful leading to results shown in Table 4 that meet the requirements for measurement of Europa's tides. Our estimated uncertainties in k, h, and $\Delta(= 1 + k - h)$, are on the order of 0.0004, 0.01, and 0.01, respectively.

[42] Each of the above solutions was obtained from 4 days of simultaneous tracking and altimetry whenever the spacecraft was visible from Earth. The results obtained herein are after the solution had been iterated to improve the gravity

Table 4. Recovered Values for the Tides and the Pole Position

Soln	Param	Recovered Value	Formal Sigma	Error
1	k	0.19959	0.22E-04	0.2%
	h	1.00276	0.22E-04	0.3%
	Pole, RA	268.8093	0.70E-05	
	Pole, DEC	65.2369	0.69E-05	0.003°
2	k	0.19964	0.18E-04	0.02%
	h	1.00944	0.22E-02	0.9%
	Pole, RA	268.8105	0.14E-04	
	Pole, DEC	65.2352	0.82E-05	0.004°

model. Initial values of the tidal parameters were zero, and initial values of the pole position were the modeled value.

8. Summary

[43] It has been argued for several years that accurate solutions for the Europan Love numbers k or h could provide definitive evidence of the presence of a liquid ocean beneath the surface ice shell [e.g., *Yoder and Sjogren*, 1996; *Edwards et al.*, 1997; *Chyba et al.*, 1998; *Moore and Schubert*, 2000; *Wu et al.*, 2001]. The issue we address in this paper is whether, if there is a liquid ocean, the Love number solutions could be further used to determine the ice thickness, d. We conclude that this cannot be done to any useful accuracy using k or h alone. Even if k or h could be measured perfectly, there would still be an unacceptably large uncertainty in any estimate of d because of the difficulty of modeling k_0 and h_0 (the values of k and h in the $d \to 0$ limit; see (3)).

[44] We find, though, that an observed value for the linear combination $\Delta = 1 + k - h$, would greatly reduce this problem, since $\Delta \to 0$ in the limit of $d \to 0$. There would still be an ambiguity in the thickness estimate owing to uncertainties in the shear modulus, μ_i, of the ice. The value of Δ is proportional to $d \times \mu_i$, implying that the uncertainty in d would be proportional to the uncertainty in $1/\mu_i$.

[45] The uncertainty, δd, in the ice thickness would also depend on the uncertainty, $\delta\Delta$, in the measured value of Δ. Our best estimate of the relation between these two uncertainties is

$$\delta\Delta = 2.7 \times \delta d / R \tag{16}$$

(see the discussion above (12)), where R is the radius of Europa. To estimate $\delta\Delta$ we have simulated a 1-month Europa mapping mission that includes both microwave spacecraft tracking and an onboard laser altimeter, and where we solve simultaneously for the Love numbers and the static gravity field and surface topography. The results suggest that Europa's gravity field can be recovered to degree and order 15, and that the static topography field can be obtained with an accuracy of 1 m. The measurement precision required for the Europa experiment has already been demonstrated in the geodetic investigation on the Mars Global Surveyor (MGS) mission [*Smith et al.*, 2001; *Neumann et al.*, 2001].

[46] This simulation suggests that $\delta\Delta$ would probably be on the order of 0.01. Using this estimate of $\delta\Delta$ in (16), we obtain an uncertainty of ±5 km in d; though the precise uncertainty depends on the ice thickness itself and on the uncertainty in μ_i.

[47] Estimates for the thickness of the Europan crust range from many tens of kilometers to (at least in places) tens or hundreds of meters [*Chyba et al.*, 1998]. The ability of a future landed mission to access the liquid water ocean for the purpose of searching for life depends sensitively on this thickness: a drilling depth of tens of kilometers is a very different challenge than a drilling depth of tens of meters. Our simulation suggests an accuracy for the recovered thickness of the crust (i.e., ±5 km) that, depending on the result, may not be sufficient to decide whether follow-on missions designed to further study the ocean are warranted. If the ice thickness is on the order of many km or larger, the observations proposed in this paper offer the most practical means of obtaining reliable numbers for the thickness of the Europan crust (and rule out drilling missions). Other techniques, such as radar probing of the crust, would be required to interpret, in terms of in situ mission feasibility, a tracking/altimeter experiment that gives (for example) 1 km ± 5 km.

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References

Anderson, D. L., and J. D. Bass (1984), Mineralogy and composition of the upper mantle, *Geophys. Res. Lett.*, *11*, 637–640.
Anderson, J. D., G. Schubert, R. A. Jacobson, E. L. Lau, W. B. Moore, and W. L. Sjogren (1998), Europa's differentiated internal structure: Inferences from four Galileo encounters, *Science*, *281*, 2019–2022.
Chyba, C. J., et al. (1998), Report of the Europa Science Definition Team, National Aeronautics and Space Administration, Washington, DC, 1998, report, pp. 2019–2022, Gov. Print. Off., Washington, D. C.
Cole, D. M., and G. D. Durell (1995), The cyclic loading of saline ice, *Philos. Mag. A*, *72*, 209–229.
Dahlen, F. A. (1976), The passive influence of the oceans upon the rotation of the Earth, *Geophys. J. R. Astron. Soc.*, *46*, 363–406.
Dorsey, N. E. (1940), *Properties of Ordinary Water-Substance*, 673 pp., Van Nostrand Reinhold, Hoboken, N. J.
Dziewonski, A., and D. L. Anderson (1981), Preliminary reference Earth model, *Phys. Earth Planet. Inter.*, *25*, 297–356.
Edwards, B. C., et al. (1997), The Europa Ocean Discovery Mission, in *Instruments, Methods, and Missions for the Investigation of Extraterrestrial Microorganisms*, edited by R. B. Hoover, *Proc. SPIE Int. Soc. Opt. Eng.*, *3111*, 249–261.
Fletcher, N. H. (1970), *Chemical Physics of Ice*, 271 pp., Cambridge Univ. Press, New York.
Hoppa, G. V., B. R. Tufts, R. Greenberg, and P. E. and Geissler (1999a), Strike-slip faults on Europa: Global shear patterns driven by tidal stresses, *Icarus*, *141*, 287–298.
Hoppa, G. V., B. R. Tufts, R. Greenberg, and P. E. Geissler (1999b), Formation of cycloidal features on Europa, *Science*, *285*, 1899–1902.
Hussmann, H., T. Spohn, and K. Wieczerkowski (2002), Thermal equilibrium states of Europa's ice shell: Implications for internal ocean thickness and surface heat flow, *Icarus*, *156*, 143–151.
Kargel, J. S., J. Z. Kaye, J. W. Head, G. M. Marion, R. Sassen, J. K. Crowley, O. P. Ballesteros, S. A. Grant, and D. L. Hogenboom (2002), Europa's crust and ocean: Origin, composition, and the prospects for life, *Icarus*, *148*, 226–265.
Kaula, W. M. (1964), Tidal dissipation by solid friction and the resulting orbital evolution, *Rev. Geophys.*, *2*, 661–685.
Kivelson, M. G., K. K. Khurana, C. T. Russell, M. Volwerk, R. J. Walker, and C. Zimmer (2000), Galileo magnetometer measurements: A stronger case for a subsurface ocean at Europa, *Science*, *289*, 1340–1343.
Konopliv, A. S., C. F. Yoder, E. M. Standish, D.-N. Yuan, and W. L. Sjogren (2006), A global solution for the Mars static and seasonal gravity, Mars orientation, and Deimos masses, and Mars ephemeris, *Icarus*, *182*, 23–50.
Lamb, H. (1945), *Hydrodynamics*, 738 pp., Dover, Mineola, N. Y.
Lambeck, K. (1990), *Geophysical Geodesy*, Cambridge Univ. Press, New York.
Lemoine, F. G., D. D. Rowlands, D. E. Smith, D. S. Chinn, D. E. Pavlis, S. B. Lutchcke, G. A. Neumann, and M. T. Zuber (1999), Orbiter determination for Mars Global Surveyor during mapping, *AAS Pap. 99-328*, Am. Astronaut. Soc., Girdwood, Alaska.
Lemoine, F. G., D. E. Smith, D. D. Rowlands, M. T. Zuber, G. A. Neumann, D. S. Chinn, and D. E. Pavlis (2001), An improved solution of the gravity field of Mars (GMM-2B) from Mars Global Surveyor, *J. Geophys. Res.*, *106*, 23,359–23,376.
Love, A. E. H. (1927), *A Treatise on the Mathematical Theory of Elasticity*, 643 pp., Cambridge Univ. Press, New York. (Reprinted Dover, Mineola, New York, 1944.)
Moore, W. B., and G. Schubert (2000), The tidal response of Europa, *Icarus*, *147*, 317–319.

Munk, W. H., and G. J. F. MacDonald (1960), *The Rotation of the Earth*, 323 pp., Cambridge Univ. Press, New York.
Neumann, G. A., D. D. Rowlands, F. G. Lemoine, D. E. Smith, and M. T. Zuber (2001), Crossover analysis of Mars Orbiter Laser Altimeter data, *J. Geophys. Res.*, *106*, 23,753–23,768.
Nimmo, F., and P. Schenk (2006), Normal faulting on Europa: Implications for ice shell properties, *J. Struct. Geol.*, in press.
Nimmo, F., B. Giese, and R. T. Pappalardo (2003), Estimates of Europa's ice shell thickness from elastically-supported topography, *Geophys. Res. Lett.*, *30*(5), 1233, doi:10.1029/2002GL016660.
Ojakangas, G. W., and D. J. Stevenson (1989), Thermal state of an ice shell on Europa, *Icarus*, *81*, 220–241.
Poirier, J.-P. (2000), *Introduction to the Physics of the Earth's Interior*, 312 pp., Cambridge Univ. Press, New York.
Rowlands, D. D., F. G. Lemoine, G. A. Neumann, and S. B. Luthcke (1999), The use of laser altimetry in the orbit and attitude determination of Mars Global Surveyor, *Geophys. Res. Lett.*, *26*, 1191–1194.
Schenk, P. M. (2002), Thickness constraints on the icy shells of the Galilean satellites from a comparison of crater shapes, *Nature*, *417*, 419–421.
Schmeltz, M., E. Rignot, and D. MacAyeal (2002), Tidal flexure along ice-sheet margins: Comparison of InSAR with an elastic-plate model, *Ann. Glaciol.*, *34*, 202–208.
Shum, C. K., B. H. Zhang, B. E. Schultz, and B. D. Tapley (1998), Altimeter crossover methods for precision orbit determination and the mapping of geophysical parameters, *J. Astronaut. Sci.*, *38*, 335–368.
Smith, D. E., W. L. Sjogren, G. L. Tyler, G. Balmino, F. G. Lemoine, and A. S. Konopliv (1999), The gravity field of Mars: Results from Mars Global Surveyor, *Science*, *286*, 94–97.
Smith, D. E., et al. (2001), Mars Orbiter Laser Altimeter: Experiment summary after the first year of global mapping of Mars, *J. Geophys. Res.*, *106*, 23,689–23,722.
Spohn, T., and G. Schubert (2003), Oceans in the icy Galilean satellites of Jupiter?, *Icarus*, *161*, 456–467.
Tapley, B. D., and G. W. Rosborough (1985), Geographically correlated orbit error and its effect on satellite altimeter missions, *J. Geophys. Res.*, *90*, 11,817–11,831.
Tobie, G., G. Choblet, and C. Sotin (2003), Tidally heated convection: Constraints on Europa's ice shell thickness, *J. Geophys. Res.*, *108*(E11), 5124, doi:10.1029/2003JE002099.
Turtle, E. P., and E. Pierazzo (2001), Thickness of a European ice shell from impact crater simulations, *Science*, *294*, 1326–1328.
Vaughan, D. G. (1995), Tidal flexure at ice shelf margins, *J. Geophys. Res.*, *100*, 6213–6224.
Wahr, J. M. (1981), Body tides on an elliptical, rotating, elastic and oceanless Earth, *Geophys. J. R. Astron. Soc.*, *64*, 677–704.
Wu, X. P., Y. E. Bar-Sever, W. M. Folkner, J. G. Williams, and J. F. Zumberge (2001), Probing Europa's hidden ocean from tidal effects on orbital dynamics, *Geophys. Res. Lett.*, *28*, 2245–2248.
Yoder, C. F., and W. L. Sjogren (1996), Tides on Europa, paper presented at Europa Ocean Conference, San Juan Capistrano Res. Inst., San Juan Capistrano, Calif.
Yuan, D.-N., W. L. Sjogren, A. S. Konopliv, and A. B. Kucinskas (2001), The gravity field of Mars: A 75th degree and order model, *J. Geophys. Res.*, *106*, 23,377–23,401.
Yuen, D. A., R. Sabadini, and E. V. Boschi (1982), Viscosity of the lower mantle as inferred from rotational data, *J. Geophys. Res.*, *87*, 10,745–10,762.
Zuber, M. T., D. E. Smith, S. C. Solomon, D. O. Muhleman, J. W. Head, J. B. Garvin, J. B. Abshire, and J. L. Bufton (1992), Mars Orbiter Laser Altimeter: Experiment summary after the first year of global mapping of Mars, *J. Geophys. Res.*, *97*, 7781–7797.

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