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Abstract

In the absence of a conducting wall, typical spheromak plasmas are unstable to tilting and/or shifting modes. The effects of the cross-sectional shape, aspect ratio, and the location of a conducting wall on the stability of these modes are investigated.

One of the most troublesome and fundamental instabilities of the spheromak configuration arises from the tendency of the plasma to tilt so as to align its magnetic moment with the externally imposed equilibrium field or to shift horizontally into a region of weaker magnetic field strength. The free energy driving this instability is large, arising from the interaction of the toroidal plasma current with the currents in the external field coils. The global, almost rigid nature of the instability makes stabilization due to magnetic shear or finite gyration radius effects seem unlikely.

Analytic studies of this mode have been confined to treatment of small departures from a spherical[1] configuration or to cylindrical[2] models. Since the mode could have serious consequences, it is important to understand its parametric dependence on the plasma shape, size of the central flux hole, the presence or strength of an external toroidal field, and the extent to which conductors in the vacuum region can provide stabilization. The application of linear ideal MHD stability computer programs[3,4] to numerically generated equilibrium[5,6] solutions should provide this understanding.

Several features of the PEST-1 stability code[3] keep this from being a straightforward exercise. This code was designed with special consideration to the numerical problems associated with MHD modes in typical low-β tokamaks. Thus, both the choice of the coordinate system and the representation of the displacement vector are not optimal for the spheromak configuration. In particular, the inaccurate representation of $\nabla \cdot \xi$ when the toroidal field is small, leads to a spurious stabilizing effect unless many Fourier components of the displacement vector ξ are retained. Tilting instabilities have been found with PEST-1 by keeping a small but finite toroidal vacuum field, keeping a modest aspect ratio, employing a high resolution equilibrium grid, and including sufficient terms in the Fourier expansion for ξ .

The difficulties associated with PEST-1 are avoided in the PEST-2 ideal MHD stability code[4] which has been developed specifically to solve the ideal MHD equations at marginal stability, and thus avoids spurious compressive stabilization by imposing $\nabla\cdot\xi = 0$ analytically for nonaxisymmetric $(n\neq0)$ modes. Representation problems are avoided by working only with the component $\xi\cdot\nabla\psi$, which also effects a factor 3 reduction in matrix size. Away from marginal stability these simplifications are obtained at the price of using a nonphysical kinetic energy, so that the growth rates obtained are not quantitatively meaningful. The use of a ψ, Θ, ζ toroidal coordinate system in which the magnetic field lines are straight with Θ chosen to be proportional to the arc length on the poloidal cross section of a magnetic surface also reduces the number of Fourier modes needed by at least a factor of 3 over that needed when the PEST-1 coordinate system is used.

Our choice of parameterizing the shape of the equilibrium we study is illustrated in Fig. 1. Motivated by the exact spherical solution,[1] the outer boundary, in spherical coordinates, is given by $r \sin\theta j_1(r)P_1^1(\cos\theta) + \delta = 0$ where $0 < \delta < 1.03$ measures the size of the "flux hole." To "flatten" (oblate) the configuration, we multiply the z coordinate by $[1 + \varepsilon^2]^{-1/2}$.

A typical spheromak result is given in Fig. 2 where the "growth rates" for n = 1 modes are given as functions of the shape of the plasma cross section. These equilibria have zero pressure and a safety factor profile varying from 1.0 at the magnetic axis to 0.01 at the edge as $q = q_o(1 - \psi)^2 + q_e$, where $0 < \psi < 1$ is the normalized poloidal flux. For a spheromak plasma with a small flux hole, $\delta = 0.1$, the system is unstable with respect to a tilting mode which is primarily a superposition of $m = \pm 1$, $n = 1$ modes $[\xi = \xi_o e^{i(m\theta - n\zeta)}]$ of equal amplitude but of opposite phase. As the plasma shape becomes oblate the growth rate of this mode decreases, but at some point a second, shifting instability sets in. This shifting instability is essentially a rigid horizontal translation or a superposition of equal amplitude $m = \pm 1$, $n = 1$ modes with the same phase. As the plasma is made more oblate the growth rate of the shifting mode continues to increase while the tilting mode growth rate decreases and finally becomes stable.

These results are qualitatively consistent with those of a simple model in which the spheromak plasma is represented by a rigid current carrying wire loop of radius r_o in an external axial field, B_e, with index $(-r_o/B_e)\, \partial B_e/\partial r$. This system is unstable to tilting for $n_i < 1$ and unstable to shifting for $n_i > 0$. The fact that these regions of instability overlap for $1 > n_i > 0$ suggests that any shape spheromak plasma will be unstable to one or both of these modes in the absence of a nearby conducting wall.

It is interesting to note the "mode mixing" that occurs in Fig. (2) for $1 \lesssim \epsilon \lesssim 3$. Examination of the two eigenfunctions for $\epsilon = 1$ shows a pure tilt mode and a pure shift mode. As ϵ is increased and the two growth rates become comparable, these modes mix to form helical $m = 1$, $n = 1$ and $m = -1$, $n = 1$ modes at $\epsilon \approx 2$. Increasing ϵ further causes the modes to change again into a pure tilt and a pure shift, but with interchanged identity.

The results of Fig. 2 were obtained without a conducting shell. It is clear that placing a perfectly conducting wall just outside the plasma should provide stabilization. In accordance with current spheromak designs we employ ae topologically spherical shell rather than the usual toroidal vacuum chamber. The shell must be fairly close, on the average, so that the eddy currents generated in the shell can affect the bulk of the plasma.

Figure 3 illustrates the parameterization of a conducting ellipsoidal shell included in the PEST-2 vacuum calculation, with a_w and b_w being dimensionless measures of the wall separation in the horizontal and in the vertical directions. In the first series of calculations, presented in Fig. 4, we set $a_w = b_w$ and plot contours of the value of this wall separation parameter for marginal stability as a function of the flux hole δ and the shape parameter ϵ. Comparison of Figs. 2 and 4 shows that for a given size flux hole, or aspect ratio, the wall separation distance needed for stabilization is roughly inversely proportional to the larger of the two growth rates without a wall, although there is a tendency for shift modes to be more easily stabilized than tilt modes. Also, the larger the flux hole, the farther away the wall can be to provide stabilization.

To estimate the effect of a purely vertical or a purely horizontal wall, we repeat the calculations of Fig. 4 but set $b_w = 10$ and solve for the critical horizontal separation, Fig. 5, and then set $a_w = 10$ and solve for the critical vertical separation, Fig. 6. Comparison of Figs. 4, 5, and 6 shows that a horizontal wall is ineffective in stabilizing tilt modes, but it can effectively stabilize shift modes if its vertical separation distance from the plasma is about half that of a nearly equidistant wall. The vertical wall, on the other hand, is totally ineffective against stabilizing shift modes, and can only stabilize tilt modes if it is very close, and if the mode's unstabilized growth rate is weak.

A qualitative understanding of the wall stabilization results of Fig. 4-6 is provided by a consideration of the pattern of eddy currents which are induced in the conducting shell to stabilize the tilt or the shift mode. We display in Figs. 7 and 8 the pattern of induced currents in a spherical wall stabilizing a tilt mode (Fig. 7) and a shift mode (Fig. 8).

The stabilization of the tilt mode requires the induced currents to produce a horizontal magnetic field. This is accomplished by eddy currents flowing in circular loops in the sides of the shell, as evidenced by Fig. 7.

The shift mode, however, requires a vertical induced field for stabilization. This is accomplished by circular loops of eddy currents flowing in the top and the bottom of the shell, as illustrated in Fig. 8.

We have also studied the effect of an axial current along the symmetry axis on tilting and shifting modes. The presence of this current, I_{axis}, causes the toroidal field in the vacuum to be non-zero. As I_{axis} is raised from zero, keeping q at the magnetic axis fixed at 1, the eigenfunctions of the unstable modes change continuously into an almost pure m = 1, n = 1 free boundary kink mode. When $I_{axis} \gtrsim 1/2\ I_{toroidal}$, q on the edge exceeds unity, and a stable configuration, the small aspect ratio tokamak, results.

Acknowledgments

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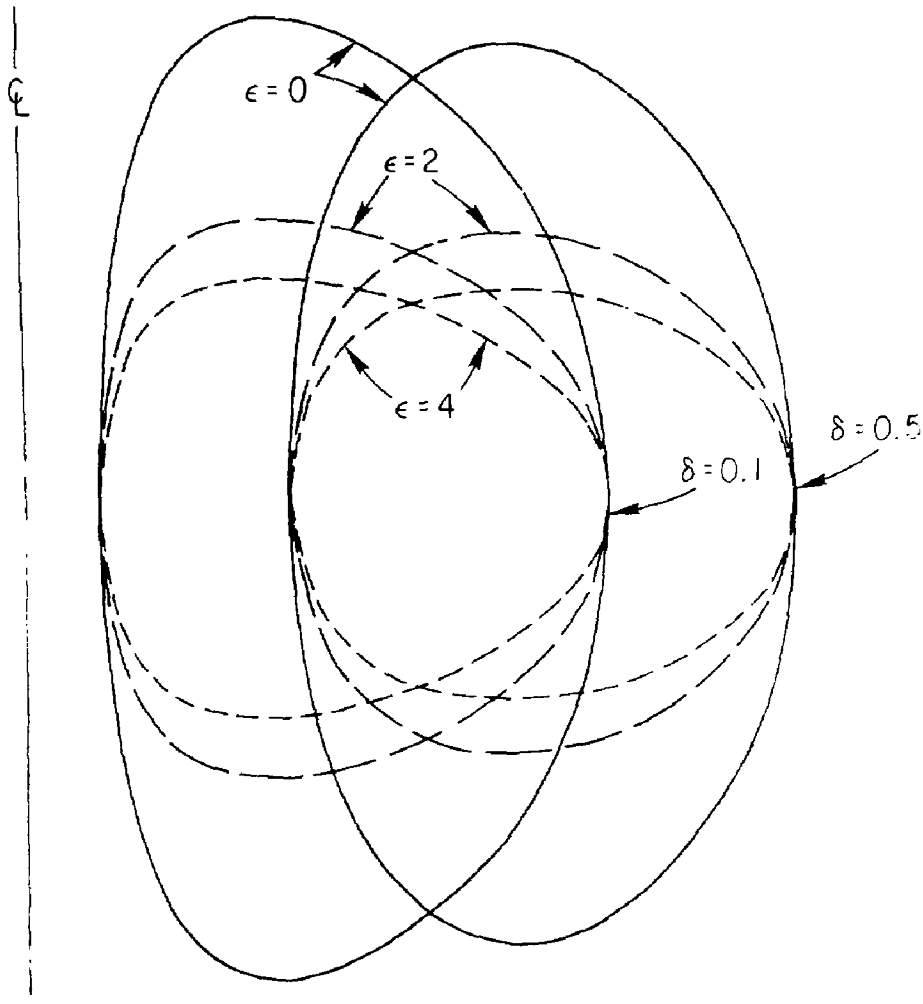
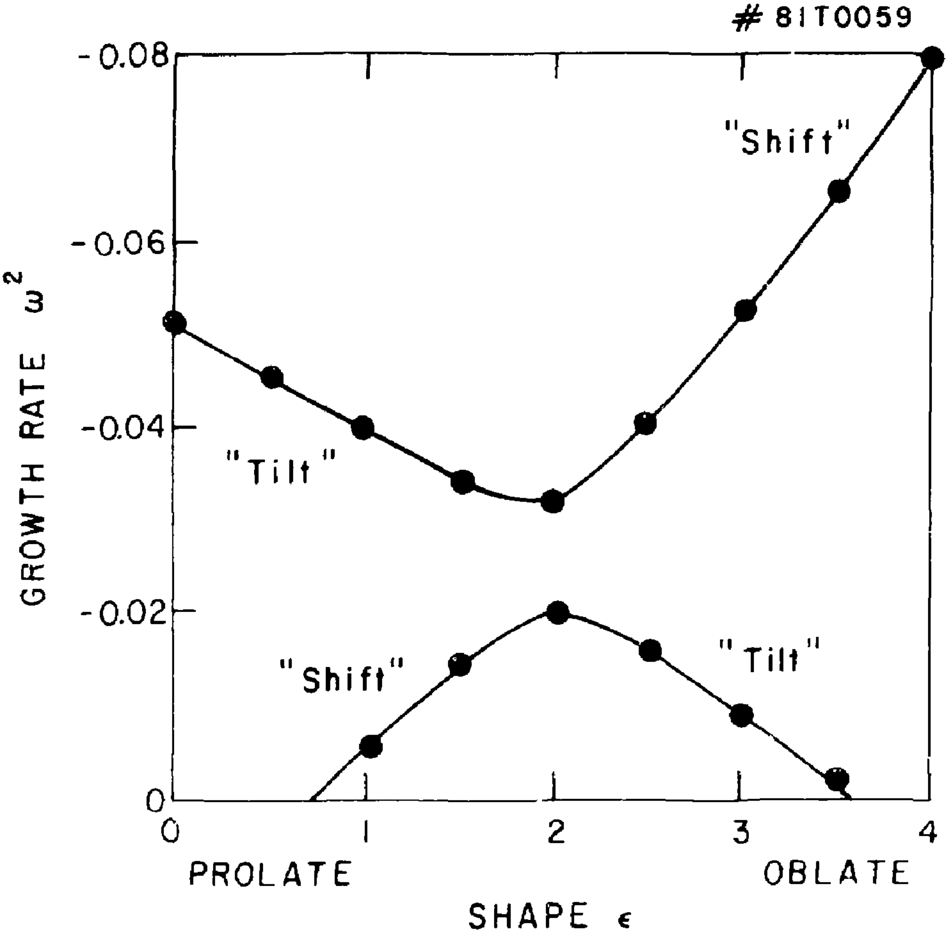
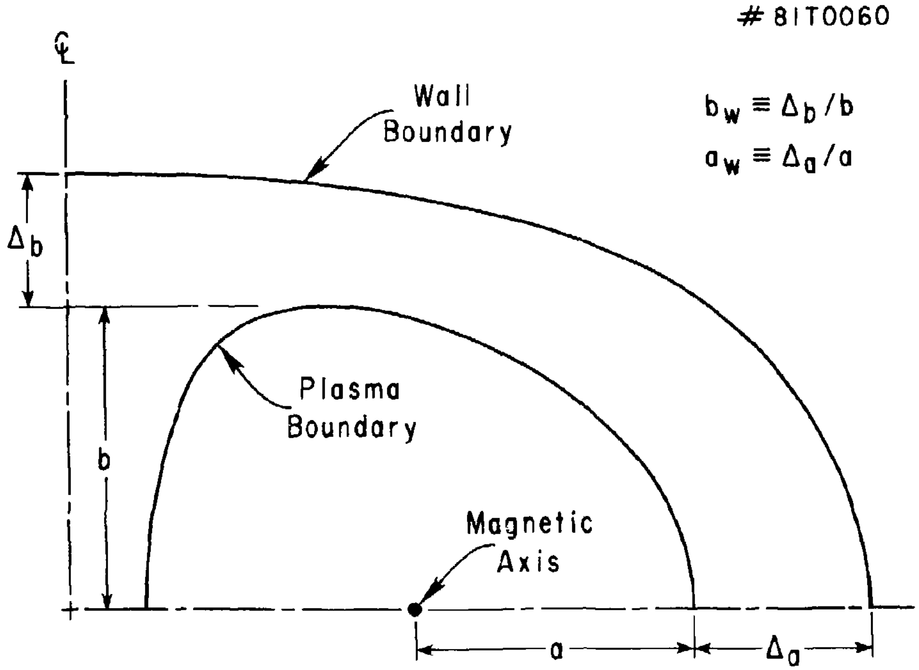
81T0048



1. Parameterization of the shape of the plasma boundary.

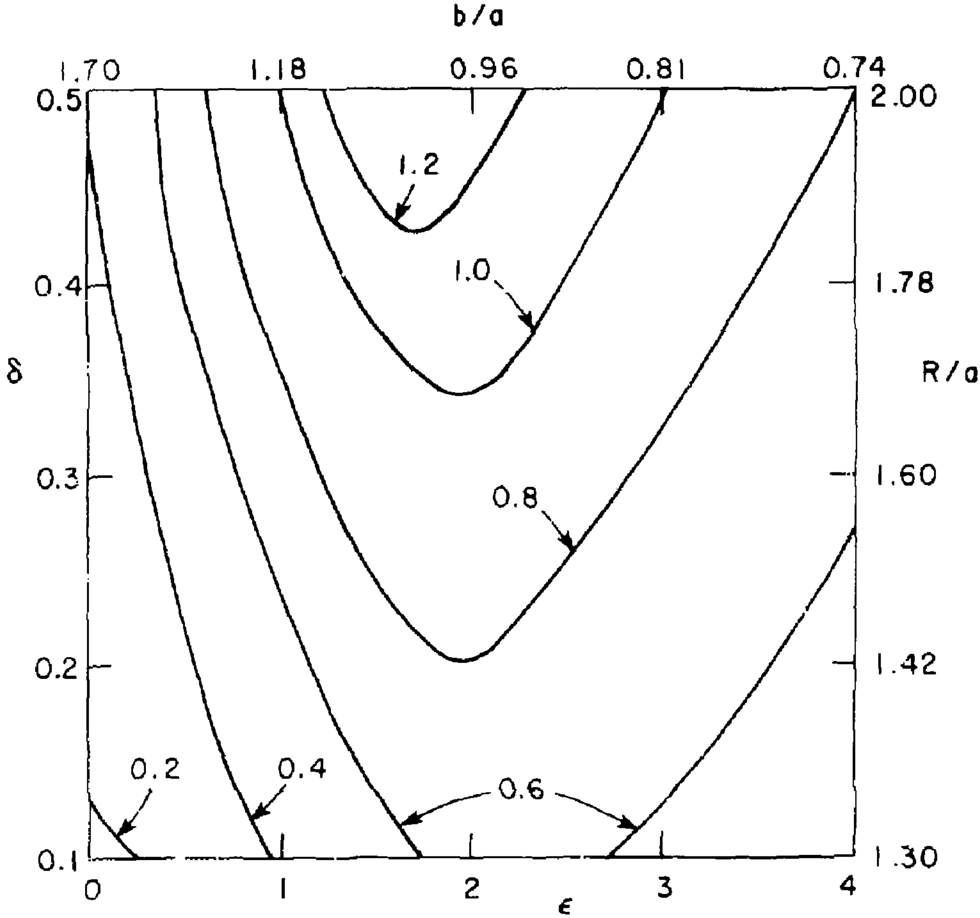


2. Growth rates of $n = 1$ modes for [illegible] = [illegible].1.



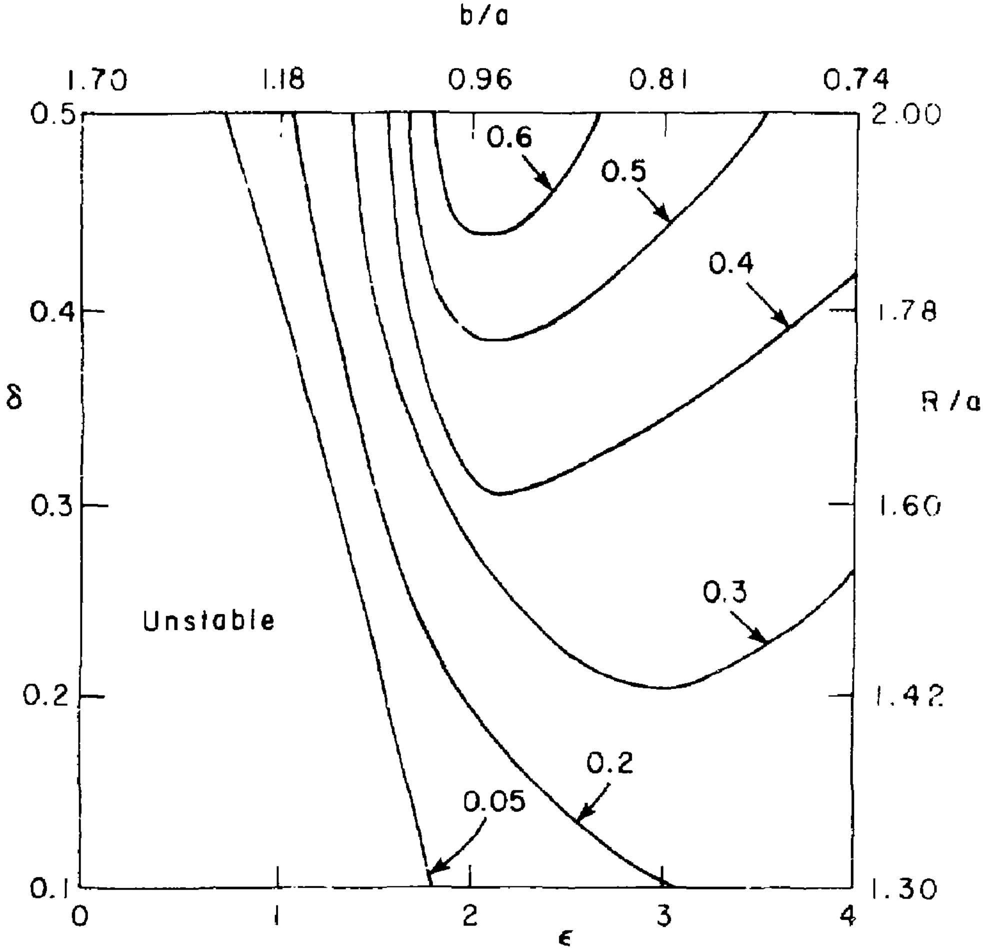
3. An ellipsoidal wall is parameterized by the dimensionless numbers a_w and b_w.

81T0063



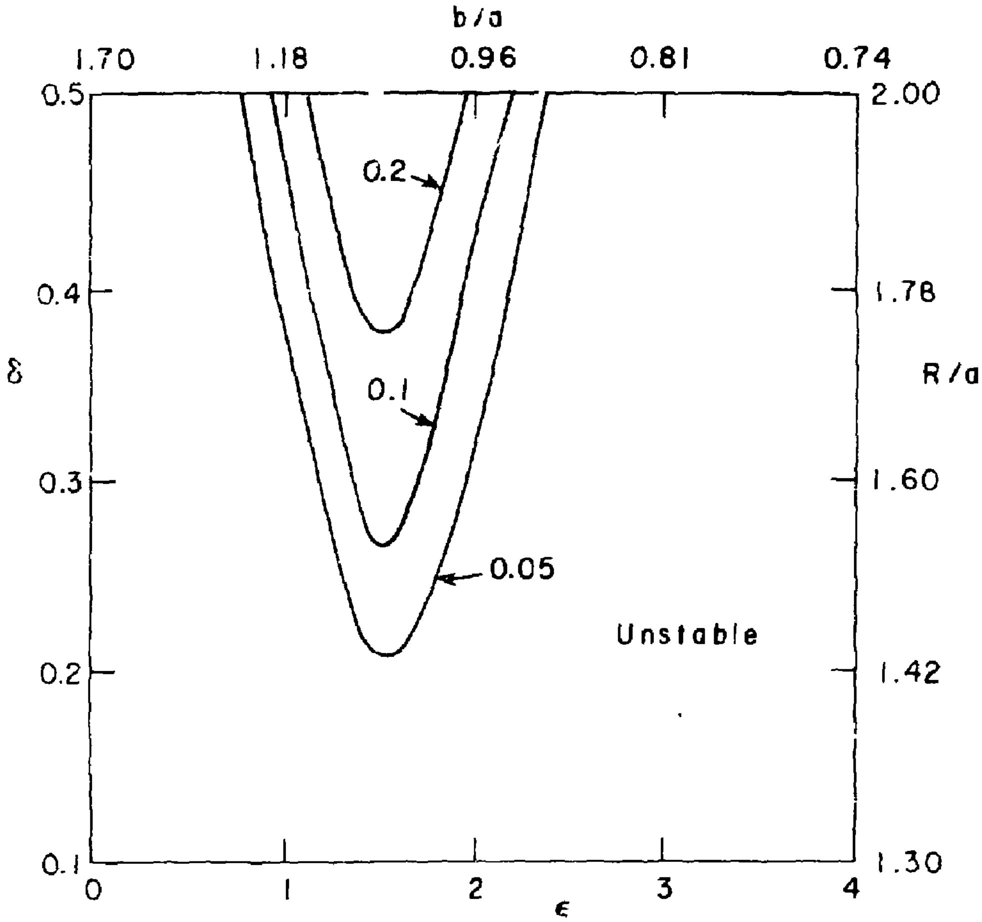
4. Contours of the critical separation distance, a_w, between the plasma boundary and a wall with $a_w = b_w$.

81T0061



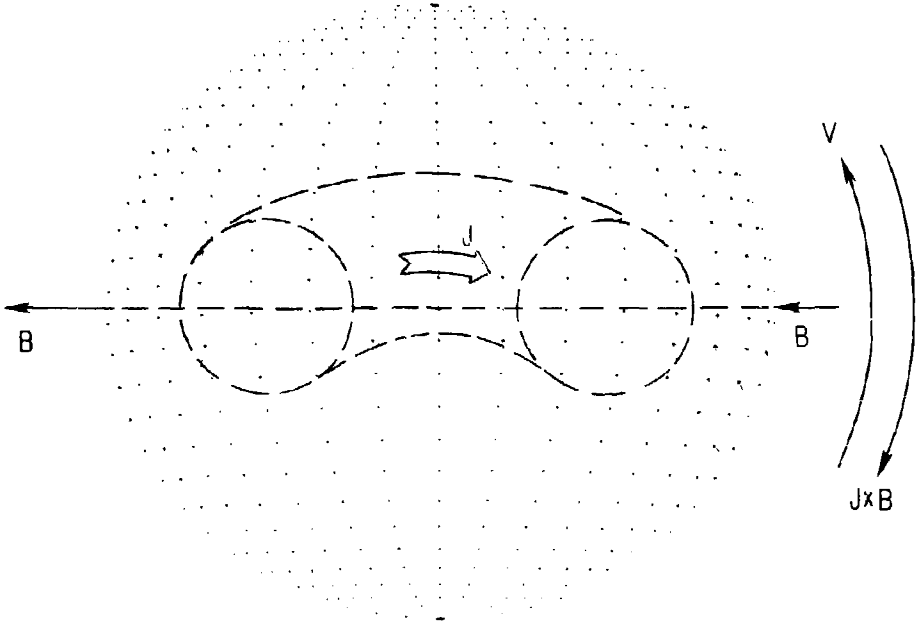
5. Contours of the critical separation distance b_w between the plasma and a nearly horizontal wall with $a_w = 10$.

81T0062

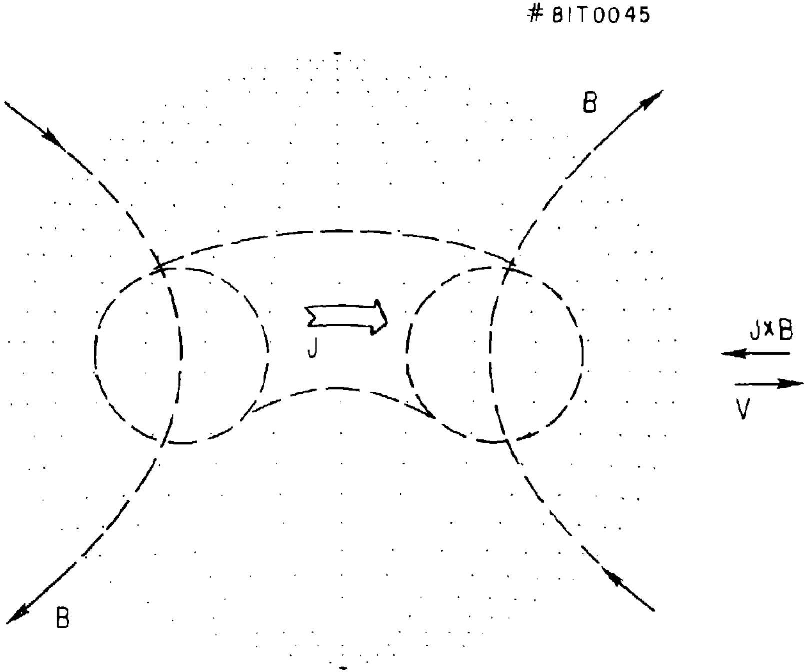


6. Contours of the critical separation distance a_w between the plasma and a nearly vertical wall with b_w = 10.

81T0044



7. Induced currents in a [illegible]



8. Induced current in a spherical wall acting to stabilize the shift mode.